For office use only	Team Control Number	For office use only
T1 ________________	**1915560**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**B**	F4 ________________

2019

MCM/ICM

Summary Sheet

Solving the Problem of Planning Through Expectation Distribution

When natural disasters occur, it is particularly important to carry out rescue missions quickly. The establishment of Aerial Disaster Relief Response System can provide the necessary material assistance and road monitoring for the disaster-stricken areas in the first time. With the help of mathematical model, we have made a comprehensive analysis and a scheme of Aerial Disaster Relief Response System to maximize its work efficiency in the event of disaster. Our thoughts in the process of modeling are as follows:

First, by comparing the size of the island with the flight range of the drone, we identify that three containers for the installation of rescue systems must be located in three different locations of the island. We made some important rationalized assumptions based on conditions and facts. For example, we assume that Aerial Disaster Relief Response System works long enough (the time of rescue work is half a year), we also assume that the flight time of drone under load decreases linearly with the increase of liability.

Secondly, we adopt the important idea of expected distribution when consider about the packing scheme. Considering the minimum packaging demand and cost, we assemble medical packages in a 7:2:5 demand ratio, optimize the solution of the model algorithm and reduce unnecessary waste caused by excessive packaging of medical packages.

Thirdly, when considering about the location of Aerial Disaster Relief Response System in Puerto Rico Island, we adopt the dynamic programming modeling method. In the process of analysis, we abstract the complex map of Puerto Rico Island into a simple topological map and establish a plane rectangular coordinate system based on longitude and latitude, so that the key points can be expressed through specific coordinates. By quantifying the constraints and considering other factors, we accurately figure out the coordinates of three system installation locations.

Finally, our model considers the attenuation of drone flight time when it is loaded, and the problem of drone's field of view and minimum resolution angle when it is flying at high altitude. The model accurately reflects the changes of these actual situations. We also assume that each drone must return after take-off to maximize the value of the drone and reduce the cost of system investment. The theoretical and practical solutions for each problem are given respectively in this paper. We have summarized these results and wrote them into the memo which will be sent to the CEO of HELP, Inc.

Contents

1. Introduction

1.1 Restatement of the Problem

The construction of Aerial Disaster Relief Response System needs us to consider about packing, transportation, distribution of medical packages, the location of the system, the optimization of road monitoring and other complex realistic issues. Meanwhile, population density, material demand and natural environment in different regions of the disaster area will also be factors affecting the construction of the system. The problem can be fundamentally divided into three aspects: packaging, location and route planning. HELP, Inc. needs us to help them solve the above problems so that Aerial Disaster Relief Response System can effectively complete the rescue mission. The above three planning problems can be simplified as follows:

- Figure out cargo containers packaging scheme to meet the requirements
- Figure out the best location of System to meet the requirements
- Figure out the packaging and flight schemes of each drone to meet requirements

1.2 Our Work

Considering the realistic factors and the simplification of the model, we have rationally simplified many details of the problem (see Assumption) to facilitate the discussion of subsequent problems. According to the data provided, we only retain three of the most powerful drones as alternatives. through simple performance comparison. In addition, based on the equations for planning problems, we consider the supply-demand relationship, and establish an original planning model of distribution according to expectations to solve problems A and B. As for Problem C, based on topology and impact of load, we use time value as the main evaluation index to complete the model.

2. Assumption

- Cargo container is equipped with shipping container, shipping container is equipped with drone, cargo bay and packages, and cargo bay is equipped with packages.
- If the volume of the container is 1.25 times of the total volume of the object, it is considered that the container can accommodate these items.
- The volume of drone can be ignored when occupying the volume of shipping container.
- Different types of drones have the same value.
- The working time of the system is half a year (180 days)
- Map can be abstracted into graphics. Important locations will be converted into coordinates.
- The flight time of drone under load decreases linearly with the increase of liabilities.
- Drone’s flight path is straight and the drone’s flight altitude can span any terrain.
- In order to maximize the scope of drone’s road monitoring, ISO containers can be transported to the wherever we need.
- Due to cost considerations, drones need to charge after each mission.
- Without considering about the power loss of drones during takeoff and landing.

3. Notations and Notes

3.1 Notations

Symbol	Description
M_X	Maximum load of drone model X
M_{MEDn}	Unit quality of class n medical packages
N_{XB_n}	Number of class n packages transported by cargo bay model X
N_{XS_n}	Number of class n packages transported by shipping container model X
N_{MEDn}	Total number of class n medical packages
N_X	The total number of shipping containers of model X
V	Volume of each cargo container
V_{X_S}	The volume of shipping container model X
V_{X_B}	The volume of cargo bay of drone model X
V_{MEDn}	Volume of class n medical package
(x_n, y_n)	Coordinates of the System site ($n = 1,2,3$)
(Hx_n, Hy_n)	Coordinates of the hospital ($n = 1,2,3,4,5$)
D_{nm}	Distance between the hospital and the system site
S_i	Service coverage area of the system site
ε	Coefficient of load affecting the flight time of drone
β	The ratio of total mass of packages transported by drone to total load
T_X	Rated flight time of drone model X under unloaded conditions
t_X	Maximum flight time of drone model X under load
S	Land area of Puerto Rico Island

3.2 Notes

For the purpose of writing, the following Notes are used:

Notes	Meaning
SC	Shipping Container
System	Aerial Disaster Relief Response System
Site	The location of the Aerial Disaster Relief Response System
ISO1,2,3	The first, second, and third system sites providing rescue services

4. Expectations Distribution and Dynamic Programming Model

4.1 Equations in Basic Model

As for problem A, considering about the volume relation between large container and small container, Three ISO cargo container not only should satisfy the demand of Puerto Rico hurricane scenario, but also need to think about the cost of HELP, Inc, too much drone equipment and medicine will result in waste of resources. The constraints of problem A are as follows：

- Bearing Capacity of Different Types of drones, volume of different types of cargo bay
- The ability of different types of shipping container accommodates cargo bay and packages
- Each shipping container should be full enough to reduce the use of buffer materials
- The Accommodation capacity of ISO container accommodates shipping container
- The number of packages should satisfy the demand of 180 days
- On the premise of meeting supply demand, the total number of drones should be as small as possible to reduce costs.

Transforming the above constraints into corresponding mathematical expressions as follows:

$$\sum_{n=1}^{3} M_{MEDn} \cdot N_{XB_n} \le M_X, X = B, C, F \tag{4.1.1}$$

$$\sum_{n=1}^{3} V_{MEDn} \cdot N_{XB_n} \le 0.8 V_{X_B}, X = B, C, F \tag{4.1.2}$$

$$V_{X_B} + \sum_{n=1}^{3} V_{MEDn} \cdot (N_{XS_n} - N_{XB_n}) \le 0.8 V_{X_S}, X = B, C, F \tag{4.1.3}$$

$$\sum_{X} V_{X_S} \cdot N_X \le 3V \tag{4.1.4}$$

$$N_{MED1} = \sum_{X} N_{XS_1} \cdot N_X \ge 1260, N_{MED2} = \sum_{X} N_{XS_2} \cdot N_X \ge 360, N_{MED3} = \sum_{X} N_{XS_3} \cdot N_X \ge 720 \tag{4.1.5}$$

$$\text{seek}: f(X) = \min[\sum_{X} N_X] \tag{4.1.6}$$

For Question B, firstly, the location of System should consider not only the natural environment, the time of transporting medical packages, but also the service range of different types of drones in each system. Location selection should meet the medical needs of five hospitals. Meanwhile, in order to monitor road conditions, the proportion of coverage of drone in the whole island area should be increased. From the analysis in Attachment 2, it can be concluded that drone B has the fastest flight speed and the farthest flight distance, so the maximum service range of drone B is taken as the service range of three System stations. The constraints of problem B are as follows:

- The service range of the system accounts for the largest proportion of the total island area.
- System Station has the smallest distance from hospitals in each service area.
- System should be built in flat plains

Secondly, we abstract the complex map of Puerto Rico Island into a simple geometric map and establish a plane rectangular coordinate system based on longitude and latitude, so that the key locations can be represented by specific coordinates. The function D_{nm} of the distance between the system site and the hospitals in the service areas is defined as follows：

$$D_{nm}=\begin{cases}\sqrt{(Hx_n-x_m)^2+(Hy_n-y_m)^2} & \text{Hospitals are within the service area of this site}\\ 0 & \text{Hospitals are without the service area of this site}\end{cases} \tag{4.1.7}$$

Transforming the above constraints into corresponding mathematical expressions as follows:

$$\text{seek}: g=\max[\frac{(S_1\cup S_2\cup S_3)}{S}] \tag{4.1.8}$$

$$\text{seek}: d=\min[\sum_{n=1}^{5}\sum_{m=1}^{3}D_{nm}] \tag{4.1.9}$$

4.2 Model Optimization

4.2.1 Expectations Distribution Optimization Model

Considering the relationship between supply and demand, we assume that packages are packaged according to the ratio of 7:2:4 of daily demand for MED1, MED2 and MED3, which can not only measure the actual situation, but also optimize multiple complex expressions of the project.

In addition, when analyzing the performance of eight different types of drones, B, C and F perform much better than the other five types of drones in all aspects. And B, C, F reflect their own unique functions so that they can be competent for different types of tasks.

- Drone B is small, files fast and long, suitable for road monitoring tasks.
- Drone F has fast flight speed, large load, can complete many package transportation tasks.
- Drone C has great performance as F and can also undertake road monitoring tasks.

Finally, we assume that the same type of shipping container has the same internal packaging scheme, and we bundle the number of B, C and F drones in a fixed proportion into a set, and then pack the set in ISO containers. The choice of sets should satisfy all the above distribution expectations and basic constraints at the same time. Therefore, each set contains at least one B, C and F drones to complete the task of medicine transportation and road monitoring effectively.

4.2.2 Dynamic Programming Optimization Model

In order to simplify the model, the idea of dynamic programming is used to optimize the solution of the model. The performance of drone B is the determinant of system service coverage. We will solve the constraints in the following order:

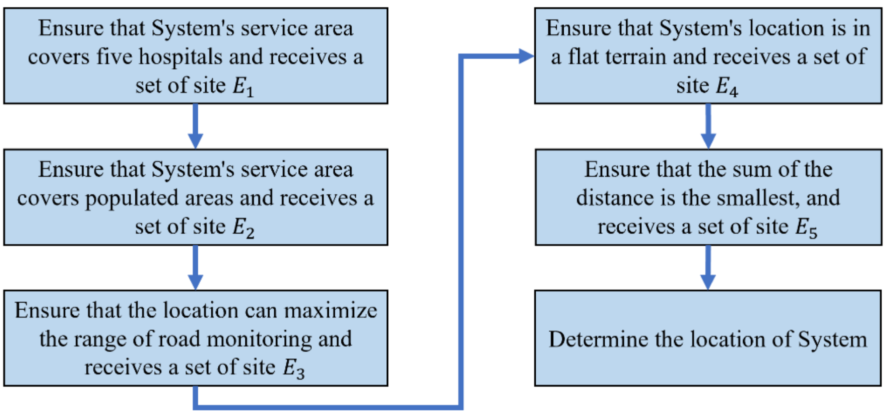

Figure 1： The Solution Process of Problem B

According to the dynamic programming optimization method, the location set obtained in each step of the flow chart should satisfy $E_1 \supseteq E_2 \supseteq \cdots \supseteq E_5$, Finally, we will select the best system locations in E_5 according to the actual situation.

The services of Aerial Disaster Relief Response System are divided into two aspects: medical package transportation and road monitoring. As for the coverage of road surveillance services, we can directly use the maximum flight time of drones under no load to determine the coverage, while for the transportation of medical packages, we need to consider the load of drones. Experience tells us that the smaller the load is, the longer drones can fly. For this reason, the determination of location set E_1 should consider the flight coverage of drone under full load.

We assume that the maximum flight time of the drone under load satisfy the following equation：

$$t_X = T_X \cdot (1 - \beta \cdot \varepsilon) \tag{4.2.1}$$

The upper formula β represents the proportion of the load to the upper limit of liability, and the upper formula ε represents the proportion coefficient of the load affecting the endurance of UAV. If $\varepsilon = 0$, it means that the maximum flight time of drone is not affected by the load, but the proportional coefficient $\varepsilon > 0$ in fact. In order to get the maximum service range of drone, in (4.2.1) set $X = B$, the properties of curve t_B under different coefficients ε are obtained as follows：

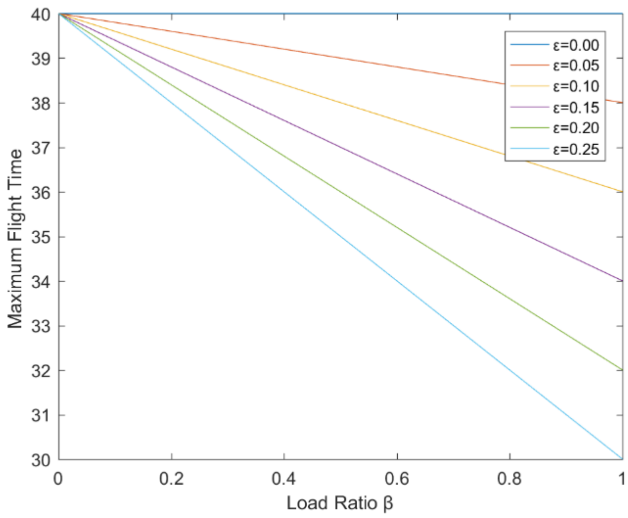

Figure 2：The Variation of t_B under Different Load Radio and Load Impact Coefficient

In order to simplify the calculation, only three cases of proportional coefficient $\varepsilon = 0$，$\varepsilon = 0.1$，$\varepsilon = 0.2$ are discussed in the following modeling process. In the reality, we set $\varepsilon = 0.2$ for the drones in Aerial Disaster Relief Response System.

4.3 Solution of the Model

4.3.1 Theoretical Solution of Problem A

Using the expected distribution model, the Set scheme satisfying the constraints is as follows:

Table 2：Packaging Set Meet Expectations Distribution Model (Ratio: 7:2:4)

Set	Number of B	Number of C	Number of F	Ratio (MED1: MED2: MED3)
I	1	1	1	140:40:80
II	2	1	1	161:46:92
III	1	2	1	224:64:128
IV	1	1	2	175:50:100
V	1	2	2	259::74:148
VI	2	1	2	196:56:112
VII	2	2	1	245:70:140

The seven schemes $\mathrm{I}-\mathrm{VII}$ mentioned above are all measured from the local point of view of shipping container and cargo bay packaging. They can satisfy $(4.1.1)\sim(4.1.3)$ of the constraints when they are solved. Next, we only need to check whether the above set meets $(4.1.4)\sim(4.1.6)$.

Firstly, according to the conditions $(4.1.5)$, we can get the number of different sets as shown in Table 3, and then check the conditions $(4.1.4)$ by using the data from Table 3. Because the volume of ISO containers is large enough, any scheme can be satisfied:

$$\sum_{X} V_{X_S} \cdot N_X \le 3V \tag{4.1.4}$$

Then we check the condition $(4.1.6)$, assume that different types of drones have the same value cost, so we can base on

$$f(X) = \sum_{X} N_X \tag{4.1.6}$$

to measure the cost of each Set and select the lowest one, the results are shown as follows:

Table 3：Demand and Value Cost of Each Packaging Set

Set	I	II	III	IV	V	VI	VII
Number of Drones	9	8	6	8	5	7	6
$f(X)$	27	32	24	32	25	35	30

To sum up, the most perfect set is Set III, which only needs 24 drones to complete the rescue mission of Puerto Rico. Specific recommended packaging solutions are as follows:

It is pointed out that: the following schemes do not take into account many practical problems (i.e. the following problems of system location and route arrangement), so the solution of the schemes is only theoretical.

Table 4: Recommended packing configuration for a drone fleet
(without considering about system locations and flights arrangement problem)

ISO1	The number of SC	B	C	F
	The number of SC	2	4	2
	Number of MED1 per SC	21	84	35
	Number of MED2 per SC	6	24	10
	Number of MED3 per SC	12	48	20
ISO2	Same as ISO1			
ISO3	Same as ISO1			

4.3.2 Theoretical Solution of Problem B

Firstly, the coordinates of five hospitals are taken as the center of the circle, and the maximum flight distance radius of drone B under different load coefficients is taken as the circle. In order to make the range of road monitoring sufficiently large, the location of System should fall on these five circles. Then, take the location of three systems as the center of the circle, and take the maximum flying distance radius of drone B as the circle, the area of the three circles is S_1, S_2, S_3, respectively. Then the service scope of the system is $S_1 \cup S_2 \cup S_3$. System coverage should cover most likely populated places. The model determines the optimal location by collecting E_4, E_5. The results of the model are shown in Table 5 (see Appendix for illustrations of problem solving):

Table 5: The Best Locations under Different Load Impact Coefficient

Load Impact Coefficient	System Number	Locations	
		Latitude	Longitude
$\varepsilon = 0.00$	ISO1	18.23333	-66.78033
	ISO2	18.26667	-66.28200
	ISO3	18.23333	-65.83333
$\varepsilon = 0.10$	ISO1	18.26667	-66.78833
	ISO2	18.31167	-66.29833
	ISO3	18.23333	-65.92583
$\varepsilon = 0.20$	ISO1	18.28333	-66.76667
	ISO2	18.30000	-66.25000
	ISO3	18.23333	-65.83333

5. Models Considering Realistic Factors

5.1 Revisions of Packing Configuration

We've discussed the recommendation of drone fleet and medical packages when we ignore the influences of systems' location and delivery routes in Section 4.3.1, but it's not the best recommendation under realistic situations according to the analysis of the model mentioned in Section 4.3.2. Take the system ISO1 on the left side of the island as an example, we only need to provide one MED1 medical package to Hospital Pavia Arecibo every day. If we still use the recommendation in Section 4.3.1, each system will take packages of MED1, MED2, MED3 at the ratio of 7:2:4, and this will bring about 2 problems:

- Packages sent to hospitals that don't need them will be a waste.
- Packages are attributed to each system in a ratio, in some systems, packages are insufficient.

Based on the above two reasons, we use the model localized processing method to analyze the requirements of specific places, then complete and fix their ISO packaging configuration. Use the method in Section 4.3.1 to analyze the ISO packaging configuration for the remaining locations. We select the case when the load impact coefficient $\varepsilon = 0.20$ in Section 4.3.2 as the actual situation and the optimal three container location options can be determined.

As to the container ISO1, there is only one hospital——Hospital Pavia Arecibo in its service area, therefore, the service site of this area only needs to provide medical packages that Hospital Pavia Arecibo needs when packaging. In the 180-day rescue mission, Hospital Pavia Arecibo needs one MED1 every day. From the perspective of road monitoring, the system ISO1 on the left side can cover largest area of the three and faces least delivery work (it only needs to deliver one medical package every day). Considering all the factors above, it's reasonable for us to choose only one kind of drones——drone B to complete the rescue mission in this area. The constraints should satisfy the following equation:

$$M_{MED1} \cdot N_{BB_1} \le M_B \tag{5.1.1}$$

$$V_{MED1} \cdot N_{BB_1} \le 0.8 V_{B_B} \tag{5.1.2}$$

$$V_{B_B} + V_{MED1} \cdot (N_{BS_1} - N_{BB_1}) \le 0.8 V_{B_S} \tag{5.1.3}$$

$$N_{MED1} \ge 180 \tag{5.1.4}$$

The solved packing configuration for ISO1 is as follows:

Table 6: Recommended packing configuration for ISO1
(within considering about system locations and flights arrangement problem)

	The type of SC	B	C	F
ISO1	The number of SC	6	0	0
	Number of MED1 per SC	32	0	0
	Number of MED2 per SC	0	0	0
	Number of MED3 per SC	0	0	0

As for the container ISO3, modeled on the ISO1 analysis method, its service coverage only includes the Caribbean Medical Center hospital, which only needs the medical packages required by the Caribbean Medical Center. In the 180-day rescue mission, Caribbean Medical Center needs one MED1 and MED3 per day. In addition, from the perspective of road monitoring and medical package transportation pressure, choosing drone B to complete the ISO3 rescue mission is also reasonable. Here we still use the expectation distribution method in the theoretical model to package the MED1 and MED3 in a 1:1 ratio. The constraints should satisfy the following equation:

$$M_{MED1} \cdot N_{BB_1} + M_{MED3} \cdot N_{BB_3} \leq M_B \tag{5.1.5}$$

$$V_{MED1} \cdot N_{BB_1} + V_{MED3} \cdot N_{BB_3} \leq 0.8 V_{B_B} \tag{5.1.6}$$

$$V_{B_B} + V_{MED1} \cdot (N_{BS_1} - N_{BB_1}) + V_{MED3} \cdot (N_{BS_3} - N_{BB_3}) \leq 0.8 V_{B_S} \tag{5.1.7}$$

$$N_{MED1} \geq 180,\ \ N_{MED3} \geq 180 \tag{5.1.8}$$

The solved packing configuration for ISO3 is as follows:

Table 7: Recommended packing configuration for ISO3
(within considering about system locations and flights arrangement problem)

ISO3	The type of SC	B	C	F
	The number of SC	10	0	0
	Number of MED1 per SC	19	0	0
	Number of MED2 per SC	0	0	0
	Number of MED3 per SC	19	0	0

As for the container ISO2, its service coverage includes three hospitals. In the 180-day rescue mission, 5 MED1s, 2 MED2s, and 3 MED3s are required every day. From the perspective of road monitoring and medical package transportation pressure, all three types of B, C, and F drones should be considered, and we still use expectation distribution method to package three medical packages in a ratio of 5:2:3. The constraints should satisfy the following equation:

$$\sum_{n=1}^{3} M_{MEDn} \cdot N_{XB_n} \leq M_X, X = B, C, F \tag{5.1.9}$$

$$\sum_{n=1}^{3} V_{MEDn} \cdot N_{XB_n} \leq 0.8 V_{X_B}, X = B, C, F \tag{5.1.10}$$

$$V_{X_B} + \sum_{n=1}^{3} V_{MEDn} \cdot (N_{XS_n} - N_{XB_n}) \leq 0.8 V_{X_S}, X = B, C, F \tag{5.1.11}$$

$$N_{MED1} = \sum_{X} N_{XS_1} \cdot N_X \geq 900, N_{MED2} = \sum_{X} N_{XS_2} \cdot N_X \geq 360, N_{MED3} = \sum_{X} N_{XS_3} \cdot N_X \geq 540 \tag{5.1.12}$$

According to the model in Section 4.3.1, the ISO2 packaging Set is obtained:

Table 8：Packaging Set Meet Expectations Distribution Model (Ratio: 5:2:3)

Set	Number of B	Number of C	Number of F	Ratio (MED1: MED2: MED3)
Set	1	0	0	20:8:12
Set	1	1	0	105:42:63
Set	1	0	1	55:22:33
Set	1	1	1	140:56:84
Set	2	1	1	160:64:96
Set	2	2	1	245:98:147
Set	2	1	2	195:78:117

Finally, we test conditions (4.1.6) to measure the cost of each plan. The results are shown in Table 9:

Table 9：Demand and Value Cost of Each Packaging Set

Set	I	II	III	IV	V	VI	VII
Number of Drones	45	9	17	7	6	4	5
$f(X)$	45	18	34	21	24	16	25

In summary, the most cost-effective packaging configuration is Package II which requires only 18 drones to complete the ISO2 rescue mission. The specific recommended packaging configuration is as follows:

Table 10：Recommended packing configuration for ISO2
(within considering about system locations and flights arrangement problem)

ISO2	The type of SC	B	C	F
	The number of SC	9	9	0
	Number of MED1 per SC	20	84	0
	Number of MED2 per SC	8	35	0
	Number of MED3 per SC	12	51	0

5.2 Medical Package Transportation Model

The distribution of medical substances is the most important work every day. Therefore, the priority of each fleet is to deliver the medical materials required by the hospitals in their service coverage to the corresponding hospitals in time, and then return. We have assumed that the route of the drone is a straight line, and the distance between each system site from each service hospital is shown in the following table:

Table 11：Distance from System sites to service hospitals

Locations	Caribbean Medical Center	Hospital HIMA	Hospital Pavia Santurce	Puerto Rico Children's Hospital	Hospital Pavia Arecibo
ISO1	N/A*	N/A	N/A	N/A	20.73
ISO2	N/A	14.04	21.07	12.85	N/A
ISO3	22.53	N/A	N/A	N/A	N/A

*Note: N/A means that hospitals are not covered by this ISO, the unit of distance in the table is kilometers.

The situation of ISO1 and ISO3 is very simple. The drones of the two service stations only need to carry the medical packages needed by the hospitals they are responsible for in the cargo bay and go straight to deliver them.

The situation of ISO2 is more complicated. It needs to be responsible for the three hospitals. In order to complete the delivery work in time, we must consider the distance from ISO2 to each hospital and the mutual distance between the three hospitals to see if we can use only one drone to complete the rescue missions of many hospitals.

The relative position of the address of ISO2 and the address of the three hospitals are as follow:

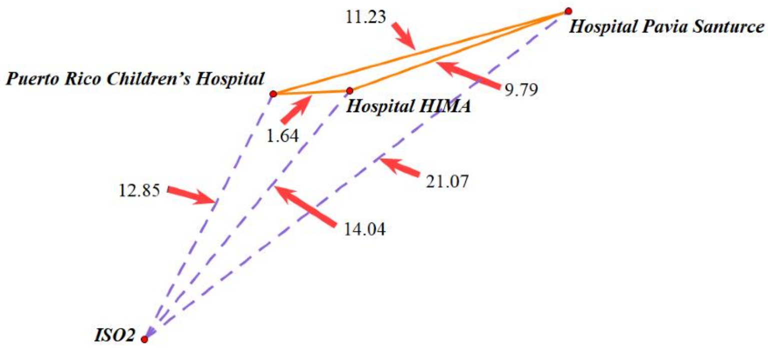

Figure 3： Relative Location Relations of ISO2 and Hospitals

Firstly, we consider the path from ISO2 to Hospital Ravia Santurce, which is 21.07km in length. It can only rely on drone B to complete the task considering the load. Hospital Ravia Santurce needs one MED1 and one MED2 per day. The carrying capacity of drone B can be satisfied, now consider the maximum flight time:

$$t_B = T_B \cdot (1-\beta\cdot\varepsilon) = 40\times(1-0.5\times0.20) = 36 \tag{5.2.1}$$

After considering the return trip, the farthest distance that drone B can fly is:

$$d_B = \frac{t_B}{2}\times\frac{1}{60}\times 79 = 23.70 > 21.07 \tag{5.2.2}$$

The drone B can complete the delivery work of the Hospital Ravia Santurce.

Then consider the path from ISO2 to the Puerto Rico Children's Hospital, which has a length of 12.85km. The Puerto Rico Children's Hospital requires 2 MED1s, 1 MED2 and 2 MEN3s per day, and if taking load influences into consideration, we can only use drone C to complete this work. Now consider the maximum flight time:

$$t_C = T_C \cdot (1-\beta\cdot\varepsilon) = 35\times(1-\frac{2\times2+1\times2+2\times3}{14}\times0.20) = 29.00 \tag{5.2.3}$$

After considering the return trip, the farthest distance that drone C can fly is:

$$d_C = \frac{t_C}{2}\times\frac{1}{60}\times 64 = 15.47 > 12.85 \tag{5.2.4}$$

It is also clear that the drone c can complete the delivery work of the Puerto Rico Children's Hospital. However, the remaining load is no longer able to undertake other delivery work.

Finally, for the delivery work of Hospital HIMA, from the perspective of load capacity, drone B and C can undertake the delivery work, but from the time value consideration, drone B can complete the delivery task faster. Consider the maximum flight time:

$$t_B = T_B \cdot (1-\beta\cdot\varepsilon) = 40\times(1-\frac{7}{8}\times0.20) = 33.00 \tag{5.2.5}$$

After considering the return trip, the farthest distance that drone B can fly is:

$$d_B = \frac{t_B}{2} \times \frac{1}{60} \times 79 = 21.73 > 14.04 \tag{5.2.6}$$

It is obvious that the drone B can complete the delivery work of the Hospital HIMA.

The results of the medical package transportation model are shown in the following table:

Table 11：Packing Configurations of Drone Fleet Delivery Solution

Origin	Drone Type	Cargo Bay Packing Configurations	
ISO1	B	MED1	1
ISO2	B_1*	MED1	2
		MED3	1
	B_2*	MED1	1
		MED2	1
	C	MED1	2
		MED2	1
		MED3	2
ISO3	B	MED1	1
		MED3	1

*Note: We use B_1, B_2 to distinguish drones in same type flying to different places

Table 12：Delivery routes and Schedule of Drone Fleet Delivery Solution

Origin	Drone Type	Termini	Departure	Waiting*	Return*	Frequency
ISO1	B	Hospital Pavia Arecibo	8:00 am	15min	8:47 am	Daily
ISO2	B1	Hospital HIMA	8:00 am	15min	8:35 am	
	B2	Hospital Pavia Santurce	8:00 am	15min	8:47 am	
	C	Puerto Rico Children's Hospital	8:00 am	15min	8:39 am	
ISO3	B	Caribbean Medical Center	8:00 am	15min	8:50 am	

*Note: Waiting means the time for drones to offload medical supplies from the drone cargo bays
Return means the moment when drone returns to its ISO cargo container along the original route

5.3 Road Monitoring Model

In order to complete the task of road monitoring, drone's photography range should cover all highways and roads on the map as much as possible. Therefore, the establishment of road monitoring model should pay attention to the following points:

- The relationship between minimum resolution angle and flight altitude in aerial photography
- The relationship between aerial imaging range, flight altitude and camera performance
- The flight time of drone
- Drones participating in the delivery of medical packages may also be able to complete road monitoring tasks

Minimum Resolution Angle Model:

When drone shoots in high altitude, the resolution ability of its camera is given by the following formula:

$$\frac{d}{L} = \tan\theta \approx \theta = 1.22\frac{\lambda}{D}(\theta \to 0) \tag{5.3.1}$$

Among them, d represents the recognition distance, L represents the lens height, λ represents the wavelength of light wave, D represents the lens diameter, and θ represents the minimum resolution angle. Because the flight altitude is far greater than the recognition distance, $\theta \to 0$ condition can be satisfied. The following figure shows the trend chart of the recognition distance d in the range of different lens height and visible wavelength ($D = 0.05m$)::

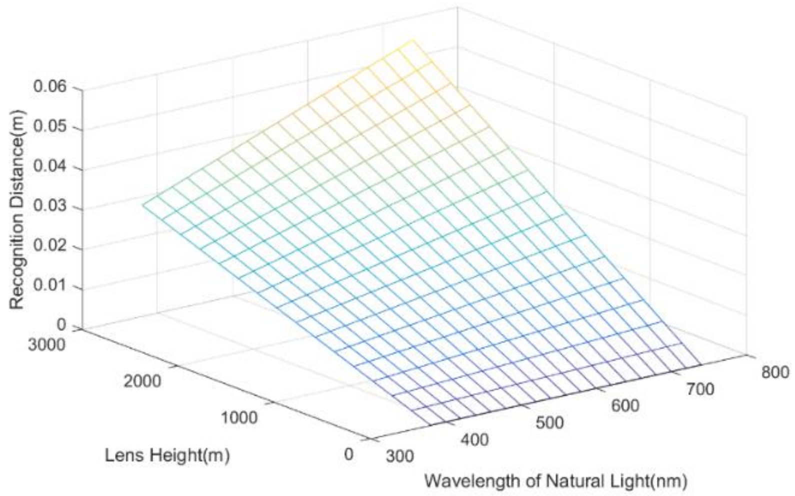

Figure 4：Trends of d at Different Heights and Wavelengths

As can be seen from the above figure, even if the flight altitude of the drone reaches 3000m for the general camera lens ($D = 0.05m$), the minimum resolution distance can still reach a high level of less than 0.1m in the visible wavelength range. Thus, in the following discussion, we assume that the lenses loaded by drones are all 0.05m in diameter and the flight altitude of drones is no more than 2000m, which can not only ensure the high recognition of aerial photographs, but also ensure that the drones won't lose too much power because of the high altitude rising or falling.

Imaging Range Model:

When drone shoots in high altitude, its angle of view is related to the focal length of lens and the nature of camera film. The expression is as follows:

$$\tan\alpha/2 = l/2f \tag{5.3.2}$$

Among them, α represents the angle of view, l represents the film width, f represents the focal length of the lens, and Figure 5 shows the trend chart of the change of the angle α of view with the film width and the lens focal length.

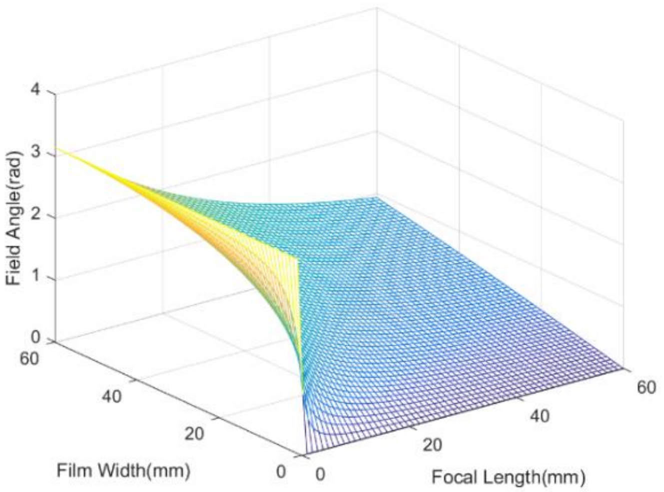

Figure 5：Trends of α at Focal Length and Film Width

We set R represent the radius of the field of view, L represent the height of the lens, and R can be denoted by the height and angle of view of the lens:

$$R = L \cdot \tan\theta \tag{5.3.3}$$

If the drone's camera is a general aerial camera, take f=20mm, l=43.27mm, and substitute the above two methods to get R≈1.08L. When the flight altitude is not more than 2000m, the drone's aerial photographic field of vision does not exceed 2160m, which means that the monitor range of the drones is actually a circular trajectory determined by the visual field angle, See the diagram below：

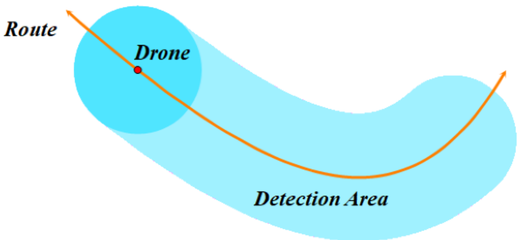

Figure 6：Diagram of Detection Area

Finally, in order to facilitate the description of drone routes, we simplify Puerto Rico's road map into a simple topological map, in which the points marked are the destination points or transit points in drone routes. The route arrangement obtained from the model solution is shown in Table 13：

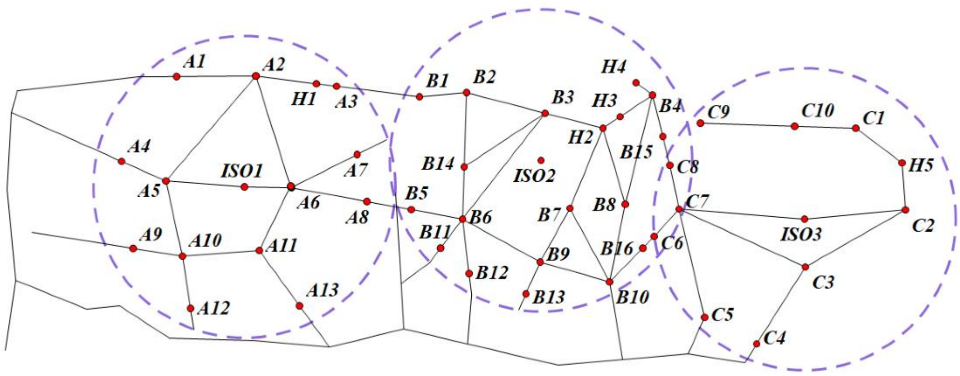

Figure 7：Simplified Road Topology

Table 13：Drone Flight Plan Schedule

Drone Number	Flight Plan		
	Task 1	Task 2	Task 3
ISO1-B_1	(D)ISO1→H→A3→ISO1	ISO1→A6→A8→ISO1	ISO1→A10→A5→ISO1
ISO1-B_2	ISO1→A2→H→ISO1	ISO1→A6→A11→ISO1	N/A
ISO1-B_3	ISO1→A2→A1→ISO1	ISO1→A11→A13→ISO1	N/A
ISO1-B_4	ISO1→A2→A5→ISO1	ISO1→ A11→A10→ISO1	N/A
ISO1-B_5	ISO1→A5→A4→ISO1	ISO1→A10→A12→ISO1	N/A
ISO1-B_6	ISO1→A6→A7→ISO1	ISO1→A10→A9→ISO1	N/A
ISO2-C_1	(D)ISO2→H2→B3→ISO2	ISO2→H2→B7→ISO2	ISO2→B7→B9→ISO2
ISO2-B_1	(D)ISO2→H2→H3→H2→B8→ISO2	ISO2→B3→B2→ISO2	ISO2→B6→B11→ISO2
ISO2-B_2	(D)ISO2→H4→B4→B15→ISO2	ISO2→B2→B1→ISO2	ISO2→B6→B12→ISO2
ISO2-B_3	ISO2→B2→B14→ISO2	ISO2→B9→B10→B7→ISO2	ISO2→B6→B9→ISO2
ISO2-B_4	ISO2→B14→B6→ISO2	ISO2→B8→B10→ISO2	ISO2→B9→B13→ISO2
ISO2-B_5	ISO2→B6→B5→ISO2	ISO2→B10→B16→ISO2	N/A
ISO3-B_1	(D)ISO3→C2→H5→ISO3	ISO3→C8→C7→ISO3	N/A
ISO3-B_2	ISO3→C1→C10→ISO3	ISO3→C7→C6→ISO3	N/A
ISO3-B_3	ISO3→H5→C1→ISO3	ISO3→C7→C5→ISO3	N/A
ISO3-B_4	ISO3→C2→C3→ISO3	ISO3→C3→C4→ISO3	N/A
ISO3-B_5	ISO3→C10→C9→ISO3	ISO3→C7→C3→ISO3	N/A

In order to make the information conveyed by the figure and table clearer, here are the necessary explanations for the unspecified parts of the above two charts:

- The dots ISO1, ISO2 and ISO3 in the topology diagram are three locations of System respectively, and three circles represent the service scope of three system sites respectively.
- The route stations within the respective service areas of ISO1, ISO2 and ISO3 are numbered with A, B and C respectively, while the highway nodes that have not been surveyed are not numbered.
- On the route of Table 13, flights with a D (Deliver) marker at the beginning indicate that medicine delivery tasks need to be completed while performing road monitoring tasks.
- N/A in Table 13 indicates that the drone has no additional mission.
- All drones in each ISO container do not necessarily need to perform tasks on the same day. System can complete road monitoring tasks by using only part of drones. In this way, a new drone can be used to replace the damaged one.
- All road monitoring tasks can be completed every morning.

6. Evaluation and Generalization

6.1 Model Evaluation

Advantage:

In the process of building and solving the model, we consider the physical factors that affect the efficiency of the system from packaging, transportation and monitoring. Additionally, we make assumptions or simplifications for many unmentioned variables, which make our models have the following advantages:

- Packages are packaged according to the demand, which will not cause extra waste.
- The idea of distribution according to expectation greatly optimizes the program algorithm and makes it easy to solve the equation
- Packing packages as much as possible in shipping container to minimize any need for buffer materials for unused space
- Maximizing the value of drone, greatly reducing the number of drone investment.
- The minimum resolution angle and imaging range in road detection are considered
- The effects of load on the maximum flight time of drone are considered

Weakness:

On the contrary, some model assumptions have some bad effects on the accuracy of the model, including the following aspects:

- Without considering the packing method of geometry, so the volume accommodation relationship is roughly estimated.
- Considering the return journey of drone, drones are unable to survey the entire island's roads.
- Without considering the power loss of drones during takeoff and landing
- Proportional distribution of medical packages may cause that the solution of the model is not the optimal solution in real circumstances.
- We assume System can be installed anywhere, it may be difficult to transport the System.
- Without considering the allocation of medical packages, if a hospital suddenly needs additional types of medical packages, the system cannot provide them.

6.2 Model Generalization

In the process of building and solving the model, many realistic factors are considered, and the most important thing is that the idea of expectation distribution is used in planning problems. Expectation distribution is used to optimize the equations of the model, which greatly reduces the complexity of solving the constraint equation by the program. For example, in solving the problem of ISO packaging scheme, before using the idea of expectation distribution, the program has three independent variables, a total of 15 iterations, which requires 3^{15} operations to complete one iteration. This is theoretically impractical. After adding the improvement of expectation distribution, the number of program's independent variable degenerates to 1 because of the proportion limitation, and the number of iterations also reduces to 5 times. Only 1^{5} operations are needed for a single traversal, which is very important in the optimization of the algorithm.

In addition, although the idea of expectation distribution narrows the scope of feasible region to a certain extent, it can guide the solution of the model to the desired direction. Therefore, even if the number of feasible solutions is reduced, the final solution of the model will not be too far away from the optimal solution in real situation. Therefore, we only need to examine the errors caused by the assumptions of expectation distribution and adjust the distribution scheme in time at each stage of the development of the model. Therefore, we can reduce the adverse effects of expectation distribution and make the results of the solution closer to the actual situation.

In all the problems of scheme design and planning, we can use the ideas and methods adopted in this paper. First, we can analyze the constraints, write the equations that meet the constraints, and use the idea of expectation distribution to optimize the multiple complex constraints of planning problems. At the same time, combined with the idea of dynamic programming model and the factors which should be considered in various realities, the local solution of the problem can improve the feasibility of the final scheme, and make the result of the solution more accurate and reliable.

7. Memo

Dear Chief Operating Officer:

After receiving your request, our team established a feasible scheme for Aerial Disaster Relief Response System. The scheme not only can meet the requirements of the Puerto Rico hurricane scenario, but also can help your company to finish the project at the lowest costs. In order to help you understand the scheme clearly, main results are as follows:

1. Because the Puerto Rico island is much larger than the maximum flight distance of the alternative drones, three cargo containers must be installed in three different locations on the island to meet the requirements of video reconnaissance.
2. Packaging of medical packages will be carried out in proportion to the needs of hospitals which can reduce the extra waste. This scheme will also contribute to the standardization of packaging.
3. Shipping containers need to pack extra medical packages while loading drone and cargo bay in order to minimize unused space. This can reduce the cost of the buffer materials.
4. Because drone B has the fastest flight speed and the longest flight distance, video reconnaissance mission is supposed to be handed over to drone B as much as possible.
5. Considering about the endurance of drones and maximization of video reconnaissance range, the flight route of drone could be set as straight as possible.
6. Considering about drone’s reduced endurance due to load and drone's return problem, the flight altitude of drone is supposed not to exceed 2000m
7. The lens configuration of drone could be as good as possible to perform the mission best. We recommend a lens with a diameter greater than 5cm and a focal length greater than 20mm, and its film width is better than 43.27mm. This allows drones to shoot at high altitudes with as wide a field of view as possible.

Additionally, we would like to emphasize the problem of locating the Aerial Disaster Relief Response System. The most critical factor in determining the location for system is to consider the load impact coefficient ε. Load impact coefficient reflects the maximum flight time of drone under different load conditions. We considered three different load impact coefficients: $\varepsilon = 0.00$, $\varepsilon = 0.10$ and $\varepsilon = 0.20$, and we figure out the best system location under these three different coefficients is determined. The results are shown in Table 1 below:

Table 1：The Best Locations under Different Load Impact Coefficient

Load Impact Coefficient	System Number	Locations	
		Latitude	Longitude
$\varepsilon = 0.00$	ISO1	18.23333	-66.78033
	ISO2	18.26667	-66.28200
	ISO3	18.23333	-65.83333
$\varepsilon = 0.10$	ISO1	18.26667	-66.78833
	ISO2	18.31167	-66.29833
	ISO3	18.23333	-65.92583
$\varepsilon = 0.20$	ISO1	18.28333	-66.76667
	ISO2	18.30000	-66.25000
	ISO3	18.23333	-65.83333

If you want to know how to pack the medical packages in different type drones and how to arrange the flight plan of medical packages delivery. The tables below answer those questions:

Table 2：Packing Configurations of Drone Fleet Delivery Solution

Origin	Drone Type	Cargo Bay Packing Configurations	
ISO1	B	MED1	1
ISO2	B_1*	MED1	2
		MED3	1
	B_2*	MED1	1
		MED2	1
	C	MED1	2
		MED2	1
		MED3	2
ISO3	B	MED1	1
		MED3	1

*Note: We use B_1, B_2 to distinguish drones in same type flying to different places

Table 3：Delivery routes and Schedule of Drone Fleet Delivery Solution

Origin	Drone Type	Termini	Departure	Waiting*	Return*	Frequency
ISO1	B	Hospital Pavia Arecibo	8:00 am	15min	8:47 am	Daily
ISO2	B1	Hospital HIMA	8:00 am	15min	8:35 am	
	B2	Hospital Pavia Santurce	8:00 am	15min	8:47 am	
	C	Puerto Rico Children's Hospital	8:00 am	15min	8:39 am	
ISO3	B	Caribbean Medical Center	8:00 am	15min	8:50 am	

*Note: Waiting means the time for drones to offload medical supplies from the drone cargo bays

Return means the moment when drone returns to its ISO cargo container along the original route

Many unknown difficulties may be encountered in the construction of Aerial Disaster Relief Response System. Because of the limitation of modeling, our results may cannot meet the requirements of this scenario, but we prefer to give some recommendations you can refer:

1. We recommend you use the long-endurance, fast flight speed drone to perform tasks. This drone has excellent advantages both in delivery and video reconnaissance.
2. We recommend you equip each cargo container with additional but small amounts of various medical packages in case of distribution adjustment problem.
3. We recommend that you do not put all drones into use at one time. Using part of the drone fleet to complete the mission through round-trip flights can provide possibility for replacement of damaged drones.
4. Under the condition that the maximum flight distance of drone can satisfy the task of video reconnaissance, we recommend that the system could be located as close as possible to the port next to the hospital. This not only facilitates the shipping of cargo containers, but also achieves delivery mission with the highest efficiency.

Sincerely,
Team#1915560

8. Reference and Appendix

Reference

[1] https://en.wikipedia.org/wiki/Unmanned_aerial_vehicle
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Appendix

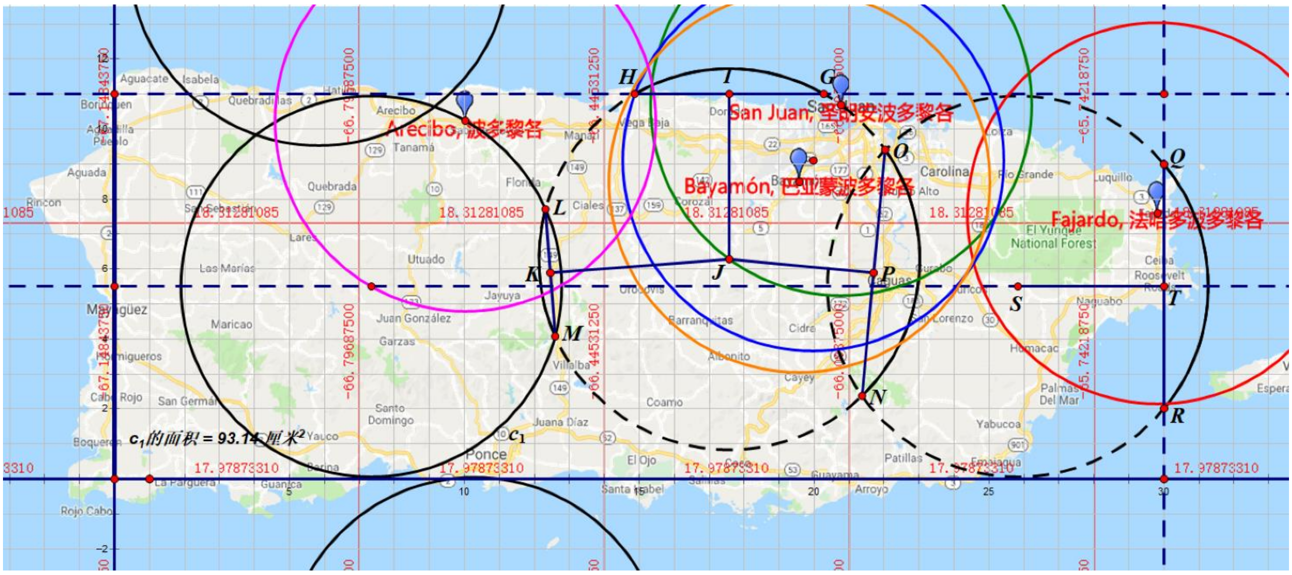

Figure 1: Case 1 of System Location

Figure 2: Case 2 of System Location

Figure 3: Case 3 of System Location

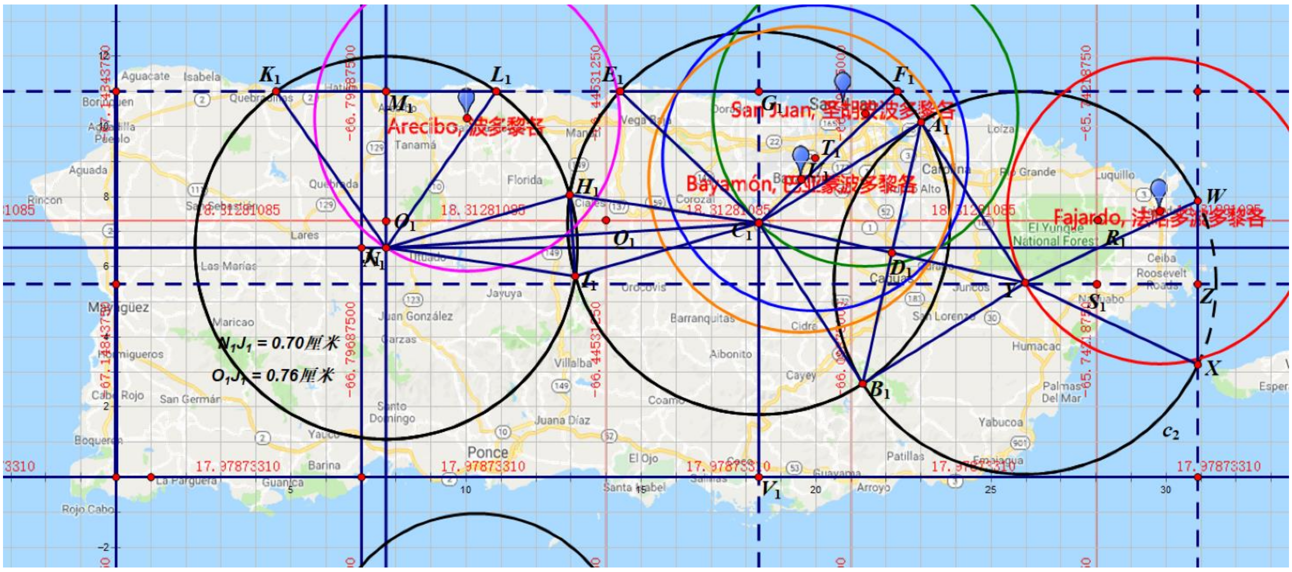

Figure 4: Case 4 of System Location

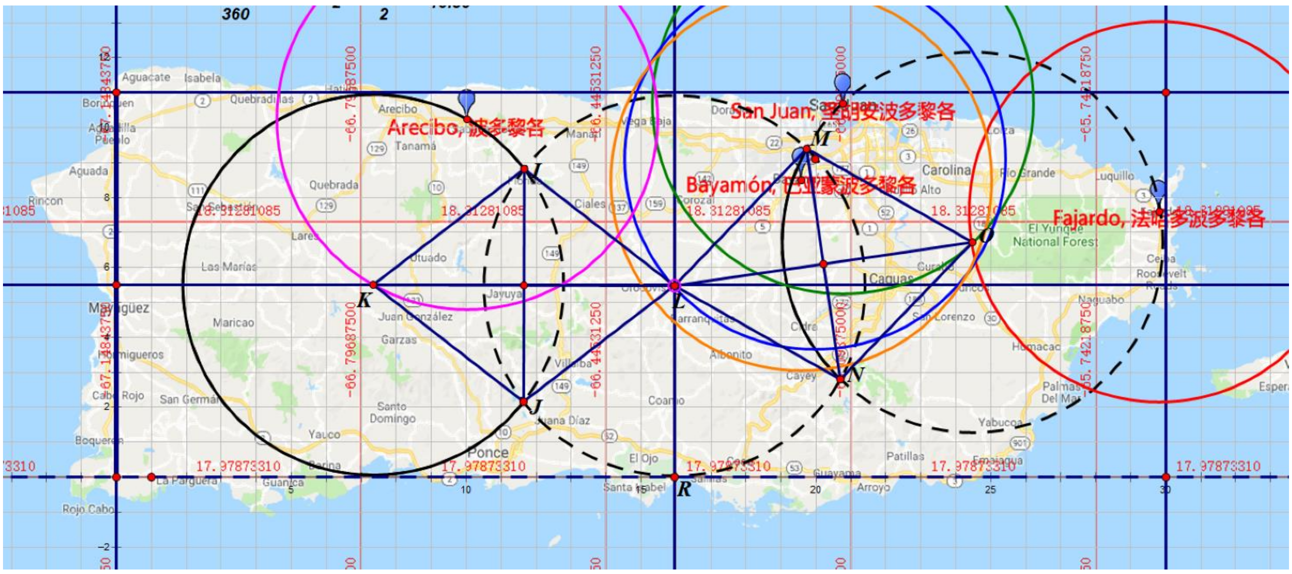

Figure 5: Case 5 of System Location