Nathan Tung
004-059-195
Discussion 1A

CS170A – Homework 2

1. Singular Value Decomposition

```
A = pascal(12,2)
[U,S,V]=svd(A)
```

- Unitary: No
- Hermitian: No
- Invertible: Yes
- Ill-Conditioned: Yes? (Condition number = 8.7639e+11 is small compared to condition numbers of B, C, D, but it is still large)
- Positive Definite: No

```
U =

  Columns 1 through 5

  -1.7401e-03   1.1416e-02  -4.9570e-02  -1.6211e-01   4.1049e-01
   1.7626e-02  -8.4924e-02   2.5153e-01   4.9172e-01  -5.4514e-01
  -8.1666e-02   2.7526e-01  -5.0343e-01  -4.3836e-01  -1.0663e-01
   2.2827e-01  -4.9029e-01   4.0985e-01  -1.4755e-01   3.5949e-01
  -4.2735e-01   4.7057e-01   9.3552e-02   3.7233e-01   1.9324e-01
   5.6236e-01  -1.0939e-01  -4.0513e-01   1.0084e-01  -2.4371e-01
  -5.3051e-01  -3.1750e-01   1.2705e-01  -3.4946e-01  -2.5081e-01
   3.5862e-01   4.6552e-01   3.1575e-01  -3.4276e-02   1.9143e-01
  -1.7019e-01  -3.2675e-01  -4.1057e-01   3.7772e-01   2.5196e-01
   5.3985e-02   1.3369e-01   2.2694e-01  -3.0356e-01  -3.3601e-01
  -1.0299e-02  -3.0661e-02  -6.3344e-02   1.0435e-01   1.4338e-01
   8.9492e-04   3.0700e-03   7.3225e-03  -1.3919e-02  -2.1946e-02

  Columns 6 through 10

  -7.1317e-01  -5.2428e-01  -1.3701e-01   1.9248e-02   1.6612e-03
   9.5489e-03  -5.3130e-01  -3.1897e-01   7.7634e-02   1.0090e-02
   3.6883e-01  -2.6718e-01  -4.6533e-01   1.8807e-01   3.5390e-02
   2.2479e-01   1.0284e-01  -4.5612e-01   3.3303e-01   9.1295e-02
  -1.4112e-01   3.0974e-01  -2.3583e-01   4.5080e-01   1.8841e-01
  -2.8596e-01   1.9101e-01   1.0517e-01   4.4740e-01   3.2061e-01
  -3.9473e-02  -1.2820e-01   3.3306e-01   2.5010e-01   4.4725e-01
   2.6024e-01  -2.7422e-01   2.2051e-01  -1.0171e-01   4.8161e-01
   1.0570e-01   2.1391e-03  -1.7589e-01  -3.7711e-01   3.1007e-01
  -3.0902e-01   3.0977e-01  -3.2736e-01  -2.1559e-01  -1.1235e-01
   1.6221e-01  -1.9853e-01   2.9369e-01   4.1062e-01  -5.1335e-01
  -2.8015e-02   3.8108e-02  -6.5749e-02  -1.1722e-01   2.1851e-01

  Columns 11 through 12

  -8.2276e-05   1.7985e-06
  -6.9433e-04   2.0015e-05
  -3.2902e-03   1.2264e-04
  -1.1403e-02   5.4558e-04
  -3.1959e-02   1.9665e-03
  -7.6062e-02   6.0858e-03
  -1.5750e-01   1.6756e-02
  -2.8558e-01   4.2032e-02
  -4.4669e-01   9.7682e-02
  -5.6533e-01   2.1295e-01
  -4.3141e-01   4.3962e-01
   4.2598e-01   8.6589e-01


S =

  Columns 1 through 5
```

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```
 9.6755e+02            0            0            0            0
          0   1.3875e+02            0            0            0
          0            0   2.9840e+01            0            0
          0            0            0   8.4219e+00            0
          0            0            0            0   2.9960e+00
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0

  Columns 6 through 10

          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
 1.3603e+00            0            0            0            0
          0   7.3514e-01            0            0            0
          0            0   3.3378e-01            0            0
          0            0            0   1.1874e-01            0
          0            0            0            0   3.3512e-02
          0            0            0            0            0
          0            0            0            0            0

  Columns 11 through 12

          0            0
          0            0
          0            0
          0            0
          0            0
          0            0
          0            0
          0            0
          0            0
          0            0
 7.2070e-03            0
          0   1.0335e-03


V =

  Columns 1 through 5

  8.6589e-01   4.2598e-01   2.1851e-01  -1.1722e-01  -6.5749e-02
  4.3962e-01  -4.3141e-01  -5.1335e-01   4.1062e-01   2.9369e-01
  2.1295e-01  -5.6533e-01  -1.1235e-01  -2.1559e-01  -3.2736e-01
  9.7682e-02  -4.4669e-01   3.1007e-01  -3.7711e-01  -1.7589e-01
  4.2032e-02  -2.8558e-01   4.8161e-01  -1.0171e-01   2.2051e-01
  1.6756e-02  -1.5750e-01   4.4725e-01   2.5010e-01   3.3306e-01
  6.0858e-03  -7.6062e-02   3.2061e-01   4.4740e-01   1.0517e-01
  1.9665e-03  -3.1959e-02   1.8841e-01   4.5080e-01  -2.3583e-01
  5.4558e-04  -1.1403e-02   9.1295e-02   3.3303e-01  -4.5612e-01
  1.2264e-04  -3.2902e-03   3.5390e-02   1.8807e-01  -4.6533e-01
  2.0015e-05  -6.9433e-04   1.0090e-02   7.7634e-02  -3.1897e-01
  1.7985e-06  -8.2276e-05   1.6612e-03   1.9248e-02  -1.3701e-01

  Columns 6 through 10

 -3.8108e-02   2.8015e-02  -2.1946e-02  -1.3919e-02   7.3225e-03
  1.9853e-01  -1.6221e-01   1.4338e-01   1.0435e-01  -6.3344e-02
 -3.0977e-01   3.0902e-01  -3.3601e-01  -3.0356e-01   2.2694e-01
 -2.1391e-03  -1.0570e-01   2.5196e-01   3.7772e-01  -4.1057e-01
  2.7422e-01  -2.6024e-01   1.9143e-01  -3.4276e-02   3.1575e-01
  1.2820e-01   3.9473e-02  -2.5081e-01  -3.4946e-01   1.2705e-01
 -1.9101e-01   2.8596e-01  -2.4371e-01   1.0084e-01  -4.0513e-01
```

```
  -3.0974e-01   1.4112e-01   1.9324e-01   3.7233e-01   9.3552e-02
  -1.0284e-01  -2.2479e-01   3.5949e-01  -1.4755e-01   4.0985e-01
   2.6718e-01  -3.6883e-01  -1.0663e-01  -4.3836e-01  -5.0343e-01
   5.3130e-01  -9.5489e-03  -5.4514e-01   4.9172e-01   2.5153e-01
   5.2428e-01   7.1317e-01   4.1049e-01  -1.6211e-01  -4.9570e-02

  Columns 11 through 12

   3.0700e-03   8.9492e-04
  -3.0661e-02  -1.0299e-02
   1.3369e-01   5.3985e-02
  -3.2675e-01  -1.7019e-01
   4.6552e-01   3.5862e-01
  -3.1750e-01  -5.3051e-01
  -1.0939e-01   5.6236e-01
   4.7057e-01  -4.2735e-01
  -4.9029e-01   2.2827e-01
   2.7526e-01  -8.1666e-02
  -8.4924e-02   1.7626e-02
   1.1416e-02  -1.7401e-03
```

```
B = magic(12)
[U,S,V]=svd(B)
```

- Unitary: No
- Hermitian: No
- Invertible: No
- Ill-Conditioned: Yes (Condition number = 7.1259e+35)
- Positive Definite: No

```
U =

  Columns 1 through 5

  -2.8868e-01   4.5993e-01   2.8868e-01   5.2632e-01  -5.1968e-01
  -2.8868e-01  -3.7631e-01  -2.8868e-01   6.9815e-01   2.8477e-01
  -2.8868e-01  -2.9268e-01  -2.8868e-01  -2.6455e-01  -4.3170e-01
  -2.8868e-01   2.0906e-01   2.8868e-01  -1.1023e-01   4.4429e-01
  -2.8868e-01   1.2544e-01   2.8868e-01  -8.9960e-02   6.0858e-02
  -2.8868e-01  -4.1812e-02  -2.8868e-01  -6.4760e-02  -1.6798e-01
  -2.8868e-01   4.1812e-02  -2.8868e-01  -1.6882e-01  -1.7094e-01
  -2.8868e-01  -1.2544e-01   2.8868e-01  -2.5801e-01   3.4939e-02
  -2.8868e-01  -2.0906e-01   2.8868e-01  -9.0409e-02  -2.0902e-01
  -2.8868e-01   2.9268e-01  -2.8868e-01  -1.9329e-01   1.9085e-01
  -2.8868e-01   3.7631e-01  -2.8868e-01  -6.7360e-03   2.9500e-01
  -2.8868e-01  -4.5993e-01   2.8868e-01   2.2297e-02   1.8862e-01

  Columns 6 through 10

  -1.9000e-02   1.2228e-01  -9.5771e-02   1.0436e-03  -1.1853e-01
   9.6892e-02  -2.0377e-01   1.8496e-01   9.5684e-03   1.3204e-01
   1.0270e-01  -1.0549e-01  -3.5595e-01  -4.1057e-01   7.8184e-02
   2.7597e-01   3.5026e-01   1.3639e-01  -5.0706e-01   2.6847e-01
  -1.9403e-01  -2.3228e-01  -1.5813e-01   3.0695e-01   5.0767e-01
   4.9015e-01   3.9478e-01  -1.0942e-01   3.3708e-01   2.1644e-01
  -4.3439e-01   3.4216e-01   6.4354e-01   2.1438e-02  -7.6891e-02
   4.2452e-01  -2.0180e-01   2.4720e-01   4.0999e-01  -4.2261e-01
  -6.9697e-02  -4.3531e-01   2.6588e-01  -3.1310e-01   2.5039e-02
  -2.7726e-01  -2.5669e-01  -1.2688e-01   2.3155e-01   1.9377e-01
   2.1912e-02  -1.7099e-01  -2.3625e-01  -1.8907e-01  -5.4355e-01
  -4.1777e-01   3.9685e-01  -3.9558e-01   1.0218e-01  -2.6004e-01

  Columns 11 through 12

   1.6560e-01  -9.3563e-02
   1.8899e-03  -1.4340e-01
   9.6064e-03  -4.1008e-01
```

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```
   1.3521e-01  -1.2317e-01
  -5.5509e-01  -1.7067e-01
  -8.3771e-02   4.7288e-01
  -1.7504e-01  -1.3018e-01
   1.2341e-01  -3.2531e-01
   3.7592e-02   6.1045e-01
   6.4925e-01   4.8352e-02
  -4.0194e-01   1.6242e-01
   9.3273e-02   1.0227e-01


S =

  Columns 1 through 5

   8.7000e+02            0            0            0            0
            0   4.9710e+02            0            0            0
            0            0   4.1425e+01            0            0
            0            0            0   5.8371e-14            0
            0            0            0            0   3.6629e-14
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0

  Columns 6 through 10

            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
            0            0            0            0            0
   2.9979e-14            0            0            0            0
            0   2.4398e-14            0            0            0
            0            0   9.8717e-15            0            0
            0            0            0   8.3151e-15            0
            0            0            0            0   6.0048e-15
            0            0            0            0            0
            0            0            0            0            0

  Columns 11 through 12

            0            0
            0            0
            0            0
            0            0
            0            0
            0            0
            0            0
            0            0
            0            0
            0            0
   3.0220e-15            0
            0   4.1906e-16


V =

  Columns 1 through 5

  -2.8868e-01   2.8868e-01   4.5993e-01   3.4488e-01  -4.0140e-01
  -2.8868e-01  -2.8868e-01  -3.7631e-01  -1.3645e-01  -2.0523e-01
  -2.8868e-01  -2.8868e-01  -2.9268e-01   1.8787e-01  -2.6995e-01
  -2.8868e-01   2.8868e-01   2.0906e-01  -6.1497e-01  -2.1311e-02
  -2.8868e-01   2.8868e-01   1.2544e-01  -6.2841e-02  -3.2256e-01
  -2.8868e-01  -2.8868e-01  -4.1812e-02  -3.9618e-01  -2.5250e-01
  -2.8868e-01  -2.8868e-01   4.1812e-02   9.2216e-02  -1.7038e-03
```

```
  -2.8868e-01   2.8868e-01  -1.2544e-01   2.9564e-01   9.7464e-02
  -2.8868e-01   2.8868e-01  -2.0906e-01  -2.3318e-01   4.7923e-01
  -2.8868e-01  -2.8868e-01   2.9268e-01   2.5421e-01   2.5966e-01
  -2.8868e-01  -2.8868e-01   3.7631e-01  -1.6672e-03   4.6971e-01
  -2.8868e-01   2.8868e-01  -4.5993e-01   2.7047e-01   1.6858e-01

  Columns 6 through 10

   1.4313e-02  -2.0181e-02  -2.4675e-01  -1.8313e-01  -4.5660e-01
  -6.4528e-01   1.8065e-01  -1.6677e-01  -3.2544e-01  -1.6829e-01
   2.4733e-01  -4.2202e-01   1.0719e-01   2.4998e-02   1.5312e-01
  -2.2499e-01  -5.8365e-02  -2.3013e-01   4.1355e-01   1.6885e-01
   1.3075e-01   2.9201e-01   4.3275e-01  -2.5120e-01   4.0271e-01
   2.9900e-01  -3.1445e-02   3.1300e-01   1.0647e-01  -2.3961e-01
   3.3879e-01   5.7776e-02  -6.1250e-01   2.1813e-01   1.5377e-01
  -3.3452e-01  -5.7320e-01   1.3814e-01   1.9347e-01   1.5875e-01
   3.1079e-01  -1.5508e-01  -1.8344e-01  -5.8286e-01  -5.4785e-02
  -1.5416e-01   2.7350e-01   4.5360e-03  -1.1503e-01   4.9026e-01
  -8.5679e-02  -5.8469e-02   3.5454e-01   9.0867e-02  -3.8926e-01
   1.0365e-01   5.1481e-01   8.9432e-02   4.1017e-01  -2.1891e-01

  Columns 11 through 12

   1.1958e-01  -1.5463e-01
  -1.0832e-01   9.0506e-02
  -2.7446e-01  -5.3791e-01
  -8.0366e-02  -3.1307e-01
  -3.4901e-01   2.6997e-01
   5.6170e-01   1.9561e-01
  -2.0321e-01   4.6613e-01
   2.0534e-01   3.9543e-01
   7.1220e-02  -5.9192e-02
   4.3931e-01  -2.6542e-01
  -4.1503e-01   5.1082e-02
   3.3231e-02  -1.3851e-01
```

```
C = hilb(12)
[U,S,V]=svd(C)
```

- Unitary: No
- Hermitian: Yes
- Invertible: Yes
- Ill-Conditioned: Yes (Condition number = 2.8142e+32)
- Positive Definite: Yes

```
U =

  Columns 1 through 5

  -6.8332e-01   6.4250e-01   3.2275e-01  -1.2097e-01   3.7092e-02
  -4.2056e-01  -3.1005e-02  -5.6141e-01   5.8798e-01  -3.6278e-01
  -3.1517e-01  -1.9491e-01  -3.9847e-01  -1.0673e-01   5.3121e-01
  -2.5544e-01  -2.4833e-01  -1.9883e-01  -3.5436e-01   2.7449e-01
  -2.1615e-01  -2.6476e-01  -4.4688e-02  -3.6041e-01  -8.1217e-02
  -1.8800e-01  -2.6629e-01   6.6706e-02  -2.6790e-01  -2.8509e-01
  -1.6671e-01  -2.6130e-01   1.4625e-01  -1.4349e-01  -3.3245e-01
  -1.4997e-01  -2.5343e-01   2.0302e-01  -1.6342e-02  -2.6791e-01
  -1.3642e-01  -2.4441e-01   2.4354e-01   1.0158e-01  -1.3442e-01
  -1.2520e-01  -2.3507e-01   2.7237e-01   2.0608e-01   3.6330e-02
  -1.1574e-01  -2.2584e-01   2.9270e-01   2.9645e-01   2.2312e-01
  -1.0766e-01  -2.1695e-01   3.0679e-01   3.7349e-01   4.1248e-01

  Columns 6 through 10

  -9.5990e-03  -2.1191e-03   3.9870e-04  -6.3212e-05  -8.2318e-06
   1.6199e-01   5.6231e-02  -1.5616e-02   3.4888e-03   6.1782e-04
  -5.3247e-01  -3.2386e-01   1.4035e-01  -4.5640e-02  -1.1216e-02
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```
 2.6245e-01   5.4734e-01  -4.4989e-01   2.3311e-01   8.3618e-02
 4.0954e-01   1.8245e-02   4.5643e-01  -5.1340e-01  -3.0318e-01
 1.6441e-01  -3.6989e-01   2.0524e-01   3.5248e-01   5.3479e-01
-1.2872e-01  -2.9797e-01  -2.8701e-01   2.9199e-01  -3.1169e-01
-3.0518e-01   8.7678e-03  -3.4919e-01  -2.5730e-01  -2.9611e-01
-3.1993e-01   2.8110e-01  -2.1624e-02  -3.5852e-01   3.4055e-01
-1.8350e-01   3.4537e-01   3.3076e-01   9.4346e-02   2.6316e-01
 7.3836e-02   1.2495e-01   3.2119e-01   4.5112e-01  -4.6629e-01
 4.1871e-01  -3.9044e-01  -3.3145e-01  -2.5168e-01   1.6576e-01

  Columns 11 through 12

 8.3442e-07  -5.7025e-08
-8.2785e-05   7.3150e-06
 2.0137e-03  -2.3217e-04
-2.0751e-02   3.1861e-03
 1.1070e-01  -2.3491e-02
-3.3057e-01   1.0370e-01
 5.4512e-01  -2.9005e-01
-3.9184e-01   5.2674e-01
-1.5546e-01  -6.1922e-01
 5.1377e-01   4.5457e-01
-3.6225e-01  -1.8938e-01
 8.9351e-02   3.4181e-02


S =

  Columns 1 through 5

 1.7954e+00            0            0            0            0
          0   3.8028e-01            0            0            0
          0            0   4.4739e-02            0            0
          0            0            0   3.7223e-03            0
          0            0            0            0   2.3309e-04
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0

  Columns 6 through 10

          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
 1.1163e-05            0            0            0            0
          0   4.0824e-07            0            0            0
          0            0   1.1229e-08            0            0
          0            0            0   2.2520e-10            0
          0            0            0            0   3.1113e-12
          0            0            0            0            0
          0            0            0            0            0

  Columns 11 through 12

          0            0
          0            0
          0            0
          0            0
          0            0
          0            0
          0            0
          0            0
          0            0
          0            0
```

```
   2.6497e-14            0
            0   1.0702e-16


V =

  Columns 1 through 5

  -6.8332e-01   6.4250e-01   3.2275e-01  -1.2097e-01   3.7092e-02
  -4.2056e-01  -3.1005e-02  -5.6141e-01   5.8798e-01  -3.6278e-01
  -3.1517e-01  -1.9491e-01  -3.9847e-01  -1.0673e-01   5.3121e-01
  -2.5544e-01  -2.4833e-01  -1.9883e-01  -3.5436e-01   2.7449e-01
  -2.1615e-01  -2.6476e-01  -4.4688e-02  -3.6041e-01  -8.1217e-02
  -1.8800e-01  -2.6629e-01   6.6706e-02  -2.6790e-01  -2.8509e-01
  -1.6671e-01  -2.6130e-01   1.4625e-01  -1.4349e-01  -3.3245e-01
  -1.4997e-01  -2.5343e-01   2.0302e-01  -1.6342e-02  -2.6791e-01
  -1.3642e-01  -2.4441e-01   2.4354e-01   1.0158e-01  -1.3442e-01
  -1.2520e-01  -2.3507e-01   2.7237e-01   2.0608e-01   3.6330e-02
  -1.1574e-01  -2.2584e-01   2.9270e-01   2.9645e-01   2.2312e-01
  -1.0766e-01  -2.1695e-01   3.0679e-01   3.7349e-01   4.1248e-01

  Columns 6 through 10

  -9.5990e-03  -2.1191e-03   3.9870e-04  -6.3212e-05  -8.2318e-06
   1.6199e-01   5.6231e-02  -1.5616e-02   3.4888e-03   6.1782e-04
  -5.3247e-01  -3.2386e-01   1.4035e-01  -4.5640e-02  -1.1216e-02
   2.6245e-01   5.4734e-01  -4.4989e-01   2.3311e-01   8.3618e-02
   4.0954e-01   1.8245e-02   4.5643e-01  -5.1340e-01  -3.0318e-01
   1.6441e-01  -3.6989e-01   2.0524e-01   3.5248e-01   5.3479e-01
  -1.2872e-01  -2.9797e-01  -2.8701e-01   2.9199e-01  -3.1169e-01
  -3.0518e-01   8.7678e-03  -3.4919e-01  -2.5730e-01  -2.9611e-01
  -3.1993e-01   2.8110e-01  -2.1624e-02  -3.5852e-01   3.4055e-01
  -1.8350e-01   3.4537e-01   3.3076e-01   9.4346e-02   2.6316e-01
   7.3836e-02   1.2495e-01   3.2119e-01   4.5112e-01  -4.6629e-01
   4.1871e-01  -3.9044e-01  -3.3145e-01  -2.5168e-01   1.6576e-01

  Columns 11 through 12

   8.3443e-07  -5.6979e-08
  -8.2786e-05   7.3101e-06
   2.0138e-03  -2.3204e-04
  -2.0752e-02   3.1847e-03
   1.1071e-01  -2.3484e-02
  -3.3058e-01   1.0368e-01
   5.4514e-01  -2.9001e-01
  -3.9188e-01   5.2671e-01
  -1.5541e-01  -6.1923e-01
   5.1373e-01   4.5461e-01
  -3.6224e-01  -1.8940e-01
   8.9348e-02   3.4188e-02
```

```
D = vander(logspace(1,4,12))
[U,S,V]=svd(D)
```

- Unitary: No
- Hermitian: No
- Invertible: Yes
- Ill-Conditioned: Yes (Condition number = 1.2610e+89)
- Positive Definite: No

```
U =

  Columns 1 through 5

            0   1.1035e-14   4.4101e-11  -1.0349e-08   9.1492e-07
  -1.0000e-30   8.7492e-42  -1.2060e-13   2.0612e-11  -2.0945e-09
  -1.0000e-27  -3.2479e-22   2.8599e-32  -2.2186e-13  -1.0517e-09
```

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```
-1.0000e-24  -1.7279e-19  -5.5125e-15  -4.1780e-11  -8.1497e-08
-1.0000e-21  -9.1682e-17  -1.5439e-12  -6.1154e-09  -6.1144e-06
-1.0000e-18  -4.8394e-14  -4.2592e-10  -8.6475e-07  -4.2581e-04
-1.0000e-15  -2.5293e-11  -1.1410e-07  -1.1409e-04  -2.5285e-02
-1.0000e-12  -1.2964e-08  -2.8804e-05  -1.2963e-02  -9.9960e-01
-1.0000e-09  -6.3850e-06  -6.3849e-03  -9.9990e-01   1.2962e-02
-1.0000e-06  -2.8738e-03  -9.9998e-01   6.3847e-03  -5.3967e-05
-1.0000e-03  -1.0000e+00   2.8738e-03  -1.1964e-05   8.5289e-08
-1.0000e+00   1.0000e-03  -1.8738e-06   6.5792e-09  -4.3284e-11

  Columns 6 through 10

-3.9508e-05  -8.9472e-04   1.1002e-02  -5.7998e-02   4.1421e-01
 8.6146e-08   1.6106e-05   1.8052e-03   6.6251e-02   4.7904e-01
-1.0458e-06   2.7957e-04   1.9300e-02   3.0900e-01   7.3303e-01
-4.1747e-05   5.4944e-03   1.7046e-01   9.3215e-01  -2.4716e-01
-1.5425e-03   9.1333e-02   9.8091e-01  -1.6661e-01   2.2947e-02
-4.8343e-02   9.9463e-01  -9.0870e-02   9.9967e-03  -5.8361e-04
-9.9851e-01  -4.8276e-02   2.8756e-03  -2.6353e-04  -1.3952e-05
 2.5276e-02   7.9684e-04  -4.0031e-05   3.2970e-06   8.5849e-07
-2.1371e-04  -5.6829e-06   2.6333e-07  -1.9738e-08  -1.3440e-08
 7.5046e-07   1.8418e-08  -8.1876e-10   5.4560e-11   8.1105e-11
-1.0946e-09  -2.5801e-11   1.1223e-12  -6.7932e-14  -1.5331e-13
 5.3353e-13   1.2317e-14  -5.3256e-16   3.4694e-17            0

  Columns 11 through 12

 3.9822e-01  -8.1631e-01
 6.6880e-01   5.6465e-01
-6.0305e-01   5.6075e-02
 1.7289e-01  -1.0501e-01
-2.3715e-02   2.5032e-02
 1.7361e-03  -2.4720e-03
-6.9675e-05   1.1762e-04
 1.5174e-06  -2.8148e-06
-1.7193e-08   3.3520e-08
 9.0798e-11  -1.8143e-10
-1.6469e-13   3.3213e-13
           0            0


S =

  Columns 1 through 5

 1.0000e+44            0            0            0            0
          0   8.7382e+36            0            0            0
          0            0   4.1118e+30            0            0
          0            0            0   8.0555e+24            0
          0            0            0            0   6.0051e+19
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0

  Columns 6 through 10

          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
          0            0            0            0            0
 1.6373e+15            0            0            0            0
          0   1.6030e+11            0            0            0
          0            0   5.6028e+07            0            0
          0            0            0   7.0652e+04            0
          0            0            0            0   3.6006e+02
```

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```
            0            0            0            0            0
            0            0            0            0            0

  Columns 11 through 12

            0            0
            0            0
            0            0
            0            0
            0            0
            0            0
            0            0
            0            0
            0            0
            0            0
 4.2193e+00            0
            0   2.8093e-01


V =

  Columns 1 through 5

  -1.0000e+00   1.0000e-04  -1.8738e-08   6.5796e-12  -4.3294e-15
  -1.0000e-04  -1.0000e+00   2.8738e-04  -1.1965e-07   8.5308e-11
  -1.0000e-08  -2.8738e-04  -1.0000e+00   6.3851e-04  -5.3980e-07
  -1.0000e-12  -6.3851e-08  -6.3851e-04  -1.0000e+00   1.2965e-03
  -1.0000e-16  -1.2965e-11  -2.8806e-07  -1.2965e-03  -1.0000e+00
  -1.0000e-20  -2.5294e-15  -1.1411e-10  -1.1413e-06  -2.5300e-03
  -1.0000e-24  -4.8400e-19  -4.2603e-14  -8.6522e-10  -4.2625e-06
  -1.0001e-28  -9.1703e-23  -1.5447e-17  -6.1217e-13  -6.1264e-09
  -1.0002e-32  -1.7287e-26  -5.5178e-21  -4.1860e-16  -8.1802e-12
  -1.0003e-36  -3.2505e-30  -1.9564e-24  -2.8142e-19  -1.0540e-14
  -1.0005e-40  -6.1052e-34  -6.9127e-28  -1.8772e-22  -1.3360e-17
  -1.0010e-44  -1.1465e-37  -2.4406e-31  -1.2496e-25  -1.6859e-20

  Columns 6 through 10

   5.3397e-18   1.2354e-20  -5.3759e-23   4.4427e-25  -7.1590e-27
  -1.0955e-13  -2.5878e-16   1.1385e-18  -9.4628e-21   1.5294e-22
   7.5108e-10   1.8472e-12  -8.2979e-15   6.9720e-17  -1.1331e-18
  -2.1389e-06  -5.6999e-09   2.6654e-11  -2.2862e-13   3.7553e-15
   2.5300e-03   7.9928e-06  -4.0491e-08   3.6143e-10  -6.0576e-12
  -9.9999e-01  -4.8440e-03   2.9082e-05  -2.8104e-07   4.8977e-09
  -4.8441e-03   9.9995e-01  -9.1993e-03   1.0523e-04  -1.9822e-06
  -1.5482e-05   9.1997e-03   9.9980e-01  -1.7487e-02   3.8869e-04
  -4.2043e-08   5.5677e-05   1.7491e-02   9.9928e-01  -3.3760e-02
  -1.0601e-10   2.8671e-07   2.0233e-04   3.3791e-02   9.9695e-01
  -2.5861e-13   1.3795e-09   2.0202e-06   7.7136e-04   7.0220e-02
  -6.2508e-16   6.5090e-12   1.9552e-08   1.5464e-05   3.8923e-03

  Columns 11 through 12

   2.5403e-28   1.5209e-29
  -5.4344e-24  -3.2577e-25
   4.0371e-20   2.4255e-21
  -1.3446e-16  -8.1121e-18
   2.1893e-13   1.3314e-14
  -1.8021e-10  -1.1123e-11
   7.5514e-08   4.7935e-09
  -1.5879e-05  -1.0631e-06
   1.6036e-03   1.1899e-04
  -7.0023e-02  -6.3907e-03
   9.8684e-01   1.4563e-01
   1.4572e-01  -9.8932e-01
```

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```
function B = isUnitary(A)
    s=size(A);
    m=s(1);
    if (A*A')==eye(m)
        B=true;
    else
        B=false;
    end

function B = isHermitian(A)
    s=size(A);
    m=s(1);
    if (A'*eye(m))==A
        B=true;
    else
        B=false;
    end

function B = isInvertible(A)
    if (det(A)~=0)
        B=true;
    else
        B=false;
    end

function B = isIllCond(A)
    B = cond(A)*cond(A'); %outputs condition number rather than boolean

function B = isPosDef(A)
%A symmetric matrix is defined to be positive definite if the real parts of
all eigenvalues are positive.
%A non-symmetric matrix (B) is positive definite if all eigenvalues of
(B+B')/2 are positive.
    [m,n]=size(A);
    if(m~=n) %matrix is non-symmetric
        e=eig((A+A')/2);
        for i=1:size(e,1)
            if(real(e(i))<0)
                B=false;
                return;
            end
        end
    else %matrix is symmetric
        e=eig(A);
        for i=1:size(e,1)
            if(e(i)<0)
                B=false;
                return;
            end
        end
    end
    B=true;
```

Using the functions I wrote above, I determined whether each of the matrices was unitary, hermitian, invertible, ill-conditioned, and positive definite (the last using Matlab's given definition). The isIllCond function actually produces the matrix's condition number rather than a boolean. Using this function, I found that matrix D has the largest condition number and is undoubtedly ill-conditioned. However, although matrix A had the smallest condition number, it was still quite large. Matrix A does indeed some to be ill-conditioned when not seen in relation to matrices B, C, and D. However, for the purpose of this assignment, I decided to say it was not ill-conditioned.

2. **Curve Fitting and PCA**
    a. **Fitting**

In order to obtain a linear regression on the Hall of Fame, we set matrix A to be the 42x21 player stats and matrix B to be the 42x1 column of whether that player is a Hall of Famer or not (that is, either 1 or 0). In Matlab, we use the least squares command `x = A\b` in order to obtain a column of coefficients. The largest coefficient is the biggest factor that affects one's "Hall of Fameness." Our x matrix is shown below:

```
x =

   1.2466e-02
  -7.2645e-04
   6.8682e-04
  -2.6663e-03
   1.2246e-03
  -5.0258e-03
  -2.5658e-04
  -6.1073e-03
   1.7874e-03
  -8.2589e-04
   2.1834e-03
   1.4419e-03
  -3.5718e-04
  -6.9118e-05
   3.3090e-03
  -6.0893e-03
  -6.4439e-03
  -5.9801e-03
  -8.4723e+00
  -5.7088e+00
   8.5764e+00
```

As shown above, the last coefficient of 8.5764 is the largest one; this one corresponds to SLG, or slugging percentage (the measure of how powerful a baseball player hits).

A negative coefficient means that the associated variable is negatively correlated with being in the Hall of Fame. As one variable increases (for example, AVG), the other variable decreases (Hall of Fameness).

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b. **PCA**

```
CorrelationMatrix = corr(PlayerStats);

[U,S,V] = svd(CorrelationMatrix);

CorrelationFirstThreePrincipalComponents = U(:,1:3);
XYZcoordinates = PlayerStats * CorrelationFirstThreePrincipalComponents;
X = XYZcoordinates(:,1);
Y = XYZcoordinates(:,2);
Z = XYZcoordinates(:,3);

figure
plot3( X, Y, Z, 'b.' )
hold on
plot3( X(1:number_of_Hall_of_Famers), Y(1:number_of_Hall_of_Famers),
Z(1:number_of_Hall_of_Famers), 'r.' )

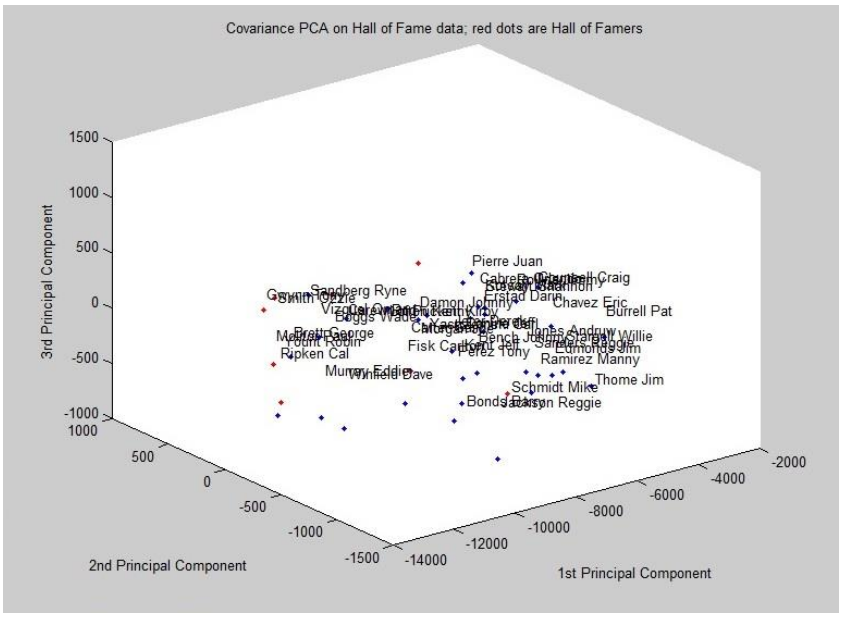
rotate3d on

title('Correlation PCA on Hall of Fame data; red dots are Hall of Famers')
xlabel('1st Principal Component')
ylabel('2nd Principal Component')
zlabel('3rd Principal Component')
```

In order to create a 3D plot demonstrating Principal Component Analysis (PCA), we first take the first three columns of the correlation matrix. Our first principal component, X, is the first column; Z is our third principal component and column.

```
CorrelationFirstThreePrincipalComponents =

  -1.5957e-03  -6.8426e-04  -7.6924e-04
  -2.3916e-01  -9.4419e-02   6.2997e-02
  -8.9912e-01   1.7776e-01  -1.5570e-01
  -1.4016e-01  -1.1375e-01   3.4405e-01
  -2.6752e-01   1.4441e-01   6.3050e-02
  -4.6173e-02  -7.2243e-03   1.1284e-02
  -7.1002e-03   2.4167e-02   1.4960e-02
  -3.1463e-02  -2.4374e-01   3.4839e-02
  -1.3843e-01  -4.0592e-01  -6.0648e-02
  -2.1049e-02   1.1139e-01   2.8182e-01
  -8.0696e-03   2.8134e-02   5.6371e-02
  -1.1484e-01  -3.4023e-01   7.6142e-01
  -7.7107e-02  -7.4969e-01  -3.8248e-01
  -2.1564e-02  -8.1127e-02   1.7592e-01
  -1.9328e-03  -2.4323e-02  -6.8039e-03
  -4.2019e-03   5.1737e-02   1.4105e-02
  -1.0659e-02  -5.3343e-03  -6.0257e-03
  -2.4385e-02   1.0482e-03  -6.5763e-02
  -1.8207e-06   1.1237e-05   1.1632e-05
  -2.3102e-06  -1.7657e-05   5.5867e-05
  -2.7183e-06  -8.5781e-05   2.3219e-05
```



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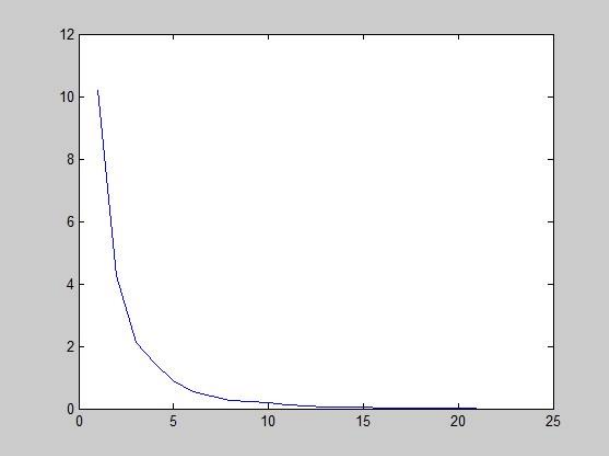
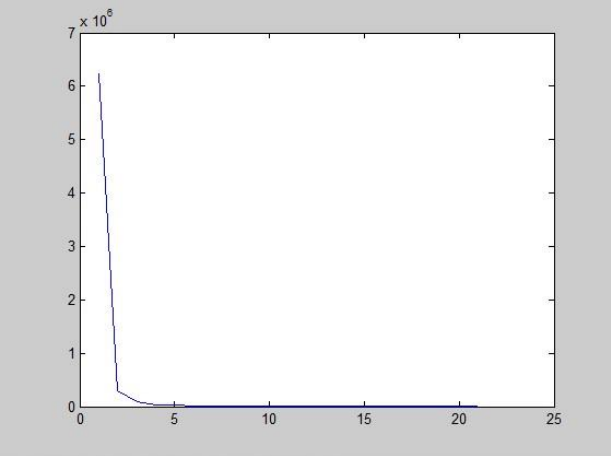
Looking at the matrix above, it seems that the largest coefficients are AB (At-Bat, the first principle component) and SO (Strike-Out, the second principle component).

It is better to have three principal components (as opposed to only two) so we can get a better understanding of which variables contribute to our result, which is Hall of Fameness in our example. We can also determine whether the third principal component plays a large role in predicting fameness or if it's largely influenced by the first two principal components only.

A correlation matrix is essentially a scaled version of the covariance matrix, and as such, provides more standardized data. Our data is a bit "all over the place," so PCA on the correlation matrix would provide results that more effectively capture outlying data, etc. This also relates to why the correlation matrix's scree plot drops off less than the covariance matrix's scree plot. As we standardize the matrices, our principal components become less affected by data at the extremes, and we see that more of our data has an effect on fameness. However, we notice that both graphs still decrease exponentially, as expected.



Correlation Matrix



Covariance Matrix

Looking at the plot of the first two principal components, it would make sense that Barry Bonds and Omar Vizquel deserve to be in the Hall of Fame. They are the only blue dots (non-Hall of Famers) amidst a bunch of red ones (Hall of Famers).

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3. **Eigenfaces**
    a. **eigenfaces.m**

```
function sigma = eigenfaces(fNames, intK, intS) %return k singular values

samplesize = intS; %number of sample faces used to find eigenfaces
k = intK;  %number of singular values to use

%Convert between images and vectors
row = 64;
col = 64;
image_vector = @(Bitmap) double(reshape(Bitmap,1,row*col));
vector_image = @(Vec) reshape( uint8( min(max(Vec,0),255) ), row, col);
vector_render = @(Vec) imshow(vector_image(Vec));

%Load all the faces
filenames = fNames;
if(samplesize>0)
    n = min(size(filenames,1),samplesize); %only look at first samplesize #
of faces
else
    n=size(filenames,1); %if s is an invalid number, use the entire sample
end

for i = 1:n
    Image_File = char(filenames(i)); %convert cell-array of string into
regular string
    Face_Matrix = imread(Image_File);
    F(i,:) = image_vector(Face_Matrix); %i-th row of F is i-th image
end

%Find average face
m = mean(F)'; %average of all rows in F
vector_render(m);
title('average face');

M = ones(n,1) * mean(F);
X = (F-M);

[U_k, S_k, V_k] = svds(cov(X), k);
sigma = diag(S_k); %output the first k singular values
Eigenfaces = V_k;

matrix_image = @(A) uint8(round ( (A-min(A))/(max(A)-min(A))*255 ));
figure

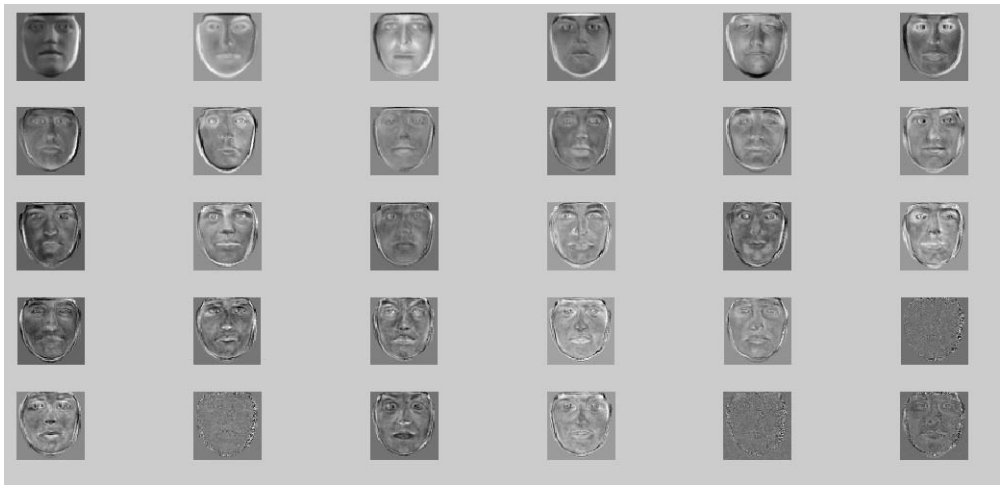
dim=ceil(sqrt(k)); %determine measurements of plot based on number of
eigenfaces
for i=1:k
    subplot(dim,dim,i)
    vector_render(matrix_image(Eigenfaces(:,i)));
end
```

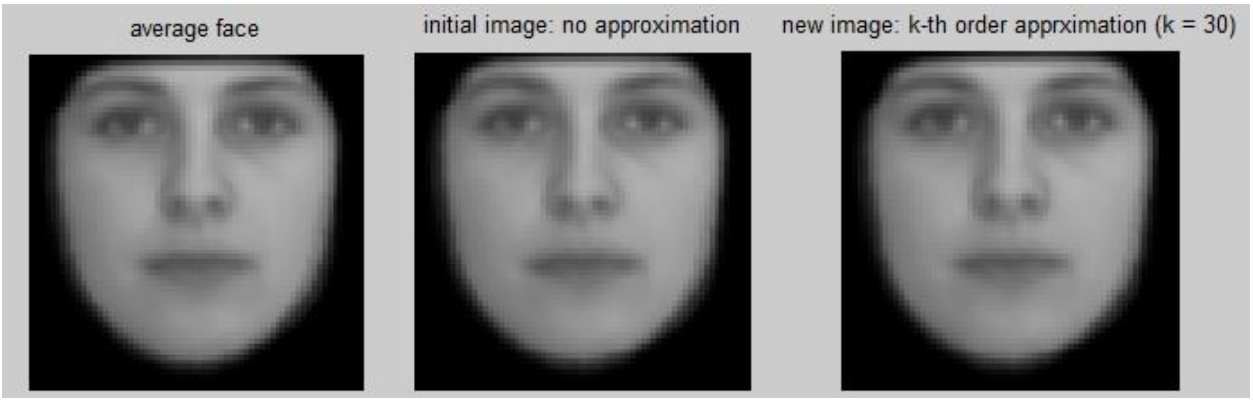
The above code takes in three parameters: an array of filenames as strings, some k value, and some s value. We fix row and col to be 64. Then we read in the first s files (provided s makes sense; otherwise, we stick with the total number of files) and average these faces out into m. We display the mean face, then perform svds to obtain the first k amount of singular values (placed in the list sigma) and the eigenfaces.

b. **face_descriptions.m**

```
females_with_blue_eyes = {
'face012.bmp' ;
'face013.bmp' ;
'face014.bmp' ;
'face031.bmp' ;
'face038.bmp' ;
'face041.bmp' ;
'face043.bmp' ;
'face070.bmp' ;
'face071.bmp' ;
'face072.bmp' ;
'face075.bmp' ;
'face078.bmp' ;
'face080.bmp' ;
'face121.bmp' ;
'face122.bmp' ;
'face131.bmp' ;
'face132.bmp' ;
'face151.bmp' ;
'face153.bmp' ;
'face154.bmp' ;
'face161.bmp' ;
'face171.bmp' ;
'face172.bmp' ;
'face176.bmp' }

eigenfaces(females_with_blue_eyes, 30, 24)
```





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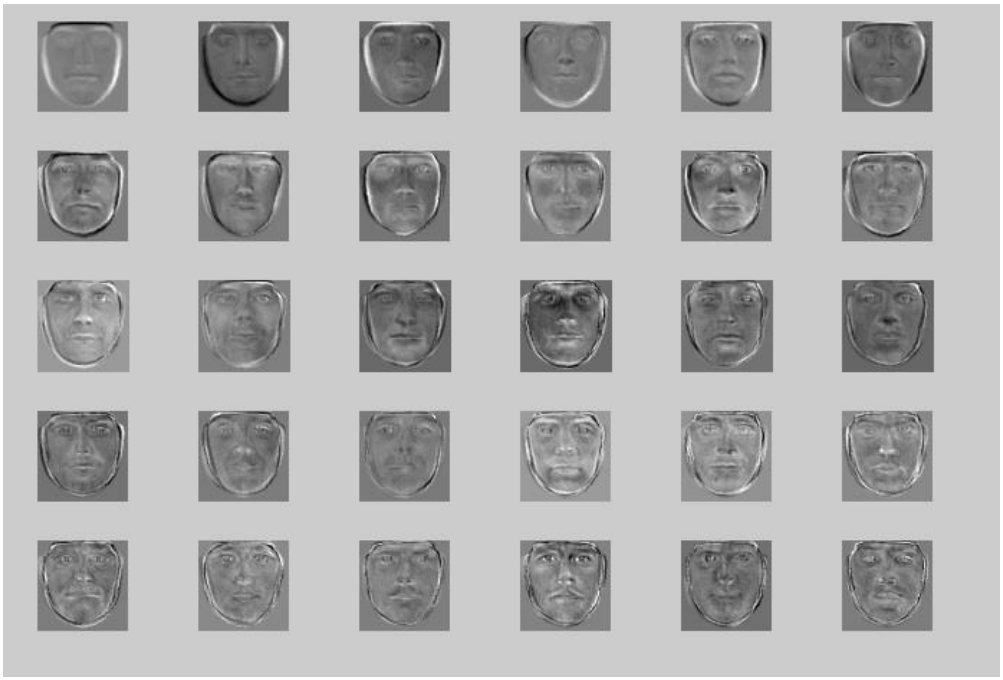
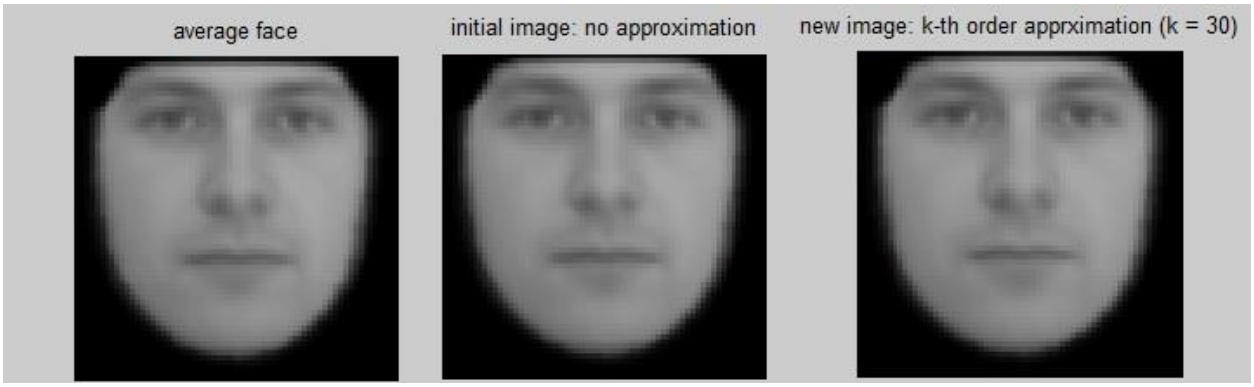
```
males_with_blue_eyes = {
'face000.bmp' ;
'face001.bmp' ;
'face002.bmp' ;
'face003.bmp' ;
'face004.bmp' ;
'face005.bmp' ;
'face006.bmp' ;
'face008.bmp' ;
'face009.bmp' ;
'face010.bmp' ;
'face039.bmp' ;
'face040.bmp' ;
'face042.bmp' ;
'face057.bmp' ;
'face058.bmp' ;
'face062.bmp' ;
'face063.bmp' ;
'face065.bmp' ;
'face066.bmp' ;
'face067.bmp' ;
'face068.bmp' ;
'face069.bmp' ;
'face090.bmp' ;
'face091.bmp' ;
'face093.bmp' ;
'face097.bmp' ;
'face099.bmp' ;
'face100.bmp' ;
'face101.bmp' ;
'face102.bmp' ;
'face103.bmp' ;
'face129.bmp' ;
'face130.bmp' ;
'face152.bmp' ;
'face158.bmp' ;
'face160.bmp' }
```



```
eigenfaces(males_with_blue_eyes, 30, 36)
```



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```
female_k_singular_values =

   1.0e+05 *

    8.7041
    5.3455
    2.7811
    2.0929
    1.6299
    1.3608
    1.0906
    0.9244
    0.7754
    0.6539
    0.5806
    0.4975
    0.4802
    0.4248
    0.3715
    0.3484
    0.3117
    0.2819
    0.2632
    0.2287
    0.1958
    0.1649
    0.1598
    0.0000
    0.0000
    0.0000
         0
         0
         0
         0
```

```
male_k_singular_values =

   1.0e+05 *

    4.2833
    3.2753
    1.7863
    1.2890
    1.0972
    0.9487
    0.6857
    0.6806
    0.5425
    0.4383
    0.4070
    0.3817
    0.3633
    0.3147
    0.2977
    0.2884
    0.2602
    0.2363
    0.2147
    0.2064
    0.1878
    0.1766
    0.1603
    0.1372
    0.1347
    0.1215
    0.1202
    0.1137
    0.1045
    0.0985
```

The average faces of females and males with blue eyes are shown in the previous few pages, respectively. Their top k=30 singular values are shown above on this page. The absolute value difference matrix between the female and male images above renders the face on the left below.

`imshow(abs(f0-f1))`

`imshow(255-abs(f0-f2))`

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c. Male/Female Face Classifier

```
function gender = classifier(fName)

%female average
f0=…
V0=…
%male average
f1=…
V1=…

%Convert between images and vectors
row = 64;
col = 64;
image_vector = @(Bitmap) double(reshape(Bitmap,1,row*col));
vector_image = @(Vec) reshape( uint8( min(max(Vec,0),255) ), row, col);
vector_render = @(Vec) imshow(vector_image(Vec));

Image_File = fName;
Face_Matrix = imread(Image_File);
f=double(Face_Matrix);

c0 = (f-f0)*V0;
c1 = (f-f1)*V1;

normc0 = norm(c0);
normc1 = norm(c1);

gender = (normc0 <= normc1);
```

In the function above, I omitted the actual female/male mean eigenfaces and projections due to size problems. However we can run the function for the blue-eyed images from before.

```
for i=1:36
classifier(char(males_with_blue_eyes(i)))
end

simplified result: 01101011111111111111011110111110101

for i=1:24
classifier(char(females_with_blue_eyes(i)))
end

simplified result: 10101001100110111110010
```

Overall, our function mistook many males as females. This might be due to the fact that the average face of all 177 images looks very much like the female mean. Perhaps using a different eigenface matrix – or leaving it out altogether – would produce to better results.