GPU Project : Batch merge and merge path sort

Arthur Zucker, Clément Apavou





14 décembre 2020

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Introduction

Introduction

Introduction 000000

		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	
		4	7	8	10	12	13	14	X
A[0]	1	.1	.1	1	.1	.1	1	.1	
A[1]	2	,1	1	1	.1	.1	1	1	
A[2]	5	0	.1	1	.1	.1	1	1	
A[3]	6	0	.1	.1	.1	1	,A**	.1	
A[4]	6	0	,1	.1	.1		1	1	
A[5]	9	0	0.	.0	,1	.1	1	.1	
A[6]	11	0	,0	.0	.0	.1		.1	
A[7]	15	0	.0	.0	.0	.0	.0	0,	
A[8]	16	0	0	0	0	0	0	0	
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Path merge: How it works

		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	
		4	7	8	10	12	13	14	X
A[0]	1	.1	.1	.1	.1	.1	.1	.1	, P
A[1]	2	.1	1	.1	.1	.1	.1	1	
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A[5]	9	0	.0	.0	.1	.1	.1	1	
A[6]	11	0	.0	.0	.0	.1	,.1	.1	
A[7]	15	0	.0	.0	.0	.0	.0	0,	
A[8]	16	0	0	0	0	0	0	0	
	y,							-	

1 Compute P and K: \rightarrow P=(9,0)

		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	
		4	7	8	10	12	13	14	X • P
A[0]	1	.1	.1	.1	.1	.1	.1	.1	, , , , , , , , , , , , , , , , , , ,
A[1]	2	.1	.1	.1	.1	.1	.1	.1	are are a second
A[2]	5	0	.1	.1	1	.1	.1	1	
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A[8]	16	0	0	0	0	0	0	0	
	y.	K						•	

$$\rightarrow P=(9,0)$$

$$\rightarrow$$
 K=(0,9)

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$$Q = (5,4)$$

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A[6]	11	0	0	.0	.0	.1	.1	.1	
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$$\rightarrow P=(9,0)$$

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$$Q = (5,4)$$

4 P =
$$(Q_X-1,Q_Y+1)=(4,5)$$

Path merge: How it works

Introduction 000000

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	V.	K							

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	V.								

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$$Q = (5,4)$$

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$$(Q_X-1,Q_Y+1)=(4,5)$$

$$Q = P = (4,5)$$

Path merged: thread 9

Introduction 000000

		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	
		4	7	8	10	12	13	14	X
A[0]	1	.1	.1	.1	.1	.1	.1	.1	
A[1]	2	,1	1	.1	.1	.1	.1	1	
A[2]	5	0	, 1	.1	.1	.1	.1	.1	
A[3]	6	0	.1	.1	.1	.1	A	.1	
A[4]	6	0	.1	.1	.1	.1	.1	1	
A[5]	9	0	.0	.0	b.	-0	.1	.1	
A[6]	11	0	0,	.0	.0	- Q	1.1	.1	
A[7]	15	0	.0	.0	.0	.0	.0	.0	
A[8]	16	0 V	0	0	0	0	0	0	
	- 1/	T 1\							

$$\rightarrow P=(9,0)$$

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$$Q = (5,4)$$

4 P =
$$(Q_X-1,Q_Y+1)=(4,5)$$

$$Q = P = (4,5)$$

Path merged: thread 9

		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	
		4	7	8	10	12	13	14	X
A[0]	1	.1	.1	.1	.1	.1	.1	.1	
A[1]	2	.1	1	.1	.1	1	.1	.1	
A[2]	5	0	.1	.1	.1	.1	.1	.1	
A[3]	6	0	.1	.1	.1	.1	A	.1	
A[4]	6	0	,1	,1	.1	1	1	1	
A[5]	9	0	.0	.0	b.		.1	.1	
A[6]	11	0	,0	.0	0	= Q	.1	.1	
A[7]	15	0	.0	.0	.0	.0	.0	.0	
A[8]	16	0	0	0	0	0	0	0	
	y.	K							, , .

$$7 A[Q_y>B[Q_x]$$

Path merged: thread 9

		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	
		4	7	8	10	12	13	14	X
A[0]	1	.1	.1	.1	.1	.1	1	.1	
A[1]	2	.1	1	.1	.1	.1	.1	.1	
A[2]	5	0	.1	.1	1	.1	.1	.1	
A[3]	6	0	.1	.1	.1	.1	.1	.1	
A[4]	6	0	,1	,.17	.1	1.1	.1	.1	
A[5]	9	0	.0	.0	D	<u></u>	.1	.1	
A[6]	11	0	.0	.0	.0	= Q	1.1	.1	
A[7]	15	0	.0	.0	.0	.0	.0	.0	
A[8]	16	0	0	0	0	0	0	0	
	y,	K			,				

7 A[
$$Q_y$$
>B[Q_x]
8 M[9]=B[Q_x]

$$8 M[9] = B[Q_X]$$

Introduction 000000

		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	
		4	7	8	10	12	13	14	X
A[0]	1	.1	.1	1	1	.1	.1	.1	7
A[1]	2	1	.1	.1	1	.1	.1	1	
A[2]	5	0	, 1	1	.1	.1	1	.1	
A[3]	6	0	.1	.1	.1	.1	.1	.1	
A[4]	6	0	,1	11	₁ ,1'	.1	.1	1	
A[5]	9	0	0.	.0	Ď.		.1	.1	
A[6]	11	0	0,	.0	.0	- Q	1	.1	
A[7]	15	0	.0	.0	.0	0.	.0	0,	
A[8]	16	0 V	0	0	0	0	0	0	
	V.								

- 7 A[$Q_y > B[Q_x]$ 8 M[9]=B[Q_x]



		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	
		4	7	8	10	12	13	14	X
A[0]	1	1	.1	.1	.1	.1	.1	.1	7
A[1]	2	11	.1	.1	.1	.1	.1	.1	
A[2]	5	0	,1	.1	.1	.1	.1	.1	
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A[6]	11	0	0	.0	.0	.1	.1	.1	
A[7]	15	0	.0	.0	.0	.0	.0	0.	
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	y,	,							

		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	
		4	7	8	10	12	13	14	X
A[0]	1	.1	.1	1	.1	.1	1	.1	
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A[7]	15	0	.0	.0	.0	.0	.0	.0	
A[8]	16	0	0	0	0	0	0	0	
	y,	_				,			

		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	
		4	7	8	10	12	13	14	X
A[0]	1	.1	.1	.1	.1	.1	.1	.1	
A[1]	2	.1	.1	.1	1	.1	.1	1	
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A[7]	15	0	.0	.0	.0	.0	.0	.0	
A[8]	16	0	0	0	0	0	0	0	
	V.								



		-7.3							1
		B[0]	<i>B</i> [1]	B[2]	B[3]	B[4]	B[5]	B[6]	
		4	7	8	10	12	13	14	X
A[0]	1	.1	1	1	.1	.1	1	1	
A[1]	2	,1	1	1	.1	.1	1	1	
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A[7]	15	0	.0	.0	.0	.0	.0	.0	
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Path merged: thread 9, path

Introduction 000000

		B[0]	B[1]	B[2]	B[3]	B[4]	B[5]	B[6]	
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A[6]	11	0	0	.0	.0	.1	.17	.1	e de la companya de
A[7]	15	0	.0	.0	.0	.0	.0	0.	
A[8]	16	0	0	0	0	0	0	0	
	V.								~

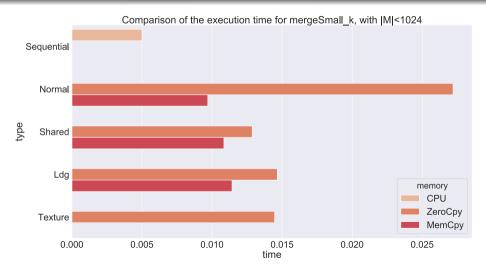
MAIN5-GPU Merge

Merge path and sort

Concept

- Two sorted arrays A and B such $|A| + |B| \le 1024$
- mergeSmall_k merges A and B
- Using one block of threads

Question 1 : Different type of memory



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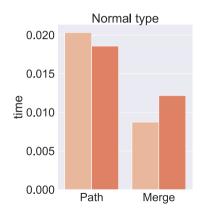
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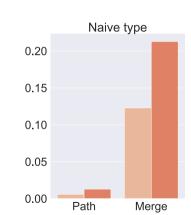
Question 2:pathBig_k and mergeBig_k

Concept

- Two sorted arrays A and B such |A| + |B| = d
- pathBig_k finds the merge path
- mergeBig_k merges A and B

Question 2 : Different type of memory and methods







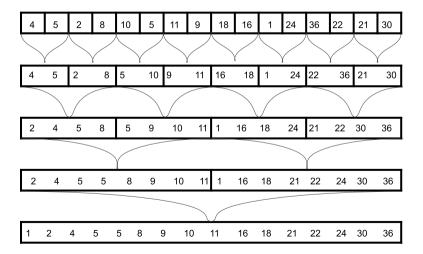
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Question 3 : Sort array

Concept

- Sort any array *M* of size *d*
- Using two previous question

Question 3 : Method



MAIN5-GPU Merge HPCA 14 décembre 2020

Question 3: The execution time for different size of M





Batch merge : Concept

Concept

- A large number N of arrays A_i and B_i with $|A_i| + |B_i| = d \le 1024$ for each i = 1, ..., N.
- Merges two by two, each i, A_i and B_i .

Batch merge: structure

Structure used

- One big array M of size N d
- $M = [A_1|B_1|...|A_i|B_i|...|A_N|B_N]$
- $|A_i| + |B_i| = d \le 1024$ for each $1 \le i \le N$
- $A_i = a_{i1}, a_{i2}, ..., a_{il}$ and $B_i = b_{i1}, b_{i2}, ..., b_{ik}$ with l + k = d
- Store sizes of A_i and B_i

```
int tidx = threadIdx.x%d;
int Ot = (threadIdx.x-tidx)/d;
int qbx = Ot + blockIdx.x*(blockDim.x/d);
```

Index

- tidx: which diagonal does the threads takes care of.
- gbx: index of the value of the size of the sub-array. Allows us to retrieve the addresses of the sub-array A and B.

With gbx, each block takes care of a sub-array.

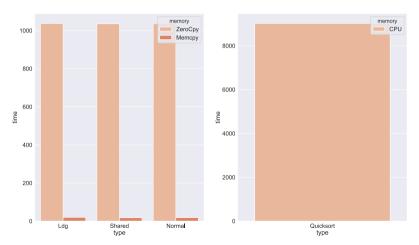
With tidx, Each thread takes care of each diagonal.





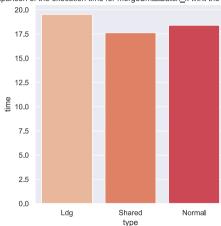
Batch merge: Different type of memory d = 500 and N = 1000000

Comparison of the execution time for mergeSmallBatch k w.r.t the type of memory



Batch merge: Different type of memory d = 500 and N = 1000000

Comparison of the execution time for mergeSmallBatch, k.w.r.t the type of memory



Batch merge: The execution time for different d and N



Merge sort applications

Improving the parallel merge sort

Based on both questions, we came up with a new version of a fast merge sort.

It is based on the observation that in question 3, we have a nested loop which is very slow. Moreover, not every thread is used.

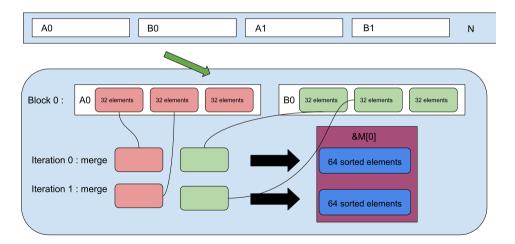
- When $d \le 1024$, mergeSmallBatch can be used in a single loop to get an array of sorted arrays of size 1024. Thus, the occupancy of the GPU is optimal since it is well schedule.
- Then, a modified version of pathBig_k and mergeSmall_k will try to use the same strategy.

A first Idea

The strategy

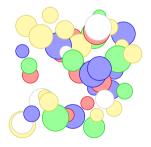
- Use block of size 32 or 64
- Each block will merge start with a window of size BlockDim
- BlockDim elements of A and B will be loaded sequentially in the shared memory
- The path is computed and the merge is also done
- Then, the action is repeated for every sub-arrays of size BlockDim in each arrays of size > 1024.

Representation



Studying species of bacteria

Question 5 can be used to sort energy values of different "cells" that can be studied in a modelling system.



We can for example consider simple cells, interacting with each other. Each cell will have an energy counter, which will vary depending on the collisions and movements of the cell We could want to know the cumulative distribution of the energy with regard to the cell kind.

Studying species of bacteria

We thus need to compute the table containing the energy of each cell. We suppose that each cell's energy is automatically modified in the table. Then, we would end up with something close to:

Species/energy	0-50	50-100	> 100
Α	10	5	0
В	60	1	1
:	:	:	:
С	30	30	30

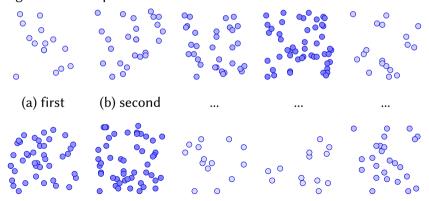
Questions such as "Which species survived the best" or "What makes a species strive?" can arise.

Merge path and sort Batch merge Merge sort applications
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Launching different simulations in parallel

We could also want to study the same dynamics but on different populations set with different types of parameters. Again, if we study groups of size \leq 1024, we can compute every histogram at a fast pace.



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Launching different simulations in parallel

mergeSmallBatch will be used on the following arary

Speed/energy	0-50	50-100	> 100
4	10	5	0
6	60	1	1
:	:	:	÷
10	30	30	30

We would loop over calls of mergeSmallBatch, changing d in order to merge and sort using the strategy from question 3. This is very efficient: at every step of the loop, each thread works on the same amount of values