WTS, without replacement ordering strategy converges to sample mean

Since the ordering index was chosen randomly and without replacement
Thus, the ordering indices are distinct in total of n, from 1 to n,
no two indices are the same.

$$x_{k+1} = x_k - \gamma_k (x_k - y_{i_k}), \quad \gamma_k = \frac{1}{k}$$

base case:

$$x_0 = 0$$

$$x_1 = 0 - 1 \cdot (0 - y_1) = y_1$$

$$x_2 = y_1 - \frac{1}{2} \cdot [y_1 - y_2] = \frac{1}{2}[y_1 + y_2]$$

$$x_3 = \frac{1}{2} y_1 + \frac{1}{2} y_2 - \frac{1}{3}\left[\frac{1}{2}(y_1 + y_2) - y_3\right] = \frac{1}{3}[y_1 + y_2 + y_3]$$

Assume $x_n = \frac{1}{n}[y_1 + \cdots + y_n]$, WTS $x_{n+1} = \frac{1}{n+1}[y_1 + \cdots + y_{n+1}]$

$$\begin{aligned}
x_{n+1} &= x_n - \frac{1}{n+1}(x_n - y_{n+1}) \\
&= \frac{1}{n}(y_1 + \cdots + y_n) - \frac{1}{n+1}\left[\frac{1}{n}(y_1 + \cdots + y_n) - y_{n+1}\right] \\
&= \frac{1}{n} y_1 + \cdots + \frac{1}{n} y_n - \frac{1}{n(n+1)} y_1 - \cdots - \frac{1}{n(n+1)} y_n + \frac{1}{n+1} y_{n+1} \\
&= \frac{1}{n+1} y_1 + \cdots + \frac{1}{n+1} y_n + \frac{1}{n+1} y_{n+1} \\
&= \frac{1}{n+1}[y_1 + \cdots + y_{n+1}], \quad \text{true}
\end{aligned}$$

Thus $x_n = \frac{1}{n}\sum_{i=1}^{n} y_i$ is true

Therefore, without replacement ordering strategy x_n converges to sample mean