Numerical validation using the CADNA library practical work

F. Jézéquel, J.-L. Lamotte
LIP6 Laboratory, Sorbonne Université,
Paris, France

November 6, 2020

Contents

Chapter 1

Getting started

Download from `http://www-pequan.lip6.fr/~jezequel/AFAE.html` the CADNA library: cadna_c-x.x.x.tar.gz, where x.x.x is the CADNA version.

Compile and install the library on your home directory.

```
tar -xzvf  cadna_c-x.x.x.tar.gz
cd cadna_c-x.x.x
./configure --prefix=$PWD
make
make install
```

Remark: on OSX gcc may be a wrapper for clang. In that case, use:

```
./configure  --prefix=$PWD CC=clang CXX=clang++
```

Download the exercises.

```
tar -xvzf ex_AFAE.tar.gz
```

Chapter 2

Exercises

2.1 Exercise 1

This example has been proposed by S. Rump.

$$f(x, y) = 333.75y^6 + x^2(11x^2y^2 - y^6 - 121y^4 - 2) + 5.5y^8 + \frac{x}{2y}$$

is computed with $x = 77617$, $y = 33096$. The 15 first digits of the exact result are -0.827396059946821.

2.1.1 Question 1

In the `ex_AFAE` directory compile and run the `rump.c` program:

```
make rump
./rump
```

Compare the result with the exact one.

2.1.2 Question 2

Copy the `rump.c` program into the `rumpd.c` program.
Change all the variable types from `float` to `double` in the `rumpd.c` program.
Compile and run the `rumpd.c` program:

```
make rumpd
./rumpd
```

Compare the result with the exact one.

2.1.3 Question 3

Implement the CADNA library in the `rump.c` and `rumpd.c` programs by creating two new programs called `rump_cad.cc` and `rumpd_cad.cc`.
In the `Makefile` uncomment and modify the following line:

```
#CADNAC=$(HOME)/cadna_c-x.x.x
```

Compile the `rump_cad.cc` and the `rumpd_cad.cc` programs. Then execute `rump_cad` and `rumpd_cad`.
What do you conclude?

2.2 Exercise 2

The determinant of Hilbert's matrix of size 11 is computed using Gaussian elimination. The determinant is the product of the different pivots. The 14 first digits of the exact determinant are $3.0190953344493\,10^{-65}$.

2.2.1 Question 1

Run the `hilbert.c` program. All the pivots and the determinant are printed out.
Compare the determinant value with the exact one.

2.2.2 Question 2

Implement the CADNA library in the `hilbert.c` program by creating a new program called `hilbert_cad.cc`.
Run the `hilbert_cad.cc` program.
What do you conclude?

2.3 Exercise 3

This example has been proposed by J.-M. Muller.
The `muller.c` program computes the first 30 iterations of the following sequence:

$$U_{n+1} = 111 - \frac{1130}{U_n} + \frac{3000}{U_{n-1} \times U_n}.$$

with $U_0 = 5.5$ and $U_1 = \frac{61}{11}$. The sequence limit is 6.

2.3.1 Question 1

Run the `muller.c` program. Is the result correct?

2.3.2 Question 2

Implement the CADNA library in the `muller.c` program by creating a new program called `muller_cad.cc`.
Run the program. Is the result correct?

Launch the program with the `gdb` debugger:

```
gdb muller_cad
```

Set a break point on the instability function (using the `break instability` command with `gdb`).
Run the program (using the `run` command with `gdb`).
What happens? What kind of instability is detected?
Use the `up` command with `gdb`.
Give the value of `i` (using the `print` command with `gdb`).
Continue the run (using the `cont` command with `gdb`).
To quit the debugger, use the `quit` command.

What do you conclude?

2.4 Exercise 4

This example deals with the improvement of an iterative algorithm by using CADNA functionalities. This program computes a root of the polynomial

$$f(x) = 1.47x^3 + 1.19x^2 - 1.83x + 0.45$$

by Newton's method. The sequence is initialized with $x = 0.5$.
The iterative algorithm $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ is stopped with the criterion

$$|x_n - x_{n-1}| < 10^{-12}.$$

2.4.1 Question 1

Run the `newton.c` program.
What is the sequence limit? How many iterations are computed?

2.4.2 Question 2

Implement the CADNA library in the `newton.c` program by creating a new program called `newton_cad.cc`.
Run the program. How many iterations are computed?
Look at the CADNA instability report.

Change the stopping criteria to `x==y`. How many iterations are computed?
Look at the new CADNA instability report.

What do you conclude?

2.5 Exercise 5

The following linear system is solved using Gaussian elimination with partial pivoting. The system is

$$\begin{pmatrix} 21 & 130 & 0 & 2.1 \\ 13 & 80 & 4.74\ 10^{8} & 752 \\ 0 & -0.4 & 3.9816\ 10^{8} & 4.2 \\ 0 & 0 & 1.7 & 9\ 10^{-9} \end{pmatrix} . X = \begin{pmatrix} 153.1 \\ 849.74 \\ 7.7816 \\ 2.6\ 10^{-8} \end{pmatrix}$$

The exact solution is $x_{sol}^{t} = \left(1, 1, 10^{-8}, 1\right)$.

2.5.1 Question 1

Run the `gauss.c` program. Are the results correct?

2.5.2 Question 2

Implement the CADNA library in the `gauss.c` program by creating a new program called `gauss_cad.cc`.
Run the program.

With the debugger, find which instructions cause the instabilities.
Modifiy the `gauss_cad.cc` program to print the values `a[2][2]` and `a[3][2]`.
Which element is chosen as a pivot? Print the line index `ll`. Il ce sont des L pas des 1
Do the same prints in the `gauss.c` program.

What do you conclude?

2.6 Exercise 6

The `jacobi.c` program implements Jacobi iteration to find the solution of a linear system. The stopping criterion is based on a value ε set to 10^{-4}.

2.6.1 Question 1

Run the `jacobi.c` program.
Are the results correct? How many iterations are performed?
Try other values for ε.

2.6.2 Question 2

Implement the CADNA library in the `jacobi.c` program by creating a new program called `jacobi_cad.cc`.
Run the program.
How many iterations are performed?
Does the stopping criterion have an impact on the number of iterations?
Print the `anorm` value.

What do you conclude?

2.7 Exercise 7

Let us consider the logistic iteration defined by $x_{n+1} = ax_n(1 - x_n)$ with $a > 0$ and $0 < x_0 < 1$.
As a remark, a mathematically equivalent sequence is: $x_{n+1} = -a(x_n - \frac{1}{2})^2 + \frac{a}{4}$.

2.7.1 Question 1

Run the `logistic.c` program which computes the logistic iteration.
In the program both sequence expressions are given. Compare the results obtained.

2.7.2 Question 2

Implement the CADNA library in the `logistic.c` program by creating a new program called `logistic_cad.cc`.
Run the program.
What should be the stopping criterion to avoid useless iterations?