Break-Even Volatility

Estimating a fair volatility skew from historical prices

1 Introduction

Commonly available historical prices are OHL (Open, High, Low, Close) prices. There are many volatility estimates using the OHLC data meant to improve the efficiency of the standard close-to-close estimator of the volatility $\sum(C_i - C_{i-1})^2$.

Different volatility estimates may be used for different purposes. One issue with the classical estimators is that they are generally not meant to improve the hedging ratio for option pricing and hedging.

Estimating volatility from price history can be necessary in the context of option pricing and hedging:

- We need to price or hedge options on an underlying with illiquid or no option market.
- We seek to find whether the market skew is *fair*, or perform a skew arbitrage on the marker.

2 Fair estimator for the historical volatility

Let us consider the following scenario.

- We sell a vanilla option of strike K and maturity T for a price corresponding to some (arbitrary) volatility σ.
- We perform a delta-hedge assuming the Black-Scholes model with the same volatility σ.
- This delta-hedge results in a final P& L that depend on the input value for σ, $P\&L_{T,K}(\sigma)$.

Using a zero-finding method to find the value $\sigma_{T,K}$ that cancels $P\&L_{T,K}(\sigma)$ results in an estimation for a "fair volatility" or break-even volatility, that is, the value for the Black-Schole volatility that would have resulted in a perfect hedge.

Repeating th processe for a range of values for T and K can be used to produce a complete volatility surface.

This method can produce a complete volatility surfaces for an underlying simply using its price history.

3 C++ Computing Project

Using the C++ programming language, compute a "break-even" volatility smile for one-year maturity options on the SPX500 index using the past year closing prices for the index.

Compare the resulting values with the current one-year volatility skew of SPX500. If our estimation is correct, how can we take advantage of it to arbitrage the implied SPX500 volatility smile?

Bonus question: using the so-called "Black-Scholes Robustness formula", the P& L from the continuous hedge of an option can be written as the time average of the instantaneous volatility weighted with the square gamma of the considered option. Use this formula to produce another estimator for the break-even volatility.