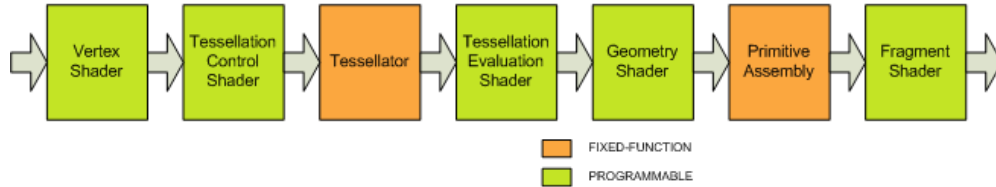


CS174A Midterm Study Guide – Nathan Tung

Lecture 1:

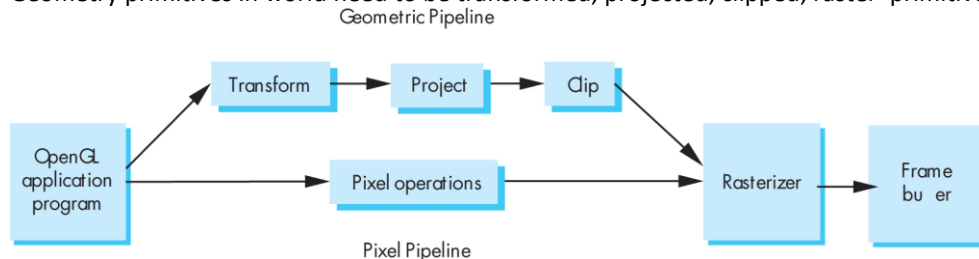
- Interaction/event loop
 - Collecting user input from external hardware (e.g. mouse, joystick, etc.)
 - Sends data down image/rendering pipeline to display and repeats
- Imaging pipeline (shaders)
 - Geometry data passed in is projected, rasterized, colored (visibility), and displayed
 - Vertex, geometry, and fragment shaders are programmable



- Camera model
 - No light rays in computer, so use math (geometry, linear algebra) to approximate
 - In a camera, moving image plane closer reduces size of image
 - Clipping/Culling (2D/3D): eliminating parts of the image/view that is out of bounds
 - We can move our camera position in space or parameters of our camera system, like size or position of window within image plane
 - Our “camera” is made up of the display “window” and the “eye-point”

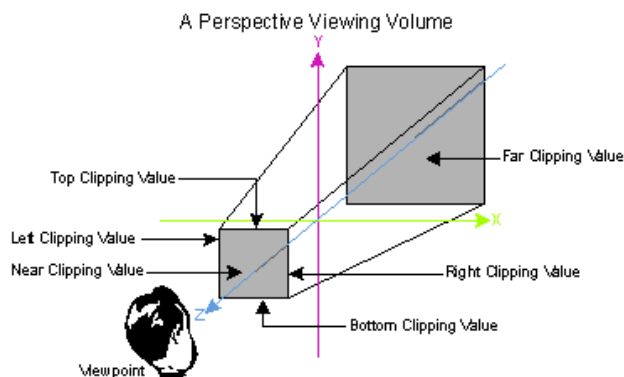
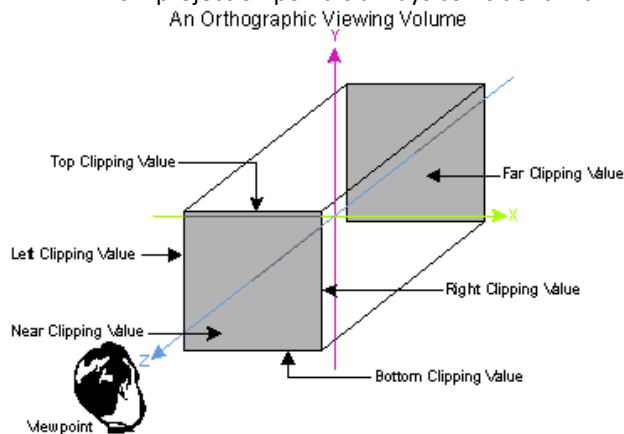
Lecture 2:

- Different Modes
 - Immediate Mode: points generated and sent to GPU each time; no storage needed
 - Lots of work
 - Retained Mode #1: points generated once, transferred to GPU each time; storage for all points needed
 - Less work, but needs storage and transfer to GPU each time
 - Retained Mode #2: points generated once, transferred to GPU once; storage for all points once
 - Lots of work, temporary storage on host and GPU, single transfer time
- Transformation Functions: translate, scale, rotate (these are used heavily by viewing)
 - Occur in application itself or in shader
- Input for interaction (i.e. keyboard, mouse)
- OpenGL is a state machine, a black box, that accepts two inputs
 - Something that alters internal state, like changing or returning state (parameter setting, enable/disable features)
 - State is persistent: stays set until you change it
 - State is not bound to primitives (“cube is yellow, grass is green, etc.” - we have to manage this ourselves)
 - Something that causes output, like specifying primitives going through pipeline
- GLUT (simple system independent windowing and input processing), GLEW (removes OS specified dependencies)
- Coordinate Systems (device independent graphics: we don’t specify pixels on display, and no units in OpenGL)
 - World: geometry is defined here, represented with floats
 - Window: physical display, width and height in pixels
- Use transformations to do world → screen coordinates
 - Geometry primitives in world need to be transformed, projected, clipped; raster primitives already in screen space



- Primitives include points, lines, triangles, triangle strips (these render very fast and can approximate complex ones)
 - Primitives made up of vertices (attributes define how GPU processes these primitives)
- CG is additive color (nothing=black, add color), as opposed to subtractive color (nothing=white, subtract from white)
 - Black is (0, 0, 0, 1) and white is (1, 1, 1, 1)
 - Red, green, blue (RGB), 8-bits per color plus another for alpha in RGBA

- Perspective vs. Orthographic Projection (for viewing)
 - Perspective: single projection point, appearance of depth like realistic vision
 - Orthographic: projection point is infinitely far away, so projection lines are parallel to image plane
 - As if projection point is always coincident with image plane



- Orthographic creates 2D data with $z=0$, like text on glass (score) or HUD (head-up display)
 - Near clip, far clip, and sides make up the viewport
- Shaders (set of shaders exist within a program that executes on GPU)
 - Vertex Shader
 - On `glDrawArrays()`, each vertex triggers, execution of vertex shader
 - `gl_position` is an OpenGL state variable to pass final transformed vertex down pipeline to rasterizer
 - Fragment shader
 - Executes on output of the rasterizer; rasterizer generates fragments for primitives in pipeline
 - `gl_frag_color` must be output for fragment at minimum per execution

Lecture 3:

- Spaces
 - Vector, Affine, Euclidian Space
 - Each builds on the last; geometric objects exist within these spaces
- Vector Space
 - Vectors only have direction and magnitude; addition/multiple allowed
 - Addition (head-to-tail axiom), multiplication (scalar, changes magnitude)
 - Two ways to understand
 - Directed line segments (helpful to understand, not practical)
 - N-tuples of real numbers like $v=(v_1, v_2, \dots, v_n)$ (OpenGL representation)
 - This space is called " R^n ", where matrix algebra can manipulate vectors
 - Linear independence defines dimension
 - Case where only set of scalars $a_1u_1 + a_2u_2 + \dots + a_nu_n = 0$ is $a_1=a_2=\dots=a_n=0$
 - In $n=3$ (or 3D), the only set of u is $(1, 0, 1), (1, 1, 0), (0, 1, 1)$
 - Largest number of linearly independent vectors in a space gives the dimension
 - A representation of a vector v is given by the B_i scalars with respect to v_1, v_2, \dots, v_n basis (head-to-tail)
 - Given a vector space of dimension n , then *any* set of n linearly independent vectors forms a *basis*
 - Basis gives us a representation (and dimension); matrix multiplication changes/transforms representations

- Can convert a frame to another
 - Vector space doesn't have location, vectors float with direction/magnitude (can have dimension though)
- Affine Space
 - Introduces concept of point to vector space; the point gives us a location
 - Operating on the point (like point subtraction – not addition – between two points) gives a vector
 - Affine space defined in terms of frames (frames: all coordinate systems existing in a basis R^n)
 - Frame consist of a point P_0 (origin) and basis; other points are defined as $P = P_0 + B_1v_1 + \dots + B_nv_n$
 - Affine space doesn't have distance or length
- Euclidian Space
 - Consists of (real) vectors and scalars only; defines dot product to combine two vectors to form a real
 - Having $u \cdot v = 0$ means u, v are orthogonal; $u \cdot u = |u|^2$ tells us the square of the length!
 - And $u \cdot v = |u||v|\cos\theta$ to give us the angle between two vectors
 - Adding in affine space's key concept of points, we can get distance between points
 - $|P - Q| = \sqrt{(P - Q) \cdot (P - Q)}$, the distance between P and Q
- Planes (an addition to vectors, points, lines)
 - Exists in affine space and defined by three points P, Q , and R
 - Those three points essentially form a triangle; notice that $(Q-P)$ and $(R-P)$ are arbitrary vectors
 - Plane is defined by two non-parallel vectors and a point
 - "Barycentric coordinate" representation is a simplified equation for planes
 - A vector n , or "normal," exists that's orthogonal to our plane using cross product: $n = u \times v$
- Homogeneous Coordinates
 - Frame in affine space: for R^3 , vector $v=[a_1, a_2, a_3, 0]$ and point $p=[b_1, b_2, b_3, 1]$; 0 is nowhere, 1 plants it in space
 - This is a 4D vector! Now we can do transformations
 - Scale: $S(a, b) = [a, 0, 0, b]$ such that $[a, 0, 0, b][x; y] = [ax; by]$
 - Rotation: $R(\theta) = [\cos \theta, -\sin \theta; \sin \theta, \cos \theta]$
 - Translation can't be done with matrix-vector multiplication, but with homogeneous coordinates and $[x; y; 1]$
 - $S(a, b) = [a, 0, 0, 0; 0, b, 0, 0; 0, 0, 1, 0]$ multiplied gives us $[ax; by; 1]$
 - Translation uses $[1, 0, j; 0, 1, k; 0, 0, 1]$, multiplied gives us $[x+j, y+k, 1]$
- Six Coordinate Frames used in OpenGL pipeline
 - Object (model), World, Eye (camera), Clip, Normalized device, Window (screen)
 - Object is represented in object/model coordinates, a local frame of the object to model geometry
 - Objects placed in world coordinates, a global frame to position relative objects via scaling, translating, rotating
 - Eye coordinates need to be placed to see object/world (whose coordinates are ideal for modeling, not viewing)
 - Use 4x4 to transform model to world to eye; this is together called the model-view transformation

```

graph LR
    subgraph Object_coordinates [Object coordinates]
        V[Vertices]
    end
    subgraph Camera_coordinates [Camera coordinates]
        MV[Model-view transformation]
    end
    subgraph Clip_coordinates [Clip coordinates]
        P[Projection transformation]
    end
    V --> MV
    MV --> P
    P --> V2[Vertices]
  
```

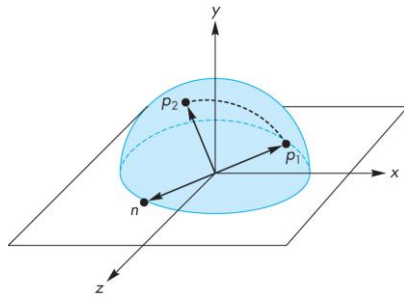
The diagram illustrates the transformation pipeline. It starts with 'Object coordinates' containing 'Vertices'. An arrow points to a box labeled 'Model-view transformation' under 'Camera coordinates'. Another arrow points to a box labeled 'Projection transformation' under 'Clip coordinates'. A final arrow points from the 'Projection transformation' box back to 'Vertices'.

 - Clip coordinates reject primitives outside of view volume after projection transformation
 - Easiest when done after transforming view volume into origin-centered cube; clipping is done quickly here
 - After projection transformation, clip coordinates are still homogenous coordinates, so we remove w component to obtain a 3D point within normalized device coordinates
 - This is finally transformed into 3D window coordinates
 - Window (screen) coordinates are 2D, so pipeline drops the z (depth) value; window defined by viewport
 - No actual camera; everything is seen by manipulating model-view transformation
 - Same thing: moving world to eye, or eye to world coordinates frames
 - Transform world frame by -2 when camera fixed at origin; Transform eye frame by 2 if world is fixed
- Transformations
 - All transformation matrices are 4x4 (OpenGL uses column vectors for matrices like $T[3][1]$)
 - Homogeneous translation matrix: $[1, 0, 0, x'; 0, 1, 0, y'; 0, 0, 1, z'; 0, 0, 0, 1]$
 - Scale: $[B_x, 0, 0, 0; 0, B_y, 0, 0; 0, 0, B_z, 0; 0, 0, 0, 1]$, Rotation $[\cos x, -\sin x, 0, 0; \sin x, \cos x, 0, 0; 0, 0, 1, 0; 0, 0, 0, 1]$
 - Matrix multiplication is associative (concatenate matrices for better performance in transformations)
 - $q = CBAp$ is the same as $q = C(B(Ap))$
 - Not commutative; order of different transformation type matters
 - $q = TRSTp$ is NOT the same as $q = TSRTp$
 - Applied last to first, since post-multiplied with current transformation matrix
 - To transform in geometry, send data once to update transformation via GPU uniform variable

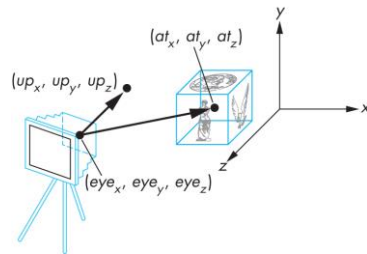
Lecture 4:

- Trackball

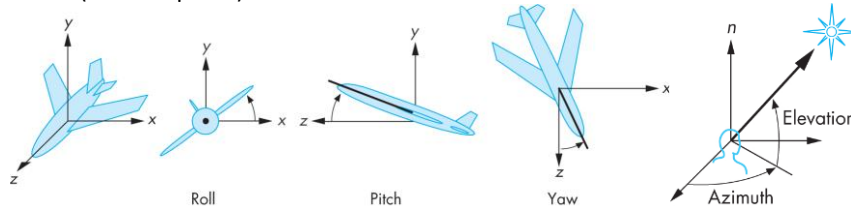
- Place unit hemisphere in xz plane, reconstructing y using mouse positions ($x^2 + y^2 + z^2 = 1$; $y = \sqrt{1 - x^2 - z^2}$)
- Given two positions on hemisphere p_1 and p_2 , we can find axis of rotation by the cross product of p_1 to p_2
 - That's the normal, or $n = p_1 \times p_2$, and for small rotations, θ and $\sin \theta$ are about equivalent



- Quaternions do the same thing with less work; $q = w + xi + yj + zk$ (imaginary terms), where w is the rotation
 - But q must be normalized to $q = |q|^2$
 - Quaternions allow smooth interpolation with rotation R and (small) increment R_1 ; renormalize R occasionally
- Viewing: perspective and parallel both have objects, projection lines and planes, and eye (center of projection – COP)
 - Model-view transformation doesn't go to clip coordinates; we need projection transformation
 - The former gets objects in front of camera; the latter defines how they appear
 - Again, transformations are specified in reverse (so specify camera position first)
 - Look-At: location of EYE, point the eye is looking AT, and the UP direction of camera $(0, 1, 0)$ or $(0, 1, 0, 0)$
 - AT and EYE points form the view-plane-normal, or vpn
 - Look-At is not smooth for moving camera

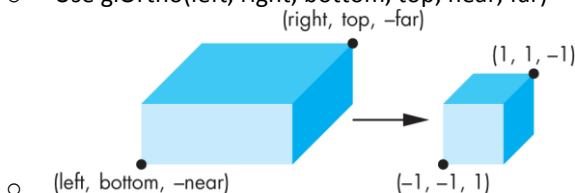


- Yaw, pitch, and roll (think airplane)



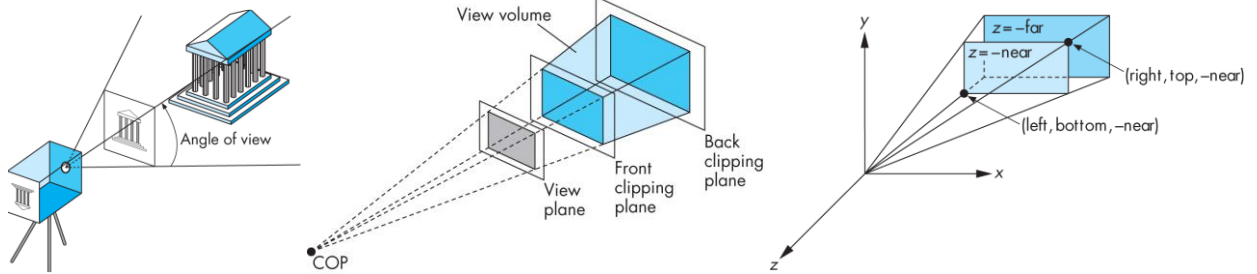
- Use polar coordinates; a simplified version uses azimuth (side to side rotation) and elevation
 - Translate camera to eye point and rotate to direction we want (reality: perform the inverse)
 - We're really moving the world to the camera

- Now we have our camera coordinates and we need a projection transformation to get clip coordinates
- Parallel (orthographic) projection/viewing
 - $M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$; this transformation is applied in hardware after vertex shader
 - Then we scale in order to fit/include within the canonical view volume $(-1, 1)$, $(-1, 1)$, $(-1, 1)$
 - Use $glOrtho(\text{left}, \text{right}, \text{bottom}, \text{top}, \text{near}, \text{far})$



- Perspective projection/viewing
 - Basic symmetrical perspective projection; project (x, y, z) through projection plane to eye point (or COP)
 - $M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$
 - The $1/d$ term means our homogeneous coordinate will be modified
 - So far we only have points on projection plane, not clip coordinates; we also need a view volume

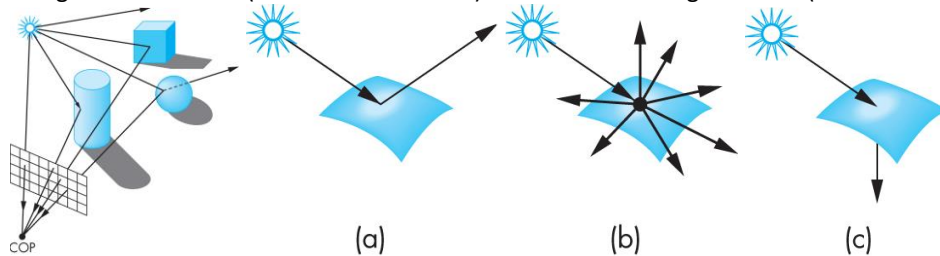
- View volume is a pyramid with apex at COP; view/clip planes don't need to be the same or coincident



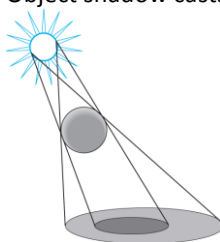
- So we use perspective(fovy, aspect, near, far), where aspect is the aspect ratio of view volume (width/height)
- Matrices are passed to vertex shader as uniform variables, as before

Lecture 5:

- Lighting and Shading
 - Shading needed in 3D to enhance effect of dimensionality; real-world color is rarely flat and uniform
 - Approximate how light interacts...
 - Different types of light sources
 - Properties of a surface that affects how light reflects off of it
 - Use local interaction, since global light models are too computationally intensive to be interactive
 - Global might be physics of light (surfaces emit, absorb, reflect; light reflects, scatters, bleeds, etc.)
 - Not fast enough (not even offline rendering); GPUs can speed this up for only simple/moderate things
 - Even approximations are too slow (radiosity – thermodynamics, ray tracing – light rays)
 - Simplify to single interactions between light source and surface
 - Need two models: light source, and how a surface reflects that light source
 - We can see light of the source (color of source itself) and reflection of light source (some absorbed/bounced)

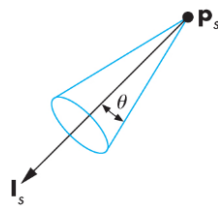


- Surface Types
 - Specular (a): most light reflected in narrow range, like a mirror
 - Diffuse (b): light is reflected everywhere, with a flat appearance
 - Translucent (c): light is refracted, some may be reflected
- Light Source Types (each have color and luminance, or brightness, each represented as RGB)
 - Ambient: uniform illumination (cloudy day or indirect lightning in house)
 - Light is scattered in all directions, modeled with luminance I_a
 - Every point receives same illumination; surfaces may reflect ambient light differently
 - Point
 - Light emitted equally in all directions, with a similar ambient luminance with location $I(p_0)$
 - Amount of light received by a point is proportional to distance from point light source
 - $d = |p - p_0|^2$, $i(p, p_0) = \frac{1}{a + bd + cd^2} I(p_0)$, where a, b, and c soften the light
 - Most real point sources are finite in size and not an actual point
 - Object shadow casts an (inside) umbra and (outside) penumbra

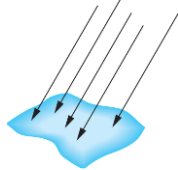


- Spotlight
 - Have a position like point lights, but these actually have a direction defined by an angle
 - Intensity of light attenuated by magnitude of angle between direction vector and point of surface

- This is the “falloff”

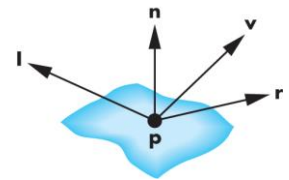


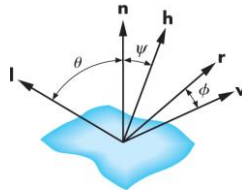
- The angle is defined as $\cos\theta = u \cdot v$
- Distant/Infinite
 - When light source is very far away (like the sun); light rays are parallel to each other in direction
 - All surface points use that direction vector instead of p0



- Shading – Phong Reflection Model

- Rather than exact physical model, use efficient approximation
- Use four vectors to enable calculation of a color value for any point on a surface
 - Vector n, normal at point P on surface
 - Vector v, from point P to eye
 - Vector l, from point P to the light source (direction)
 - Vector r, the perfectly reflected ray from l (computed from n and l)
- Supports ambient, diffuse, and specular light types
 - These 3 “color” values are stored for a surface
 - Generally, define a material and associate with surface so surface can have multiple colors
- To compute light (color) at some surface point...
 - Sum contributions from each source interacting with surface (R) and global ambient term
 - $I = \sum (I_{a_i} + I_{d_i} + I_{s_i}) + I_a$, where $(I_{a_i} + I_{d_i} + I_{s_i}) = (L_{a_i}R_a + L_{d_i}R_d + L_{s_i}R_s)$
- Ambient Reflection
 - Intensity of ambient light at each point on surface is equal
 - Amount that’s reflected is given by coefficient k_a
 - For any light source, resulting reflected light intensity is $I_a = k_a L_a$ where $0 \leq k_a \leq 1$
- Diffuse Reflection
 - No preferred direction of reflection; behavior modeled by Lambert’s law
 - Only see vertical component of reflected light, but light gets dimmer as angle becomes larger
 - Reason: light is spread out over larger area
 - Direction of light source l and surface normal n determine amount of reflected light
 - Take into account reflection coefficient k_d
 - Also include quadratic attenuation term to account for surface’s distance from light
 - $I_d = \frac{k_d}{a+bd+cd^2} \max((l \cdot n)L_d, 0)$, where max accounts for light when it’s below the horizon
- Specular Reflection
 - Adding shininess to a smooth surface; reflection of part of light source itself
 - Smoother surface means more concentrated
 - Mirror is a perfect specular surface
 - Phong approximates this (can’t mirror)
 - Depends on angle between perfect reflection vector r, viewer direction v, k_s (reflection coefficient of specular reflection), and α (the shininess coefficient which controls how narrow/broad the reflection is)
 - $I_s = k_s L_s \max((r \cdot v)^\alpha, 0)$
- Full Phong Model
 - Combines diffuse, specular, ambient terms and computes it for each light source in scene
- Blinn-Phong Model
 - Usually we compute the reflection vector r for each point on surface
 - When l, n, r, and v are in same plane, halfway vector h can be used instead of computing r





- $h = \frac{l+v}{|l+v|}$ and $\phi = 2\psi$
- The angle between n and h is smaller than angle between r and v, however
- So shininess coefficient α has to be raised to get approximately the same result as pure Phong

- Normal

- For flat surfaces we mean normal to plane
- For three non-collinear points, normal $n = (p_2 - p_0) \times (p_1 - p_0)$, where order matters; determines direction of n
- For curved surfaces, depends on how surface represented; normal at point on a unit sphere is $n = p$
 - Approximations are needed (often using points/planes) since we represent objects using primitives

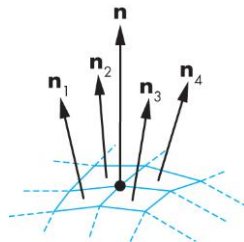
- Flat

- Vectors l, n, and v vary as we move across surface
 - But flat surface with far light source and viewer means vectors can be held constant
- Then every point on surface primitive receives the same shading, or “flat shading”
- Flat shading is called per primitive in OpenGL, since a single color (light) is applied to entire primitive

- Smooth

- Provide color and normal information per vertex to achieve a smoother shading
- Let rasterizer interpolate color values across surface of primitive
 - How do we compute normal for a surface?
- Gouraud shading tells us to use the average of surface normal of the primitive they are a part of
 - For a vertex shared by four surfaces, we compute the vertex normal as

$$n = \frac{n_1 + n_2 + n_3 + n_4}{|n_1 + n_2 + n_3 + n_4|}$$



- Phong shading, a variation of Gouraud, interpolates the normals themselves (one step further)
 - Requires us to compute lighting at vertex or fragment level

Lecture 6:

- Frame (Color) Buffer

- Manages RGB or RGBA color values on screen

- Depth Buffer

- Stores normalized z values from clip volume
- Used for hidden surface elimination; allows drawing objects without respect to space position
 - Order independent
- When enabled, z value written in buffer for every pixel
 - If incoming pixel with z value less than value already in buffer, pixel written into frame (color) buffer, and z value is updated in depth buffer
 - Otherwise, pixel is rejected (not written to frame buffer) and depth buffer remains unchanged
- Without depth buffer, last write to frame buffer wins; drawing becomes order dependent (render back-to-front)
- Problem cases: inter-penetrating objects, moving objects
- Entails requesting depth buffer (initialization time) and enabling it; also clear buffer (bit) before new image

- Back to Frame Buffer

- Uses double buffering: front and back color buffers
 - Rendering happens on back buffer, front/back can be swapped, and front gets displayed
- Transparent object rendering
 - Enable blending to consider alpha values
 - Blending involves RGBA values of pixels in pipeline (“source”) and current frame buffer (“destination”)
 - Blending adds these two pixels together after multiplying each by its blending factor
 - Prevents saturating (going above alpha of 1.0)

- Don't want to update depth buffer; set `gl.depthMask` to false to make it read-only
 - Example: to render a frosted window, see what's behind the window and not see the window if it's behind other opaque objects
 - This requires depth test, but not writing depth value of window
 - Therefore, we render transparent objects AFTER opaque objects
 - Also sort and render transparent objects back-to-front (remember, no depth buffer here!)
 - Once transparent objects rendered, reenable depth buffer writing (mask) and disable blending
 - Leaving blending on just makes it unnecessarily slow
- Mapping Methods
 - Texture, Environment, and Bump Mapping
 - All three can be combined
 - All are performed at fragment stage
 - All can be stored in 1, 2, or 3 dimensional buffers (maps)
 - Three variations of same mechanism to add detail and complexity
 - More complex geometry, details assigned to vertex colors, etc.
 - Imagery adds detail and appearance of complexity to geometry without actually modeling it
 - Maps can modify shading directly by altering color values, or alter surface material or normal properties
- Texture Mapping: use image to alter surface color values
 - Consider 2D form; individual pixels of the image are called texels
 - Texture map described by $T(s,t)$ - texture coordinates s and t defined over $[0,1]$ - and mapped onto point $p(u,v)$
 - Assign texture coordinates to vertices (like color or normal), which are interpolated over rendered surface and made available in fragment shader
 - Define/download image to GPU → assign texture coordinates to geometry → apply texture to fragments
 - First, create texture object (container object describing texture)
 - After it's created and bound(active), define image data itself (as raw RGB) and copy into GPU
 - Texture is defined and coordinates are assigned to vertices statically; they can be used like colors/normal
 - Vertex shader: pass texture coordinates through to fragment shader, allowing s and t to be interpolated
 - Fragment shader: set up a way to control texture object to use; then set it
 - Define texture reference as a type `sampler2D`
 - Built in function `texture 2D` samples referenced texture at coordinate specified
 - Color and texture can be combined with each other and with the normal
 - For (s,t) values outside $[0,1]$, use texture wrapping by either...
 - Clamp (reuse edge texel, smears)
 - Repeat
 - These are set separately for s and t , performed when texture is defined
 - Mapping between $[0,1]$ and discrete texels
 - Sampling
 - Point, nearest neighbor
 - Linear, bi-linear interpolation
 - Minification and magnification
- Texture Mapping Overview
 - What kind of sampling to use? Depends on quality and type of image used in texture map.
 - Avoidance of sampling artifacts (Moiré patterns)
 - Use experimentation
 - Consider resource usage (mipmaps use 1/3 extra storage)
 - Multi-texturing GPUs today can process several textures at once
 - Activate hardware texture unit, then bind texture to it (may need multiple sets of texture coordinates)
 - Up to fragment shader to determine how to process/combine result
 - Usually texture mapping is used as a tool to make fine adjustments
 - Load image from disk (no OpenGL mechanism for this – use a library; but WebGL is pretty easy)
- Environment Mapping: give objects the appearance of reflection
 - This reflection of fixed environment is sometimes called “reflection mapping”
 - Can be computed via ray tracing, but usually too slow for real-time; texture map can approximate the effect
 - Reflection vector is calculated by eye → surface → reflect → world
 - Intersect that ray with scene and compute the color (shading) value; but this is essentially ray tracing
 -
- Bump Mapping: distort normal vectors during shading process