## Sorbonne Université

Calcul haute performance : programmation et algorithmique avancées

# Projet: Batch merge and merge path sort

Code source et prsentation du travail le 10/12/2020

Le code source doit tre lisible et suffisamment comment La dure des prsentations est de  $\sim 15$  minutes par binme

Based on the merging sort presented in [1], this project has three parts. The first part deals with sorting an array on GPU. The second part deals with batch merging various small arrays at the same time. The last part is left to students to propose applications involving merging arrays. In the following, given a set A, the cardinal |A| denotes the number of its elements.

### 1 Merge path and sort

We start with the merge path algorithm. Let A and B be two ordered arrays (increasing order), we want to merge them in an M sorted array. The merge of A and B is based on a path that starts at the top-left corner of the  $|A| \times |B|$  grid and arrives at the down-right corner. The Sequential Merge Path is given by Algorithm 1 and an example is provided in Figure 1.

		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	
		4	7	8	10	12	13	14	
A[0]	1	1	1	.1	1	1	1	1	
A[1]	2	1	1	1	1	1	1	1	
A[2]	5	0	1	1	1	1	1	1	A
A[3]	6	0	1	1	1	1	1	1	$\Delta_{11}$ , .
A[4]	6	0	.1	.1	1	1	1	1	
A[5]	9	0	0	0	1	1	.1 	1	
A[6]	11	0	0	$0 - K_0$	0	1	$Q_0$ 1	1	
A[7]	15	0	$\begin{bmatrix} 1 - T_2 \\ 0 \end{bmatrix}$	$=K_2$	$\sim$	0	0	0	
A[8]	16	0	0	0	$\begin{bmatrix} Q_1 \\ 0 \end{bmatrix}$	0	0	0	
			$K_0$ =	$=K_1$					-

Figure 1: An example of Merge Path procedure

#### Algorithm 1 Sequential Merge Path

```
Require: A and B are two sorted arrays
Ensure: M is the merged array of A and B with |M| = |A| + |B|
  procedure MergePath (A, B, M)
     j = 0 and i = 0
     while i + j < |M| do
        if i \geq |A| then
            M[i+j]=B[j]
            j = j + 1
                                                                                 \triangleright The path goes right
         else if j \ge |B| or A[i] < B[j] then
            M[i+j]{=}A[i]
                                                                                 ▶ The path goes down
            i = i + 1
         else
            M[i+j]=B[j]
            j = j + 1
                                                                                  ▶ The path goes right
        end if
     end while
  end procedure
```

#### **Algorithm 2** Merge Path (Indexes of n threads are 0 to n-1)

```
Require: A and B are two sorted arrays
Ensure: M is the merged array of A and B with |M| = |A| + |B|
  for each thread in parallel do
     i=index of the thread
     if i > |A| then
         K = (i - |A|, |A|)
                                                                                  P = (|A|, i - |A|)
                                                                                 ▶ High point of diagonal
     else
         K = (0, i)
         P = (i, 0)
     end if
     while True do
         offset = abs(K_y - P_y)/2
         Q = (K_x + offset, K_y - offset)
         if Q_y \geq 0 and Q_x \leq B and
           (Q_y = |A| \text{ or } Q_x = 0 \text{ or } A[Q_y] > B[Q_x - 1]) \text{ then}
            if Q_x = |B| or Q_y = 0 or A[Q_y - 1] \le B[Q_x] then
                if Q_y < |A| and (Q_x = |B|) or A[Q_y] \le B[Q_x] then
                   M[i] = A[Q_y]
                                                                                            \triangleright Merge in M
                else
                   M[i] = B[Q_x]
                end if
                Break
            else
                K = (Q_x + 1, Q_y - 1)
            end if
         else
             P = (Q_x - 1, Q_y + 1)
         end if
     end while
  end for
```

Each point of the grid has a coordinate  $(i,j) \in [0,|A|] \times [0,|B|]$ . The merge path starts from the

point (i, j) = (0, 0) on the left top corner of the grid. If A[i] < B[j] the path goes down else it goes right. The array  $[0, |A| - 1] \times [0, |B| - 1]$  of boolean values A[i] < B[j] is not important in the algorithm. However, it shows clearly that the merge path is a frontier between ones and zeros.

To parallelize the algorithm, the grid has to be extended to the maximum size equal to  $\max(|A|, |B|) \times \max(|A|, |B|)$ . We denote  $K_0$  and  $P_0$  respectively the low point and the high point of the ascending diagonals  $\Delta_k$ . On GPU, each thread  $k \in [0, |A| + |B| - 1]$  is responsible of one diagonal. It finds the intersection of the merge path and the diagonal  $\Delta_k$  with a binary search described in Algorithm 2.

- 1. For  $|A| + |B| \le 1024$ , write a kernel mergeSmall\_k that merges A and B using only one block of threads.
- 2. For any size |A| + |B| = d sufficiently smaller than the global memory, write two kernels that merge A and B using various blocks: The first kernel pathBig\_k finds the merge path and the second one mergeBig\_k\_merges A and B.
- 3. Looping on appropriate calls of pathBig\_k and of mergeBig\_k, write a function that sorts any array.

  M of size d sufficiently smaller than the global memory. Give the execution time with respect to d.

# 2 Batch merge

In this part, we assume that we have a large number  $N(\geq 1e3)$  of arrays  $\{A_i\}_{1\leq i\leq N}$  and  $\{B_i\}_{1\leq i\leq N}$  with  $|A_i|+|B_i|=d\leq 1024$  for each i. Using some changes on mergeSmall\_k, we would like to write mergeSmallBatch\_k that merges two by two, for each i,  $A_i$  and  $B_i$ .

Given a fixed common size  $d \leq 1024$ , mergeSmallBatch\_k is launched using the syntax

mergeSmallBatch\_k<<<numBlocks, threadsPerBlock>>>(...);

with threadsPerBlock is multiple of d but smaller than 1024 and numBlocks is an arbitrary sufficiently big number.

- 4. Explain why the indices
   int tidx = threadIdx.x%d;
   int Qt = (threadIdx.x-tidx)/d;
   int gbx = Qt + blockIdx.x\*(blockDim.x/d);
   are important in the definition of mergeSmallBatch\_k.
- 5. Write the kernel mergeSmallBatch k that batch merges two by two  $\{A_i\}_{1 \leq i \leq N}$  and  $\{B_i\}_{1 \leq i \leq N}$ . Give the execution time with respect to (d = 4, 8, ..., 1024).

# 3 Merge sort applications

You are free to give a specific application of either question 3. or question 5.

## References

[1] O. Green, R. McColl and D. A. Bader GPU Merge Path - A GPU Merging Algorithm. 26th ACM International Conference on Supercomputing (ICS), San Servolo Island, Venice, Italy, June 25-29, 2012.