A paper on Hyperbolics as a Reference for the Future

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1 Definitions and Pythagorean Identities

The basic $\sinh(x)$ and $\cosh(x)$ functions are defined as follows.

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \tag{1}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \tag{2}$$

We can see that summing and finding the difference of (1) and (2) will show the following identities.

$$\cosh(x) + \sinh(x) = e^x$$
$$\cosh(x) - \sinh(x) = e^{-x}$$

Squaring (1) and (2) will yield the Pythagorean identity for $\cosh$ and $\sinh$, which is $\cosh^2(x) - \sinh^2(x) = 1$. Note that $RHS = 1$

$$\begin{aligned}
LHS &= \cosh^2(x) - \sinh^2(x) \\
&= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 \\
&= \frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{4} - \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4} \\
&= \frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4} \\
&= \frac{1}{4}\left(e^{2x} + 2 + e^{-2x} - \left(e^{2x} - 2 + e^{-2x}\right)\right) \\
&= \frac{1}{4}(4) \\
&= 1
\end{aligned}$$

Since $LS = RS$, we have proved the basic Pythagorean identity. As a result, we know that $(x, y) = \left(\cosh^2(t), \sinh(t)\right)$ is a parametric representation of the unit hyperbola $x^2 - y^2 = 1$. The relationship between the remaining hyperbolic functions with respect to each other, is the same as the relationship between the trigonometric functions with respect to each other.

$$\begin{aligned}\operatorname{sech}(x) &= \frac{1}{\cosh(x)} \\ &= \frac{2}{e^{x}+e^{-x}}\end{aligned} \tag{3}$$

$$\begin{aligned}\operatorname{csch}(x) &= \frac{1}{\sinh(x)} \\ &= \frac{2}{e^{x}-e^{-x}}\end{aligned} \tag{4}$$

$$\begin{aligned}\tanh(x) &= \frac{\sinh(x)}{\cosh(x)} \\ &= \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\end{aligned} \tag{5}$$

$$\begin{aligned}\coth(x) &= \frac{\cosh(x)}{\sinh(x)} \\ &= \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}\end{aligned} \tag{6}$$

We can now extend the hyperbolic Pythagorean identities like we do with the trigonometric Pythagorean identities. We can either divide both sides by $\cosh(x) or \sinh(x)$.

$$\begin{aligned}\cosh^2(x) - \sinh^2(x) &= 1 \\ \frac{\cosh^2(x)}{\cosh^2(x)} - \frac{\sinh^2(x)}{\cosh^2(x)} &= \frac{1}{\cosh^2(x)} \\ 1 - \tanh^2(x) &= \operatorname{sech}^2(x) \\ \operatorname{sech}^2(x) + \tanh^2(x) &= 1\end{aligned}$$

$$\begin{aligned}\cosh^2(x) - \sinh^2(x) &= 1 \\ \frac{\cosh^2(x)}{\sinh^2(x)} - \frac{\sinh^2(x)}{\sinh^2(x)} &= \frac{1}{\sinh^2(x)} \\ \coth^2(x) - 1 &= \operatorname{csch}^2(x) \\ \operatorname{csch}^2(x) + \coth^2(x) &= 1\end{aligned}$$

2 Inverse Hyperbolic Functions

In general, $y = f^{-1}(x)$ means $x = f(y)$. We can apply this concept to the definition of the hyperbolic functions to find the definitions of their inverse functions.

$\boldsymbol{y = \operatorname{arcsinh}(x)}$ **means**:

$$x = \frac{e^{y}-e^{-y}}{2}$$

We can then rearrange for y which is $\operatorname{arcsinh}(x)$.

$$\begin{aligned}2x &= e^{y}-e^{-y} \\ e^{y}-e^{-y}-2x &= 0\end{aligned}$$

We can then multiply both sides by e^{y}. This puts the equation in a form where we can use the quadratic equation to find e^{y}.

$$\begin{aligned}e^{2y} - 1 - 2xe^{y} &= 0 \\ (e^{y})^2 - 2xe^{y} - 1 &= 0 \\ e^{y} &= \frac{2x \pm \sqrt{(-2x)^2 - (4)(1)(-1)}}{2} \\ e^{y} &= \frac{2x \pm \sqrt{4x^2+4}}{2} \\ e^{y} &= \frac{2x \pm 2\sqrt{x^2+1}}{2} \\ y &= \ln\left(x \pm \sqrt{x^2+1}\right)\end{aligned}$$

We must take note however that the domain of $\ln x$ is $x > 0$. In the above expression, we note that $\sqrt{x^2+1} > x$ for all x (I'm not putting a rigorous proof but it's because the square root of something squared plus some constant is always greater than that something), meaning that we can't take the negative option because that will yield a valueless than 0. Therefore we have

$$\begin{aligned} y &= \ln\left(x + \sqrt{x^2+1}\right) \\ \operatorname{arcsinh}(x) &= \ln\left(x + \sqrt{x^2+1}\right) \end{aligned} \tag{7}$$

$\boldsymbol{y = \textbf{arccosh}\,(x)}$ **means**:

$$x = \frac{e^y + e^{-y}}{2}$$

We can then rearrange for y which is $\operatorname{arccosh}(x)$.

$$\begin{aligned} 2x &= e^y + e^{-y} \\ e^y + e^{-y} - 2x &= 0 \end{aligned}$$

We can then multiply both sides by e^y. This puts the equation in a form where we can use the quadratic equation to find e^y.

$$\begin{aligned} e^{2y} + 1 - 2xe^y &= 0 \\ (e^y)^2 - 2xe^y + 1 &= 0 \\ e^y &= \frac{2x \pm \sqrt{(-2x)^2 - (4)(1)(1)}}{2} \\ e^y &= \frac{2x \pm \sqrt{4x^2-4}}{2} \\ e^y &= \frac{2x \pm 2\sqrt{x^2-1}}{2} \\ y &= \ln\left(x \pm \sqrt{x^2-1}\right) \end{aligned}$$

The situation here in $\operatorname{arccosh}(x)$ is different than $\operatorname{arcsinh}(x)$ because $\sqrt{x^2-1} < x$ for all x, so both the positive and negative square roots can work. This is actually expected, because $\cosh(x)$ is an even function meaning that it fails the horizontal line test leading to no inverse FUNCTION. Like the inverse functions of other even functions (such as x^2), we have to restrict our initial domain. So, we chose the domain to be $x > 0$ meaning that $y > 0$ in the inverse. For this to be true, we have to pick the positive variant of the square root. This is what calculators will output when we use the inverse hyperbolic cosine function. However, when graphing, it is okay to graph both square root signs. Therefore we have

$$\begin{aligned} y &= \ln\left(x + \sqrt{x^2-1}\right) \\ \operatorname{arccosh}(x) &= \ln\left(x + \sqrt{x^2-1}\right) \end{aligned} \tag{8}$$

$\boldsymbol{y = \textbf{arctanh}\,(x)}$ **means**:

$$\tanh(x) = \frac{e^y - e^{-y}}{e^y + e^{-y}}$$

We can then rearrange for y which is $\operatorname{arctanh}(x)$.

$$xe^y + xe^{-y} = e^y - e^{-y}$$

We can then multiply both sides by e^y. This allows to explicitly express e^y as a function of x.

$$\begin{aligned} xe^{2y} + x &= e^{2y} - 1 \\ (x-1)\, e^{2y} &= -(x+1) \\ e^{2y} &= \frac{-(x+1)}{x-1} \\ e^{y} &= \sqrt{\frac{x+1}{1-x}} \\ y &= \ln\left(\sqrt{\frac{x+1}{1-x}}\right) \end{aligned}$$

It wasn't explicitly expressed mathematically, but we have to take the positive square root because the inside of a logarithm has to be positive. Thus, we have

$$\operatorname{arctanh}(x) = \frac{1}{2}\ln\left(\frac{x+1}{1-x}\right) \tag{9}$$

Note that the above function has the following domain: $|x| < 1$

$\boldsymbol{y = \operatorname{arccoth}(x)}$ **means**:

$$\coth(x) = \frac{e^y + e^{-y}}{e^y + e^{-y}}$$

We can then rearrange for y which is $\operatorname{arccoth}(x)$.

$$xe^y + xe^{-y} = e^y + e^{-y}$$

We can then multiply both sides by e^y. This allows to explicitly express e^y as a function of x.

$$\begin{aligned} xe^{2y} + x &= e^{2y} + 1 \\ (x-1)\, e^{2y} &= x+1 \\ e^{2y} &= \frac{x+1}{x-1} \\ e^{y} &= \sqrt{\frac{x+1}{x-1}} \\ y &= \ln\left(\sqrt{\frac{x+1}{x-1}}\right) \end{aligned}$$

It wasn't explicitly expressed mathematically, but we have to take the positive square root because the inside of a logarithm has to be positive. Thus, we have

$$\operatorname{arccoth}(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right) \tag{10}$$

Note that the above function has the following domain: $|x| > 1$

$\boldsymbol{y = \operatorname{arccsch}(x)}$ **means**:

$$x = \frac{2}{e^y - e^{-y}}$$

We can then rearrange for y which is $\operatorname{arccsch}(x)$.

$$\begin{aligned} xe^y - xe^{-y} &= 2 \\ xe^y - xe^{-y} - 2 &= 0 \end{aligned}$$

We can then multiply both sides by e^y. This puts the equation in a form where we can use the quadratic equation to find e^y.

$$\begin{aligned}
xe^{2y} - x - 2e^y &= 0 \\
xe^{2y} - 2e^y - x &= 0 \\
e^y &= \frac{2 \pm \sqrt{(-2)^2 - 4(x)(-x)}}{2x} \\
e^y &= \frac{2 \pm \sqrt{4 + 4x^2}}{2x} \\
e^y &= \frac{1 \pm \sqrt{1 + x^2}}{x} \\
y &= \ln\left(\frac{1 \pm \sqrt{1 + x^2}}{x}\right)
\end{aligned}$$

Technically, the derived equation is correct (even with the plus-minus sign). We will remove the plus minus sign by squaring the $\pm\frac{1}{x}$ term into the square root sign. What I would suggest to help visualize, is to go into a graphing software, graph the above expression with plus, and then graph the expression with a minus. Finally, graph $\operatorname{arccsch}(x)$. The final equation is the combination of the two.

$$\begin{aligned}
y &= \ln\left(\frac{1}{x} + \left(\pm\frac{1}{x}\right)\sqrt{1 + x^2}\right) \\
y &= \ln\left(\frac{1}{x} + \sqrt{\left(\pm\frac{1}{x}\right)^2 (1 + x^2)}\right) \\
y &= \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2}(1 + x^2)}\right) \\
y &= \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1}\right)
\end{aligned}$$

Thus we have:

$$\operatorname{arccsch}(x) = \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1}\right) \tag{11}$$

$y = \operatorname{arcsech}(x)$ means:

$$x = \frac{2}{e^y + e^{-y}}$$

We can then rearrange for y which is $\operatorname{arcsech}(x)$.

$$\begin{aligned}
xe^y + xe^{-y} &= 2 \\
xe^y + xe^{-y} - 2 &= 0
\end{aligned}$$

We can then multiply both sides by e^y. This puts the equation in a form where we can use the quadratic equation to find e^y.

$$\begin{aligned}
xe^{2y} + x - 2e^y &= 0 \\
xe^{2y} - 2e^y + x &= 0 \\
e^y &= \frac{2 \pm \sqrt{(-2)^2 - 4x^2}}{2x} \\
e^y &= \frac{2 \pm \sqrt{4 - 4x^2}}{2x} \\
e^y &= \frac{1 \pm \sqrt{1 - x^2}}{x} \\
y &= \ln\left(\frac{1 + \sqrt{1 - x^2}}{x}\right)
\end{aligned}$$

With $\text{arcsech}\,(x)$, we have a similar situation to $\text{arccosh}\,(x)$ as they are both even functions. With similar logic we only use the positive square root. However, we will continue with our algebra so that the function looks similar to that of $\text{arccsch}\,(x)$

$$\begin{aligned}
y &= \ln\left(\frac{1}{x} + \left(\pm\frac{1}{x}\right)\sqrt{1 - x^2}\right) \\
y &= \ln\left(\frac{1}{x} + \sqrt{\left(\pm\frac{1}{x}\right)^2 (1 - x^2)}\right) \\
y &= \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2}(1 - x^2)}\right) \\
y &= \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right)
\end{aligned}$$

Thus we have:

$$\text{arcsech}\,(x) = \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right) \tag{12}$$

Again, I want to note that the domain of this function is $0 < x \leq 1$

3 Derivatives

For all of the derivatives, we can use the definitions to find the derivatives. Many of the derivatives are the same as the regular trigonometric functions. However, some are different, so we have to be careful. For $\sinh(x)$

$$\begin{aligned}
\frac{d}{dx}\sinh(x) &= \frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right) \\
&= \frac{e^x + e^{-x}}{2}
\end{aligned}$$

And thus,

$$\frac{d}{dx}\sinh(x) = \cosh(x) \tag{13}$$

For $\cosh(x)$

$$\begin{aligned}\frac{d}{dx}\cosh(x) &= \frac{d}{dx}\left(\frac{e^x+e^{-x}}{2}\right)\\ &= \frac{e^x-e^{-x}}{2}\end{aligned}$$

And thus,

$$\frac{d}{dx}\cosh(x) = \sinh(x) \tag{14}$$

For $\tanh(x)$

$$\begin{aligned}\frac{d}{dx}\tanh(x) &= \frac{d}{dx}\left(\frac{\sinh(x)}{\cosh(x)}\right)\\ &= \frac{\frac{d}{dx}(\sinh(x))\cosh(x) - \frac{d}{dx}(\cosh(x))\sinh(x)}{\cosh^2(x)}\\ &= \frac{\cosh^2(x)-\sinh^2(x)}{\cosh^2(x)}\\ &= \frac{1}{\cosh^2(x)}\end{aligned}$$

And, thus

$$\frac{d}{dx}\tanh(x) = \operatorname{sech}^2(x) \tag{15}$$

For $\coth(x)$

$$\begin{aligned}\frac{d}{dx}\coth(x) &= \frac{d}{dx}\left(\frac{\cosh(x)}{\sinh(x)}\right)\\ &= \frac{\frac{d}{dx}(\cosh(x))\sinh(x) - \frac{d}{dx}(\sinh(x))\cosh(x)}{\sinh^2(x)}\\ &= \frac{\cosh^2(x)-\sinh^2(x)}{\sinh^2(x)}\\ &= \frac{1}{\sinh^2(x)}\end{aligned}$$

And, thus

$$\frac{d}{dx}\coth(x) = \operatorname{csch}^2(x) \tag{16}$$

For $\operatorname{csch}(x)$

$$\begin{aligned}\frac{d}{dx}\operatorname{csch}(x) &= \frac{d}{dx}\left(\frac{1}{\sinh(x)}\right)\\ &= -\frac{1}{\sinh^2(x)}\frac{d}{dx}\sinh(x)\\ &= -\frac{\cosh(x)}{\sinh(x)}\frac{1}{\sinh(x)}\end{aligned}$$

And thus,

$$\frac{d}{dx}\operatorname{csch}(x) = -\coth(x)\operatorname{csch}(x) \tag{17}$$

For $\operatorname{sech}(x)$

$$\begin{aligned}
\frac{d}{dx}\operatorname{sech}(x) &= \frac{d}{dx}\left(\frac{1}{\cosh(x)}\right) \\
&= -\frac{1}{\cosh^2(x)}\frac{d}{dx}\cosh(x) \\
&= -\frac{\sinh(x)}{\cosh(x)}\frac{1}{\cosh(x)}
\end{aligned}$$

And thus,

$$\frac{d}{dx}\operatorname{sech}(x) = -\tanh(x)\operatorname{sech}(x) \tag{18}$$

For $\operatorname{arcsinh}(x)$

$$\begin{aligned}
\frac{d}{dx}\operatorname{arcsinh}(x) &= \frac{d}{dx}\left(\ln\left(x+\sqrt{x^2+1}\right)\right) \\
&= \frac{1}{x+\sqrt{x^2+1}}\left(1+\frac{1}{\cancel{2}\sqrt{x^2+1}}\cancel{2}x\right) \\
&= \frac{1}{x+\sqrt{x^2+1}}\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right) \\
&= \frac{1}{\sqrt{x^2+1}}
\end{aligned}$$

And thus,

$$\frac{d}{dx}\operatorname{arcsinh}(x) = \frac{1}{\sqrt{x^2+1}} \tag{19}$$

For $\operatorname{arccosh}(x)$

$$\begin{aligned}
\frac{d}{dx}\operatorname{arccosh}(x) &= \frac{d}{dx}\left(\ln\left(x+\sqrt{x^2-1}\right)\right) \\
&= \frac{1}{x+\sqrt{x^2-1}}\left(1+\frac{1}{\cancel{2}\sqrt{x^2-1}}\cancel{2}x\right) \\
&= \frac{1}{x+\sqrt{x^2-1}}\left(\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}\right) \\
&= \frac{1}{\sqrt{x^2-1}}
\end{aligned}$$

And thus,

$$\frac{d}{dx}\operatorname{arccosh}(x) = \frac{1}{\sqrt{x^2-1}} \tag{20}$$

For arctanh (x)

$$
\begin{aligned}
\frac{d}{dx}\operatorname{arctanh}(x) &= \frac{d}{dx}\left(\frac{1}{2}\ln\left(\frac{x+1}{1-x}\right)\right)\\
&= \frac{1}{2}\left(\frac{1-x}{x+1}\right)\frac{d}{dx}\left(\frac{x+1}{1-x}\right)\\
&= \frac{1}{2}\left(\frac{1-x}{x+1}\right)\left(\frac{f'g-g'f}{g^2}\right)\\
&= \frac{1}{2}\left(\frac{1-x}{x+1}\right)\left(\frac{(1)(1-x)-(-1)(x+1)}{(1-x)^2}\right)\\
&= \frac{1}{2}\left(\frac{1-x}{x+1}\right)\left(\frac{2}{(1-x)^2}\right)\\
&= \frac{1}{(1+x)(1-x)}\\
&= \frac{1}{1-x^2}
\end{aligned}
$$

For arctanh (x)

$$
\begin{aligned}
\frac{d}{dx}\operatorname{arccoth}(x) &= \frac{d}{dx}\left(\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)\right)\\
&= \frac{1}{2}\left(\frac{x-1}{x+1}\right)\frac{d}{dx}\left(\frac{x+1}{x-1}\right)\\
&= \frac{1}{2}\left(\frac{x-1}{x+1}\right)\left(\frac{f'g-g'f}{g^2}\right)\\
&= \frac{1}{2}\left(\frac{x-1}{x+1}\right)\left(\frac{(1)(x-1)-(1)(x+1)}{(x-1)^2}\right)\\
&= \frac{1}{2}\left(\frac{x-1}{x+1}\right)\left(\frac{-2}{(x-1)^2}\right)\\
&= \frac{-1}{(x+1)(x-1)}\\
&= \frac{-1}{x^2-1}\\
&= \frac{1}{1-x^2}
\end{aligned}
$$

At the surface, it may seem that arctanh (x) and arccoth (x) have the same derivative. However, we have to consider that a function's derivative has the same domain as the original function. Taking into consideration the domains mentioned below (9) and (10), we get the following:

$$\frac{d}{dx}\operatorname{arctanh}(x) = \frac{1}{1-x^2};\quad |x|<1 \tag{21}$$

$$\frac{d}{dx}\operatorname{arccoth}(x) = \frac{1}{1-x^2};\quad |x|>1 \tag{22}$$

For arccsch (x)

$$\frac{d}{dx}\operatorname{arccsch}(x) = \frac{d}{dx}\left(\ln\left(\frac{1}{x}+\sqrt{\frac{1}{x^2}+1}\right)\right)$$

For ease of algebra, I will rewrite the reciprocals using negative exponents of x. However, I will continue to use the square root sign

$$\begin{aligned}
\frac{d}{dx}\operatorname{arccsch}(x) &= \frac{d}{dx}\left(\ln\left(x^{-1}+\sqrt{x^{-2}+1}\right)\right)\\
&= \frac{1}{x^{-1}+\sqrt{x^{-2}+1}}\left(-x^{-2}+\frac{1}{2\sqrt{x^{-2}+1}}\left(-2x^{-3}\right)\right)\\
&= \frac{-x^{-3}}{x^{-1}+\sqrt{x^{-2}+1}}\left(x^{-1}+\frac{1}{\sqrt{x^{-2}+1}}\right)\\
&= \frac{-1}{x^{3}\left(x^{-1}+\sqrt{x^{-2}+1}\right)}\left(\frac{x\sqrt{x^{-2}+1}+1}{\sqrt{x^{-2}+1}}\right)\\
&= \frac{-x}{x^{3}\left(x^{-1}+\sqrt{x^{-2}+1}\right)}\left(\frac{x^{-1}+\sqrt{x^{-2}+1}}{\sqrt{x^{-2}+1}}\right)\\
&= \frac{-1}{x^{2}\sqrt{x^{-2}+1}}\\
&= \frac{-1}{x^{2}\sqrt{x^{-2}\left(1+x^{2}\right)}}\\
&= \frac{-1}{x^{2}\left|x^{-1}\right|\sqrt{1+x^{2}}}\\
&= \frac{-1}{|x|\sqrt{1+x^{2}}}
\end{aligned}$$

The final equation that we reached seems to be what the math community agrees to be the ”simplist” form, despite the absolute value, so I simply decided to reach the agreed answer. Though personally, I probably would’ve stopped earlier to avoid the absolute value. Thus, we have

$$\frac{d}{dx}\operatorname{arccsch}(x) = \frac{-1}{|x|\sqrt{1+x^{2}}} \tag{23}$$

An extremely similar process is used for $\operatorname{arcsech}(x)$

$$\begin{aligned}
\frac{d}{dx}\operatorname{arcsech}(x) &= \frac{d}{dx}\left(\ln\left(x^{-1}+\sqrt{x^{-2}-1}\right)\right)\\
&= \frac{1}{x^{-1}+\sqrt{x^{-2}-1}}\left(-x^{-2}+\frac{1}{2\sqrt{x^{-2}-1}}\left(-2x^{-3}\right)\right)\\
&= \frac{-x^{-3}}{x^{-1}+\sqrt{x^{-2}-1}}\left(x^{-1}+\frac{1}{\sqrt{x^{-2}-1}}\right)\\
&= \frac{-1}{x^{3}\left(x^{-1}+\sqrt{x^{-2}-1}\right)}\left(\frac{x\sqrt{x^{-2}-1}+1}{\sqrt{x^{-2}-1}}\right)\\
&= \frac{-x}{x^{3}\left(x^{-1}+\sqrt{x^{-2}-1}\right)}\left(\frac{x^{-1}+\sqrt{x^{-2}-1}}{\sqrt{x^{-2}-1}}\right)\\
&= \frac{-1}{x^{2}\sqrt{x^{-2}-1}}\\
&= \frac{-1}{x^{2}\sqrt{x^{-2}\left(1-x^{2}\right)}}\\
&= \frac{-1}{x^{2}\left|x^{-1}\right|\sqrt{1-x^{2}}}\\
&= \frac{-1}{|x|\sqrt{1-x^{2}}}
\end{aligned}$$

We could just leave the equation like this. However, since we know that the domain of $\operatorname{arcsech}(x)$ is $0 < x \leq 1$, the absolute value is not needed, and we can simply remove it. Furthermore, it is important that we mention the domain when declaring the derivative

$$\frac{d}{dx}\operatorname{arcsech}(x) = \frac{-1}{x\sqrt{1-x^2}} \tag{24}$$

4 Conclusion

There are a total of 24 labelled equations in this document. They are as follows:

- definitions of the 6 hyperbolic functions
- definitions of the 6 inverse hyperbolic functions
- derivatives of the 6 hyperbolic functions
- derivatives of the 6 inverse hyperbolic functions