Analysis of DC Transients and Components

S. Ali
Data Analyst

G. Paraschiv
Lab Technician

A. Xu
Lab Technician

(Received 5 November 2020)

Abstract

A resistor network circuit, RC circuit, PN diode circuit, and NPN transistor circuit were constructed to investigate the properties of components in a DC circuit. Van Bemmel's algorithm was used to solve the currents in the resistor network circuit, which were compared to experimental values. The charging and discharging curves of the RC circuit were fitted and the time constant was found to be symmetrical in accordance with theory at (1.039±.005) s and (1.040±.002) s for the charging and discharging curves. The current through the PN diode was fitted with respect to the potential across the diode. A fit was computed for 4 different base currents in the NPN transistor circuit. The β_f of the transistor was found to be (197±9), which was within the manufacturer range.

I Theory

The purpose of this experiment was to demonstrate the function and operation of resistors, capacitors, diodes, and transistors in a circuit. Furthermore, the observed electrical values at the components sometimes differ from learned theory, which can be explained and mitigated by appropriate changes to methodology.

Circuits can be analyzed to calculate relevant values at all components. These values can then be compared with observed values.

Ohm's Law shows that potential is directly proportional to current as given by,

$$V = iR \qquad [1]$$

(Halliday et al, 2010)

Where:
V = potential (V)
i = current (A)
R = resistance (Ω)

Kirchhoff's Voltage Law (KVL) states that in a circuit the total electric potential increase at the source(s) is equal to the total electric potential decrease throughout the circuit,

$$\sum V_k = 0 \qquad [2]$$

(van Bemmel, 2020)

Where:
V_k = kth potential in a closed loop

Kirchhoff's Current Law (KCL) states that the amount of current entering a junction is equal to the amount of current exiting the junction. A corollary of Kirchhoff's Current Law is that all points in a closed loop all have the same current. It can be expressed as,

$$\sum i_k = 0 \qquad [3]$$

(van Bemmel, 2020)

Where:
i_k = kth current going in or out of a junction

When a potential difference exists across a capacitor, it will charge at a rate controlled by the resistance. When a charged capacitor is not connected to a power source, it will discharge. The rate at which a capacitor charges and discharges is not instantaneous; as such, the capacitor is known as transient.

The potential difference of a charging capacitor in an RC-circuit as a function of time is given by,

$$V_C(t) = \mathcal{E}\left(1 - e^{\frac{-t}{RC}}\right) \quad [4]$$

(Halliday et al, 2010)

Where:

$V_C(t)$ = potential difference of capacitor (V)
t = time (s)
$\mathcal{E}$ = electromotive force (V)
C = capacitance (F)

The potential difference of a discharging capacitor in an RC-circuit as a function of time is given by,

$$V_C(t) = \mathcal{E}e^{\frac{-t}{RC}} \quad [5]$$

(Halliday et al, 2010)

In an RC circuit, the capacitor functions as a variable resistor as it charges and discharges. When the capacitor is nearing full charge, it will possess a very large resistance. In which case the LabQuest will become the path of least resistance and thus the primary circuit if placed across the capacitor, giving erroneous readings.

When a diode is positioned in forward bias, the current as a function of potential is represented by the diode equation,

$$i_D = i_s\left(e^{\frac{q}{NkT}V_D} - 1\right) \quad [6]$$

(Kuphaldt, T. R., 2001)

Where:

i_D = current through diode (A)
i_s = saturation current (A)
q = fundamental charge (C)
N = emission coefficient
k = Boltzmann's constant (JK^{-1})
T = temperature (K)
V_D = potential across diode (V)

When a diode is in reverse bias, the diode stops almost all the current from passing through. An extremely small amount of current, usually in nA or µA, still passes through the diode. This value is expected to be constant until the peak inverse potential. When the potential drops below the peak inverse potential, the current will drastically decrease and will have damaging effects on the diode. The peak inverse potential is constant and specific to the diode (Kuphaldt, T. R., 2001). That of the diode used in the experiment is -1000V.

By considering the Ebers-Moll model of the transistor, it can be shown that the collector current can be accurately approximated as a logistic function of the potential across the collector and emitter,

$$i_C = \beta_f i_B\left(1 - \frac{\frac{1}{\alpha_R} - \frac{\beta_f}{\beta_R}}{e^{\frac{V_{CE}}{V_t}} - \frac{\beta_f}{\beta_R}}\right) \quad [7]$$

(Smith, R., J., 1976)

Where:

i_C = collector current (A)
V_{CE} = potential across collector-emitter (V)
i_B = base current (A)
β_f = CE current gain in forward active mode
β_R = CE current gain in reverse active mode
α_R = CB current gain in reverse active
V_T = thermal potential (V)

Uncertainties were derived by method of quadratures,

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 {\sigma_{x_1}}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 {\sigma_{x_2}}^2 + \cdots} \quad [8]$$

(van Bemmel, 2020)

Where:

f = The value of the function
x_k = The k^{th} variable in the function
σ_f = The uncertainty of the function
σ_{x_k} = The uncertainty of the k^{th} variable

When the LINEST function was unable to produce the uncertainties of regression, the jackknife method was used (McIntosh, n.d).

II METHOD

Four circuits were constructed on a breadboard. The lab technicians have experience with the breadboard, and this was deemed to be the most organized manner of constructing the circuits.

A multimeter and a LabQuest were used to measure voltage in parallel with components. The values given were correct to 3 decimal places for both devices. The

LabQuest was used where multiple values were required over a period of time as it produced dynamic graphs for the measured values. The uncertainty of the multimeter and the LabQuest was given by the manufacturer.

All resistor values and their uncertainty were given by the manufacturer.

The same power source potential was used for circuits 0 and 1, and was measured to be (18.5±.4) V.

Circuit 0 contains 6 randomly selected resistors oriented in a manner that exceeds the basic requirements of a S-P circuit, shown in Figure 1. The potential was measured for the battery and for each resistor.

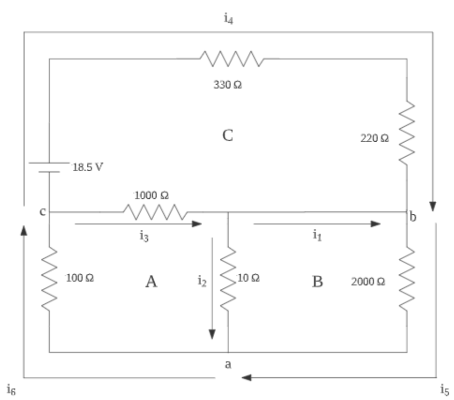

Fig. 1 Network Circuit Diagram. Three current loops and three junctions are labelled for a total of six currents.

Circuit 1 contains a simple RC circuit, consisting of a resistor in series with a capacitor, shown in Figure 2.

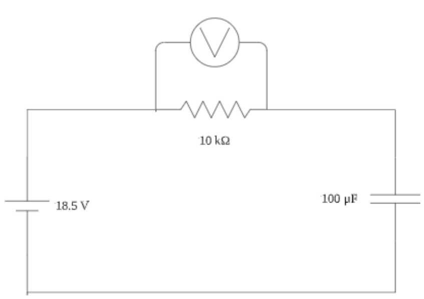

Fig. 2 RC Circuit Diagram. The LabQuest is only placed across the resistor. The simplicity of the circuit allows for calculation of the potential across the capacitor.

To counteract the erroneous reading given by a voltmeter in parallel with a capacitor, the potential difference across the resistor in series was measured instead. Similarly, the potential curves were produced using the LabQuest at the resistor.

The capacitor was allowed to charge until the potential across the resistor ceased to decrease significantly. Then, the power source was removed from the circuit, as opposed to turning it off and leaving a possible path for current. The capacitor was allowed to discharge until the potential across the resistor reached its asymptote again. A time interval of 0.05 seconds was chosen for higher accuracy.

A capacitance of (100±20) μF and a resistance of (10±0.5) kΩ were chosen for a time constant of (1.0±.2) s in [4] and [5] for readability in the LabQuest operational window.

Circuit 2 contains a resistor in series with a PN diode, shown in Figure 3. The potential difference across the resistor and the diode was measured using multimeters. The power supply used allowed for varying potentials using a dial.

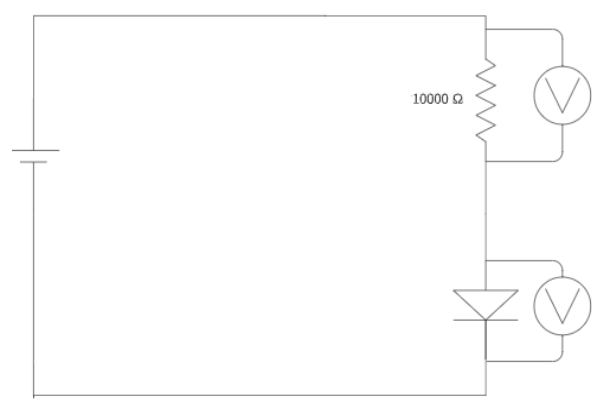

Fig. 3 Diode Circuit Diagram. Two multimeters are placed across the resistor and diode.

The potential of the resistor was found in order to calculate current through the diode using [1] and [3]. A large resistance was chosen because in [6], for small values of potential, the current will be in mA or µA. The multimeter can only measure millivolts at the least. Using a resistance of 10 kΩ ensures that the potential across the resistor will be in millivolts.

Multiple readings were taken for different potential differences across the power supply to construct the Vi plot of the diode from a reverse bias to a forward bias. The dial of the power supply was adjusted, and the lab technicians waited until the value on both multimeters stabilized.

The diode used had a saturation current of approximately 10^{-12} A and an emission coefficient of approximately 1.

After 22 readings, it was determined that an adequate graph can be constructed to observe the curvature.

Circuit 3 was constructed using an NPN transistor with a β_f of 100-300, shown in Figure 4.

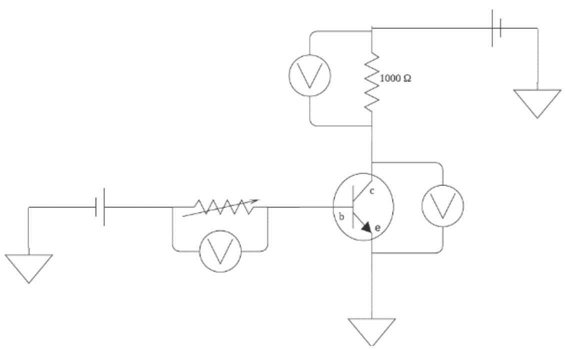

Fig. 4 Transistor Circuit Diagram. A potentiometer was not actually used in the experiment, it is shown to indicate that different resistances were used.

Three multimeters were placed to measure the current of the collector across a resistor in series and the base current across a resistor in series using [1] as well as the potential of the collector-emitter. Four different resistors were used to provide four different base currents. Another power supply was adjusted to changes the current of the collector and the potential of the collector-emitter to collect data points for each of the four base currents.

III DATA

The potential across each resistor in circuit 0 is given in Figure 5.

ID	R (Ω)	V (V)
1	10±.5	0.266±.004
2	100±5	2.66±.04
3	220±10	6.53±.06
4	330±20	9.82±.08
5	1000±50	2.94±.04
6	2000±100	0.266±.004

Fig. 5 Resistance and Potential in Circuit 0. [caption].

The potential across the resistor in circuit 1 for the charging and discharging capacitor is given in Figures 6-7. Only the data from the first 2.5 seconds of charging and discharging was used since the LabQuest is inaccurate at small values.

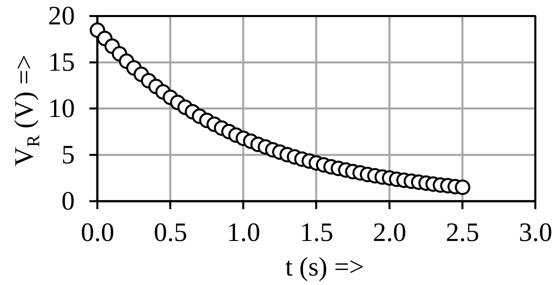

Fig. 6 Potential across Resistor vs Time for Charging Capacitor. Error bars were too small to be shown.

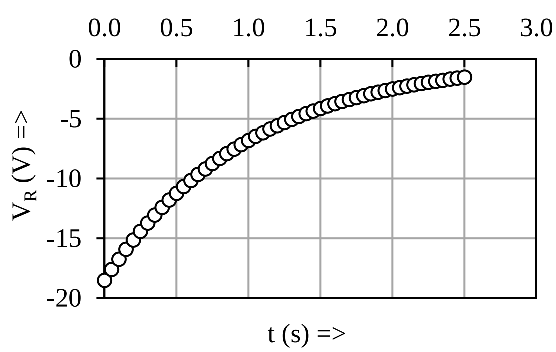

Fig. 7 Potential across Resistor vs Time for Discharging Capacitor. Error bars were too small to be shown.

The potential across the diode in reverse bias and the resistor in circuit 2 is given in Figure 8.

ID	V_D (V)	Potential across Resistor (V_R) (V)
1	-0.427±.005	-0.007±.003
2	-0.412±.005	-0.007±.003
3	-0.306±.005	-0.007±.003
4	-0.167±.003	-0.007±.003
5	-0.386±.005	-0.007±.003
6	-0.418±.005	-0.007±.003
7	-0.148±.003	-0.007±.003
8	-0.665±.006	-0.007±.003

Fig. 8 Potential Across Resistor and Diode in Reverse Bias. The potential across the resistor is constant.

The 4 measured potential and resistor values used to calculate the base current using [1] are shown in Figure 9.

ID	V_B (mV)	R (kΩ)	i_B (μA)
1	297±4	10.0±.5	30±2
2	315±5	20±1	15.7±.5
3	324±5	30±2	10.8±.2
4	331±5	40±2	8.3±.2

Fig. 9 Base Current in Transistor Circuit. The resistance was modified to yield slightly different base currents. The potential along the base current changed slightly.

IV Analysis

Using van Bemmel's algorithm for solving circuits, the currents in circuit 0 were calculated. Using [2] and [3] at the 3 loops and 3 junctions labelled in Figure 1, a total of 6 currents were calculated. The following matrix equation was used to calculate current,

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \\ 0 & -10 & -1000 & 0 & 0 & -100 \\ 0 & 10 & 0 & 0 & -2000 & 0 \\ 0 & 0 & 100 & -550 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -18.5 \end{bmatrix} \quad [9]$$

Where:

i_k = current as labelled in Figure 1

The current that applies to each resistor can be observed in Figure 1 depending on which loop they are situated in. The experimental currents were determined using [1] and the data in Figure 5, while the theoretical currents were determined using [9].

ID	R (Ω)	Experimental Current (i_e) (mA)	Theoretical Current (i_c) (mA)	Residuals (mA)
1	10	27±1	26	1
2	100	27±1	26	1
3	220	30±1	29	1
4	330	30±2	29	1
5	1000	2.9±.2	2.8	.1
6	2000	0.133±.007	0.128	.005

Fig. 10 Current for each Resistor in Circuit 0. The residuals are higher for smaller resistors. The experimental current was consistently higher.

The residuals are all less than or equal to the uncertainty of the experimental current, verifying the calculations. The slight residual can be attributed to the high uncertainty of the resistors used. The higher experimental currents indicate that the true resistance was likely greater than the manufacturer value.

Knowing the potential of the battery, the potential difference across the charging capacitor was calculated in Figure 11 using [2] and the data in Figure 6.

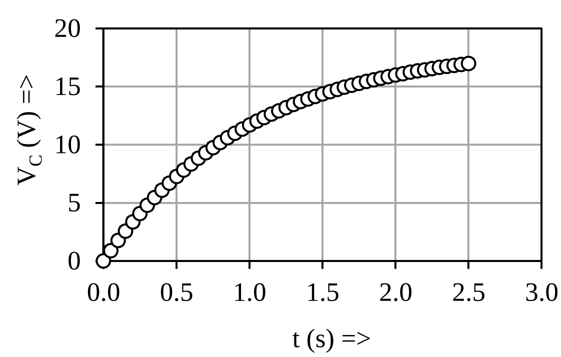

Fig. 11 Charging Curve of Capacitor. The concentration of data points obscures the trendline while still providing a clear superimposed line.

The curve in Figure 11 was regressed to match [4]. The following equation was found with an r^2 value of 0.99984,

$$V_C(t) = (18.50 \pm .009)\left(1 - e^{\frac{-t}{(1.039 \pm .005)}}\right) \quad [10]$$

Using [2], the potential difference across the discharging capacitor was calculated as the negative of the potential difference across the resistor in Figure 7. The results are shown in Figure 12.

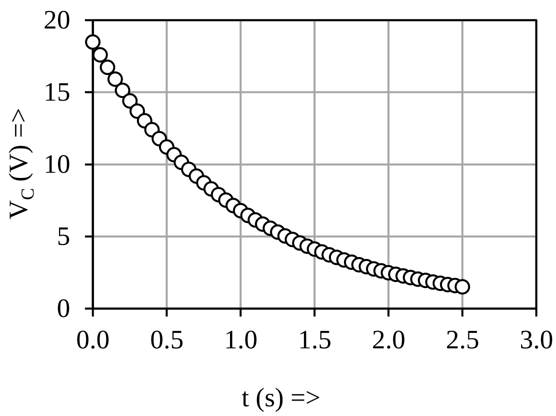

Fig. 12 Discharging Curve of Capacitor. The concentration of data points obscures the trendline while still providing a clear superimposed line.

The curve in Figure 12 was regressed to match [5]. The following equation was found with an r^2 value of 0.99976,

$$V_C(t) = (18.50 \pm .07)e^{\frac{-t}{(1.040 \pm .002)}} \quad [11]$$

The discrepancy between the charging and discharging time constant and the theoretical time constant is most likely due to discrepancy between the true values of resistance and capacitance and the manufacturer values. This follows from the high accuracy of the electromotive force in the regressed equations, high S/N ratios, and high r^2 values, which indicates that the data collected was accurate and that the discrepancy is likely not due to error in data.

It should also be noted that the experimental time constant is within the range of uncertainty of the theoretical time constant.

The Vi curve of the PN junction diode in forward bias is modelled in Figure 13. The V_D values that were recorded were all above 0.29 V. When values below that amount are used, the corresponding V_C becomes too small for the multimeter to record. The reverse bias values were not graphed as the constant current was so small relative to the current in forward bias.

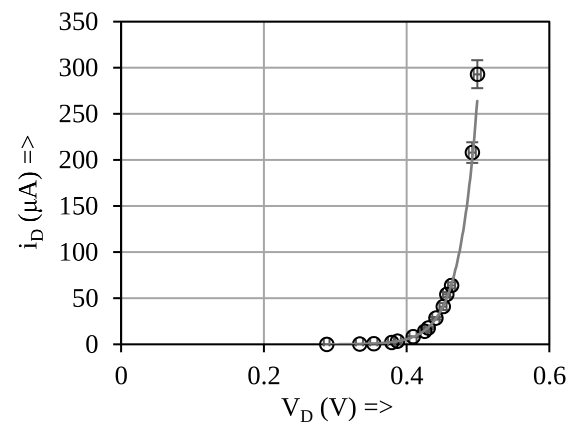

Fig. 13 Diode Current vs Potential in Forward Bias. The behaviour resembles an exponential as expected due to [6]. Error bars are too small to be seen.

In [6], the equation has an extra constant term that prevents a regression effected by taking the natural logarithm of both sides. However, if the i_s term were to be distributed, the constant would have a minimal effect as the value was expected to be ~10^{-12} A, which is negligible compared to the data values at ~10^{-6} A. Thus, the constant term can be ignored when effecting the regression. The data in Figure 13 was regressed to yield the following equation,

$$i_D = \left((1.1 \pm .2)E - 12\right)\left(e^{(38.8 \pm .6)V_D} - 1\right) \quad [12]$$

[12] is only true when the diode is in forward bias. The residuals of [12] and the data in Figure 12 are shown in Figure 14.

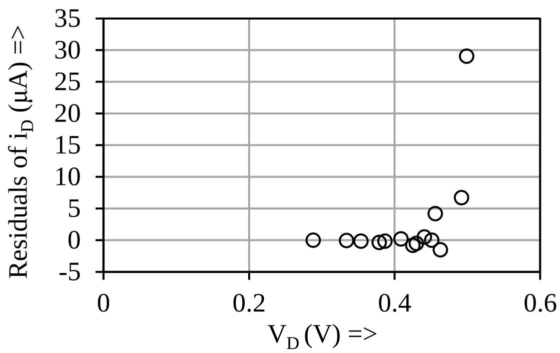

Fig. 14 Diode Current Residuals vs Potential. As the potential increases, the residuals increase due to the increase in the value of current, which allows greater room for error.

The regression yielded the i_s term to be (1.1±.2)E-12 A, which is close to the predicted value of 1E-12 A. Furthermore, the coefficient of 38.8 implies that the product of the temperature and emission coefficient should be 299.2K. This is reasonable as room temperature is 298K which would mean that the emission constant was very close to 1. This suggests that the data was accurate with only a small margin of error.

In reverse bias, the data supports the pre-established theory that the current is extremely small and constant. All potential values were (-0.007±.003) V. Using [1] gives a current of (-7±3)E-7 A. The low S/N ratio suggests inaccurate values. This can be explained by the fact that the current leaked by the diode in reverse bias is extremely small, resulting in equally small potential and thus the multimeter is unable to produce precise results.

The current through the collector is graphed against the potential across the collector-emitter in Figure 15.

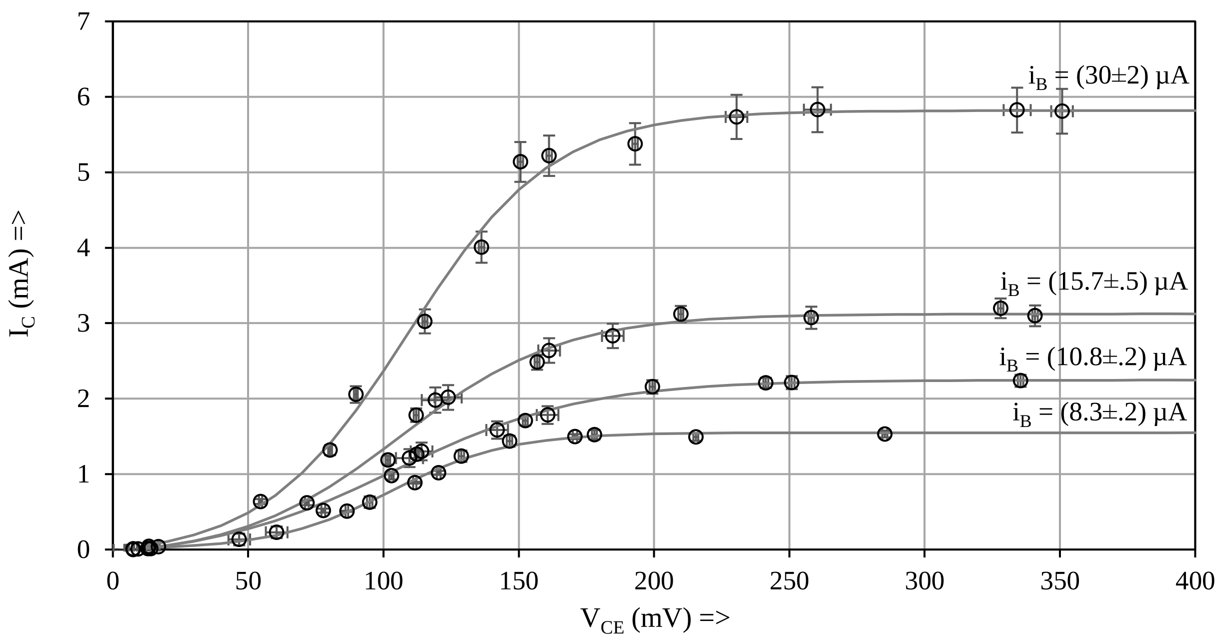

Fig. 15 Collector Current vs Potential of Collector-Emitter. As the base current decreases, the collector current plateaus earlier.

The plot in Figure 15 was regressed for each separate base current to match [7]. The parameters of the equation are displayed in Figure 16. V_T was taken to be a constant at 0.026 V (Kuphaldt, T. R., 2001).

i_B (µA)	$\beta_f i_B$ (µA)	$\frac{1}{\alpha_R} - \frac{\beta_f}{\beta_R}$	$\frac{\beta_f}{\beta_R}$	r^2
30±2	5.81±.001	71±3	-71±3	0.9959
15.7±.5	3.12±.003	67±2	-66±2	0.9933
10.8±.2	2.245±.0008	64±2	-63±2	0.9903

i_B (μA)	$\beta_f i_B$ (μA)	$\frac{1}{\alpha_R} - \frac{\beta_f}{\beta_R}$	$\frac{\beta_f}{\beta_R}$	r^2
8.3±.2	1.549±.0005	54±2	-53±2	0.9950

Fig. 16 Collector Current vs Potential of Collector-Emitter Regression. The high r^2 values indicate that the data closely aligns with the fitted equation.

The regression can be used to plot the residuals, shown in Figure 17. The plot of the residuals is reflective of the curvature in Figure 15. The graph is initially dynamic in the saturation zone. It then becomes relatively constant and plateaus in the active zone. As a result, it is expected that the residuals are higher in the dynamic area but decrease as the graph begins to plateau.

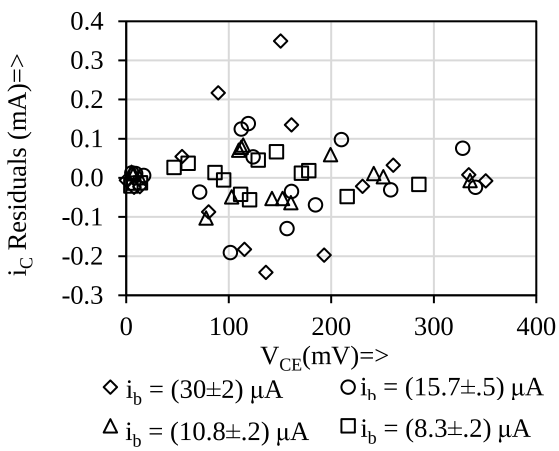

Fig. 17 Collector Current Residuals vs Potential of Collector-Emitter. Note that the residuals are not strictly increasing as usual.

From [7], it is shown that the CE current gain of the transistor in forward mode can be determined if the leading coefficient of the regression is divided by the base current. This can be done for each separate regression and averaged, which yielded a β_f of (197±9). This is within the manufacturer range of 100 to 300. Solving a system of equations for each base current, β_R and α_R were calculated to be (3.1±.2) and (1.0±.1), respectively. These values were not provided by the manufacturer and cannot be compared.

V Sources of Error

Much of the error in the observations and calculations stems from limits in methodology.

Internal resistance in all components of the circuit, including the battery and wires, could have caused the deviation from learned theory, where other resistances were assumed to be negligible. As such, resistance was not measured for these components; however, the additional resistance was likely to be small enough to truly be negligible and is thus not a likely cause of deviation.

The S/N ratios for manufacturer values were very low and could have led to erroneous calculations. To mitigate this, the resistances should have been measured as well but were not due to time constraints.

VI Conclusion

In the resistor network circuit, the measured values of the current were compared with the theoretical values and were found to be in an acceptable range. The charging and discharging curves of the RC circuit were regressed and the time constant was found to be (1.039±.005) s for the charging curve, and (1.040±.002) s for the discharging curve. The current across the PN diode was regressed and compared to pre-established theory. In the NPN transistor circuit, the data was regressed for 4 different base currents. The β_f of the transistor was found to be (197±9) which matches the manufacturer value.

VII Sources

Halliday, D., Resnick, R., Walker, J. (HRW), Fundamentals of Physics, John Wiley & Sons Inc., 2010.

Kuphaldt, T. R., https://www.allaboutcircuits.com/textbook/, 2001.

McIntosh, A. I., The Jackknife Estimation Method, n.d.

Smith, R. J., Circuits Devices and Systems, John Wiley & Sons Inc., 1976.

van Bemmel, H., Electromagnetism Notes, MGCI Physics Department, 2020.

van Bemmel, H., Handling of Experimental Data, MGCI Physics Department, 2020.

FORMAL REPORT MARKING RUBRIC – LABORATORY PAPERS APC2021

Lab: 1 2 3 4 5 **Submit signature page on reverse**

Student1 : Sarah Ali Student2 : George Paraschiv Student3 : Arthur Xu

Student4 : ____________________ Student5 : ____________________

SUB CATEGORY	DESCRIPTION	WT%	LETTER GRADE
SYNTAX	Spelling and Grammar including proper tense of expression. Was jargon used appropriately?	**10**	
FORM – GRAPHS AND CHARTS	Do your graphs and tables follow the guidelines? Did you put a caption beneath each one? Are they numbered? Are they well presented? Are they necessary? Your tables should not break at the end of a page. Did you remember the index column? Are your graphs scaled properly and with the proper format? Have you included error bars as required?	**7**	
FORM – EQUATIONS AND MARGINS	Are your equations properly presented? Are the variables defined with the expected units included? Are variables only defined on first use? Are your equations numbered with the number well off to the right? Did you source them as required? Are subscripts used correctly? Do your margin and columns conform to the rules? Is the font properly chosen? Are you using the correct size of paper and did you set up the first page as per the exemplar. Does your report conform to the length rules? Is your report free from silly computer glitches such as widows, orphans, and large gaps in the text? You must fix all of this. Did you email the .pdf copy of your work to your instructor prior to the due time? Did you print your work on BOTH sides of the paper? Did you submit a single page on the back of the marking rubric, detailing the work of the team members and include signatures of all?	**7**	
FORM – CITATIONS AND SOURCES	Have you cited any facts not fairly earned by your team? Are they cited in the proper manner described in the lab manual? Did you list all your sources? Are the sources conveyed in the proper format? Did you include at least TWO text sourcings?	**7**	
UNCERTAINTY	Have you indicated the uncertainty of your measured, computed and regressed values? Has this been done according to the rules as stipulated in the manual? Have your uncertainties been stated in the proper format?	**10**	
ABSTRACT	Is this abstract written in the proper form? Is it a fair description of this work and its accomplishments? Is it a reasonable length? Does the abstract use some of the most impressive numerical values to buttress its claims?	**5**	
METHODOLOGY	How have you used the equipment available to you? Have you maximized the precision and possibly the accuracy of your work? Did you know that you were doing this? Did you waste undue time on lengthy, but not effective or productive techniques?	**17**	
BASIC CONTENT	Were the basic questions of the experiment answered? Were all the stated requirements met?	**17**	
ADVANCED ANALYSIS	How aggressively did the report discuss the subtle relationships? How well were numerical relationships between variables developed and explained? How creative was the work presented via written text, graphs, tables and such like. (I really care about this folks!)	**20**	
NOTEBOOK	See Notebook Rubric – page 27	**2**	

PAPER	%	NOTEBOOK	/2	TOTAL	%

Signature Page

Please sign and indicate what each person did.

SIGNATURE	NAME	CONTRIBUTION
SA	Sarah Ali	- RC Circuit - Formatting - Data Analysis - Editing
GP	George Paraschiv	- Diode Circuit - Editing - Data Analysis - Transistor Circuit
AX	Arthur Xu	- Transistor Circuit - Circuit 0 - Data Analysis - Editing