APPENDIX 3: TRIGONOMETRIC REGRESSION FOR FINDING CYCLES

The following computer programs and examples solve the cycle problems in Chapter 11.

Single-Frequency Trigonometric Regression

The FORTRAN program *TRIG1* and the subroutine *LINREG* are used to find the single-frequency representation of the copper cycle. The program output is clearly separated into the following information:

1. Input data, where each time period is the average cash price for a calendar quarter.
2. The solution to the linear regression, giving the detrending line. With $b = .267$, there is an inflationary bias of +.267¢ per quarter.
3. The detrended data resulting from subtracting the line values (2) from the original data (1).
4. Intermediate values for α, ω, and T.
5. The constant values a and b for the normal equations solving the single-frequency problem.
6. The cycle resulting from the detrended data.
7. The final cycle with the trend added back.

The results show a copper cycle of approximately 8.4 quarters, or slightly more than 2½ years.

An additional test was run on monthly cash corn prices from 1964 through 1983 to see if the seasonal cycle dominated the detrended pattern. The linear regression equation used for detrending was calculated as:

$$y = .939 + .01x$$

showing only a 1¢ per bushel per month rate of inflation, despite the bull markets in 1973 and 1980 through 1981. The cycle showed a period of 21.4 months, with the last highs in the cycle in August 1983 and the last lows in

September 1982. Because this is clearly not a seasonal pattern, it must be either:

1. Dominated by other supply-demand characteristics, such as stocks, or,
2. Distorted by the nonseasonal rallies of 1973 and 1980, which each took three years to return to the traditional seasonal pattern.

```
      PROGRAM TRIG1
C---- Single Frequency Trigonometric Regression
C---- Copyright 1986 PJ Kaufman
      DIMENSION X(250), Y(250), D(250), R(250)
      DOUBLE PRECISION SC2,SCD
      DATA MAX/25O/
      OPEN(6,FILE='PRN')
C---- Read data, set X to incremental time
C---- I = 1
C---- WRITE(*,7000)
  7000 FORMAT(' SINGLE FREQUENCY TRIGONOMETRIC REGRESSION'/
     +       ' Enter data 1 per line'/' Extra <return> ends input'//
     +       ' Enter below:')

    10 WRITE(*,7001)I
  7001 FORMAT(14,'>'\)
       READ(*,5000)Y(I)
  5000 FORMAT(BN,F8.2)
       X(I) = I
       IF(Y(I).NE.0)THEN
         IF(I.GT.MAX)STOP 'Maximum data exceeded'
         I = I+1
         GOTO 10
         ENDIF
       N = I-1
       WRITE(6,6000)
  6000 FORMAT('1Single Frequency Trigonometric Regression',2OX,
     +       'Data Input'//' Time',16X,'Prices')
       DO 40 J = 1,N,4
    40    WRITE(6,6001)J,(Y(I),I=J,J+3)
  6001    FORMAT(14,4F8.2)
C----  Linear regression analysis for detrending
       CALL LINREG(N,Y,A,B,SD)
       WRITE(6,6002)A,B
  6002 FORMAT(/'Linear regression results: A =',F8.3,', B =',F8.3)
C----  Detrend data into D and computer sums for equation (4)
       DO 60 I = 1,N
    60 D(I) = Y(I) - (A+B*I)
C----  Print detrended data
       WRITE(6,6003)
  6003 FORMAT(/' Detrended data'/)
       DO 65 I = 1,N,4
    65    WRITE(6,6001)I,(D(J),J=I,I+3)
       SC2 = 0
       SCD = 0
C----  Solve equation (4) using detrended data
       DO 70 I = 2,N-1
          DI = D(I)
          SC2 = SC2 + DI*DI
    70    SCD = SCD + DI*(D(I-1)+D(I+1))

       ALPHA = SCD/SC2
       WRITE(6,6004)SC2,SCD,ALPHA
  6004 FORMAT(/'Sum C-squared =',F8.1,', Sum C x D =',F8.1,', Alpha =',
     +       F8.3)
C----  Solve for omega
       W = ACOS(ALPHA/2)
       T = 36O/W
       WRITE(6,6O14)W,T
  6014 FORMAT(/'Omega (W) =',F5.1,' degrees, Period (T) =',F6.2,
```

```
     +        'time units')
C----  Sums for normal equations
       COS2 = 0
       COSSIN = 0
       YCOS = 0
       SINCOS = 0
       SIN2 = 0
       YSIN = 0
       DO 8O I = 1,N
          C = COS(W*I)
          S = SIN(W*I)
          COS2 = COS2 + C*C
          COSSIN = COSSIN + C*S
          YCOS = YCOS + Y(I)*C
          SINCOS = SINCOS + S*C
          SIN2 = SIN2 + S*S
    80    YSIN = YSIN + Y(I)*S
C----  Solve normal equations
       TB = (YSIN*COS2 - YCOS)/(SIN2*COS2 - COSSIN)
       TA = (YCOS - B*COSSIN)/COS2
       WRITE(6,6005)TA,TB
  6005 FORMAT(/' Solution to normal equations: A =',F8.3,', B =',F8.3)
C----  Values of fitted curve using detrended data
       DO 90 I = 1,N
    90    R(I) = TA*COS(W*I) + TB*SIN(W*I)
       WRITE(6,6006)
  6006 FORMAT(/' Trigonometric regression results using detrended data',
      +       /)
       DO 100 I = 1,N,4
   100 WRITE(6,6001)I,(R(J),J=I,I+3)
C----  Add trend back to result
       DO 110 I = 1,N
   110    R(I) = R(I) + A + B*I

       WRITE(6,6007)
  6007 FORMAT(/' Final regression results with trend added back'/)
       DO 120 I = 1,N,4
   120    WRITE(6,6001)I, (R(J),J=I,I+3)
       CALL EXIT
       END

       SUBROUTINE LINREG(N,DATA,A,B,SD)
C----  Generalized simple linear regression
       DIMENSION DATA(2)

C----  Initialize sums
       SX=0.
       SY=0.
       SXY=0.
       SX2=0.
       A=0.
       B=0.
       SD=0.
       IF(N.LE.2)RETURN

       DO 100 I=2,N
          X = I
          Y = DATA(I)
          SX=SX+X
          SY=SY+Y
          SXY=SXY+Y*X
          SX2=SX2+X*X
   100    CONTINUE
       M=N-1
       B=(M*SXY-SX*SY) / (M*SX2-SX*SX)
       A=(SY-B*SX) / M

C----- Residuals
       SSR=0
       DO 200 I=2,N
          Y=DATA(I)
```

```
         R=Y-(A+B*I)
         SSR=SSR+R*R
  200    CONTINUE
       SD=SQRT(SSR/M)
       RETURN
       END
```

```
Single Frequency      Trigonometric Regression
Time          Prices
1       22.12   22.46   22.17   22.00
5       23.18   24.56   25.57   30.59
9       28.23   33.77   35.90   40.05
13      46.22   51.48   40.76   40.16
17      36.51   29.30   30.36   36.42
21      39.75   30.07   29.08   32.13
25      38.94   42.95   43.38   46.23
29      47.70   46.98   35.78   27.35
33      25.40   29.45   27.15   28.48
37      32.74   33.53   30.01   29.25
41      36.82   45.07   55.13   65.51
45      66.56   70.06   27.30   35.62
49      32.06   31.46   35.75   36.46
53      38.22   43.24   45.46   38.96
57      37.08   38.72   34.01   33.00
61      35.07   40.23   41.63   44.95
65      51.12   63.71   59.56   63.38
Linear regression results: A = 28.889, B = .267 Detrended data
1       -7.04   -6.96   -7.52   -7.96
5       -7.05   -5.93   -5.19   -.44
9       -3.06   2.21    4.07    7.95
13      13.86   18.85   7.86    7.00
17      3.08    -4.40   -3.61   2.19
21      5.25    -4.70   -5.96   -3.17
25      3.37    7.11    7.28    9.86
29      11.06   10.07   -1.39   -10.09
33      -12.31  -8.52   -11.09  -10.03
37      -6.04   -5.51   -9.30   -10.33
41      -3.02   4.96    14.75   24.86
45      25.65   28.88   -14.15  -6.10
49      -9.92   -10.79  -6.77   -6.32
53      -4.83   -.08    1.87    -4.89
57      -7.04   -5.67   -10.64  -11.92
61      -10.12  -5.23   -4.09   -1.04
65      4.86    17.19   12.77   16.32
Sum C-squared = 6338.4, Sum C x D = 9282.2, Alpha = 1.464
Omega (W) =    .7 degrees, Period (T) =480.5O time units
Solution to    normal equations: A = -.6O3, B = 1.831
Trigonometric regression results using detrended data
1       .81     1.78    1.80    .86
5       -.54    -1.66   -1.88   -1.10
9       .27     1.50    1.92    1.32
13      .01     -1.31   -1.92   -1.51
17      -.28    1.09    1.88    1.66
21      .56     -.85    -1.80   -1.79
25      -.82    .59     1.68    1.87
29      1.06    -.32    -1.53   -1.92
33      -1.28   .04     1.34    1.92
37      1.47    .23     -1.13   -1.89
41      -1.64   -.51    .89     1.82
45      1.77    .77     -.64    -1.71
49      -1.86   -1.02   .37     1.56
53      1.91    1.24    -.09    -1.38
57      -1.93   -1.44   -.19    1.17
61      1.90    1.61    .46     -.94
65      -1.83   -1.75   -.72    .69
Final regression results with trend added back
1       29.96   31.21   31.50   30.82
5       29.68   28.83   28.88   29.93
9       31.57   33.06   33.75   33.41
13      32.37   31.32   30.98   31.66
17      33.15   34.79   35.85   35.90
```

```
21      35.06   33.92   33.23   33.52
25      34.75   36.43   37.79   38.24
29      37.70   36.58   35.64   35.52
33      36.43   38.02   39.59   40.43
37      40.25   39.28   38.18   37.69
41      38.21   39.60   41.27   42.46
45      42.68   41.95   40.81   40.01
49      40.12   41.23   42.89   44.34
53      44.96   44.56   43.49   42.47
57      42.19   42.95   44.47   46.09
61      47.09   47.07   46.18   45.05
65      44.42   44.78   46.07   47.74
```

Two-Frequency Trigonometric Regression

The FORTRAN program *TRIG2* and its subroutines *LINREG* (found in Appendix 4) and *MTX* are used to find the two-frequency representation of the copper cycle. The program output is clearly separated into the following steps.

1. Input data, where each time period is the average cash price for a calendar quarter.
2. The solution to the linear regression, giving the detrending line.
3. The detrended data resulting from subtracting the line values (2) from the original data (1).
4. Intermediate values for α_1, α_2, ω_1, and ω_2.
5. Resulting values a_1, b_1, a_2, and b_2, which are derived from the matrix solution using Gaussian elimination.
6. The cycle resulting from the detrended data.
7. The final cycle with the trend added back.

```
       PROGRAM TRIG2
C---- 2-Frequency Trigonometric Regression
C---- Copyright 1986 PJ Kaufman
      DIMENSION X(250), Y(250), D(250), R(250), A(4,4), B(4)
      EQUIVALENCE (A(1,1),C2W1),(A(1,2),CW1SW1),(A(1,3),CW1CW2),
      +            (A(1,4),CW1SW2),(A(2,1),DUP1),(A(2,2),S2W1),
      +            (A(2,3),SW1CW2),(A(2,4),SW1SW2),(A(3,1),DUP2),
      +            (A(3,2),DUP3),(A(3,3),C2W2),(A(3,4),CW2SW2),
      +            (A(4,1),DUP4),(A(4,2),DUP5),(A(4,3),DUP6),
      +            (A(4,4),S2W2),(B(1),YCW1),(B(2),YSW1),(B(3),YCW2),
      +            (B(4),YSW2),(B(1),A1),(B(2),B1),(B(3),A2),(B(4),B2)
      DATA MAX/250/,NDIM/4/
      OPEN(6,FILE='PRN')
C---- Read data, set X to incremental time
      I = 1
      WRITE(*,7000)
 7000 FORMAT(' 2-FREQUENCY TRIGONOMETRIC REGRESSION'/
     +       ' Enter data 1 per line'/'Extra <return> end input'//
     +       ' Enter below:')

   10 WRITE(*,7001)I
 7001 FORMAT(14,'>'\)
      READ(*,5000)Y(I)
 5000 FORMAT(BN,F8.2)
      X(I) = I
```

```
      IF(Y(I).NE.0)THEN
        IF(I.GT.MAX)STOP 'Maximum data exceeded'
        I = I + 1
        GOTO 10
        ENDIF
     N = I-1
     WRITE(6,6000)
 6000 FORMAT('12-Frequency Trigonometric Regression'//
     +       'Time',16X,'Prices')
      DO 40 J = 1,N,4
   40    WRITE(6,6001)J,(Y(I),I=J,J+3)
 6001    FORMAT(14,4F8.2)
C---- Linear regression analysis for detrending
      CALL LINREG(N,Y,ALIN,BLIN,SD)
      WRITE(6,6002)ALIN,BLIN

 6002 FORMAT(/' Linear regression results: A =',F8.3,', B =',F8.3)

C---- Detrend data into D and computer sums for equation (4)
      DO 60 I = 1,N
   60    D(I) = Y(I) - (ALIN+BLIN*I)
C---- Print detrended data
      WRITE(6,6003)
 6003 FORMAT(/' Detrended data'/)
      DO 65 I = 1,N,4
   65    WRITE(6,6001)I,(D(J),J=I,I+3)
      SC2 = 0
      SCD = 0
      SCP = 0
      SD2 = 0
      SDP = 0
C---- Solve for alpha1 and alpha2 using detrended data
      DO 70 I = 2,N-3
         C = D(I) + D(I+2)
         T = D(I+1)
         P = D(I-1) + D(I+3)
         SC2 = SC2 + C*C
         SCD = SCD + C*T
         SCP = SCP + C*P
         SD2 = SD2 + T*T
   70    SDP = SDP + T*P
      ALPHA2 = (SDP*SC2-SCP)/(SD2*SC2-SCD)
      ALPHA1 = (SCP-ALPHA2*SCD)/SC2
      T = SQRT(ALPHA1*ALPHA1+8*(1+ALPHA2/2))
      W1 = ACOS((ALPHA1+T)/4)
      W2 = ACOS((ALPHA1-T)/4)
      WRITE(6,6004)SC2,SCD,SCP,SD2,SDP,ALPHA1,ALPHA2,W1,W2
 6004 FORMAT(/' Intermediate values:'//
     +        ' SUMS C2 =', F8.1,', C*D =',F8.1,', C*P =',F8.1/
     +        '      D2 =', F8.1,', D*P =',F8.1//' Alpha1 = ',F8.3,
     +        '  Alpha2 =', F8.3,' Omega1 =',F5.2,', Omega2 =',F5.2)
C---- Sums for normal equations. . .to be used for matrix solution
      C2W1 = 0
      CW1SW1 = 0
      CW1CW2 = 0
      CW1SW2 = 0
      YCW1 = 0
      S2W1 = 0
      SW1CW2 = 0
      SW1SW2 = 0
      YSW1 = 0
      C2W2 = 0
      CW2SW2 = 0
      YCW2 = 0
      S2W2 = 0
      YSW2 = 0

      DO 100 I = 1,N
         DI = D(I)
         SW1 = SIN(W1*I)
         CW1 = COS(W1*I)
```

```
         SW2 = SIN(W2*I)
         CW2 = COS(W2*I)
         C2W1 = C2W1 + CW1*SW1
         CW1SW1 = CW1SW1 + CW1*SW1
         CW1CW2 = CW1CW2 + CW1*CW2
         CW1SW2 = CW1SW2 + CW1*SW2
         YCW1 = YCW1 + DI*CW1
         S2W1 = S2W1 + SW1*SW1
         SW1CW2 = SW1SW2 + SW1*CW2
         SW1SW2 = SW1SW2 + SW1*SW2
         YSW1 = YSW + DI
         C2W2 = C2W2 + CW2*CW2
         CW2SW2 = CW2SW2 + CW2*SW2
         YCW2 = YCW2 + DI*CW2
         S2W2 = S2W2 + SW2*SW2
  100    YSW2 = YSW2 + DI*SW2
C---- Duplicate calculations for matrix
      DUP1 = CW1SW1
      DUP2 = CW1CW2
      DUP3 = SW1CW2
      DUP4 = CW1SW2
      DUP5 = SW1SW2
      DUP6 = CW2SW2

      WRITE(6,6009)
 6009 FORMAT(/' Coefficient matrix:'/)
      DO 110 I = 1,NDIM
  110    WRITE(6,6010)(A(I,J),J=1,4),B(I)
 6010    FORMAT(5F8.3)
C---- Solve using matrix Gaussian Elimination
      CALL MTX(A,B,NDIM)

C---- Solution vector
      WRITE(6,6011)(B(I),I=1,NDIM)
 6011 FORMAT(/' Solution vector:'/4F8.3)
C---- Values of fitted curve using detrended data
      DO 90 I = 1,N
   90    R(I) = A1*COS(W1*I) + B1*SIN(W1*I) + A2*COS(W2*I) +
     +          B2*SIN(W2*I)
WRITE(6,6006)
 6006 FORMAT(/' Trigonometric regression results using detrended data',
      +      /)
       DO 105 I = 1,N,4
  105 WRITE(6,6001)I,(R(J),J=I,I+3)
C---- Add trend back to result
      DO 115 I = 1,N
  115    R(I) = R(I) + ALIN + BLIN*I
      WRITE(6,6007)
 6007 FORMAT(/' Final results with trend added back'/)
      DO 120 I = 1,N,4
  120    WRITE(6,6001)I,(R(J),J=I,I+3)
      CALL EXIT
      END

      SUBROUTINE MTX(A,C,N)
C---- Matrix solution to simultaneous linear equations
C---- Copyright 1986 PJ Kaufman

      DIMENSION A(4,4), C(4), A1(4,4), C1(4)

C---- Process row by row (Gaussian Elimination)
      DO 100 I = 1,N
         DIV = A(I,I)

         DO 40 J = 1,N
   40       A(I,J) = A(I,J)/DIV
         C(I) = C(I)/DIV
C---- Zero out column I for each row
         DO 60 J = 1,N
            IF(J.EQ.I)GOTO 60
            FACTOR = A(J,I)
```

```
            DO 50 K = I,N
   50          A(J,K) = A(J,K) - A(I,K)*FACTOR
            C(J) = C(J) - C(I)*FACTOR
   60       CONTINUE

  100    CONTINUE
      RETURN
      END
```

```
2-Frequency Trigonometric Regression
Time    Prices
1       22.12   22.46   22.17   22.00
5       23.18   24.56   25.57   30.59
9       28.23   33.77   35.90   40.05
13      46.22   51.48   40.76   40.16
17      36.51   29.30   30.36   36.42
21      39.75   30.07   29.08   32.13
25      38.94   42.95   43.38   46.23
29      47.70   46.98   35.78   27.35
33      25.40   29.45   27.15   28.48
37      32.74   33.53   30.01   29.25
41      36.82   45.07   55.13   65.51
45      66.56   70.06   27.30   35.62
49      32.06   31.46   35.75   36.46
53      38.22   43.24   45.46   38.96
57      37.08   38.72   34.01   33.00
61      35.07   40.23   41.63   44.95
65      51.12   63.71   59.56   63.38

Linear regression results: A = 28.889, B = .267
Detrended data
1       -7.04   -6.96   -7.52   -7.96
5       -7.05   -5.93   -5.19   -.44
9       -3.06   2.21    4.07    7.95
13      13.86   18.85   7.86    7.00
17      3.08    -4.40   -3.61   2.19
21      5.25    -4.70   -5.96   -3.17
25      3.37    7.11    7.28    9.86
29      11.06   10.07   -1.39   -10.09
33      -12.31  -8.52   -11.09  -10.03
37      -6.04   -5.51   -9.30   -10.33
41      -3.02   4.96    14.75   24.86
45      25.65   28.88   -14.15  -6.10
49      -9.92   -10.79  -6.77   -6.32
53      -4.83   -.08    1.87    -4.89
57      -7.04   -5.67   -10.64  -11.92
61      -10.12  -5.23   -4.09   -1.04
65      4.86    17.19   12.77   16.32

Intermediate values:
SUMS C2 = 17396.7, C*D = 8753.0, C*P = 10475.1
            D2 = 6126.9, D*P = 5499.2
Alpha1 = .151, Alpha2 = .898, Omega1 = .47, Omega2 = 2.52
Coefficient matrix:
34.206  .790    -.461   .426    124.829
.790    33.794  -.454   .479    -7.036
-.461   -.454   33.742  -.781   -33.500
.426    .479    -.781   34.258  28.219

Solution vector:
3.635    -.317  -.930   .762

Trigonometric regression results using detrended data
1       4.29    .84     .69     -1.20
5       -3.72   -2.38   -4.57   -2.18
9       -1.06   -.43    3.23    2.12
13      4.21    3.45    1.39    1.82
17      -2.17   -1.93   -3.36   -4.39
21      -1.80   -2.78   .67     1.72
25      2.23    4.76    2.31    3.24
29      1.05    -1.35   -1.09   -4.45
```

33	-2.90	-3.25	-2.67	.70
37	.12	3.49	3.41	2.97
41	3.91	.27	.59	-2.04
45	-3.65	-2.57	-4.56	-1.50
49	-1.06	.38	3.44	2.19
53	4.54	2.79	1.38	1.20
57	-2.70	-1.91	-3.91	-3.91
61	-1.68	-2.44	1.44	1.69
65	2.86	4.60	2.08	3.19

Final results with trend added back

1	33.44	30.26	30.38	28.75
5	26.50	28.12	26.19	28.85
9	30.23	31.13	35.05	34.21
13	36.58	36.08	34.29	34.98
17	31.26	31.77	30.61	29.85
21	32.71	31.99	35.70	37.02
25	37.80	40.60	38.42	39.61
29	37.69	35.56	36.08	32.99
33	34.81	34.72	35.57	39.21
37	38.89	42.54	42.72	42.54
41	43.76	40.38	40.97	38.60
45	37.26	38.61	36.89	40.22
49	40.93	42.63	45.96	44.98
53	47.59	46.11	44.96	45.06
57	41.42	42.47	40.75	41.01
61	43.50	43.01	47.16	47.68
65	49.12	51.13	48.87	50.25