Team Number :	90350

Problem Chosen :	B

2019APMCM summary sheet

Analysis and Research on Regional Economic Vitality

Regional economic vitality is the reflection of the comprehensive strength and the symbol of the future development potential of a region. It is a necessity to study the influencing factors of economic vitality to propose decisions and future planning with the aim of increasing the economic vitality. The paper discusses the relation model among influencing factors of economic vitalities, reveals short effects and long effects on economic vitality from policy transformation, establishes the comprehensive assessment index of regional economic vitality, and provides some future planning and suggestions for the sustainable development of regional economy.

For Question 1, the paper selects permanent residents, quantity of enterprises, GDP, quantity of employed population, total retail sales and development policies as influencing factors of economics vitality. The paper figures out the positive correlation among factors. The paper establishes the relation model among influencing factors of economic vitality by the relation matrix in Discrete Mathematics.

For Question 2, the paper chooses Wuhan as the research object of the model. The model assumes that economic growth has a fixed growth rate and economic development is affected by environmental resistance (such as resource consumption, policies, international situation). Therefore, the paper abstracts the economic vitality and it's influencing factors as the population in the biosphere and uses the population growth model to describe the change trend of regional economic vitality. The results show that: considering the short-term impact of positive policy transformation on economic vitality, economic vitality will continue to grow, meanwhile, the growth rate will continue to increase. The economic vitality will reach 4.5 times of the current level in 20 years; considering the long-term impact, economic vitality will continue to grow, it will tend to be stable until 300 years later.

For Question 3, the paper uses PCA to analyze the influencing factors of regional economic vitality and constructs a comprehensive evaluation index of regional economic vitality by using the obtained principal components and their contribution rate. The results show that the top three cities in Attachment 3 are Beijing, Shanghai and Chongqing, and the last three are Changsha, Shenyang and Kunming.

For Question 4, the paper provides suggestions for the future planning of Wuhan economic development from four aspects: Promote the implementation of preferential policies for college students, increase support for enterprises, attach importance to the development of urban science and technology, and improve social construction.

Key word: Relation Matrix, Population Growth Model, Principal Component Analysis (PCA)

Contents

1. Introduction

1.1 Restatement of the Problem

Background: The powerful regional economic vitality can drive the overall urban development. It can not only stimulate the establishment and development of local enterprises to provide more employment opportunities for urban residents, but also promote the expand of high-technology industry to attract more talented people. Economic vitality can provide a virtuous circle of the regional growth, with sustainable growth of comprehensive strength. However, the key to help regional development enter a virtuous circle is to find out the important factors affecting the regional economic vitality. Therefore, the following problems should be solved:

1. Build a relational model of influencing factors. Study the effects of changing trend of influence factors and find out the solution to improve the economic vitality.
2. Analyze the effects of economic policies transformation on the economic vitality.
3. Establish a mathematical model to represent economic vitality of regions. Rank the economic vitality of cities in Attachment 3.
4. Provide a development proposal for the region discussed in Problem 2

1.2 Our Work

First, we determine the influencing factors of regional economic vitality, and collect relevant data from the statistical yearbooks of each city. Then, the relation model of influencing factors of economic vitality is established through correlation coefficient, logical reasoning and relation matrix. The population growth model and logistic regression model are used to fit the change trend of economic vitality. PCA is used to analyze the data characteristics of influencing factors of economic vitality, and we construct a comprehensive evaluation index of economic vitality. The cities in Attachment 3 are ranked by the index. Finally, we provide some proposals for the future development planning of the city based on the characteristics of it.

2. Assumption and Notations

2.1 Model Assumption

（1） Assumption of Positive Development Policy:

Assume that all changes in development policies are in a positive direction. Because the theme of this paper is how to improve economic vitality and only positive economic development policies can improve economic vitality

（2） Assumption of Population Growth Model:

It is assumed that the economic vitality and its influencing factors can be abstracted as the biological population in the ecosystem, and their growth trend can be described by the population growth model

2.2 Notations

Symbol	Description
P	Urban permanent population
Q	Quantity of established enterprises
E	Quantity of employed population
G	GDP (Gross Domestic Product)
S	Total Retail Sales
D	Development Policy
$r_{X,Y}$	Correlation coefficient of variables X and Y
M_R	Relation matrix of relation R in Discrete Mathematics
$t(R)$	Transitive closure of relation R in Discrete Mathematics
$N(\mathrm{t})$	Statistical value of influencing factors over time
N_0	Initial statistics of influencing factors
λ	Fixed growth rate in model
K	Maximum environmental capacity
$\|\|X\|\|_\infty$	Infinite norm of vector X
V	Statistics of economic vitality
$e(V)$	Residual statistic of economic vitality
$X_{n\times p}$	Statistical matrix for PCA
$\tilde{X}_{n\times p}$	Normalized statistical matrix for PCA
Σ_X	Covariance matrix of matrix X
λ_i	The $i-\mathrm{th}$ eigenvalue of a matrix
u_i	Eigenvector corresponding to the $i-\mathrm{th}$ eigenvalue of matrix
F_i	The $i-\mathrm{th}$ principal component from PCA
a_i	Weight coefficient of comprehensive evaluation index
w	Comprehensive evaluation index of economic vitality

3. Relational Model of Influencing Factors

3.1 Problem Analysis

There are many variables influencing economic vitality. In order to figure out the basic relational model of influencing factors of economic vitality, the paper analyzes the four basic aspects of population, market, economic and development policy. The paper selects six variables of urban permanent population, quantity of established enterprises, quantity of employed population, GDP, total retail sales and development policy as the influencing factors of economic vitality. The paper chooses Wuhan as the research object of the model. In general, the changing trend of each influencing factor will induce the positive or negative development of economic vitality. Besides the direct or indirect transmission relationship, factors may influence each other by feedback. The model will examine the interaction among the selected influencing factors, and then establish the relationship model of the influencing factors of economic vitality.

The modeling process of Question 1 is as follows:

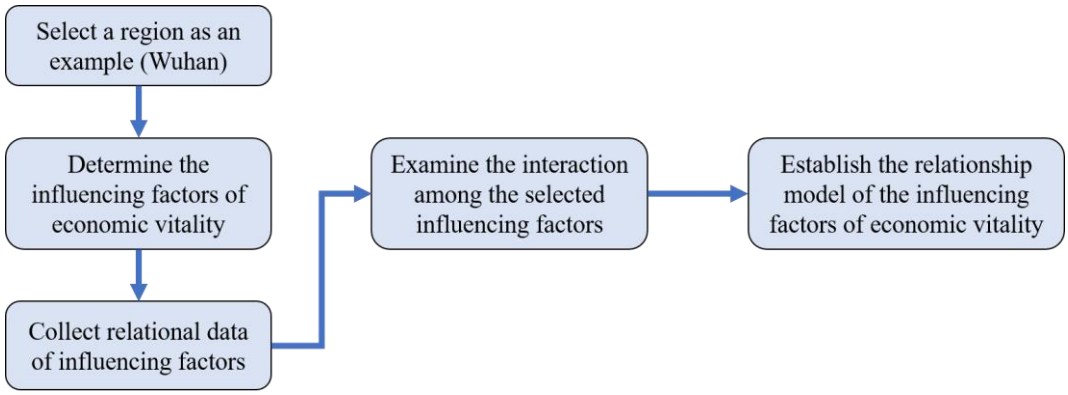

Figure 1: The modeling process of Question 1

3.2 Data Collection

The paper chooses Wuhan as an example to figure out the relational model of regional economic vitality, meanwhile the paper considers permanent residents, quantity of established enterprises, quantity of employed population, GDP, total retail sales and development policy as factors affecting economic vitality. The paper collected the relative data from Wuhan Bureau of Statistics from 2009 to 2018.

As a non-numerical influencing factor, development policy will be analyzed separately after numerical data. Although there are various development policies, they all play a positive or negative role in the changing trend of the influencing factors of economic vitality.

Part of the statistical data of the collected influencing factors of economic vitality over the years are as follows, and the completed statistical data can be found in the Appendix.

Table 1: Part of the statistical data of the collected influencing factors

Influencing Factors	2009	2010	…	2017	2018
Permanent Resident Population (unit: 10,000)	897.00	910.00	…	1076.62	1089.29
Quantity of Established Enterprises (unit: 10,000)	13.32	15.99	…	38.12	46.17
Quantity of Employed Population (unit: 10,000)	456.00	468.60	…	550.37	564.08
GDP (unit: 100 million)	4115.51	4620.86	…	11912.61	13410.34
Total Retail Sales (unit: 100 million)	1850.05	2164.09	…	5610.59	6196.30

3.3 Model Establishment

In this part, the paper will study five numerical factors, including the permanent resident population, the quantity of established enterprises, the quantity of employed population, GDP and the total retail sales in the whole year. The paper will examine the relationship between them and observe whether their influence is positive or negative. After that, the paper will figure out the contact between policies and other influencing factors based on comprehending the development policy in recent years. Finally, according to the previous research, the paper will give the relationship model of the influencing factors of economic vitality.

The paper focuses on the feature of relationships among the influencing factors of economic vitality. The relationship can be divided into positive, negative or irrelevant. The linear correlation coefficient is used to describe correlation of influencing factors in this paper. For the vectors corresponding to two influencing factors:

$$X=(x_1,x_2,\cdots,x_n),\ \ Y=(y_1,y_2,\cdots,y_n) \tag{3.3.1}$$

The correlation coefficient $r_{X,Y}$ between them is defined as follows:

$$r_{X,Y}=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\cdot\sum_{i=1}^{n}(y_i-\bar{y})^2}} \tag{3.3.2}$$

Therefore, in the following discussion of this section, the paper will focus on the qualitative analysis of the relationship between the influencing factors affecting economic vitality, not the quantitative analysis.

3.3.1 Relationship between Permanent Residents & Enterprises

Taking the permanet resident population as the independent variable and the quantity of established enterprises as the dependent variable, the discrete function images are drawn as follows：

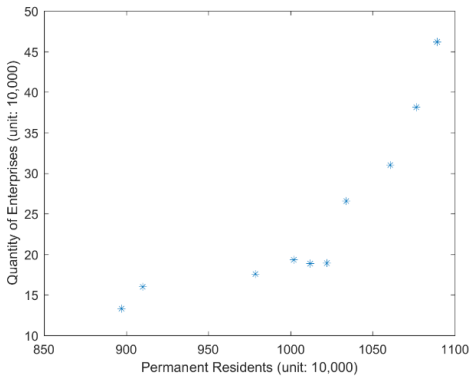

Figure 2: The discrete function images of permanent resident population & quantity of established enterprises

Using the correlation coefficient formula, the correlation coefficient $r_{P,Q}$ between the Permanent residents P and the quantity of established enterprises Q is calculated:

$$r_{P,Q} = 0.846463 > 0$$

It can be seen from the above that there is a positive correlation between them.

3.3.2 Relationship between Enterprises & Employment Opportunities

Taking the quantity of established enterprises as the independent variable and the quantity of employed population as the dependent variable, the discrete function images are drawn as follows：

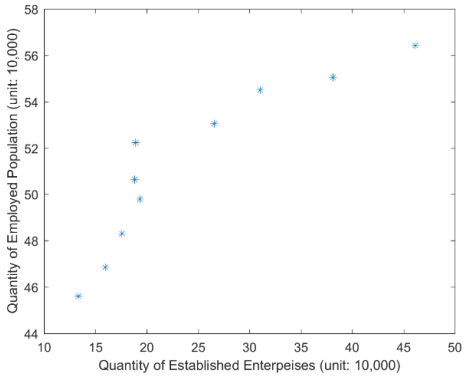

Figure 3: The discrete function images of quantity of established enterprises & quantity of employed population

The correlation coefficient $r_{Q,E}$ between the quantity of established enterprises Q and the quantity of employed population E is calculated:

$$r_{Q,E} = 0.899302 > 0$$

It can be seen from the above that there is a positive correlation between them.

3.3.3 Relationship between GDP & Permanent Residents

Taking the permanet resident population as the independent variable and the GDP as the dependent variable, the discrete function images are drawn as follows:

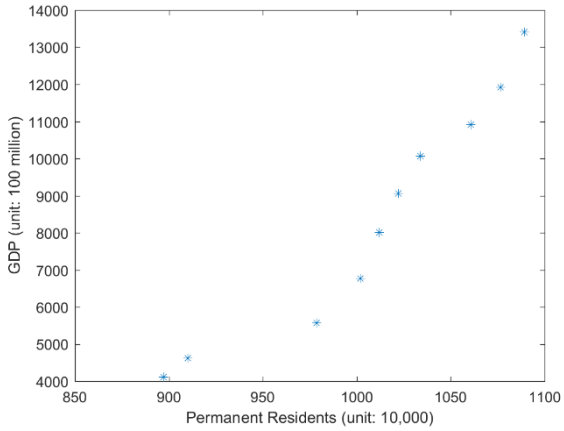

Figure 4: The discrete function images of permanet resident population & GDP

The correlation coefficient $r_{P,G}$ between the permanet resident population P and the GDP G is calculated:

$$r_{P,G} = 0.957461 > 0$$

It can be seen from the above that there is a positive correlation between them.

3.3.4 Relationship between GDP & Enterprises

Taking the quantity of established enterprises as the independent variable and the GDP as the dependent variable, the discrete function images are drawn as follows:

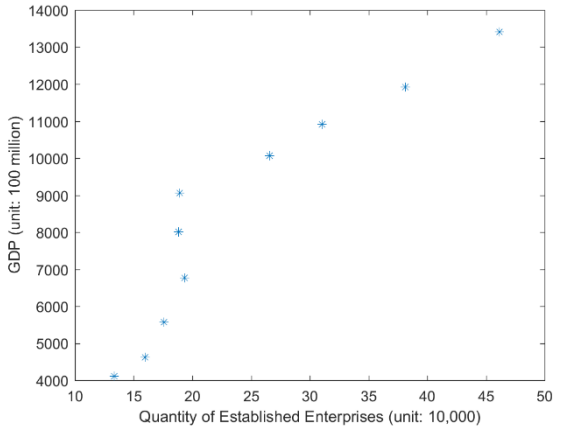

Figure 5: The discrete function images of quantity of established enterprises & GDP

The correlation coefficient $r_{Q,G}$ between the quantity of established enterprises Q and the GDP G is calculated:

$$r_{Q,G} = 0.923706 > 0$$

It can be seen from the above that there is a positive correlation between them.

3.3.5 Relationship between GDP & Total Retail Sales

Taking the total retail sales as the independent variable and the GDP as the dependent variable, the discrete function images are drawn as follows:

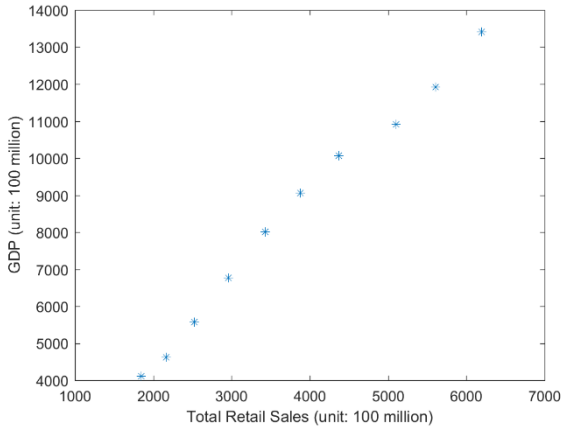

Figure 6: The discrete function images of total retail sales & GDP

The correlation coefficient $r_{S,G}$ between the total retail sales S and the GDP G is calculated:

$$r_{S,G} = 0.994887 > 0$$

It can be seen from the above that there is a positive correlation between them.

3.3.6 Other Relationships in Influencing Factors

The paper selects 5 numerical influencing factors of economic vitality. In order to investigate their contact, there are a total of $C_5^2 = 10$ combination kinds of relationships among them. In general, the relationships between variables can be divided into direct or indirect relation. Several relationships among influencing factors have been discussed from Section 3.3.1 to 3.3.5, now the paper will consider the other relationships among these influencing factors.

Make the following logical reasoning: the increase in the permanent residents means the growth of the local labor force, which will be more attractive for the establishment of enterprises. The increase in the quantity of enterprises results in the increase in the employed population for the employment needs of enterprises. Therefore, there is an indirect relationship between the permanent residents and employment opportunities. Besides, more quantity of employment help to the increase of consumption in that the Burden Coefficient of Family Employees decreases, and per capita income of families increases. This can be shown in the boost of total retail sales. Therefore, the permanent residents, the quantity of established enterprises and employment opportunities are indirectly positively related to total retail sales. At the same time, the growth of GDP is an inevitable result of the increase of consumption level, so the influencing factors above are ultimately positively related to GDP.

To sum up, the above discussion about the relationships among influencing factors of economic vitality is enough.

3.3.7 Policy Influence

Because development policy is not a numerical variable, the correlation between development policies and other influencing factors is complex and cannot be studied quantitatively. In this part, the paper analyzes and studies some recent development policies in Wuhan as a typical example, then the paper discusses the relationship between development policies and other influencing factors in qualitative way.

Through the collection and analysis of Wuhan's development policies in recent years, the paper finds that: In order to better implement the project named "One Million College Students to Stay in Wuhan for Entrepreneurship and Employment", Wuhan has successively issued several new policies, i.e. housing and income improvement for college students, taking the lead in providing the greatest discount for college students to buy houses in Wuhan, at the same time improving the income level of college students, increasing employment positions and encouraging college students to start their own businesses. The positive development policies increase the permanent population and employment opportunities, further improve the consumption level.

These are just some policies launched by Wuhan in terms of attracting talents. There are many other development policies in Wuhan which have a positive role in promoting economic vitality. In general, these positive policies can increase the permanent residents in Wuhan, increase the quantity of established enterprises, increase the quantity of employment population, encourage consumption, promote the steady growth of economy and stimulate economic vitality.

On the contrary, considering China's development history, the government will implement relevant policies such as reducing private enterprises and controlling population growth in order to solve some urgent problems in some special periods. In the long run, these development policies will lead to the reduction of the permanent residents and the quantity of enterprises, but when there is an aging population and a shortage of labor force, the development of social economy will slow down or even reverse. the government must adjust the development strategy according to the background of the times to ensure the long-term prosperity of economic vitality.

To sum up, development policies can be divided into two categories: positive and negative. In order to develop economy and promote economic vitality, positive development policies are necessary. From long experience and logic deduction, the positive development policy and the changing trend of influencing factors of economic vitality shows a positive correlation.

The following figure is a summary of the relationship among development policies, influencing factors and economic vitality:

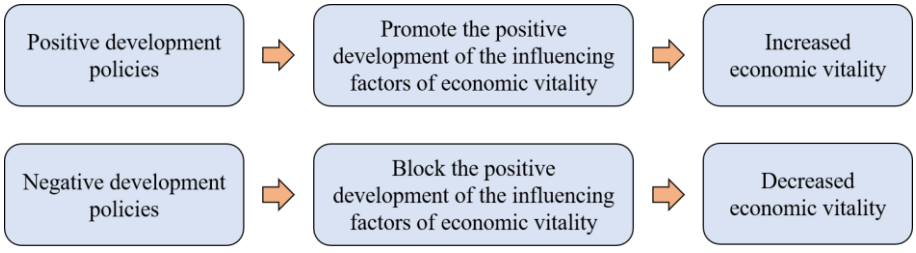

Figure 7: The relationship among policies, influencing factors and economic vitality

3.4 Solution of the Model

In this part, according to results in Section 3.3, the paper will give the relational model of the influencing factors. According to the relational model, the paper will analyze the effect of the changing trend of the influencing factors on the economic vitality and give the solution to improve the economic vitality.

3.4.1 The Relational Model of Influencing Factors

In this paper, the relationship matrix in Discrete Mathematics [1] is used to build the relational model of the influencing factors of economic vitality. In order to clearly describe the information contained in the model, some symbols and operations are defined as follows:

Table 2: Symbolic definition of influencing factors

Symbol	Description
P	Permanent resident population
Q	Quantity of established enterprises
E	Quantity of employed population
S	Total retail sales
G	GDP (Gross Domestic Product)
D	Development policy

Based on the research in Section 3.3, it can be concluded that all the influencing factors of economic vitality are positively correlated. According to Assumption 1, under the consideration of the actual situation, the model only considers that development policies are all positive. Therefore, development policies and variables also show a positive correlation. Through the above analysis, the operation is defined as follows:
Mark a set X as follows:

$$X = \{P, Q, E, S, G, D\} \tag{3.4.1}$$

Define a relation R on set X as follows:

$$\text{If } x \text{ is directly positively related to } y, \text{ then } (x, y) \in R \tag{3.4.2}$$

Based on the work above, the relation matrix M_R of the relation R is as follows:

$$M_R = \begin{array}{c|cccccc} & P & Q & E & S & G & D \\ \hline P & 0 & 0 & 0 & 0 & 0 & 1 \\ Q & 1 & 0 & 0 & 0 & 0 & 1 \\ E & 0 & 1 & 0 & 0 & 0 & 1 \\ S & 0 & 0 & 1 & 0 & 0 & 1 \\ G & 0 & 0 & 0 & 1 & 0 & 0 \\ D & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \tag{3.4.3}$$

For the above relation matrix M_R, it is necessary to explain the value of its elements. There are two examples for the explanation:

(1) Observe the rows of the relation matrix. There is a direct positive correlation between the quantity of established enterprises (Q) and the permanent population (P) based on the results of Section 3.3, so the ordered pairs (Q,P) comes to $(Q,P) \in R$. Corresponding to the position of the relation matrix, the value of the corresponding element of relation (Q,P) is 1. Therefore, the element of the second row and the first column of the matrix is 1.
(2) Based on the results of Section 3.3, considering the GDP (G) and the permanent population (P), there is no direct positive correlation between them, so the ordered pairs (G,P) comes to $(G,P) \notin R$. Corresponding to the position of the relation matrix, the value of the corresponding element of relation (G,P) is 0. Therefore, the element of the fifth row and the first column of the matrix is 0.

Obviously，if x is directly positively related to y, y is directly positively related to z, then x is positively related to z. In order to know the complete positive correlation relation among them, it's necessary to calculate the transitive closure of the relation R. According to the knowledge of Discrete Mathematics, the transitive closure of a relation R can be calculated as follows:

$$t(R) = R \bigcup R^2 \bigcup \cdots \bigcup R^n \qquad (3.4.4)$$

in which n is the cardinal number of the set X. $R^k(k = 1,2,\cdots,n)$ represents the relation matrix M_R to do logical multiplication. The operation rules of logical multiplication $(\otimes)$ are as follows:

$$0 \otimes 0 = 0,\ 0 \otimes 1 = 0,\ 1 \otimes 0 = 0,\ 1 \otimes 1 = 1 \qquad (3.4.5)$$

So, the matrix of the transitive closure of relation R is as follows:

$$M_{t(R)} = \begin{array}{c|cccccc} & P & Q & E & S & G & D \\ \hline P & 0 & 0 & 0 & 0 & 0 & 1 \\ Q & 1 & 0 & 0 & 0 & 0 & 1 \\ E & 1 & 1 & 0 & 0 & 0 & 1 \\ S & 1 & 1 & 1 & 0 & 0 & 1 \\ G & 1 & 1 & 1 & 1 & 0 & 1 \\ D & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \qquad (3.4.6)$$

Furthermore, based on logical reasoning and fact analysis, we can define a feedback relation R^{-1} as follows:

$$M_{R^{-1}} = \begin{array}{c|cccccc} & P & Q & E & S & G & D \\ \hline P & 0 & 0 & 1 & 0 & 1 & 0 \\ Q & 0 & 0 & 0 & 0 & 1 & 0 \\ E & 0 & 0 & 0 & 0 & 0 & 0 \\ S & 0 & 0 & 0 & 0 & 0 & 0 \\ G & 0 & 0 & 0 & 0 & 0 & 0 \\ D & 1 & 1 & 1 & 1 & 1 & 0 \end{array} \qquad (3.4.7)$$

Also, it is necessary to explain the value of its elements in order to give a clear introduction to the model:

(1) The explanation of $(P, E) \in M_{R^{-1}}$:

Where there are more employment opportunities, there are more permanent population. This is because people tend to live in the city, they work in. Therefore, a city with more employment opportunities easily attracts more people to work and live in, with a result of more permanent population.

(2) The explanation of $(P, G) \in M_{R^{-1}}$:

A city with high GDP often represents high level of development while people, especially young people prefer economically developed areas because of better development opportunities.

(3) The explanation of $(Q, G) \in M_{R^{-1}}$:

A city with high GDP often has a better development potential, so enterprises choose a city based on the GDP to a largely extent.

(4) The explanation of $(D, P), (D, Q), (D, E), (D, S), (D, G) \in M_{R^{-1}}$:

The government will initiate suitable policy for a better development according to the actual situation.

Finally, the complete relation matrix is defined as follows:

$$M_C = M_R \oplus M_{t(R)} \oplus M_{R^{-1}} = \begin{array}{c|cccccc} & P & Q & E & S & G & O \\ \hline P & 0 & 0 & 1 & 0 & 1 & 1 \\ Q & 1 & 0 & 0 & 0 & 1 & 1 \\ E & 1 & 1 & 0 & 0 & 0 & 1 \\ S & 1 & 1 & 1 & 0 & 0 & 1 \\ G & 1 & 1 & 1 & 1 & 0 & 1 \\ O & 1 & 1 & 1 & 1 & 1 & 0 \end{array} \quad (3.4.8)$$

The addition $(\oplus)$ between the above matrices follows the principle of logical addition. The operation rules of logical addition are as follows:

$$0 \oplus 0 = 0,\ 0 \oplus 1 = 1,\ 1 \oplus 0 = 1,\ 1 \oplus 1 = 1 \quad (3.4.9)$$

This is the complete relation model of influencing factors of economic vitality. All the positive correlation among all influencing factors of economic vitality can be found in relation matrix M_C. The paper will study the interaction among influencing factors and economic vitality based on this model.

3.4.2 The Effects of Changing Trend of Influence Factors

GDP is the representation of comprehensive strength. As an index, GDP can also represent the economic vitality of a region to some extent. In the absence of quantitative analysis of economic vitality and its influencing factors, the paper considers the change trend of GDP to represent that of economic vitality based on the relationship model of the influencing factors of economic vitality.

Whether it is the trend of population change or the trend of enterprise vitality change, the trend can be divided into two categories: upward trend and downward trend. The two trends will be discussed and analyzed respectively as follows:

(1) Impact of the changing trend of population

If talents are introduced, or the region's excellent conditions attract more people to come for development, which means the permanent residents increases. Then the number of labor force increases, which is helpful for enterprises to increase the quantity of enterprises, expand the scale of enterprises and provide more employment opportunities. consumption level can be improved based on those changes. Ultimately it promotes the growth of regional GDP to enhance economic vitality.

On the contrary, if the brain drains, urban population choose to move out one after another and the permanent residents decreases, also the number of labor force decreases. For enterprises, it may lead to the reduction of the quantity of enterprises, the reduction of scale, marketing amount and employment opportunities. Finally, those changes lead to the reduction of regional consumption level and GDP. Therefore, economic vitality will fall into depression.

(2) Impact of the changing trend of enterprise vitality

If the quantity of newly-established enterprises increases, the scale of enterprises expands, so the vitality of enterprises increases, it will help enterprises increase their marketing volume, which will attract more people to the region for development. The expansion of enterprise scale will provide more employment opportunities for the new people, the consumption level of the region will increase, the economy will continue to grow. All of these will drive the growth of GDP. Ultimately the economic vitality will be improved.

On the contrary, if many enterprises are cancelled, the scale of enterprises is reduced and enterprise vitality goes downhill, it will lead to the loss of talents, the migration of migrant workers. Those factors will cause the reduction of permanent residents, regional consumption level, which is not conducive to the growth of regional economy. Finally, it will cause the decline of economic vitality.

Summarize the above analysis process into a flowchart as follows:

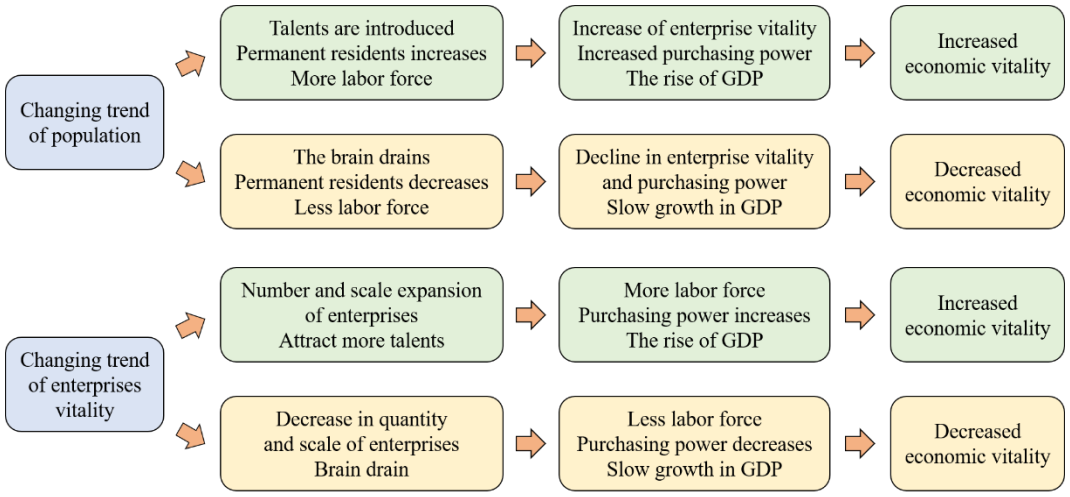

Figure 8: Impact of the changing trend of population and enterprises vitality

3.4.3 The Solution to Improve the Economic Vitality

To improve the economic vitality, the paper provides the following solutions from four aspects: permanent residents, quantity of enterprises, employment opportunities and level of consumption power.

(1) Permanent residents:

The permanent population is something worth considering. For example, lowering the settlement threshold is an effective method, especially for those developing cities. In Wuhan, there are more than a million college students studying here. If a policy can greatly reduce the cost of staying in Wuhan, many of these students are willing to stay here. Although other cities like Beijing, Shanghai are superior then Wuhan, it is expensive to stay in these cities. So, such a policy can give full play to comparative advantages of Wuhan to attract more people.

(2) Quantity of enterprises:

The increase in the quantity of enterprises is also important. To achieve this goal, the policy should encourage entrepreneurship and attract none-local enterprises. On the on hand, the government can provide start-up funds for start-ups or at least reduce their difficult of raising funds, to create a good soil for mass entrepreneurship. On the other hand, reducing the investment attraction approval steps is necessary. Long approval circle leads to extra cost and time for enterprises. Efficiency approval will improve the attraction of Wuhan to none-local enterprises.

(3) Quantity of employed population:

The quantity of employed population is a key factor. In general, more enterprises bring more jobs, so increase the quantity of enterprises which is mentioned above is effective. Besides, the government should establish a complete talent market mechanism to provide free employment services for job-seekers. This is helpful to connect the talent demand of the enterprise with the work demand of the talent, which ensures full employment.

(4) Level of consumption power

The level of consumption power should be taken in consideration. Lowering taxes is an excellent policy. Lowering taxes can stimulate consumption both of individual and enterprises. Less money is payed to the government; more money is used for consumption. Meanwhile, the relevant supplementary measures ought to be set up. For example, to improve the consumption on cars, road construction, parking lots and gas stations had better be completed, then people will show more interest in car purchase.

The above solutions are summarized in the chart as follows:

Figure 9: The solution to improve the economic vitality

4. Changing Trend Model of Economic Vitality

4.1 Problem Analysis

In view of problem 2, the paper will describe the relationship between economic vitality and its influencing factors through the ecological characteristics of biosphere. In the same ecosystem, there may be mutual inhibition of competition between various organisms, and there will also be cooperative relations of common growth. According to Assumption 2, the paper abstracts the influencing factors of economic vitality into population and describes the changing trend of economic vitality by studying the changing characteristics of population quantity. The research results of population change trend are mature, the most typical is the S-shaped curve of population growth.

As a special influencing factor, development policy can directly or indirectly affect the change of influencing factors. Development policy is not a population, it plays the role of environmental resistance in the model. When the government promulgates the positive development policy which is equivalent to the reduction of environmental resistance, then the ecosystem is suitable for the growth of various groups, so positive development policies promote the enhancement of economic vitality. However, the negative development policy will increase the environmental resistance, hinder the population development. Therefore, it is reasonable for the above population model to describe the changes of economic vitality and its influencing factors. The establishment and solution of the model will be analyzed and introduced in detail later.

The modeling process of Question 2 is as follows:

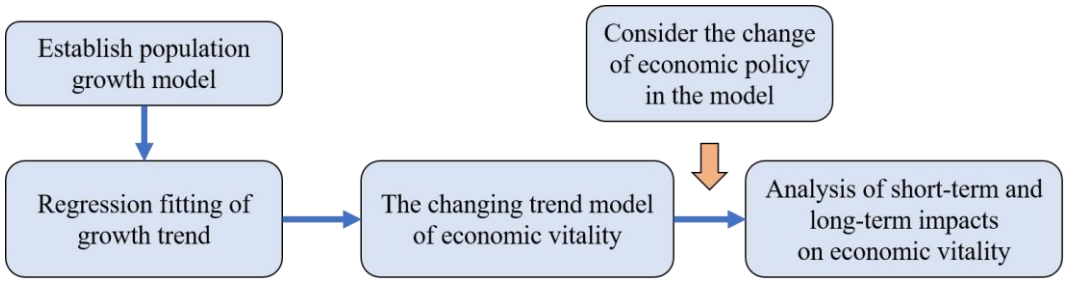

Figure 10: The modeling process of Question 2

4.2 Population Growth Model

In this part, the paper will explain the rationality of using population growth model to describe the change trend of influencing factors and give the expression of the model.

For any kind of influence factor, let its initial quantity be N_0. If model does not consider any external influence factors and assume that the growth change of influence factors has a fixed growth rate λ, then it can figure out the following relationship between the number of influence factors N and the differential of time t:

$$\frac{dN}{dt} = \lambda \cdot N \qquad (4.2.1)$$

The solution of the above differential equation is:

$$N=N_0 \cdot \exp(\lambda t) \tag{4.2.2}$$

The curve of this function is a typical J-shaped curve, which has the characteristics of infinite growth and accelerated growth. However, there must be environmental resistance in fact. For the biological population, the maximum capacity of the environment has an upper limit. With the growth of population, the food pressure and living space pressure will gradually increase. The changing trend of the influencing factors of regional economic vitality will be similar. Based on not considering the factors such as urban expansion, the maximum population that a region can accommodate, the number of jobs that a region can provide, and the productivity and purchasing power of a region will always be saturated because of the space resources or material resources. Therefore, it is reasonable to use S-shaped curve population growth model to describe the growth of economic vitality.

Common population growth models have the following differential forms:

$$\frac{\mathrm{d}N}{\mathrm{d}t}=\lambda \cdot N \cdot \left(1-\frac{N}{K}\right) \tag{4.2.3}$$

Where λ is the fixed growth rate, N is the population number, K is the maximum capacity of the environment and the solution of the above equation is in the form of:

$$N=\frac{K}{1+\exp(-\lambda t-C)} \tag{4.2.4}$$

Where $N_{t=0}=N_0$, parameter C is calculated as follows:

$$C=\ln\left(\frac{N_0}{K-N_0}\right) \tag{4.2.5}$$

The above is the most basic population growth model. Take the case of $K=1$, $\lambda=1$ as an example, the function is plotted as follows:

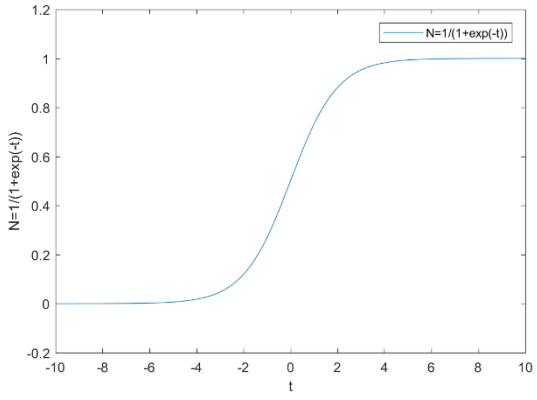

Figure 11: The function curve of the population growth

The characteristics of the population growth model can be analyzed from the above figure. The establishment and analysis of the later model will be based on the population growth model.

4.3 Changing Trend Model of Economic Vitality

When the economic policy changes, the implementation of the policy will not directly affect the economic vitality. The policy will directly affect the change of the permanent population, the quantity of enterprises, and the employment situation. Through these factors, the economic vitality will be indirectly affected. Any change of influencing factors will affect the change of economic vitality. Therefore, this paper will construct a statistic V about economic vitality, which can reflect the change trend of any influencing factors, to show the real situation of economic vitality. The specific construction process is as follows:

First, the influencing factors of economic vitality are standardized to remove the influence of units on the model calculation and only retain their change trend characteristics. For the corresponding variable X of influencing factors, the method of standardization is as follows:

$$X = \frac{X}{\|X\|_\infty} \tag{4.3.1}$$

Take the year as the independent variable (and translate the independent variable year to start with 1), draw the change of each influencing factor with the year as follows:

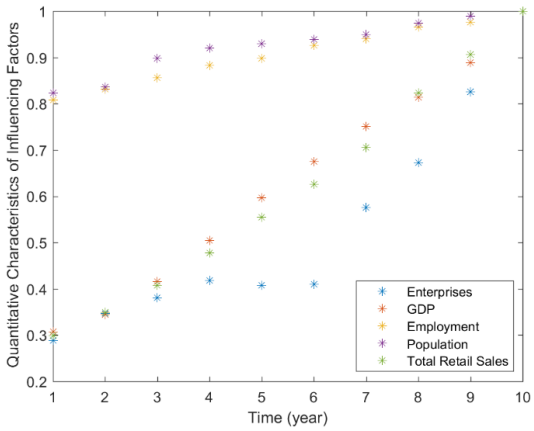

Figure 12: The change trend of each influencing factor with the year

$a_{mn}(m = 1,\cdots,5,\ n = 1,\cdots,10)$ is used to represent the trend data of the $n-$th year of the $m-$th influencing factor. In order to make the statistics of economic vitality $V = (v_1, v_2, \cdots, v_n)$ better reflect the change trend of any influencing factor, the following residual statistic $e(V)$ are constructed:

$$e(V) = \sum_{n=1}^{10}\sum_{m=1}^{5}(a_{mn} - v_n)^2 \tag{4.3.2}$$

The solution principle of the above model is defined as follows:

$$\text{Seek } V_0\text{: } e(V_0) = \min_{V\in R^n} e(V) = \min_{V\in R^n}\sum_{n=1}^{10}\sum_{m=1}^{5}(a_{mn} - v_n)^2 \tag{4.3.3}$$

The sum of the squares of the distance between the vector V and the variables is the smallest. The paper considers that the statistics of economic vitality V_0 can better reflect the change trend of any influencing factors.

Note that for each fixed value of $k = 1,2,\cdots,n$, the value of v_k won't affect the residual of other columns. Therefore, the minimum value point of the residual statistics of each column can be calculated separately. The calculation formula of the minimum value point v_n is as follows:

$$v_n = \frac{1}{5}\sum_{m=1}^{5} a_{mn} \qquad (4.3.4)$$

From formula (4.3.4), the calculation formula of the optimal statistical vector V_0 of economic vitality can be solved and mark the expression vector V_0 of the change trend of economic vitality in the change trend chart of each influencing factor. The image is as follows:

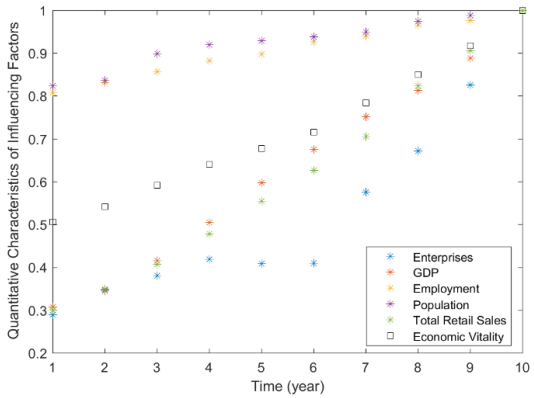

Figure 13: The change trend of eeconomic vitality with the year

Next, based on the population growth model, using vector V and regression fitting, the paper figures out the logistic model of the trend of economic vitality [3]. Using the data from 2009 to 2018, the fitting effect of the regression model is as follows:

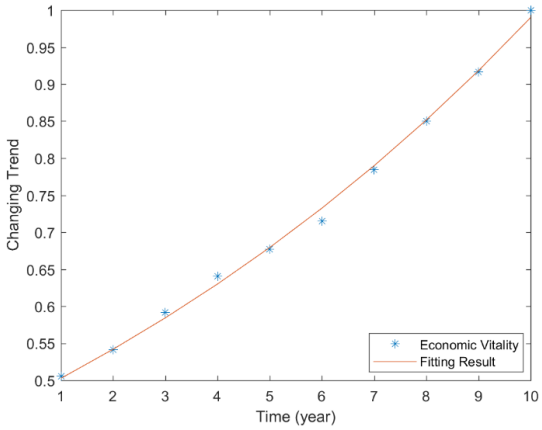

Figure 14: The fitting result of the regression

The above model has achieved good regression approximation effect, The following discussion and analysis will be based on this model.

4.4 The Effects of Policies Transformation on the Economic Vitality

According to Assumption 1, considering the actual social development situation, the model assumes that the change of development policy is always in a good direction. Therefore, the transformation of economic development policy plays a positive role in promoting the growth of permanent population, the increase of the quantity and scale of enterprises, the employment opportunities, the improvement of consumption level and the growth of GDP.

The analysis of the short-term and long-term impact of economic policy changes on economic vitality is as follows:

4.4.1 Short-term Effects on Economic Vitality

Considering the impact of positive economic policies on economic vitality in the next 20 years, the economic development policies of Wuhan are all positive in recent years. It is assumed that the change trend of economic vitality in Wuhan can maintain the change trend shown in Section 4.3.

Using the logistic model curve analysis based on population growth model, the change trend of economic vitality is drawn as follows:

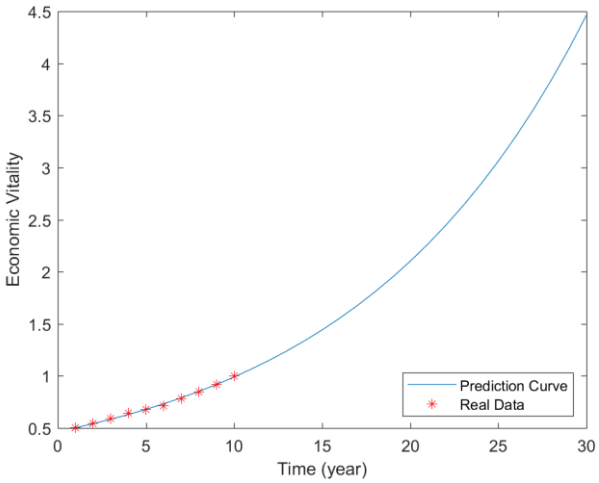

Figure 15: Trends of economic vitality in the next 20 years

From the above figure, the economic vitality data from 1 to 10 years present the economic vitality data from 2009 to 2018. As the statistical data has been standardized before, the economic vitality in 2018 is just one unit.

In the next 20 years, the economic vitality will continue to grow, and the growth rate will accelerate. It is estimated that in 2038 (20 years later), the economic vitality of Wuhan will be 4.5 times of the current economic vitality.

This shows that positive economic development policy changes will promote the continuous growth of Wuhan's economic vitality in the short term. Wuhan will be in a good stage of urban development with huge development potential. The development of Wuhan will not be limited by space and resources in a short period of time.

4.4.2 Long-term Effects on Economic Vitality

Considering the impact of positive economic policies on economic vitality in the next 200 years, it is assumed that the change trend of economic vitality in Wuhan can maintain the change trend shown in Section 4.3. Using the logistic model curve analysis, the change trend of economic vitality is drawn as follows:

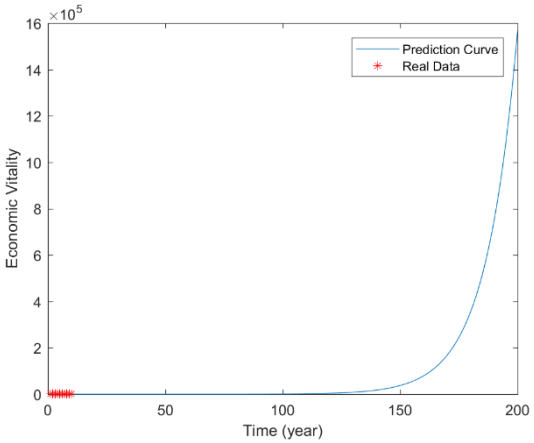

Figure 16: Trends of economic vitality in the next 200 years

From the above figure. In the next 200 years, the economic vitality of Wuhan will continue to grow. In the first 150 years, the growth rate will gradually accelerate. For the period from 150 to 200 years, the economic vitality of Wuhan will grow rapidly. It is expected that by 2200 years, the economic vitality of Wuhan will be 16×10^5 times of the present.

In addition, the paper use the model to investigate when the economic vitality of Wuhan reaches the development bottleneck, and draw the forecast curve for the next 400 years as follows：

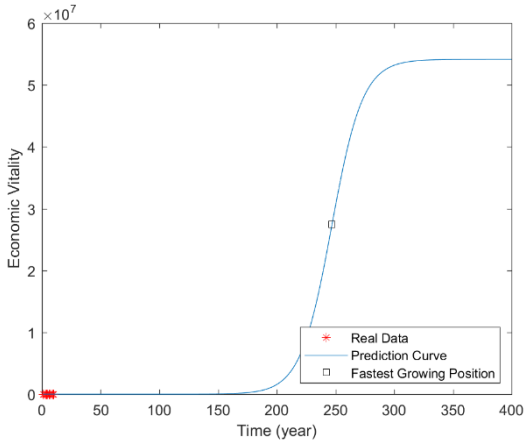

Figure 17: Trends of economic vitality in the next 400 years

It can be seen from the figure that in the next 300 years, the economic vitality of Wuhan will show an increasing trend. After 300 years, the economic vitality of Wuhan will tend to be stable due to the limitation of space and resources. The fastest period of economic vitality growth will be reached in about 250 years. Moreover, the final stable economic vitality will be 10^7 times of the current economic vitality.

5. Evaluation Model of Economic Vitality

5.1 Problem Analysis

In view of Question 3, the paper will build a comprehensive evaluation model of urban economic vitality by Principal Component Analysis (PCA). The economic vitality of a city is affected by various factors. Each influencing factor of economic vitality is regarded as a variable, PCA can effectively reduce the dimension of variables without losing too much useful information, and then the comprehensive evaluation model can be established.

The modeling process of Question 1 is as follows:

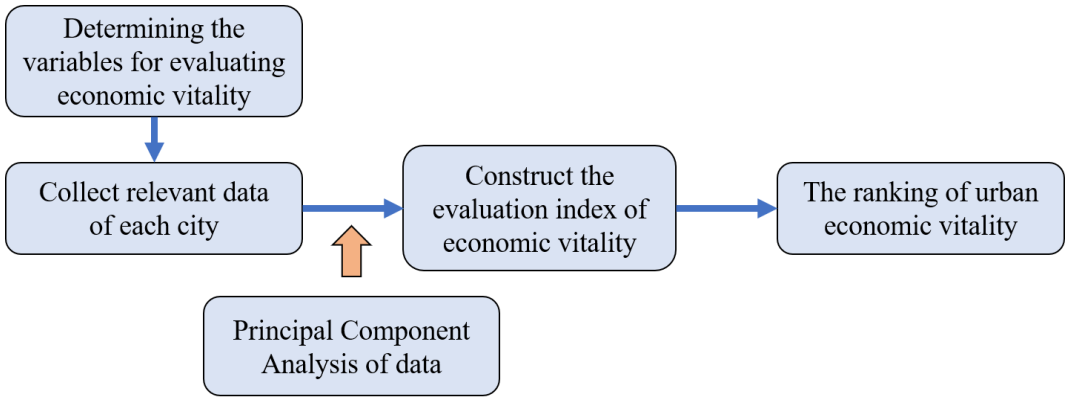

Figure 18: The modeling process of Question 3

5.2 Principal Components Analysis (PCA)

In this part, the paper will introduce the basic algorithm of PCA and the establishment method of comprehensive evaluation index.

With n samples and p indexes observed for each sample, the original data matrix is obtained:

$$X_{n\times p}=\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{bmatrix} \tag{5.2.1}$$

First, the original matrix is standardized as follows:

$$\tilde{X}_{n\times p}=\begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1p} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{n1} & \tilde{x}_{n2} & \cdots & \tilde{x}_{np} \end{bmatrix}, \begin{cases} \tilde{x}_{ij}=\dfrac{x_{ij}-\overline{x}_j}{\sigma_j} \\ \overline{x}_j=\dfrac{1}{n}\sum_{i=1}^{n}x_{ij}, \sigma_j^2=\dfrac{1}{n-1}\sum_{i=1}^{n}(x_{ij}-\overline{x}_j)^2 \end{cases} \tag{5.2.2}$$

Figure out the covariance matrix of the sample Σ_X:

$$\Sigma_X=\frac{1}{n}\tilde{X}^T\times\tilde{X} \tag{5.2.4}$$

Σ_X is a symmetric and semi positive definite matrix. Calculate the eigenvalue and eigenvector of the covariance matrix Σ_X, solve the eigenequation:

$$\left|\lambda E - \sum_X\right| = 0 \tag{5.2.5}$$

Obtain eigenvalues and arrange them in order of size:

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p \geq 0 \tag{5.2.6}$$

The eigenvectors corresponding to the eigenvalue λ_{i} are obtained:

$$u_i(i = 1, 2, \cdots, p) \tag{5.2.7}$$

Then, the contribution rate and cumulative contribution rate of principal components are calculated, and the principal components are selected. The calculation method of contribution rate is as follows:

$$\frac{\lambda_k}{\sum_{i=1}^{p} \lambda_i}(k = 1, 2, \cdots, p) \tag{5.2.8}$$

The calculation method of cumulative contribution rate is as follows:

$$\frac{\sum_{j=1}^{k} \lambda_j}{\sum_{i=1}^{p} \lambda_i}(k = 1, 2, \cdots, p) \tag{5.2.9}$$

Generally, the first and second values corresponding to the characteristic values with cumulative contribution rate of 85% ~ 95% are taken. Then calculate the scores of the principal components:

$$F_i = \tilde{X} u_i(i = 1, 2, \cdots, p) \tag{5.2.10}$$

The comprehensive evaluation index can be defined as:

$$w = \sum_{j=1}^{m} \frac{\lambda_j}{\sum_{i=1}^{p} \lambda_i} F_j \tag{5.2.11}$$

5.3 Evaluation Model of Economic Vitality

Based on the data given in the previous studies and Attachment 3, the paper will focus on the permanent residents, the quantity of employment population, and the consumption level, GDP， quantity of newly-established enterprises from 2009 to 2018, quantity of surviving enterprises in 2019. Use these variables as indicators to evaluate the impact of economic vitality. Through the PCA of 19 cities and 6 variables of each city, the comprehensive evaluation index of economic vitality is determined.

Part of the data of the impact indicators of each city obtained on each database website are as follows and the complete data can be found in the Appendix.

Table 3: Partial data of impact indicators of each city

City	Permanent Residents	GDP	…	Total Retail Sales	Quantity of Employment
unit	**10,000**	**100 million**	…	**100 million**	**10,000**
Shanghai	2423.78	32679.87	…	11745.96	1430.82
⋮	⋮	⋮	⋮	⋮	⋮
Kunming	685.0	5206.9	…	2787.4	461.2

The cumulative contribution rate of the principal component of the model is 85%, and the comprehensive evaluation index w of economic vitality is obtained as follows:

$$w = a_1 \cdot F_1 + a_2 \cdot F_2 \qquad (5.3.1)$$

F_1 and F_2 are the two main components with the largest proportion, a_1 and a_2 are weight coefficients. The values of a_1 and a_2 are:

$$a_1 = \frac{\lambda_1}{\sum_{i=1}^{6} \lambda_i} = 0.7617, \quad a_1 = \frac{\lambda_2}{\sum_{i=1}^{6} \lambda_i} = 0.1289 \qquad (5.3.2)$$

The comprehensive evaluation index w of economic vitality is used to calculate the economic vitality score of each city. The cities in Attachment 3 are ranked as follows:

Table 4: Ranking table of urban economic vitality

Ranking	City	Economic Vitality Score w
1	Shanghai	3.599643
2	Beijing	3.189219
3	Chongqing	2.507832
4	Shenzhen	1.832115
5	Guangzhou	0.696709
6	Chengdu	0.257801
7	Tianjin	0.169547
8	Suzhou	-0.27013
9	Hangzhou	-0.57192
10	Wuhan	-0.61358
11	Qingdao	-0.87857
12	Nanjing	-0.88051
13	Zhengzhou	-0.91865
14	Xi’an	-1.02937
15	Dongguan	-1.12886
16	Ningbo	-1.2217
17	Changsha	-1.30103
18	Shenyang	-1.64122
19	Kunming	-1.79733

6. Evaluation and Generalization of the Model

6.1 Model Evaluation

Advantage:

1. The model considers influencing factors of economic vitality from three aspects of industrial vitality, talent vitality and policy vitality. Permanent residents, quantity of enterprises, GDP, the level of consumption and the quantity of employed population are selected as influencing factors of economic vitality in the paper. Besides, the model investigates economic development policies of cities in recent years and proposes the most suitable development strategies for a city combined with characteristics of the city.
2. All the data selected can be checked through the statistics yearbook of cities, so they are accurate. It is representative for these data to describe the economic vitality.
3. The model investigates the correlation coefficient among influencing factors of economic vitality, resulting in the discover of the positive correlation among factors.
4. Inspired by the population growth model, the model makes use of the logistic model to fit out the trend chart of economic vitality. Then, the fitting curve is compared with the existing data to ensure the reliability of the result.
5. In the establishment of the comprehensive assessment index, the model chooses PCA, by which the original complex variables are represented by fewer variables including as many features of the original variables as possible. This is an effective method to simplify the complexity of the problem.

Weakness:

1. When considering the influencing factors of economic vitality, the model is unable to include factors such as capital vitality and innovative vitality, which leads to certain error in the assessment of economic vitality.
2. In PCA, the model inevitably decreases or ignores the influences from some ingredients, meanwhile, the result fluctuates according to different contribution rate.

6.2 Model Generalization

Three models are used in the paper: relation matrix model, population growth model and PCA comprehensive evaluation index. Each method can be extended to different application directions. When we study the relationship between variables, we can use the relation matrix in Discrete Mathematics to describe the relationship quickly and concisely. The population growth model can be used to study the quantity change with fixed growth rate under the condition of environmental resistance. PCA can help the model extract useful features from many complex variables and give comprehensive evaluation indexes of these variables. The comprehensive evaluation index constructed by PCA can be extended to the comprehensive evaluation of students' performance and enterprise strength.

7. Development Proposal

Based on the research results, combined with the characteristics of Wuhan, the paper provides the following proposals for the development decision-making of Wuhan:

1. Promote the implementation of preferential policies for college students:
    a) Reduce the threshold for college students to buy houses in Wuhan. Buying houses is a major problem for the young generation in China. To solve the problem of young people's demand can attract more talents to choose to develop in Wuhan.
    b) Provide social welfare for college students, such as free subway card. Strong social welfare support can attract more college students in Wuhan and retain development objects.
    c) Strengthen the construction of social security system, such as building and improving the medical insurance policy for college students. Good social security is not only an important guarantee to improve the city's happiness, but also the key to attract talents.
    d) Increase publicity of preferential policies to let more college students know about the implementation of preferential policies.
2. Increase support for enterprises:
    a) Optimize the audit process and establishment of enterprises, make entrepreneurship more "civilian" and reduce the cost of entrepreneurship.
    b) Establish an investment fund pool to provide financial support for innovative and entrepreneurial talents and enhance the innovation vitality of enterprises.
    c) Strengthen the guidance work for the construction and development of enterprises, expand the scale of enterprises, enhance the vitality of enterprises.
    d) Promote the cooperation between enterprises and universities, utilize the talent resources of universities, ensure the vitality of enterprises, and establish the export channels for local graduates to local enterprises.
3. Attach importance to the development of urban science and technology:
    a) Promote the establishment and development of emerging industries. As the first productivity in the 21st century, science and technology can make a breakthrough in urban development only if we attach importance to the development of science and technology.
    b) Increase the support for scientific research in universities and increase the investment in scientific research funds. Ensure the high output of scientific and technological research results and attract more top talents to develop in Wuhan.
4. Improve social construction:
    a) Improve Wuhan's urban traffic construction to make Wuhan's urban travel more convenient for people. Excellent transportation network can promote urban consumption and improve the level of purchasing power.
    b) Pay attention to urban education investment, which is the basis of development. To ensure that the children of each family can receive compulsory education is the key to increase the permanent population and ensure sustainable development.

8. Reference and Appendix

8.1 Reference

[1] Xiaoling Zuo, Wei Li, Yongcai Liu. Discrete mathematics [M]. 1988
[2] A first course in mathematical modeling: Frank R. Giordano and Maurice D. Weir. Monterey, CA: Brooks/Cole Publishing Company, 1985. 382 pp. $00.00 ISBN 0-53-403367-9[J]. 8(2):250-0.
[3] iLoveMATLAB, How to fit logistic function with MATLAB
https://www.ilovematlab.cn/thread-477055-1-1.html?tdsourcetag=s_pcqq_aiomsg
visit time: Nov. 30, 2019

8.2 Appendix

Appendix 1:

Table 1: The statistical data of cities in Attachment 3 in 2018

City	Permanent Resident (unit: 10,000)	Quantity of Employed Population (unit:10,000)	GDP (unit:100 million)	Total Retail Sales (unit:100 million)
Shanghai	2423.78	1430.82	32679.87	11745.96
Shenzhen	1252.83	943.29	24221.98	6168.87
Beijing	2154.20	1105.37	30320.00	25405.90
Guangzhou	927.69	896.54	22859.35	9256.19
Chongqing	3101.79	1714.55	20363.19	8067.67
Chengdu	1604.47	892.70	13889.39	6403.53
Nanjing	833.50	457.60	12820.40	5832.46
Hangzhou	980.60	681.06	13509.00	5715.33
Suzhou	1068.36	691.60	18597.47	5746.90
Tianjin	1559.6	896.6	18809.6	5533.0
Qingdao	939.5	605.3	12001.5	4842.5
Dongguan	839.2	660.4	8278.6	2905.6
Zhengzhou	1008.0	629.6	10143.3	4268.1
Wuhan	1089.3	564.1	13410.3	6196.3
Xi’an	1000.4	621.2	8349.9	4658.7
Ningbo	820.2	544.4	10746.0	4154.9
Changsha	815.5	487.1	11003.4	4765.0
Shenyang	831.6	433.0	6292.4	4051.2
Kunming	685.0	461.2	5206.9	2787.4

Note: Relevant data on the quantity of enterprises are given in Attachment 3.

Appendix 2:

Table 2: The statistical data of the collected influencing factors of Wuhan

Year	Permanent Residents (unit: 10,000)	Quantity of Established Enterprises (unit: 10,000)	Quantity of Employed Population (unit:10,000)	GDP (unit:100 million)	Total Retail Sales (unit:100 million)
2009	897.00	13.32	456.00	4115.51	1850.05
2010	910.00	15.99	468.60	4620.86	2164.09
2011	978.54	17.56	483.00	5565.93	2523.20
2012	1002.00	19.31	498.00	6762.20	2959.04
2013	1012.00	18.82	506.40	8003.82	3432.43
2014	1022.00	18.90	522.24	9051.27	3878.60
2015	1033.80	26.55	530.44	10069.48	4369.32
2016	1060.77	31.02	544.92	10905.60	5102.24
2017	1076.62	38.12	550.37	11912.61	5610.59
2018	1089.29	46.17	564.08	13410.34	6196.30

Appendix 3:

The MATLAB code used in the paper is as follows:

1. The Matlab program for the calculation of correlation coefficient in Section 3:

```
r1=corrcoef(P,Q)
%P is permanent resident people, Q is quantity of established enterprises.
```

```
r2=corrcoef(Q,E)
%Q is quantity of established enterprises，E is quantity of employed population.
```

```
r3=corrcoef(P,G)
%P is permanent resident people, G is GDP.
```

```
r4=corrcoef(Q,G)
%Q is quantity of established enterprises, G is GDP.
```

```
r5=corrcoef(S,G)
%S is total retail sales, G is GDP.
```

2. The Matlab program for the image in Section 4

Figure 11:

```
Plot(t,1/(1+exp(-t)),'-')
```

Figure 12:

```
Plot(t,P,'*');
hold on;
Plot(t,Q,'*');
hold on;
Plot(t,E,'*');
hold on;
Plot(t,S,'*');
hold on;
Plot(t,G,'*');
```

Figure 13: (Based on the Figure 12)

```
Plot(t,V,'ks')
```

The Matlab program for the logistic model

```
logimod=fitnlm(t,y,'y~b1/(1+b2*exp(-b3*x))',[5000 5 1])
```

3. The Matlab program for PCA in Section 5

```
clear
load('data.txt');
%Save all data in a plain text file 'data.txt'
sum=0;  num=1;
% Numerical initialization.
%'sum' is the cumulative contribution rate.
%'num' is the number of main components.
data=zscore(data);
%Data standardization.
r=cov(data);
% Calculate covariance matrix.
[x,y,z]=pcacov(r);
% Principal component analysis.
%the column of X is the eigenvector of R.
%y is the eigenvalue of R.
%z is the contribution rate of principal component.
while sum<85
    sum=sum+z(num);
    num=num+1;
end
```

```
%Determine the number of principal components by cumulative contribution rate.
num=num-1;
% The final number of principal components
pcc=data*x(:,1:num);
%calculate principal component score
cs=pcc*z(1:num)/100;
%calculate comprehensive score
[scs,posi]=sort(cs,'descend');
%Descending order.
%’scs’ is    the comprehensive score in descending order.
%’posi’is    the corresponding position before sequence change.
scs=scs'; posi=posi';
fprintf('Rate of contribution:\n');disp(z);
fprintf('Comprehensive score:\n');disp(scs);
fprintf('Position:\n');disp(posi);
```