A Quick Dice Probability Problem

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1 Problem

Three six-sided die (that are numbered 1-6) are rolled. What is the probability that the largest roll is exactly twice the smallest roll?

2 Solution

There are $6^3 = 216$ total possibilities. Of these 216 possibilities, we just need to count how many of these satisfy the conditions of the problem. First, we note that there are only 3 ways in which the max roll is double the smallest roll. This is when (min,max) = (1,2), (2,4), and (3,6). I can write the probability as follows,

$$P = \frac{P(1,2) + P(2,4) + P(3,6)}{216} \tag{1}$$

Where $P($(min,max) simply denotes the amount of cases as opposed to the probability.

For the first case, there are only 2 possibilities: having two 2s and one 1, or one 2 and two 1s. In both these cases, we can say that there are $\binom{3}{2}$ permutations as were picking 2 of the 3 dice slots for the duplicate to occur in. Thus,

$$\begin{aligned} P(1,2) &= (2)\binom{3}{2} \\ &= 6 \end{aligned} \tag{2}$$

For the second case, there are 3 possibilities. Similarly, we have the case where there are two 2s and one 4 or one 2 and two 4s. These both have $\binom{3}{2}$ permutations like case 1. However, this case is different as we can have a 2, a 3 and a 4. There are 3 permute 3 or $\frac{3!}{(3-3)!}$ ways to do this. Thus,

$$\begin{aligned} P(2,4) &= (2)\binom{3}{2} + \frac{3!}{(3-3)!} \\ &= 6 + 6 \\ &= 12 \end{aligned} \tag{3}$$

Doing similar case work for the third case, there are $\binom{3}{2}$ possibilities when there are two 3s and one 6 or one 3 and two 6s. There are also 3 permute 3 possibilities when there is a 3,4, and 6 or 3,5, and 6. Thusm

$$\begin{aligned} P(3,6) &= (2)\binom{3}{2} + (2)\frac{3!}{(3-3)!} \\ &= 6 + 12 \\ &= 18 \end{aligned} \tag{4}$$

Substituting our results from (2), (3), and (4) back into (1), we have

$$\begin{aligned} P &= \frac{P\left(1,2\right)+P\left(2,4\right)+P\left(3,6\right)}{216} \\ &= \frac{6+12+18}{216} \\ &= \frac{1}{6} \end{aligned}$$

3 The Computer Program

I ended up writing a computer program for this problem to verify my answer. It is simply a probability simulator that runs through the scenarios thousands of times. If you found this paper in a google drive folder, the code for this problem is found on my GitHub (@ArthurXu17). If you found this paper in my GitHub repository "Math Simulations", the corresponding program is titled Threedie.java.