

Machine Learning

By Ghazal Lallooha

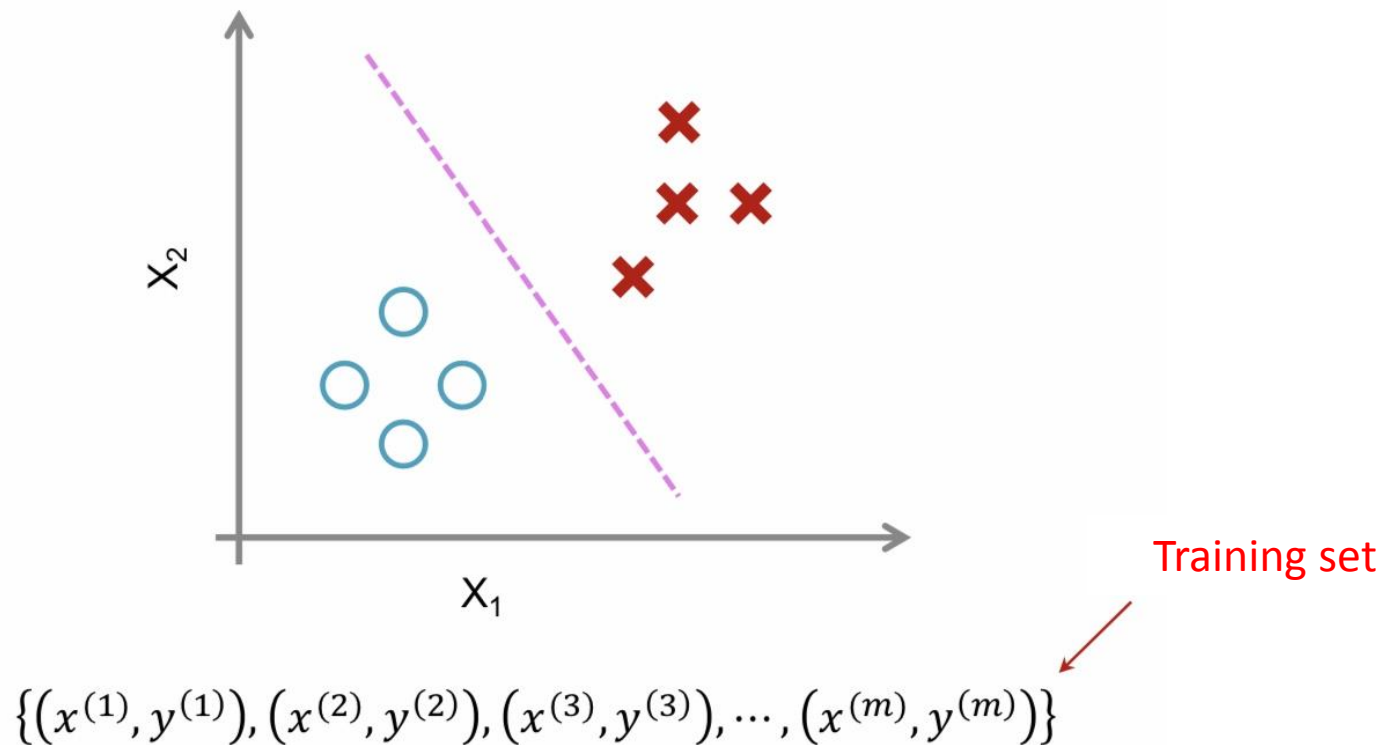
Unsupervised Learning

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- applications
- Clustering
- K-means algorithm
- Improved clustering
- Two-part generator algorithm
- Hierarchical clustering

Supervised Learning

- Supervised Learning: For each example, the correct answer is given.

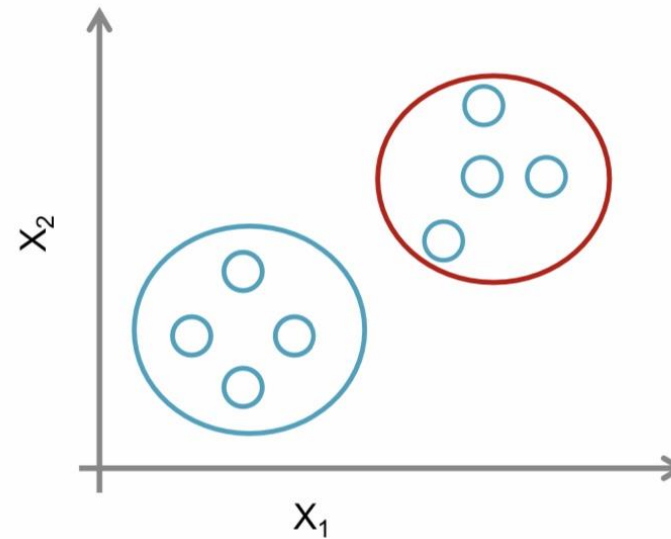


Unsupervised Learning

- Unsupervised learning: Not knowing the correct answers.

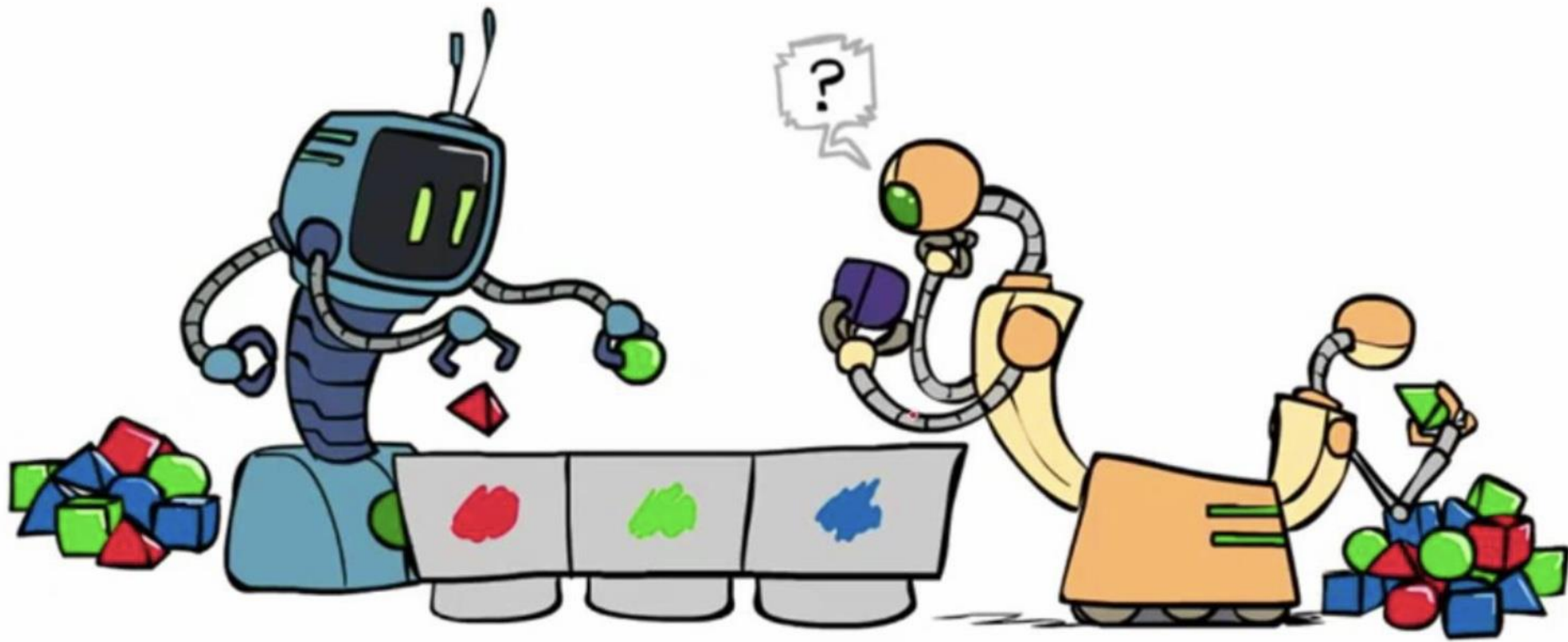
$$\{x^{(1)}, x^{(2)}, x^{(3)}, \dots, x^{(m)}\}$$

Training set

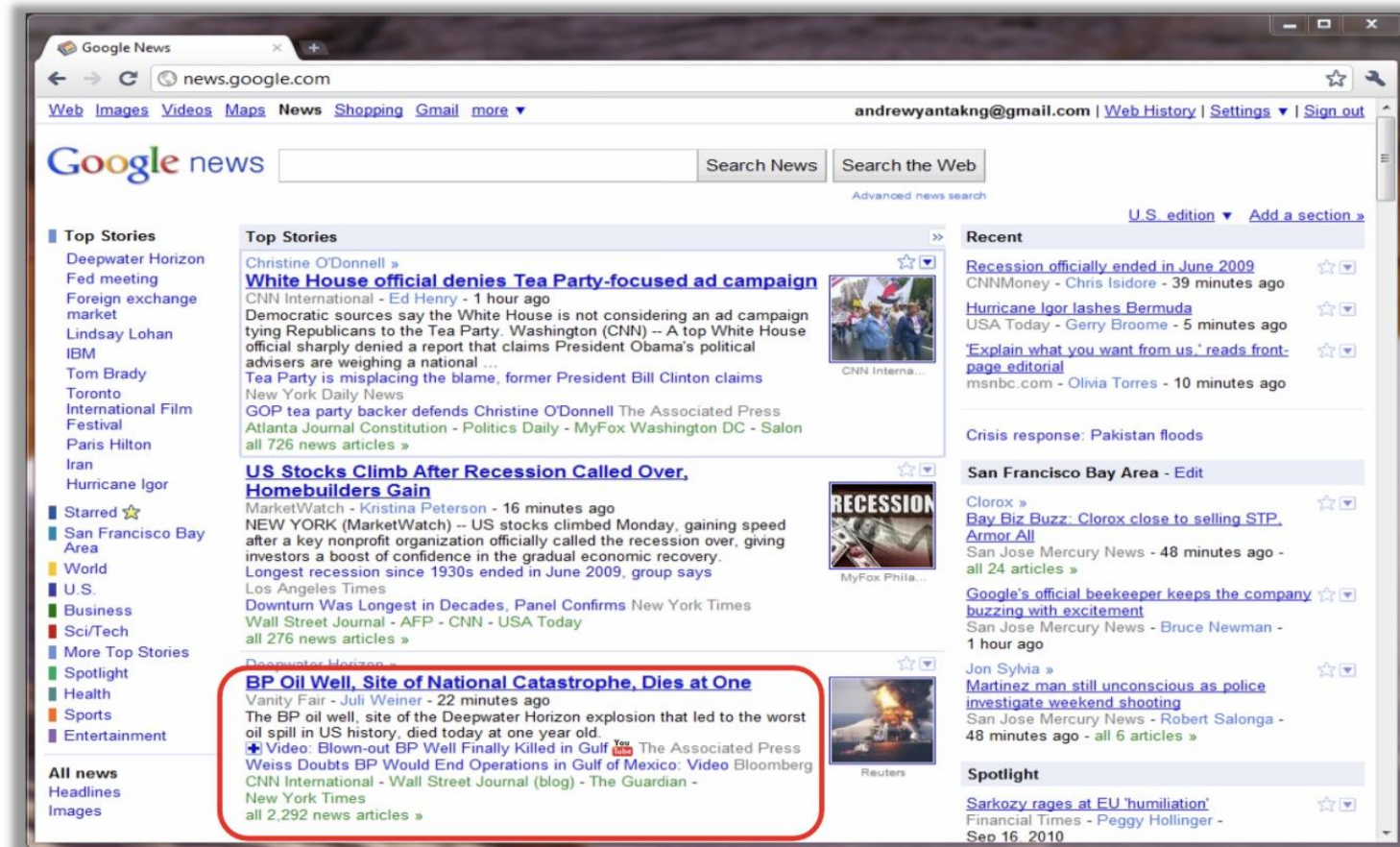


- Goal: recognizing the structure in the input data (grouping similar data)

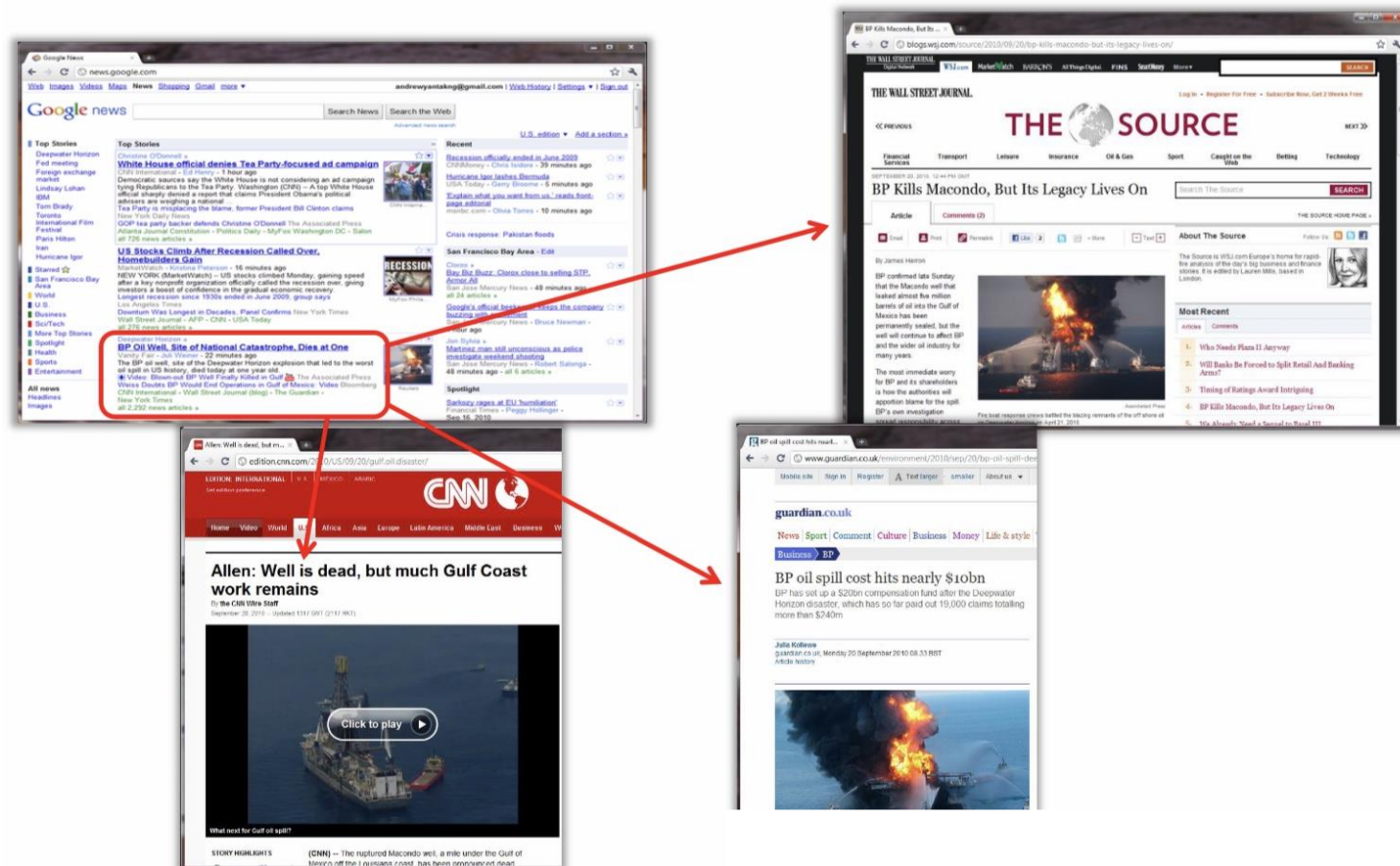
Clustering



Application of clustering: grouping related news



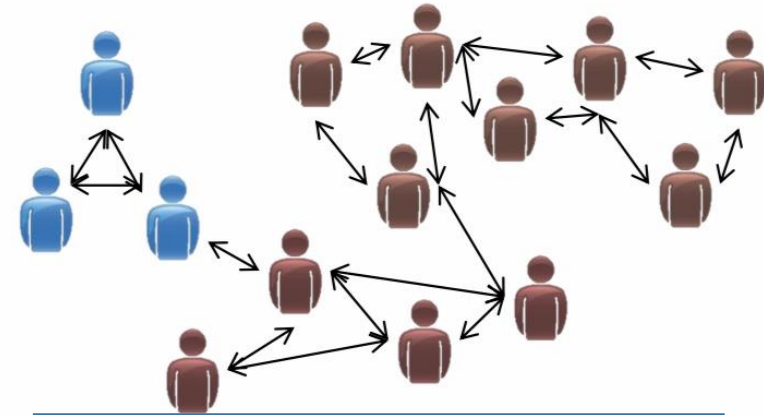
Application of clustering: grouping related news



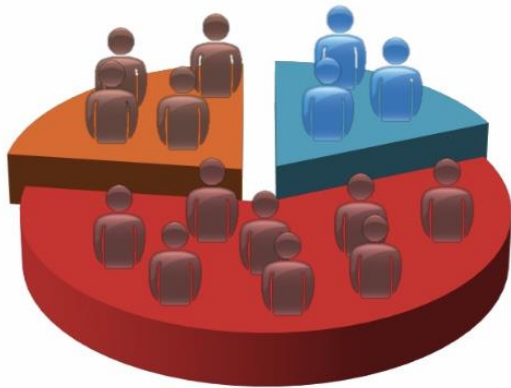
Some other applications of unsupervised learning



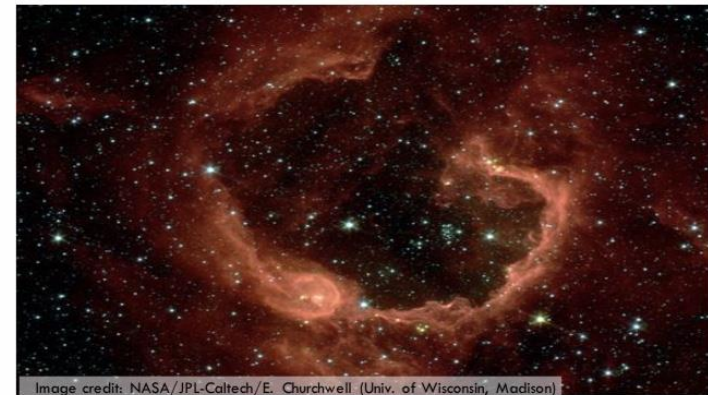
Organization of computing clusters (data center)



Social networks analysis

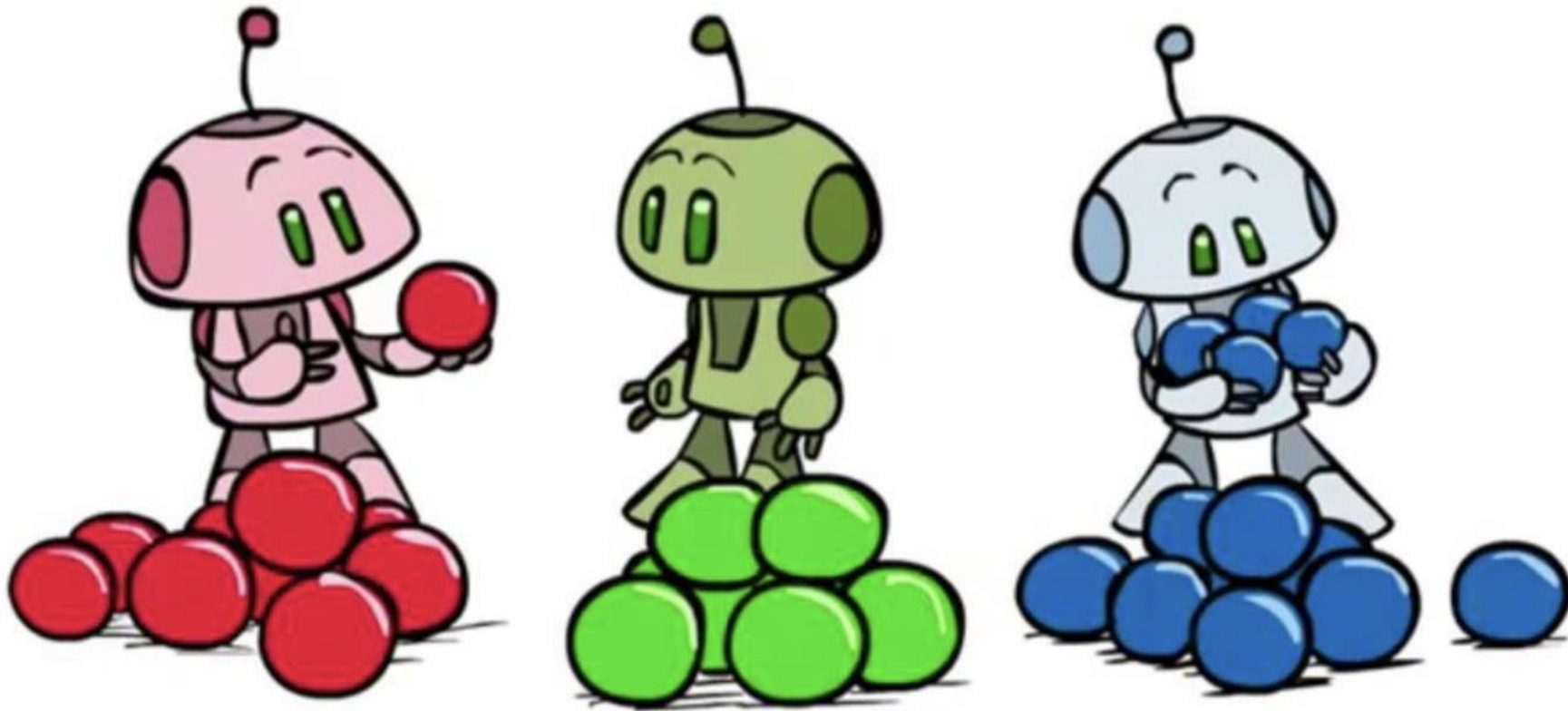


Market segmentation



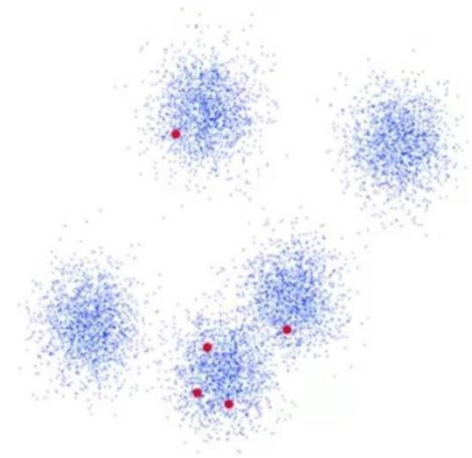
Analysis of astronomical data (how galaxies form)

K-means clustering algorithm



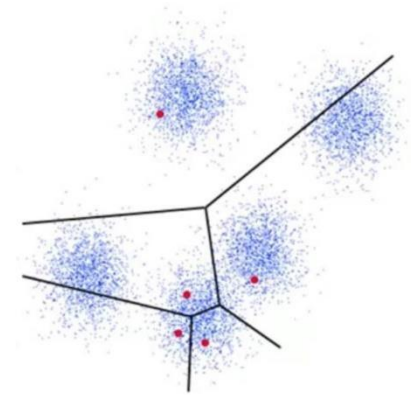
K-means clustering algorithm

- An iterative clustering algorithm:
 - Choose K points randomly as centers of clusters.
 - Repeat the following steps:
 - Assign each data to a cluster with the closest center.
 - Update the center of each cluster by averaging the data assigned to that cluster.
 - Stop: when no data changes its cluster in an iteration.



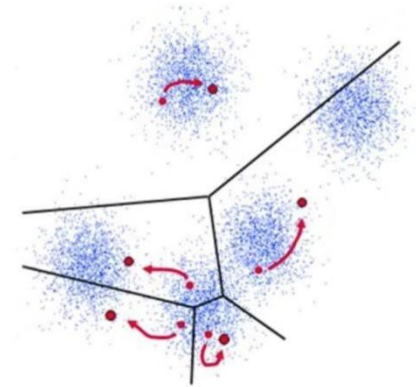
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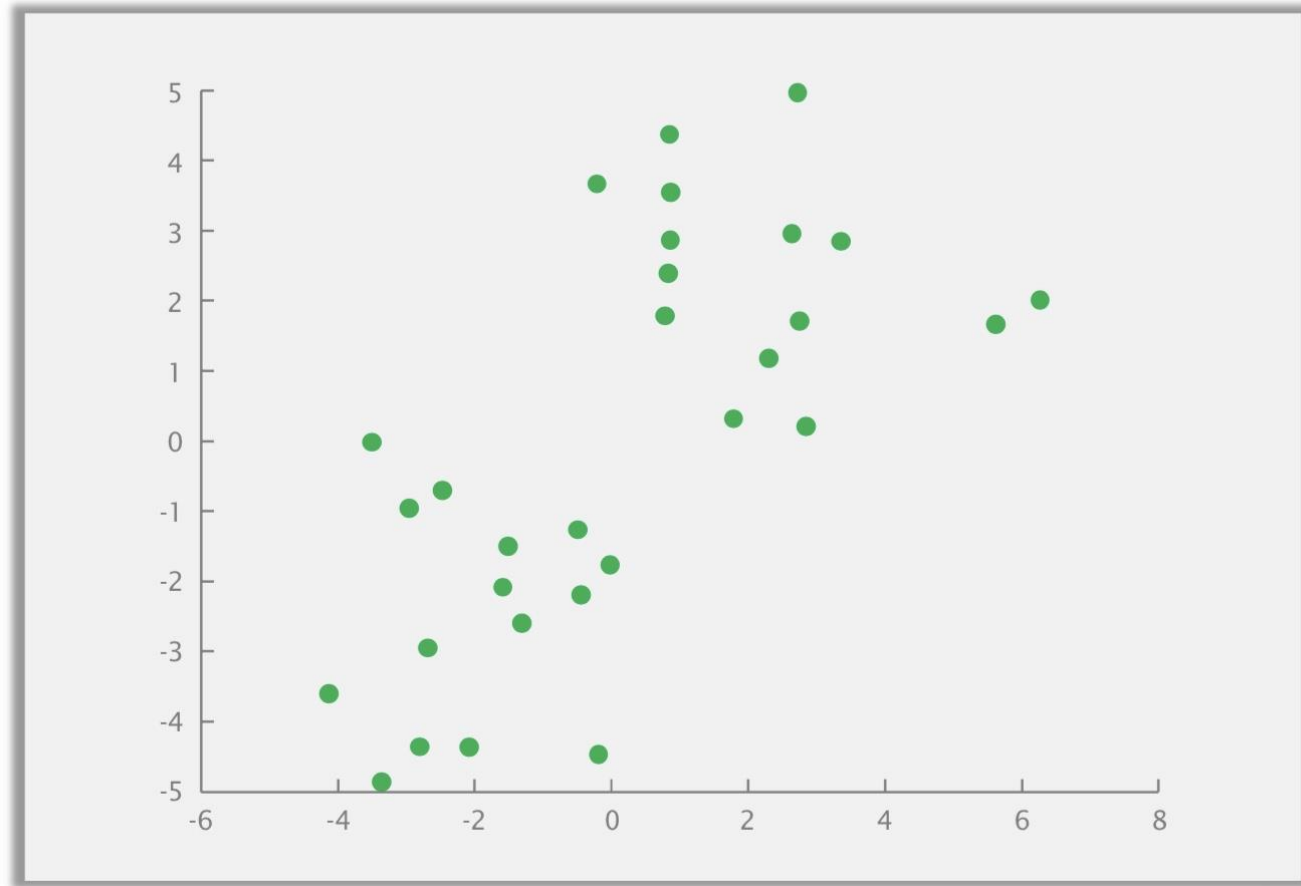


K-means clustering algorithm

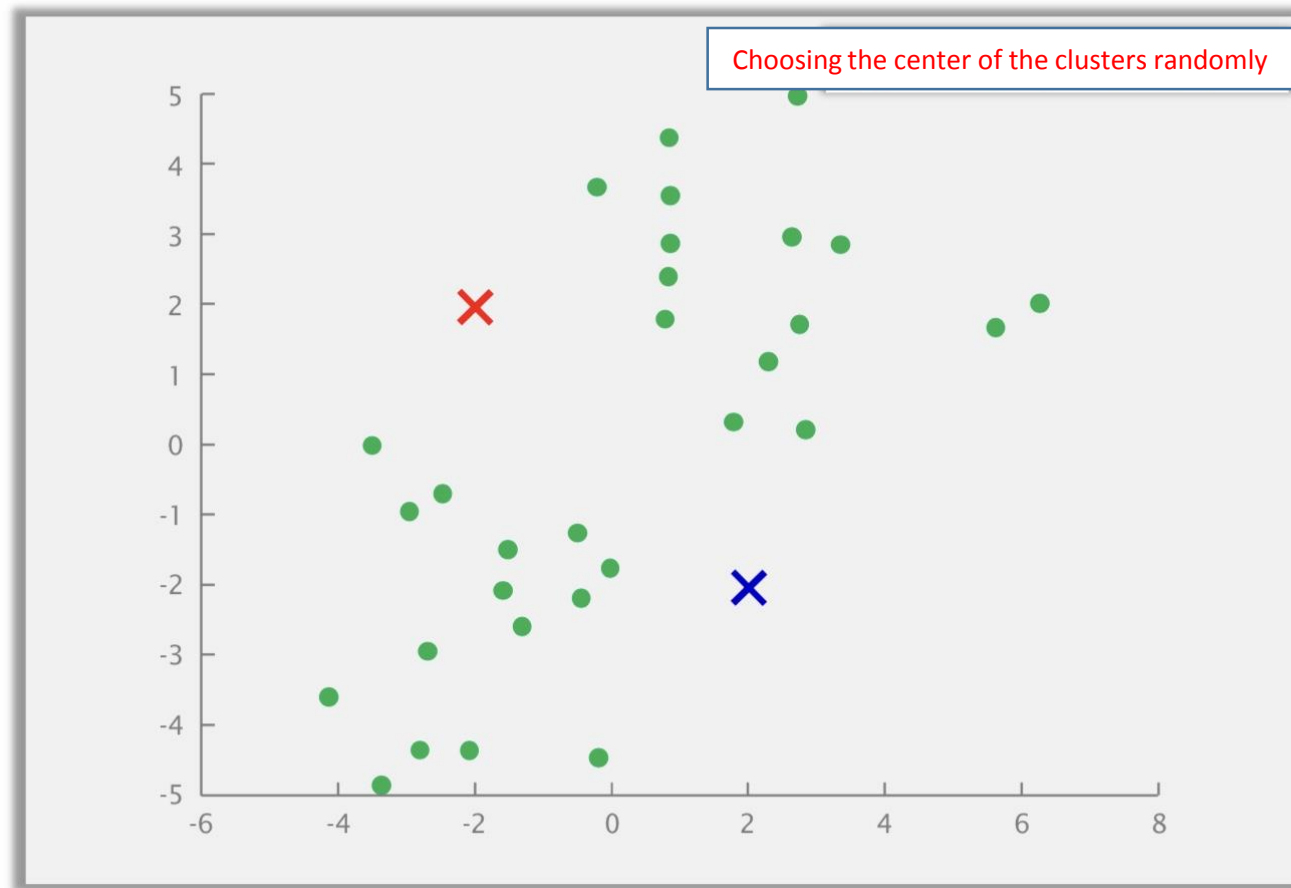
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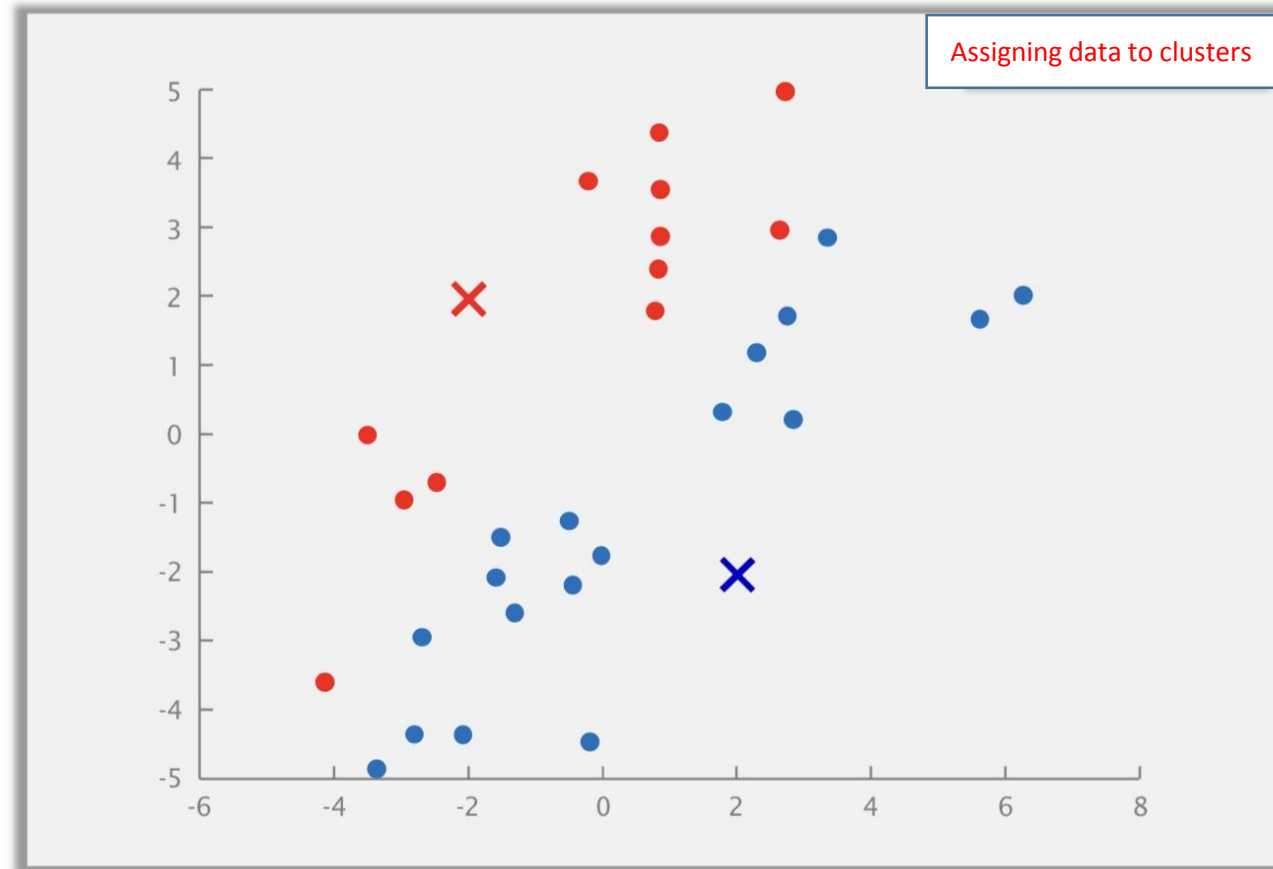
K-means algorithm: demonstration implementation



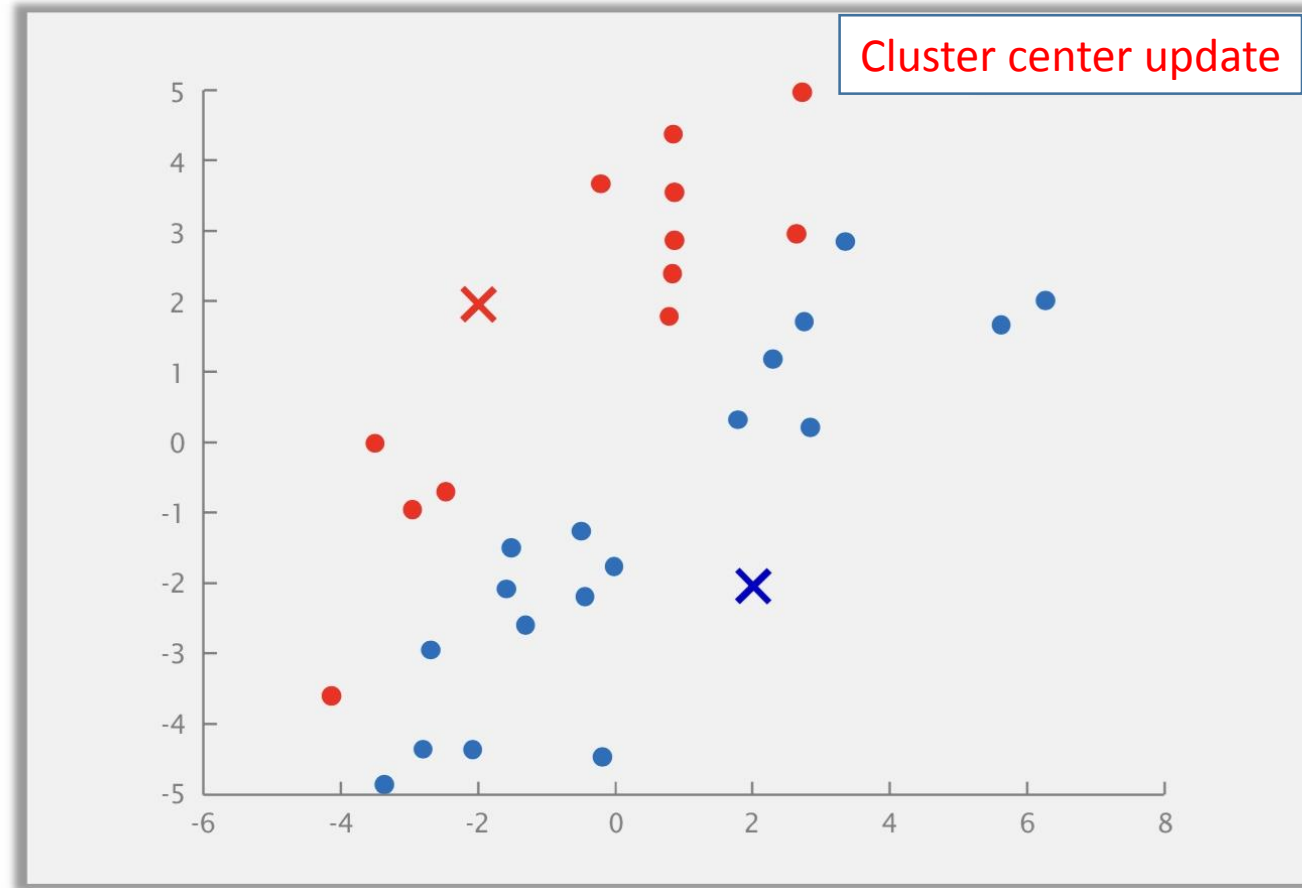
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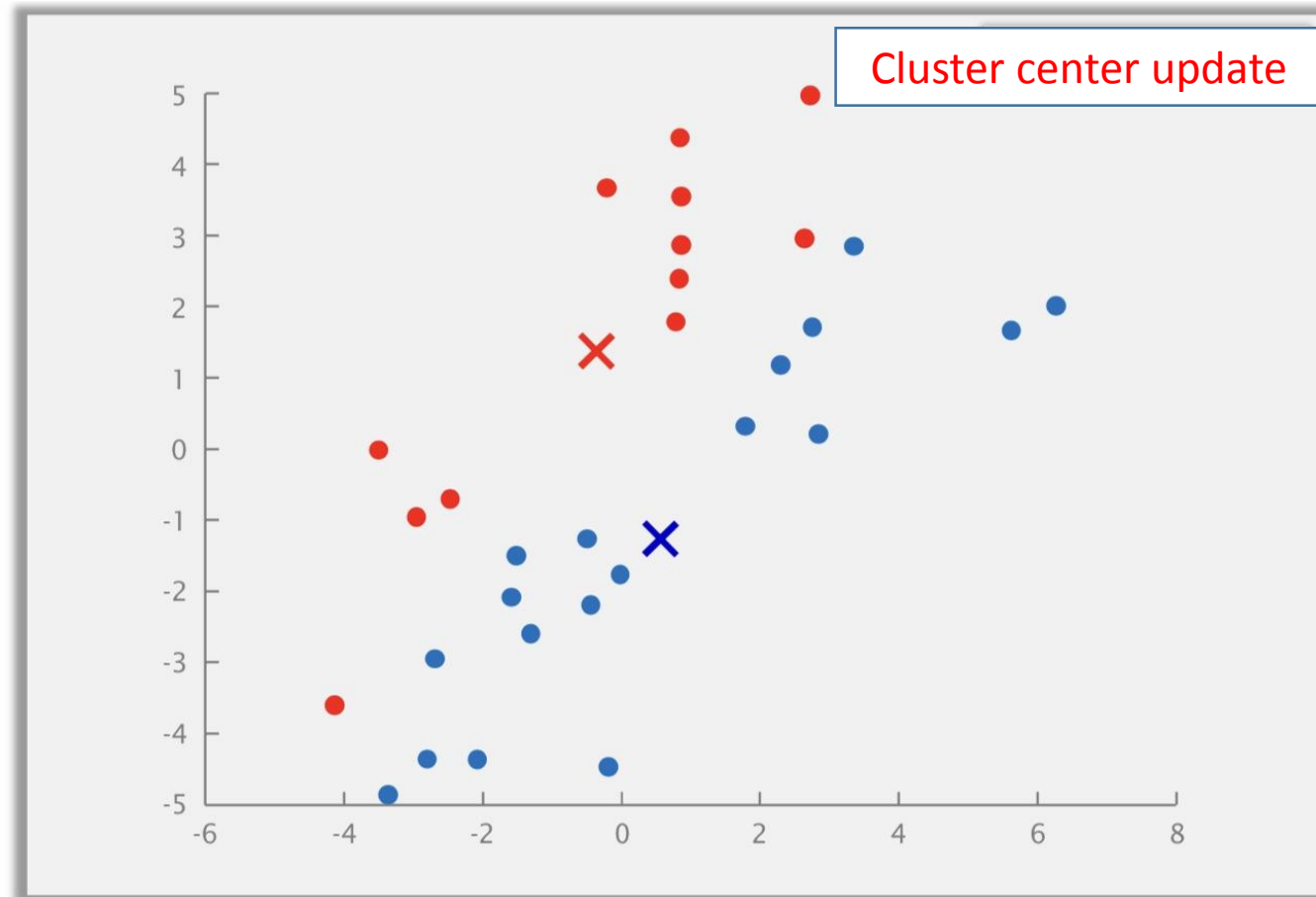
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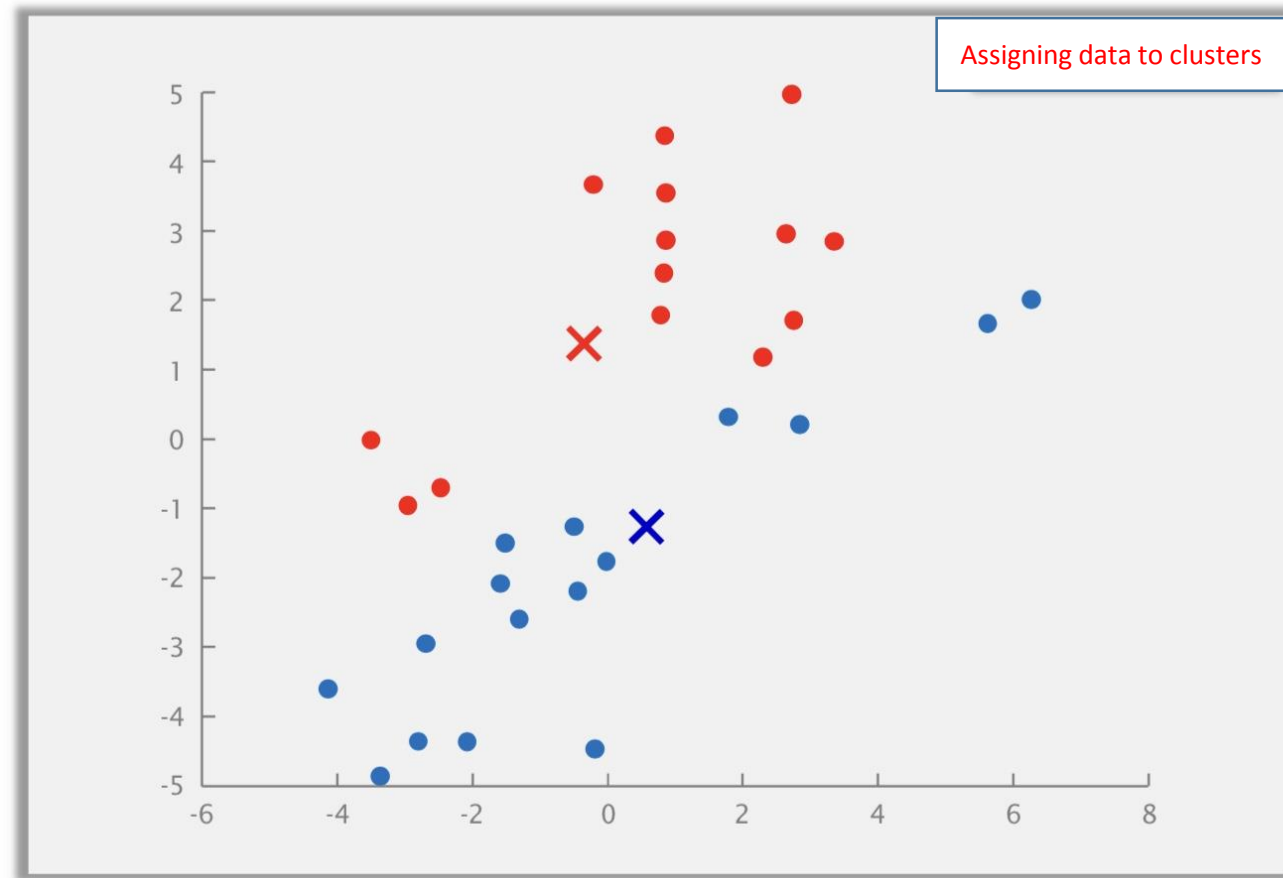
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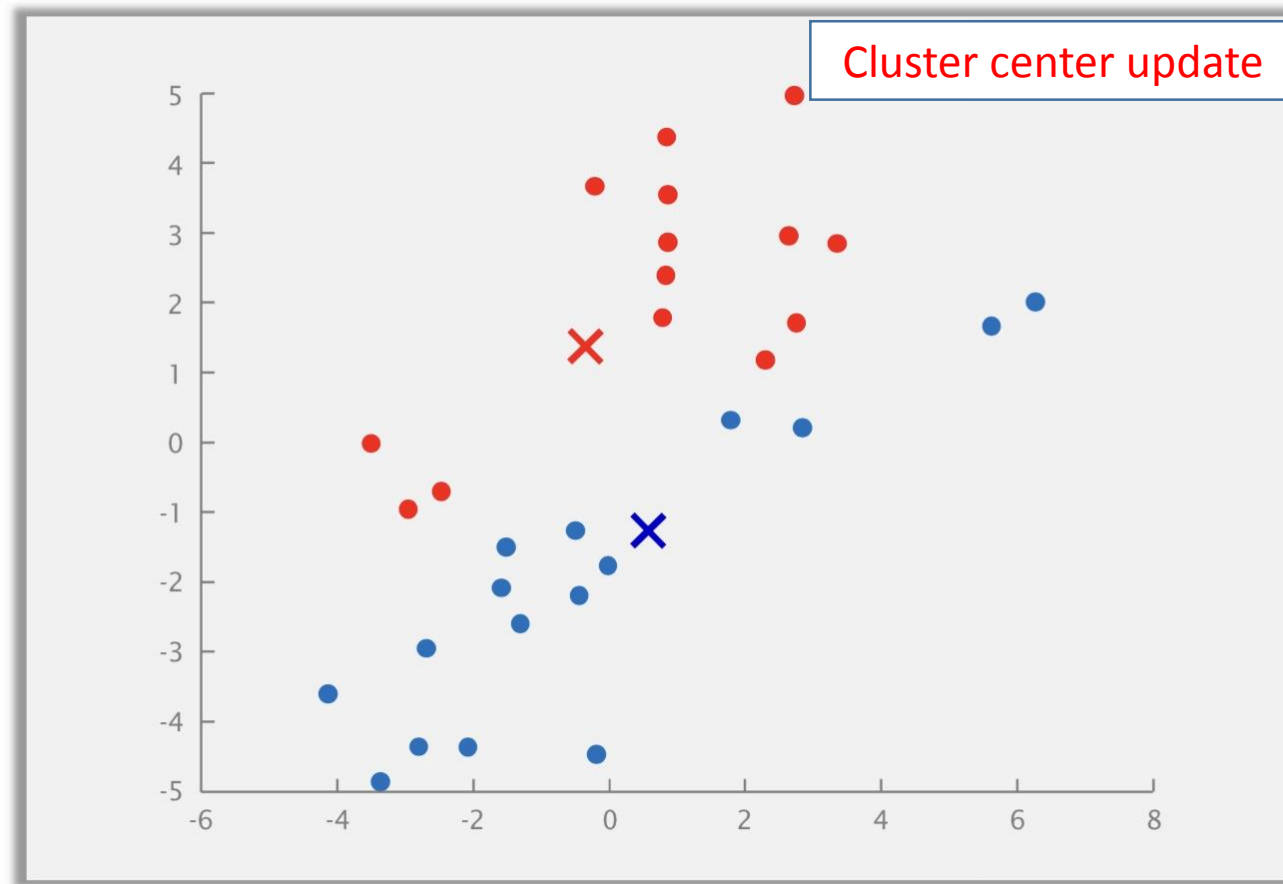
K-means algorithm: demonstration implementation



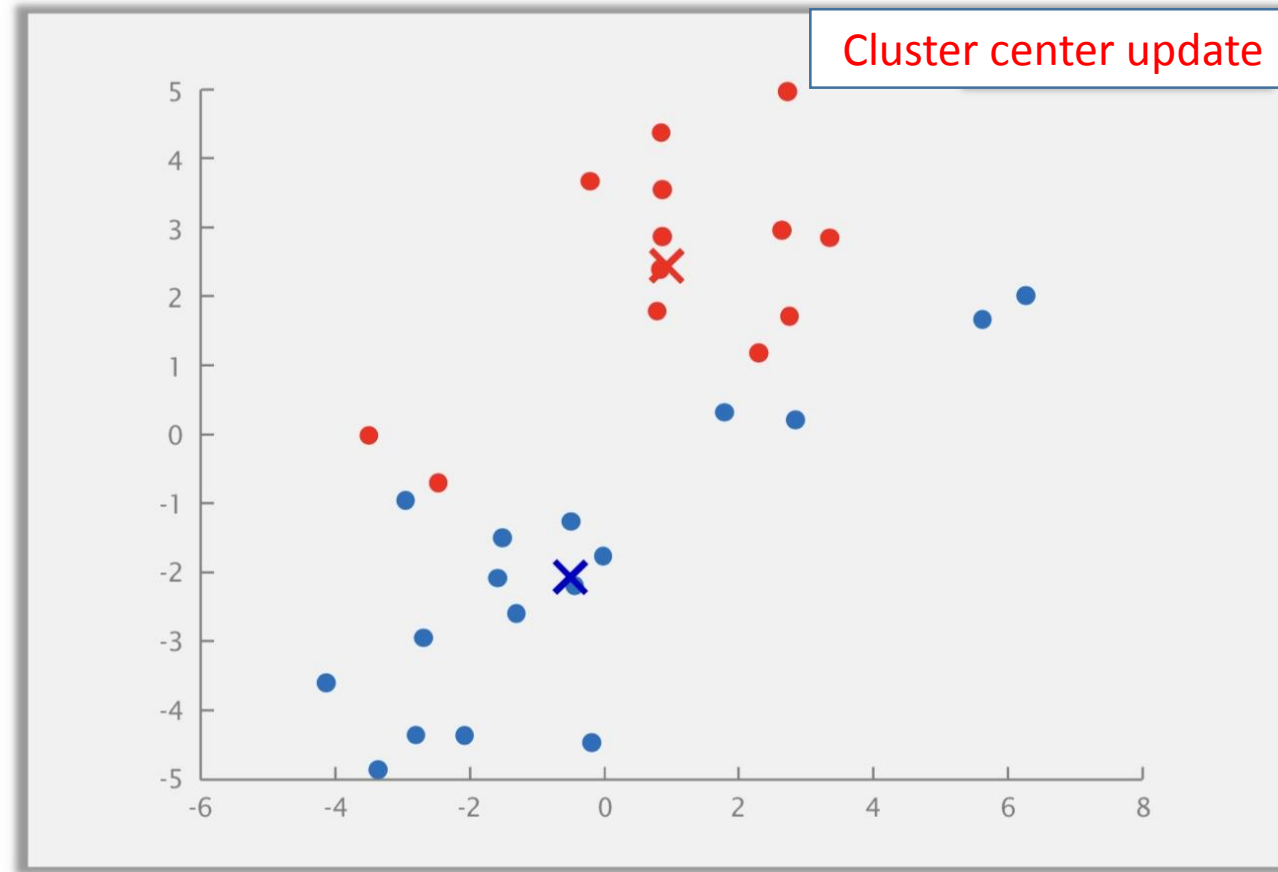
K-means algorithm: demonstration implementation



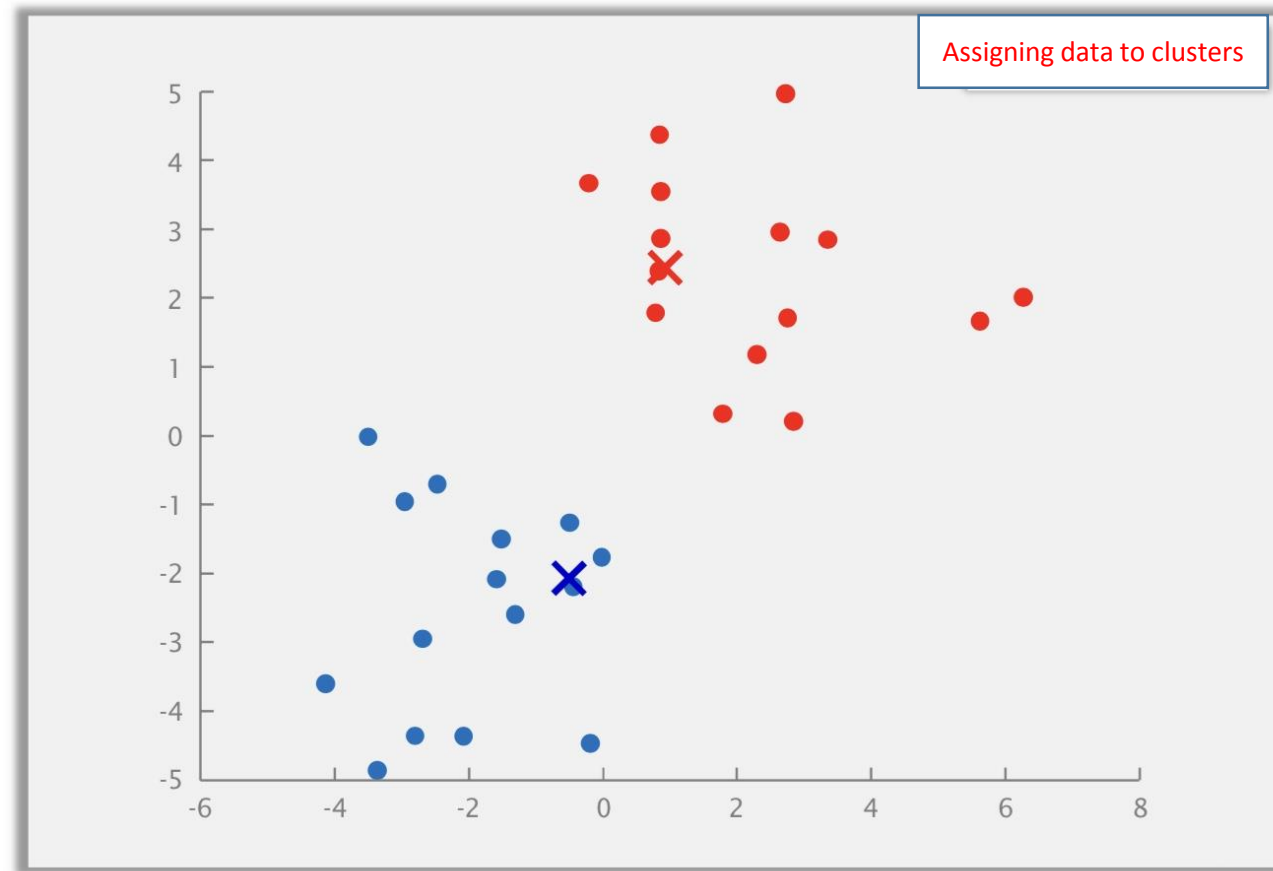
K-means algorithm: demonstration implementation



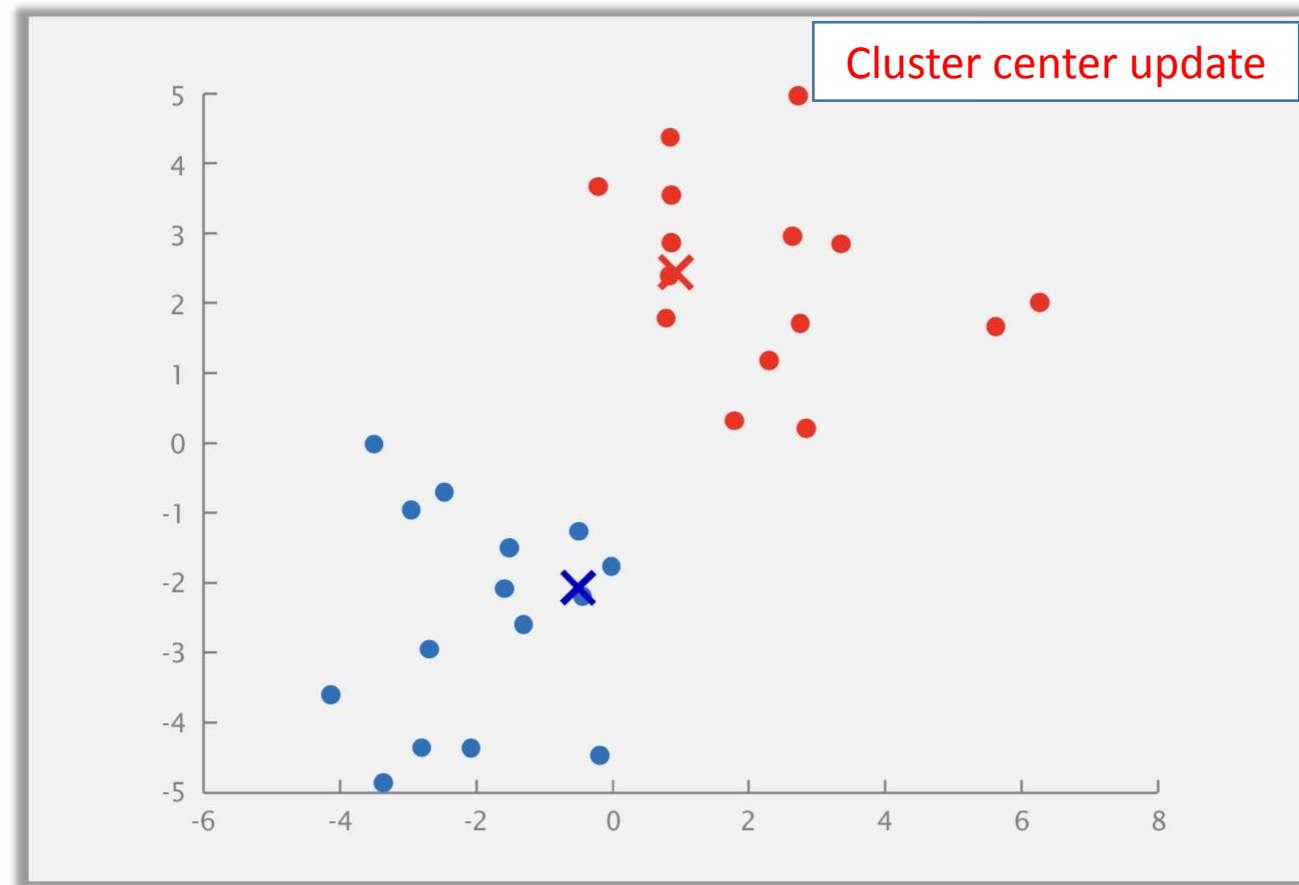
K-means algorithm: demonstration implementation



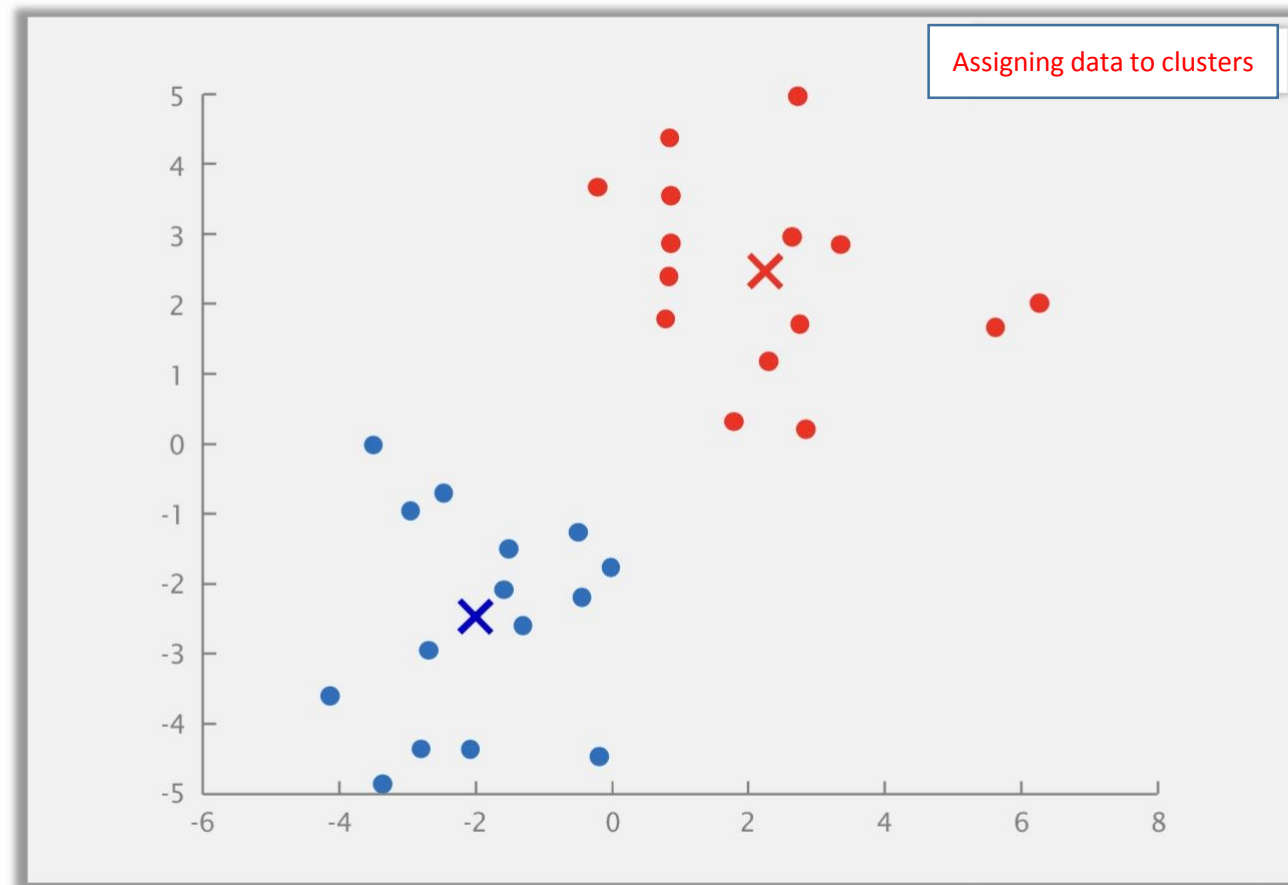
K-means algorithm: demonstration implementation



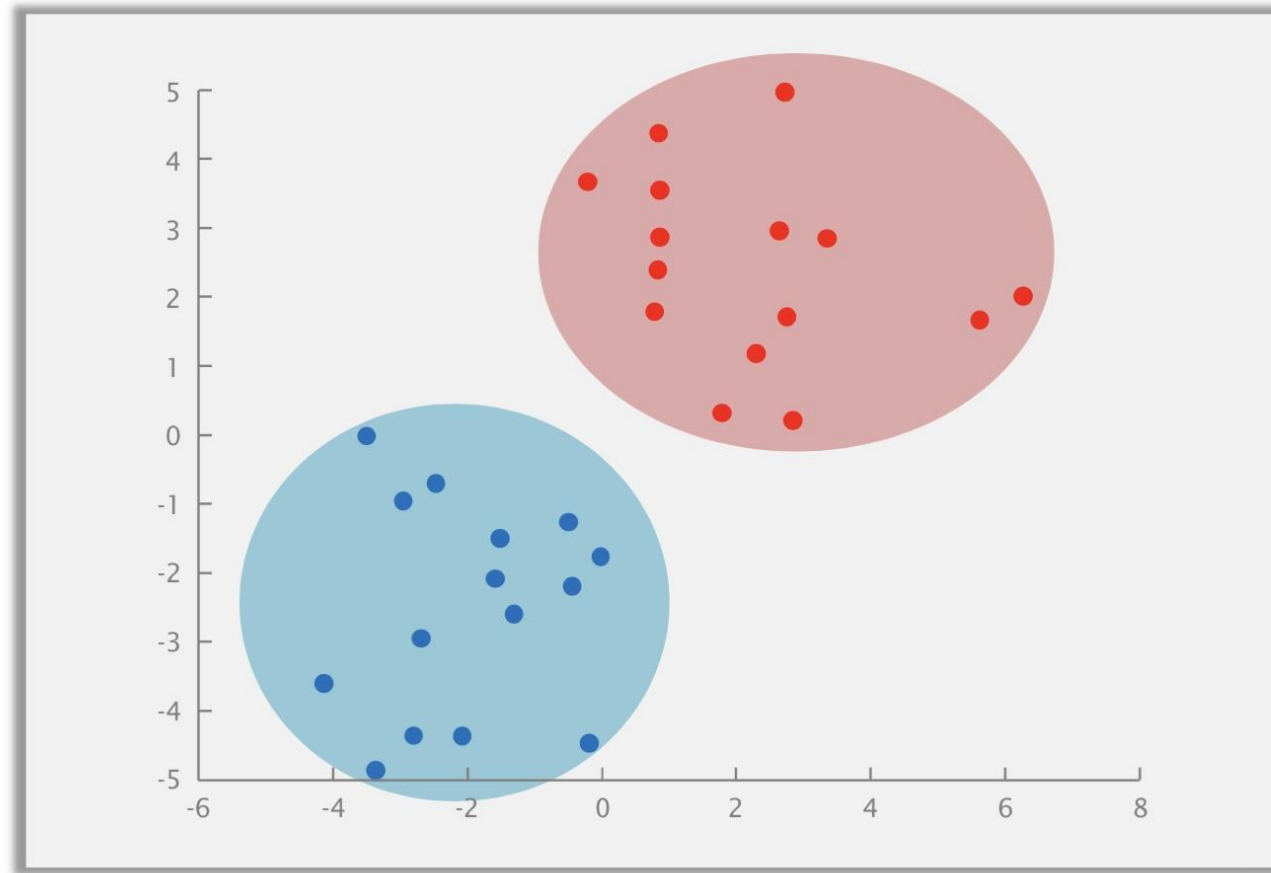
K-means algorithm: demonstration implementation



K-means algorithm: demonstration implementation



K-means algorithm: demonstration implementation



K-means algorithm

- Entrance:
 - Number of clusters: k
 - Training set: $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$
- Note: In the training set, no label is assigned to the data.
- Note: There is no need to add the attribute $x_0 = 1$ in clustering.

K-means algorithm

randomly initialize K cluster centroids $\mu_1, \mu_2, \dots, \mu_K \in \mathbb{R}^n$

repeat

{

for $i = 1$ **to** m

$$c^{(i)} = \arg \min_k \|x^{(i)} - \mu_k\|$$

Assigning data to clusters

for $k = 1$ **to** K

Cluster center update

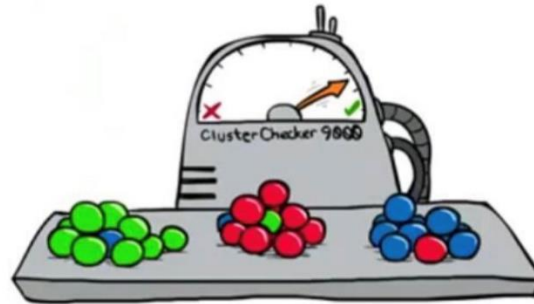
μ_k = average of points assigned to cluster k

}

Clustering: objective function

objective function

- symbols:
 - μ_k : cluster center k
 - $c^{(i)}$: the number of the cluster assigned to the data $x^{(i)}$
 - $\mu_{c^{(i)}}$: the center of the cluster assigned to the data $x^{(i)}$
- The objective function



$$J(c^{(1)}, c^{(2)}, \dots, c^{(m)}, \mu_1, \mu_2, \dots, \mu_k) = \frac{1}{m} \sum_{i=1}^m \|x^{(i)} - \mu_{c^{(i)}}\|^2$$

K-means algorithm

randomly initialize K cluster centroids $\mu_1, \mu_2, \dots, \mu_K \in \mathbb{R}^n$

repeat

{

for $i = 1$ **to** m

$$c^{(i)} = \arg \min_k \|x^{(i)} - \mu_k\|$$

Minimization of the
objective function with
respect to parameters $c^{(i)}$

for $k = 1$ **to** K

μ_k = average of points assigned to cluster k

Minimization of the
objective function with
respect to parameters μ

}

Clusters' centers initializing

K-means algorithm

randomly initialize K cluster centroids $\mu_1, \mu_2, \dots, \mu_K \in \mathbb{R}^n$

repeat

{

for $i = 1$ **to** m

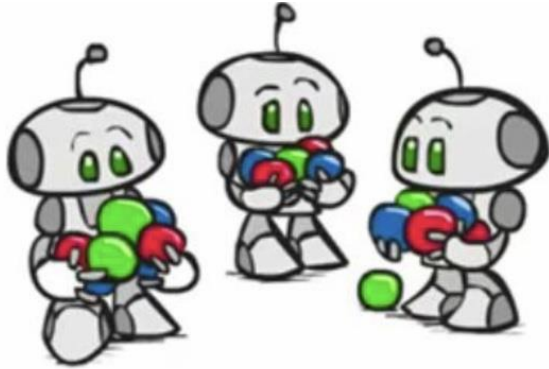
$$c^{(i)} = \arg \min_k \|x^{(i)} - \mu_k\|$$

for $k = 1$ **to** K

μ_k = average of points assigned to cluster k

}

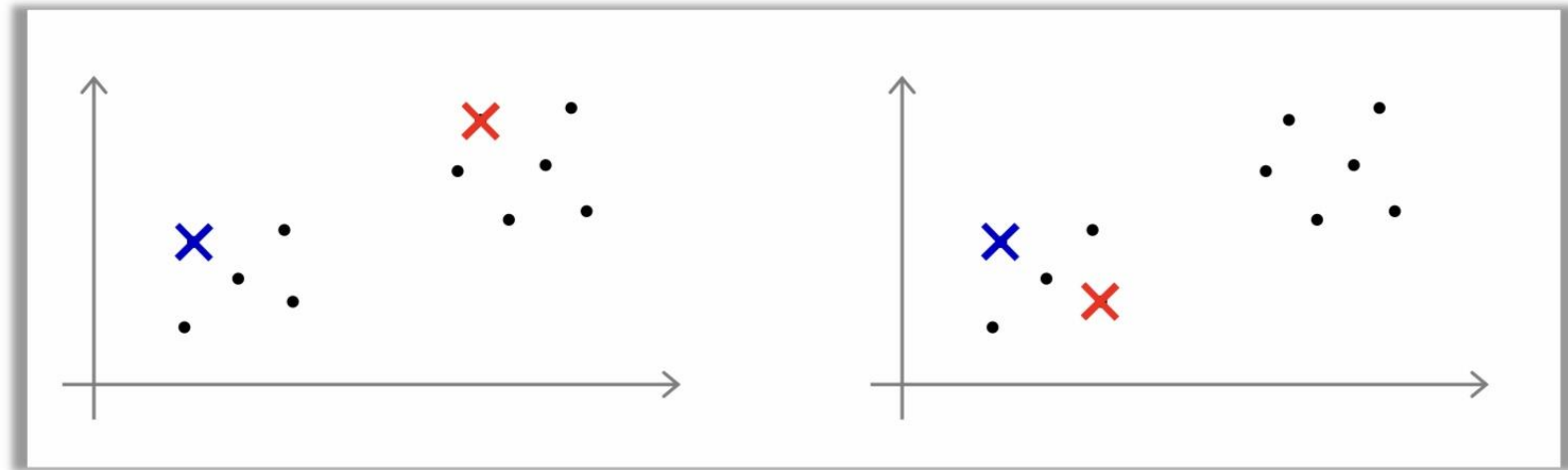
Clusters' centers initializing



Initial initialization ($K \leq m$):

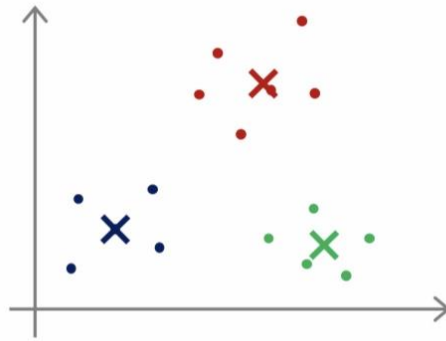
Selection of K training sample randomly

Assigning cluster centers to K selected samples

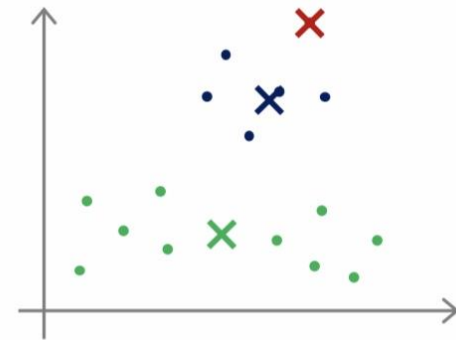
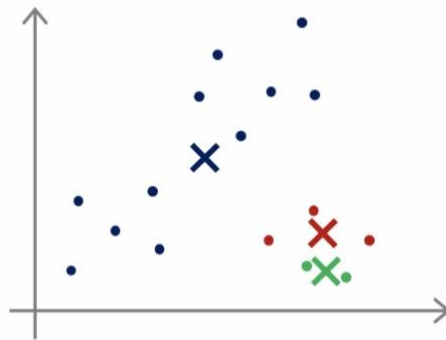


Local Optimum

- Global Optimum



- Local Optimum



Local Optimum Avoidance

```
for  $t = 1$  to  $MAX$ 
```

```
{
```

```
    randomly initialize cluster centroids  $\mu_1, \mu_2, \dots, \mu_k$ 
```

```
    run K-means to get  $c^{(1)}, c^{(2)}, \dots, c^{(m)}, \mu_1, \mu_2, \dots, \mu_k$ 
```

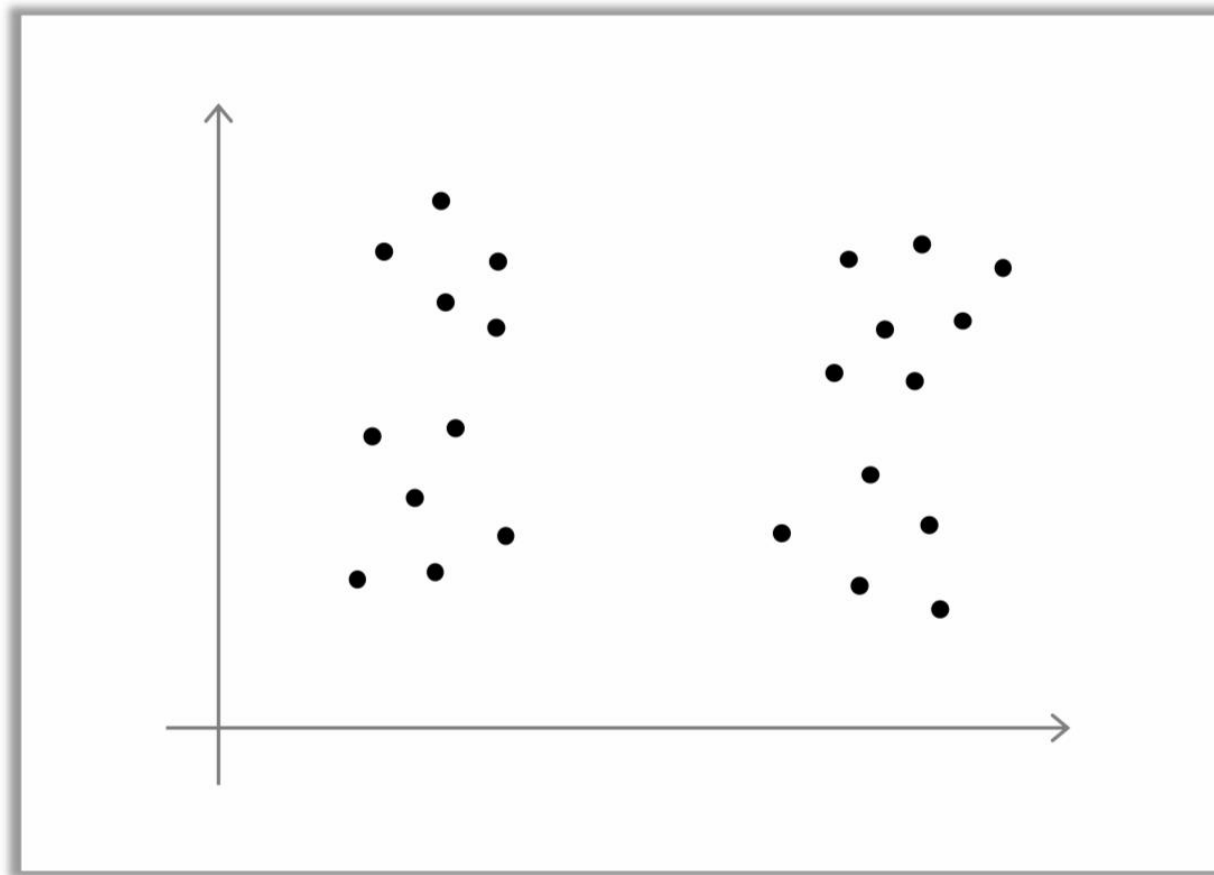
```
    compute cost function  $J(c^{(1)}, c^{(2)}, \dots, c^{(m)}, \mu_1, \mu_2, \dots, \mu_k)$ 
```

```
}
```

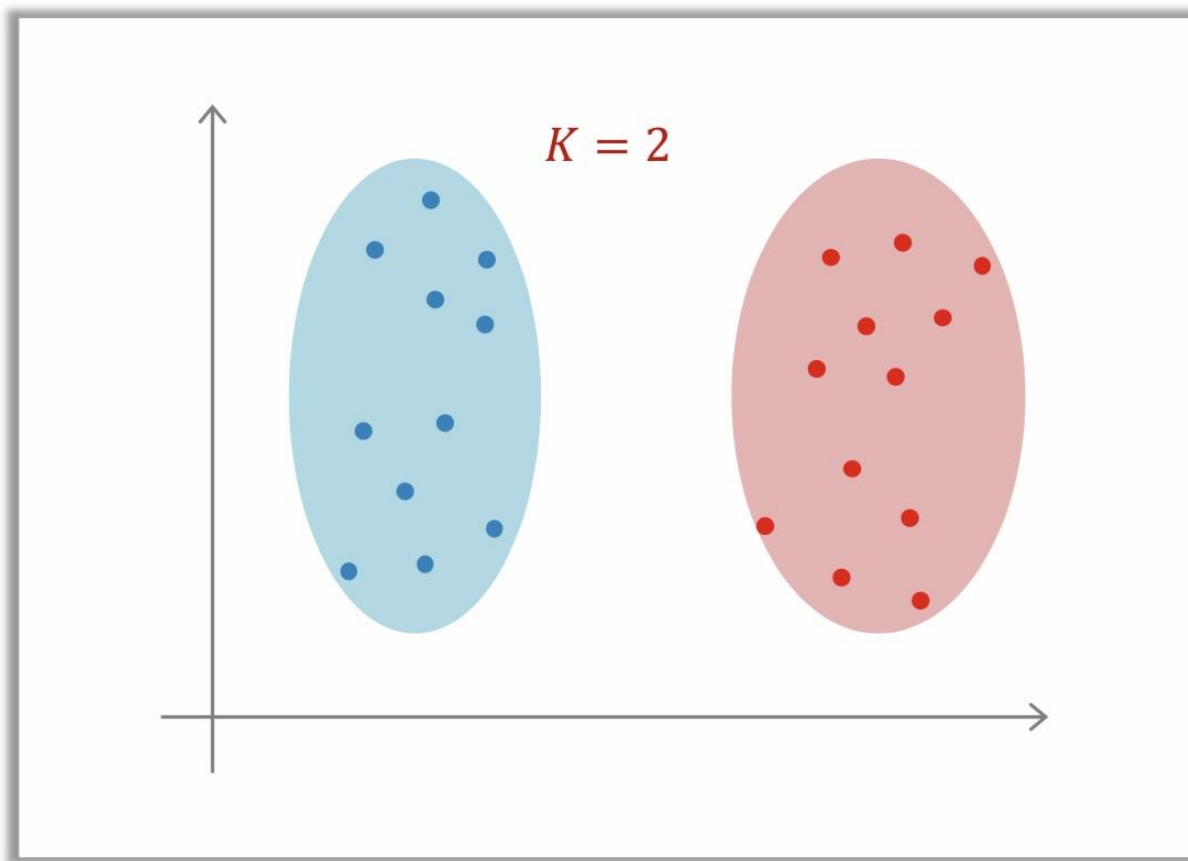
```
pick clustering with minimum cost
```

Determine the number of clusters

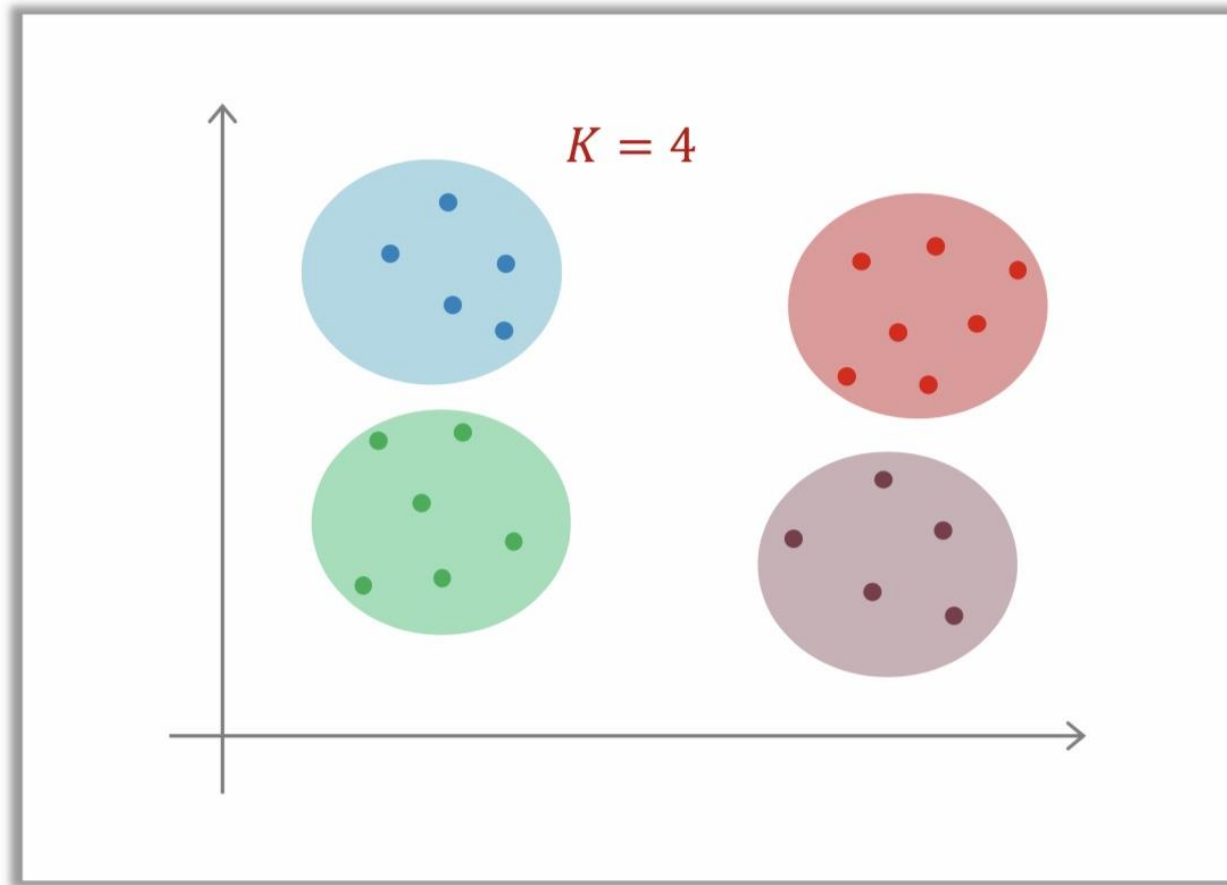
What is the right value for K?



What is the right value for K?

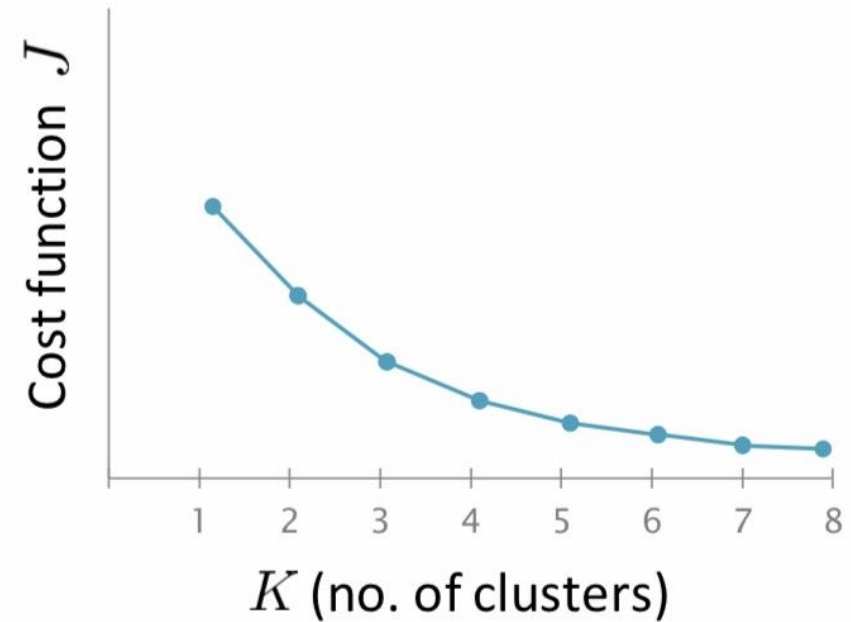
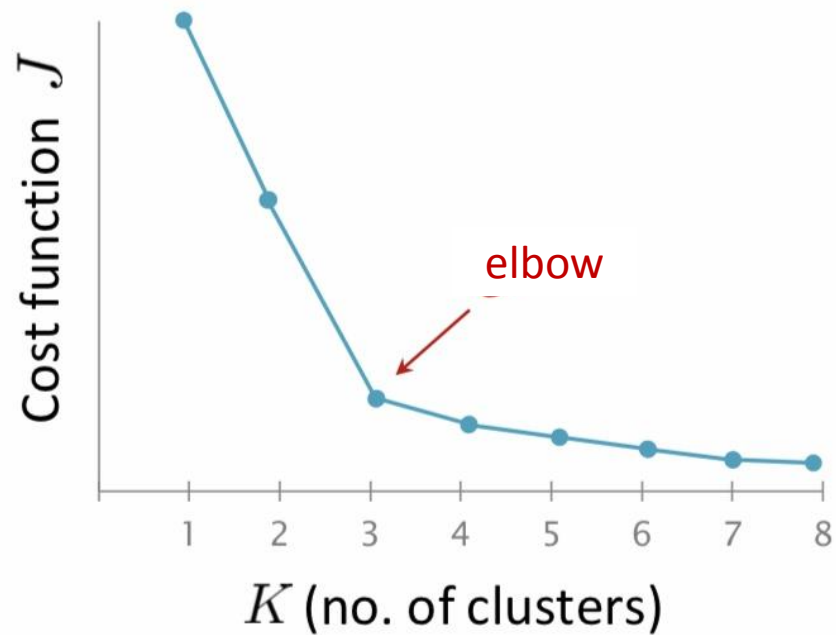


What is the right value for K?



Determine the number of clusters

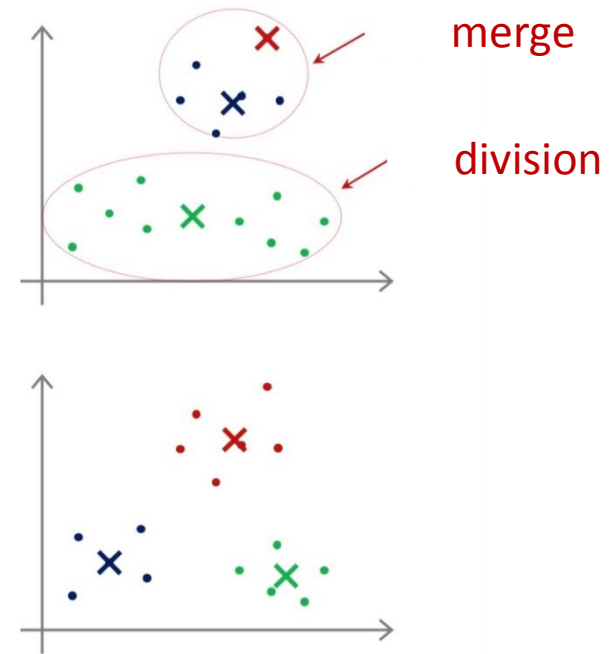
- “elbow” method:



Clustering improvement

Clustering improvement by clusters post-processing

- Division:
 - Splitting a cluster with the highest error into two clusters
 - merge:
 - Merge the two closest clusters
 - Merging two clusters with minimal increase in total error
- By running K-means on the data of this cluster with a value of $K = 2$



Two-part K-means algorithm

- Algorithm of two parts:
 - Start with a cluster containing all the data
 - Choose one cluster at a time:
 - Divide the selected cluster into two clusters using the K-means algorithm.
 - Calculate the total clustering error.
 - Choose the clustering with the least error.
 - Repeat the above process until you reach the desired number of clusters.

Two-part K-means algorithm

Start with all the points in one cluster

while the number of clusters is less than K

 measure the total error

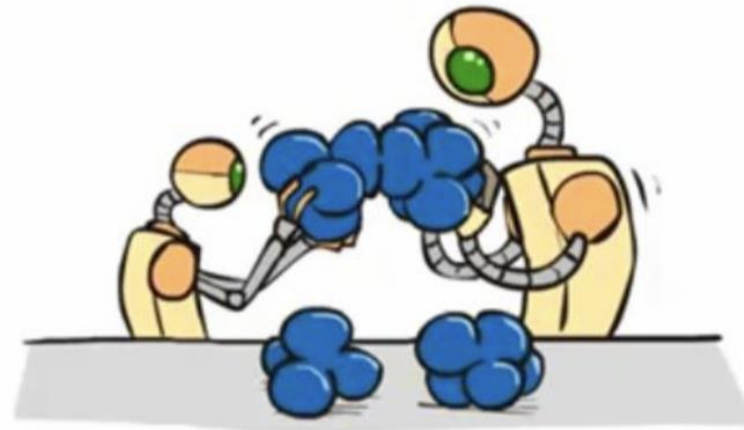
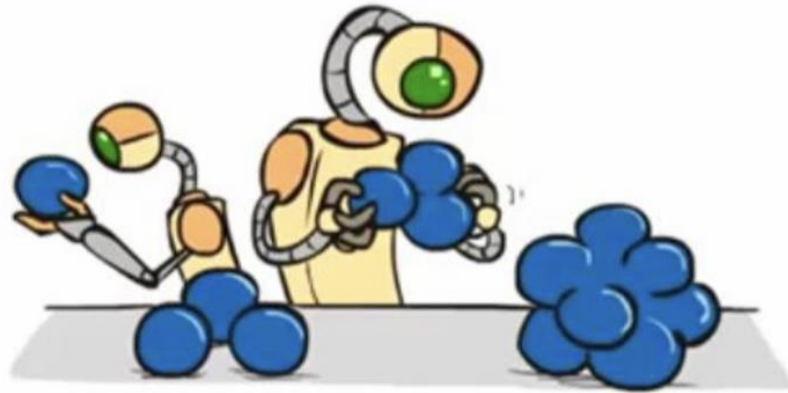
 for every cluster

 perform K-means clustering with $k = 2$ on the given cluster

 measure the total error after splitting

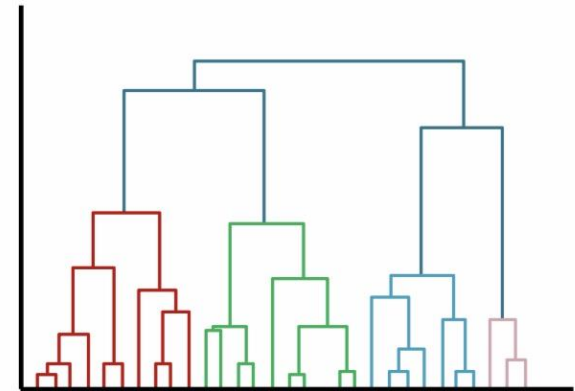
 choose the cluster split that gives the lowest error

Hierarchical clustering



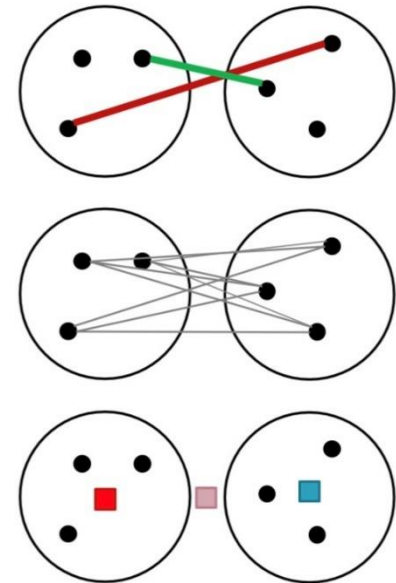
Hierarchical clustering

- Hierarchical clustering:
 - First, merge very similar data.
 - Gradually create larger clusters by merging smaller clusters.
- Algorithm:
 - At first, each data represents a cluster.
 - Repeat the following steps:
 - Choose the two closest clusters each time.
 - Merge those two clusters into a new cluster.
 - Stop: when there is only one cluster left.
- Create a tree diagram containing a wide range of clusters.



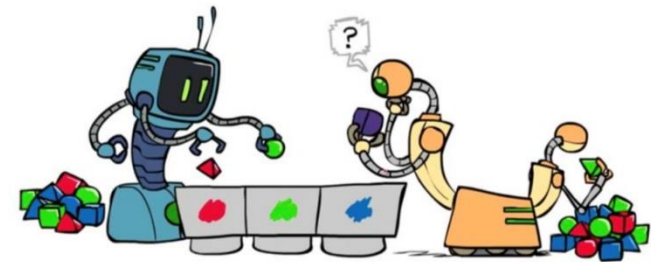
Hierarchical clustering

- Question: How to define the closest two clusters?
- Criteria for determining the similarity of clusters:
 - Nearest pair (one-link clustering)
 - Farthest pair (all-link clustering)
 - Average distance of all pairs
 - "WARD" method (least dispersion like K-means)
- Different criteria create different clusters.



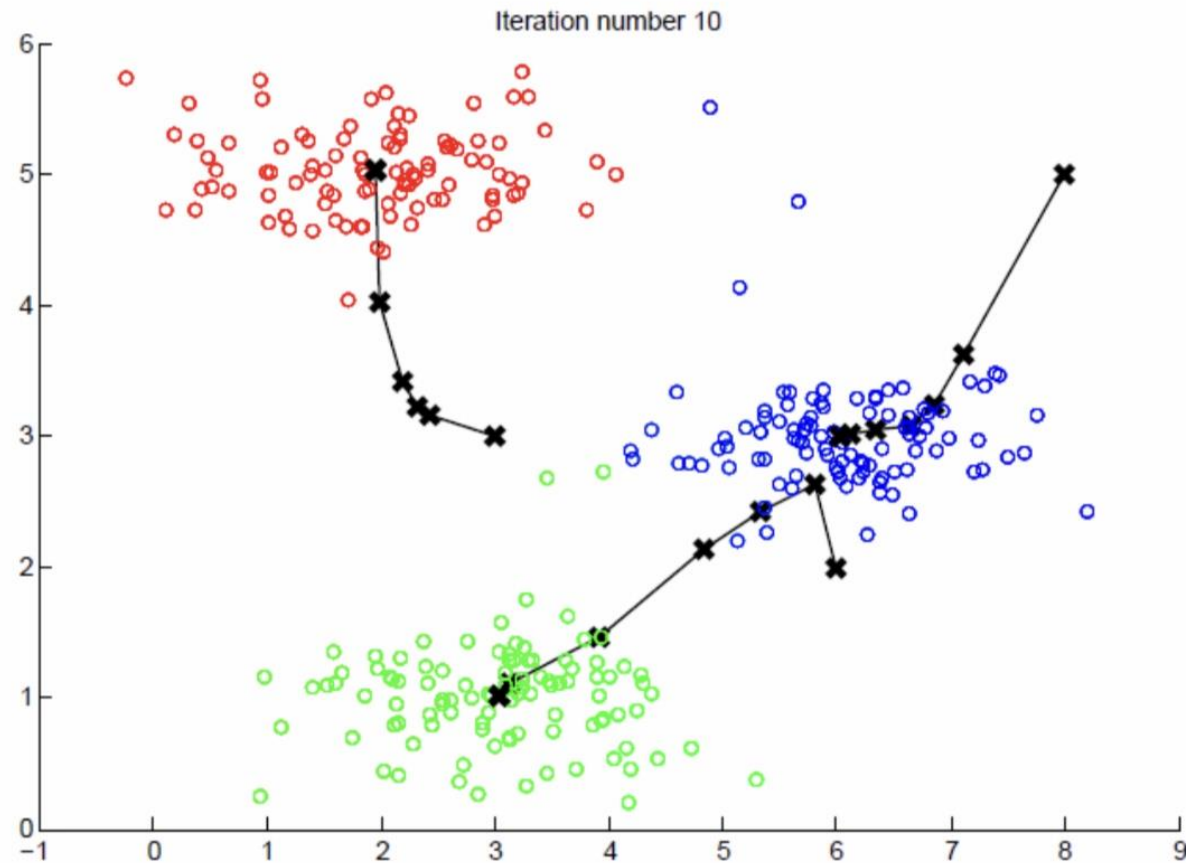
Summary

- Unsupervised learning: finding structure in data
- Clustering: grouping similar data
 - K-means clustering algorithm
 - Easy implementation
 - Slow for very large data sets
 - Possibility of getting stuck in the local optimum
 - Post-processing of clusters: splitting and merging of clusters
 - Two-part K-means algorithm
 - Better clustering than K-means algorithm
 - Hierarchical clustering algorithms



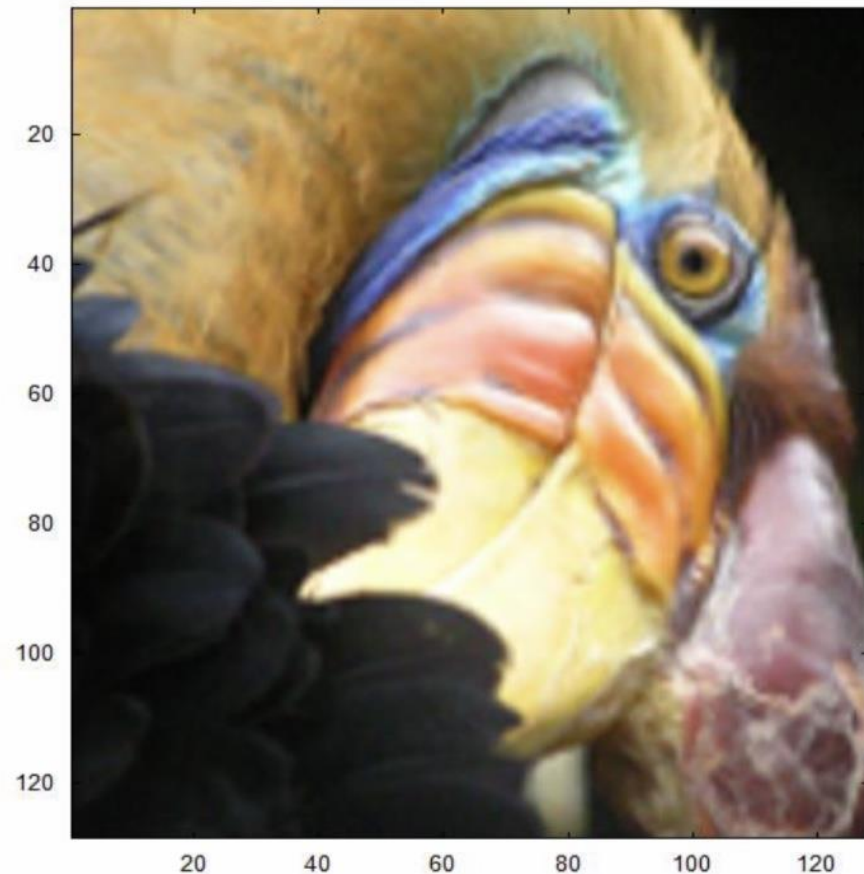
Practices

Practice1: K-means algorithm implementation



Exercise 2: Image compression using K-means

Main image



compressed image (16 colors)

