# INVESTIGATION OF QUANTUM MECHANICS, DIFFRACTION, AND BLACK BODIES


S. ALI
*Lab Technician, Data Analyst*

G. PARASCHIV
*Lab Technician, Data Analyst*

A. XU
*Lab Technician, Data Analyst*


(Received 17 November 2020)


## ABSTRACT

A circuit was constructed using different coloured LEDs to derive Planck's constant using the properties of light. The activation potential of the LED was regressed with respect to frequency, and Planck's constant was found to be (6.89±.03)E-34 Js, which has a deviation of 3.98% from the established value. An intensity distribution was found for a single slit diffraction pattern using a red laser. The distribution agreed with learned theory, and the width of the slit was found to be (7.9±.5)E-5 m. A black body curve was produced using a white light spectrum. The wavelength of max spectral radiance was found to be (541.22±.05) nm. The temperature of the light bulb filament was found to be (5354.1±.5) K which clashes with pre-established theory.


## I THEORY

The purpose of this experiment is to investigate the wave properties of light and consolidate results with known theory in quantum mechanics. One of the fundamental values in quantum mechanics, Planck's constant, can be measured using electrical theory. Light interference patterns, which display the probabilistic nature of quantum mechanics, can be observed with controlled diffraction. The phenomenon known as black-body radiation describes the process in which black bodies begin to emit significant amounts of visible light above certain temperatures. Black body curves can thus be used to measure the temperature of objects that cannot be measured with most apparatus.

Wavelength of light is related to frequency by the wave equation,

$$f = \frac{c}{\lambda} \qquad [1]$$

*(Halliday et al, 2010)*

Where:

$c$ = speed of light ($ms^{-1}$)
$\lambda$ = wavelength (m)

By combining Planck's Law with the definition of electrical potential energy, an expression can be produced to relate the activation potential of the LED and its frequency,

$$V = \frac{h}{e}f + \frac{k}{e} \qquad [2]$$

*(Halliday et al, 2010)*

Where:

$h$ = Planck's constant (Js)
$e$ = fundamental charge (C)
$V$ = activation potential (V)
$f$ = frequency ($s^{-1}$)
$k$ = energy when wavelength is 0 (J)

The relative intensity of the single slit diffraction pattern as a function of the angle from the central axis of the slit is given by,

$$\frac{I(\theta)}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad [3]$$

*(Halliday et al, 2010)*

Where:

$I_m$ = maximum intensity ($Wm^{-2}$)
$\theta$ = angle from central axis of slit (˚)
$\frac{I(\theta)}{I_m}$ = relative intensity

The value $\alpha$ is a convenient connection between the angle $\theta$ and the intensity at a point,

$$\alpha = \frac{\pi a \sin\theta}{\lambda} \quad [4]$$

*(Halliday et al, 2010)*

Where:
$a$ = slit width (m)

The angle $\theta$ can be found using,

$$\tan\theta = \frac{x}{D} \quad [5]$$

*(Halliday et al, 2010)*

Where:
$x$ = distance from absolute maximum (m)
$D$ = distance between screen and slit (m)

For experiments where $x \ll D$, the width of the slit can be approximated using,

$$a \approx \frac{m\lambda D}{x_m} \quad [6]$$

*(Nave,2008)*

Where:
$m$ = order of maximum
$x_m$ = position of maximum (m)

Heisenberg's Uncertainty Principle states that the momentum and position of a particle cannot be simultaneously known with absolute certainty. As such, the uncertainties are inversely proportional to each other, as shown by the following inequality,

$$\sigma_p \sigma_s \geq \frac{h}{4\pi} \quad [7]$$

*(Halliday et al, 2010)*

Where:
$\sigma_p$ = uncertainty of momentum (Ns)
$\sigma_s$ = uncertainty of position (m)

De Broglie related a particle's wavelength with its momentum,

$$p = \frac{h}{\lambda} \quad [8]$$

*(Halliday et al, 2010)*

Where:
$p$ = magnitude of momentum (Ns)

The spectral radiance is the derivative of the intensity with respect to wavelength,

$$L = \frac{d}{d\lambda}(I) \quad [9]$$

*(Elert, 2020)*

Where:
$L$ = spectral radiance ($Wsr^{-1}m^{-3}$)
$I$ = intensity ($Wm^{-2}$)

Wien's displacement law is as follows,

$$\lambda_{max} = \frac{b}{T} \quad [10]$$

*(Elert, 2020)*

Where:
$\lambda_{max}$ = wavelength at maximum L (m)
$b$ = Wien's displacement constant (m K)
$T$ = temperature of the light bulb filament (K)

## II METHOD

A circuit was constructed consisting of a power supply, resistor, and LED in series. 5 different coloured LEDs were used to produce 5 different wavelengths, which were given by the manufacturer. A multimeter measured the electric potential difference across the LED.

A potentiometer is needed to allow the minimum potential difference to occur across the LED to produce the minimum amount of photons. This is imperative as Planck's law only stands for a singular photon.

A single slit diffraction pattern was produced using a red laser and a vertical single slit. A red laser was used since a monochromatic light produces a more stable interference pattern due to its singular wavelength. The pattern was projected onto a screen consisting of black paper taped to the wall for higher contrast with the light pattern. The distance between the slit and the screen as well as the length of the captured pattern were measured. A large distance was chosen in order to increase the spacing of the pattern for greater clarity. A large distance was also required to effect [6].

Several pictures were taken of the diffraction pattern in a dark room to minimize interference from other light sources. The intensity profile of the image was produced in an algebra engine. The image was converted to grayscale without affecting the results since the background was black and the light pattern was monochromatic. The average intensity was taken at vertical cross sections since the pattern was roughly symmetrical across its horizontal axis. The positions were then scaled to place the absolute maximum at the position $x = 0$ m.

A white light spectrum was obtained using a prism and a diffraction grating held against each other in front of a white light bulb. This was done within a makeshift lightbox made with binders. However, the binder at the top of the box was removed for ease of access during imaging. The spectrum was projected on a white paper for clarity of colour.

Several pictures were taken of the spectrum in a dark room to minimize interference from other light sources. The intensity and colour profile of the image was produced in an algebra engine. The image was converted to grayscale when analyzing intensity without affecting the results. The average intensity and wavelength were taken at horizontal cross sections since the spectrum consisted of coloured strips.

## III DATA

The sourced wavelengths of the 5 LED colours and their measured forward potentials are given in Figure 1.

| ID | LED Colour | λ (nm) | V (V) |
|---|---|---|---|
| 1 | Red | 640±5 | 1.887±.008 |
| 2 | Orange | 601±5 | 1.939±.006 |
| 3 | Yellow | 590±5 | 1.980±.008 |
| 4 | Green | 525±5 | 2.32±.02 |
| 5 | Blue | 465±5 | 2.60±.03 |

**Fig 1. LED Table.** The S/N ratios for the potential were consistently low, suggesting high precision.

The speed of light and fundamental charge were taken as (299 792 458±5) $ms^{-1}$ and (1.602 176 634±.000 000 005)E-19 C.

The distance between the single slit and the screen was found to be (3.38±.06) m. The length of the interference pattern analyzed was found to be (19.35±.04) cm. The wavelength of the red laser was taken as the approximate wavelength of red light (700±5) nm.

## IV ANALYSIS

The frequency of each LED wave was determined using [1] and the data in Figure 1. This allowed for the regression of activation potential with respect to frequency.

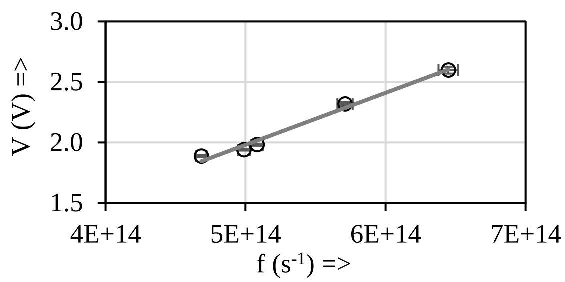


**Fig 2. Potential vs Frequency.** A close linear fit can be observed in the data.

A linear regression yielded the following equation,

$$V = (4.3 \pm .3)E - 15f + (0.4 \pm .3) \qquad [11]$$

By considering [2], Planck's constant was determined to be (6.89±.03)E-34 Js. This has a deviation of 3.98% from the accepted value of 6.626E-34 Js.

The intensity profile of the single slit diffraction pattern as a function of the position is shown in Figure 3. Using [6], the slit width was found to be (7.9±.5)E-5 m. This value was then used to regress relative intensity as a function of position using [3], [4], and [5], as shown in Figure 3. A meaningful regression cannot be produced by other means since the minima did not have relative intensities of 0.

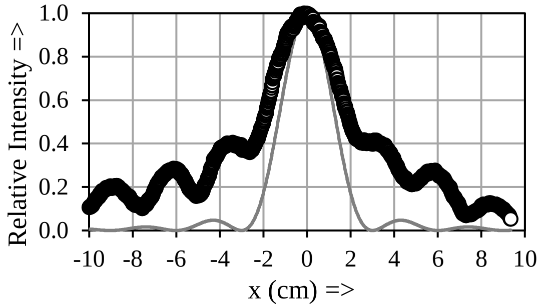


**Fig 3. Relative Intensity vs Position.** The data is consistently higher than the theoretical value in the regression. Error bars are not visible.

It is observed in Figure 3 that the minima of the data do not show a relative intensity of 0 as in learned theory. This is likely due to the diffraction pattern of the lens image leading to overlap between maxima, resulting in low resolvability. However, the positions at which minima occur are periodic as expected. It follows that the minima would be higher between larger maxima, which is observed in Figure 3. The maxima otherwise still match the expected single slit intensity distribution; however, the data has a smaller period compared to the regressed equation and is not symmetrical.

To derive an expression for Heisenberg's Uncertainty Principle, an x-axis and a y-axis are set up that are perpendicular and parallel to the width of the slit, respectively. Observing the photon at the moment it enters the slit, the uncertainty of its position is based solely on the y axis since it is established that its x coordinate is in the slit. Considering the first minimum, the uncertainty of its position is the width of the slit and is given by,

$$\sigma_s = \frac{\lambda}{\sin(\theta)} \qquad [12]$$

The horizontal component of the momentum will be the same regardless of its angle after diffraction. The vertical component of the momentum is then what contributes to uncertainty. Again, by considering the first minimum, the photon can travel in either direction from the slit, so the vertical momentum must be doubled. Using [8] and a basic understanding of vector mathematics, the following equation can be produced,

$$\begin{aligned}\sigma_p &= 2p_y \\ &= 2p\sin(\theta) \\ &= \frac{2h\sin(\theta)}{\lambda}\end{aligned} \qquad [13]$$

By multiplying [12] and [13], an expression for Heisenberg's Uncertainty Principle is derived,

$$\sigma_y \sigma_p = 2h \qquad [14]$$

Due to this equality, if the uncertainty of one of the terms were to decrease, the other term must increase to satisfy the equality, as per Heisenberg's Uncertainty Principle. This equation also satisfies [7].

The idea that the single slit diffraction pattern can be recreated over a long period of time with single photon emission can be realized using probability. If a single emitted photon becomes chaotic and not continuous as wave propagation suggests, then it must follow a seemingly random path. The randomness of this path can de described as a probability, where its distribution would be directly proportional to the intensity distribution (Halliday et al, 2010).

The relative intensity of the white light spectrum as a function of the wavelength was graphed.

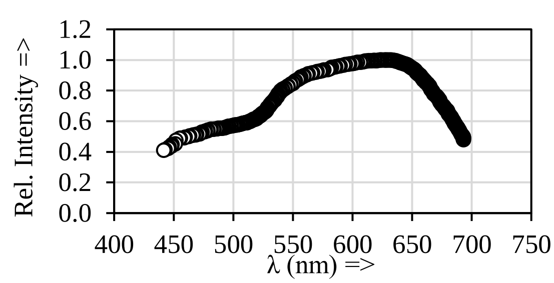


**Fig 4. Relative Intensity vs Wavelength** Until λ~540nm, the graph seems to be concave up while after it seems to be concave down.

Due to [9], the derivative of the relative intensity was taken which gives a multiple of the spectral radiance. This was rescaled to produce a graph for the relative spectral radiance in Figure 5.

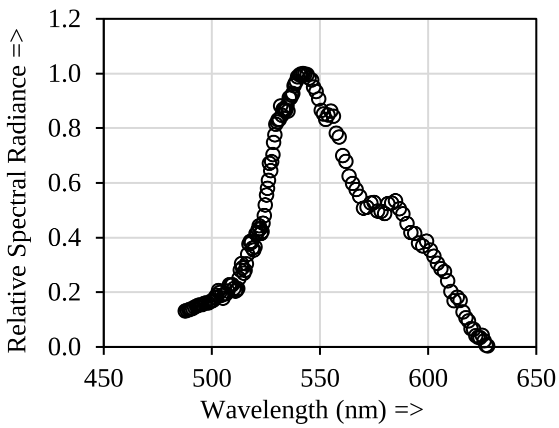


**Fig 5. Relative Spectral Radiance vs Wavelength.** Another local maximum occurs at approximately 575 nm due to error.

Due to the high density of points, it was determined that a regression was not necessary to determine $\lambda_{max}$. Instead, it was directly determined to be $\lambda_{max}$ = (541.22±.05) nm. Using [10], the temperature of the light bulb filament is found to be (5354.1±.5) K. This is much higher than the expected value as light bulb filaments are generally unable to maintain temperatures above 2900 K (Elert, 2020). The large margin of error is likely because at a temperature of 2900 K, $\lambda_{max}$ would occur in the infrared spectrum; however, the camera was only able to detect visible light. It should still be expected that the intensity would be strictly increasing in the visible spectrum, so that spectral radiance is strictly positive. As seen in Figure 4, a sharp drop begins at about $\lambda$ = 650 nm. This is because at the edge of the spectrum, the colour's intensity as detected by a photo decreases as it eventually becomes black.

## V Sources of Error

When measuring Planck's constant, not all the potential that is measured by the multimeter is used to produce light due to the inefficiency of the light bulb. This results in the measured value of Planck's constant being higher than the accepted value.

When imaging the diffraction pattern, the camera was off to the side, resulting in asymmetrical imaging, as suggested in the asymmetrical intensity distribution in Figure 3.

Lastly, some equipment, such as the lightbulbs and LEDs, could not have their properties, like wavelength and temperature, confirmed via measurement, due to lack of appropriate testing equipment. These values were instead sourced from the manufacturer, which could differ from the actual values.

## VI Conclusion

Planck's constant was found to be (6.89±.03)E-34 Js. The intensity distribution of a single slit diffraction pattern was produced and found to match with learned theory. The width of the slit was found to be (7.9±.5)E-5 m. The expression $\sigma_y\sigma_p = 2h$ was derived, which agrees with Heisenberg's Uncertainty Principle. The blackbody curve of the white light spectrum was produced. It was determined that the maximum spectral radiance occurs when the wavelength is (541.22±.05) nm. Using Wien's displacement law, the temperature of the light bulb filament was determined to be (5354.1±.5) K.

## VII Sources

*Elert, G. The Physics Hypertextbook. https://physics.info/planck/, 2020*

*Halliday, D., Resnick, R., Walker, J. (HRW), Fundamentals of Physics, John Wiley & Sons Inc., 2010.*

*Kuntzleman, T., Jacobson, C., "Teaching Beer's Law and Absorption Spectrophotometry with a Smart Phone: A Substantially Simplified Protocol", https://pubs.acs.org/doi/suppl/10.1021/acs.jchemed.5b00844/suppl_file/ed5b00844_si_001.pdf,2016*

*Nave, R., http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sinslit.html#c1, 2008.*

# FORMAL REPORT MARKING RUBRIC – LABORATORY PAPERS APC2021

Lab: 1 2 3 4 5 **Submit signature page on reverse**

Student1 : Sarah Ali Student2 : George Paraschiv Student3 : Arthur Xu

Student4 : ____________________ Student5 : ____________________

| SUB CATEGORY | DESCRIPTION | WT% | LETTER GRADE |
|---|---|---|---|
| SYNTAX | Spelling and Grammar including proper tense of expression. Was jargon used appropriately? | **10** | |
| FORM – GRAPHS AND CHARTS | Do your graphs and tables follow the guidelines? Did you put a caption beneath each one? Are they numbered? Are they well presented? Are they necessary? Your tables should not break at the end of a page. Did you remember the index column? Are your graphs scaled properly and with the proper format? Have you included error bars as required? | **7** | |
| FORM – EQUATIONS AND MARGINS | Are your equations properly presented? Are the variables defined with the expected units included? Are variables only defined on first use? Are your equations numbered with the number well off to the right? Did you source them as required? Are subscripts used correctly? Do your margin and columns conform to the rules? Is the font properly chosen? Are you using the correct size of paper and did you set up the first page as per the exemplar. Does your report conform to the length rules? Is your report free from silly computer glitches such as widows, orphans, and large gaps in the text? You must fix all of this. Did you email the .pdf copy of your work to your instructor prior to the due time? Did you print your work on BOTH sides of the paper? Did you submit a single page on the back of the marking rubric, detailing the work of the team members and include signatures of all? | **7** | |
| FORM – CITATIONS AND SOURCES | Have you cited any facts not fairly earned by your team? Are they cited in the proper manner described in the lab manual? Did you list all your sources? Are the sources conveyed in the proper format? Did you include at least TWO text sourcings? | **7** | |
| UNCERTAINTY | Have you indicated the uncertainty of your measured, computed and regressed values? Has this been done according to the rules as stipulated in the manual? Have your uncertainties been stated in the proper format? | **10** | |
| ABSTRACT | Is this abstract written in the proper form? Is it a fair description of this work and its accomplishments? Is it a reasonable length? Does the abstract use some of the most impressive numerical values to buttress its claims? | **5** | |
| METHODOLOGY | How have you used the equipment available to you? Have you maximized the precision and possibly the accuracy of your work? Did you know that you were doing this? Did you waste undue time on lengthy, but not effective or productive techniques? | **17** | |
| BASIC CONTENT | Were the basic questions of the experiment answered? Were all the stated requirements met? | **17** | |
| ADVANCED ANALYSIS | How aggressively did the report discuss the subtle relationships? How well were numerical relationships between variables developed and explained? How creative was the work presented via written text, graphs, tables and such like. (I really care about this folks!) | **20** | |
| NOTEBOOK | See Notebook Rubric – page 27 | **2** | |

| PAPER | **%** | NOTEBOOK | **/2** | TOTAL | **%** |
|---|---|---|---|---|---|

# Signature Page

Please sign and indicate what each person did.

| SIGNATURE | NAME | CONTRIBUTION |
|---|---|---|
| SA | Sarah Ali | - Experiment 2<br>- Algebra Engine |
| GP | George Paraschiv | - Experiment 1<br>- Uncertainty Principle |
| AX | Arthur Xu | - Experiment 1<br>- Experiment 3 |
| | | |
| | | |