Sample Quiz
CS170A – Mathematical Models and Methods for Computer Science
Spring 2013
OPEN BOOK
Wednesday, April 17, 2013, 5:00pm-5:50pm

1. **Matrices**
For each of the following matrices, determine (yes/no) whether the matrix is: unitary, hermitian, invertible, positive definite. (Recall A is positive definite if it is hermitian and all of its eigenvalues are positive real values.) Assume that $i = \sqrt{-1}$.

matrix	invertible?	hermitian?	skew-hermitian?	unitary?
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	**True** ☐ **False** ☐	**True** ☐ **False** ☐	**True** ☐ **False** ☐	**True** ☐ **False** ☐
$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	**True** ☐ **False** ☐	**True** ☐ **False** ☐	**True** ☐ **False** ☐	**True** ☐ **False** ☐
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	**True** ☐ **False** ☐	**True** ☐ **False** ☐	**True** ☐ **False** ☐	**True** ☐ **False** ☐
$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	**True** ☐ **False** ☐	**True** ☐ **False** ☐	**True** ☐ **False** ☐	**True** ☐ **False** ☐

For each of the following equations, mark whether the equation is True (valid for all specified matrices A) or False. Assume that A' denotes the transpose of A, A' denotes the hermitian transpose of A, and $i = \sqrt{-1}$.

For full credit, if you mark it True, you must show how you derived this. If you mark it False, you must give a *counterexample* (values for these sets for which the equation does not hold).

(a) **True** ☐ **False** ☐
Every product of two 2×2 rotation matrices is orthogonal, if we assume that the rotation matrices have the form $\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$ where θ is real.

Explanation:

(b) **True** ☐ **False** ☐
If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then the eigenvalues of A are $\lambda = \left((a+d) \pm \sqrt{(a+d)^2 - 4(a\,d - b\,c)}\right)/2$.

Explanation:

(c) **True** ☐ **False** ☐
If $A = U\ D\ U'$, where D is a diagonal matrix of positive real values and U is a real unitary matrix, then A is symmetric and U is orthogonal.

(d) **True** ☐ **False** ☐
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(e) **True** ☐ **False** ☐
If the eigenvalues of a square matrix A are all real and positive, then A is invertible.

(f) **True** ☐ **False** ☐
If a matrix is hermitian and also unitary, then it is the identity matrix.

2. MATLAB

Suppose you are given the following program:

```
n = 20
rotate3d on
zoom on
colormap(summer)
for i=1:n
   for j=1:n ; u(i,j) = sin(i+j) ; end
   end
for iteration = 1:10
    for i=2:n-1
        for j=2:n-1
            u(i,j) = (u(i-1,j)+u(i,j-1)+u(i+1,j)+u(i,j+1))/4;
        end
    end
surf(u)
M(iteration) = getframe;
end
movie(M,5)
```

Please answer the following:

- show how to turn the for-loops portion of this program into a MATLAB function that takes the number of iterations (fixed at 10 above) and a square initial matrix `u` as input (defined by `u(i,j) = sin(i+j)` above), and then – after finding the value of `n` as the size of `u` – returns the movie `M` resulting the iteration. In other words the function should take two arguments as input, and yield `M` as output.

- consider the function

```
function B = shift(A, k)
  m = size(A,1);
  n = size(A,2);
  B = zeros(m,n);
  for i=1:m
     for j=max(1,1-k):min(n,n-k)  %  1:(n-k) if k>0,  (1-k):n if k<0
        B(i,j) = A(i,j+k);
     end
  end
```

  First, what does this function do? For $k = -1$ and $m = n = 3$, give an example matrix `A` and the matrix `B` it produces.

- By using Matlab vector sequences (like '`1:n`'), replace both of the loops with a single statement of the form `B(...) = A(...);`

- Similarly, convert the inner two for-loops and assignment `u(i,j) = ...` in the middle of the movie program above into a single matrix assignment `U = ...`.