The First Home on the Moon

Summary

Since ancient times, people have been full of fantasy about infinite cosmos. With the development of intelligent devices, unlimited communication and remote control, 3D printing and other technologies, the dream of space travel has come a step closer. Astronauts want to live and work on the moon, to resist the extremely harsh space environment, a reliable "home" is essential.

Due to the absence of atmosphere, the surface temperature of the moon varies greatly. In the daytime, the surface temperature can reach $160℃$ under direct sunlight, and in the night, the temperature will drop sharply to $-180℃$. Through consulting the data, we assume that the lunar soil used for the construction of the space station is basalt. We consider the heat conduction along the station wall and indoor air under the condition of high and low temperature constant heat source, and divide the heat transfer process into three parts: the heat transfer on the space station wall, the convective heat transfer between the inner surface of the wall and the indoor air, and the heat transfer of the indoor air. By using the forward difference scheme to solve the heat conduction equation, we obtain the relationship between the temperature distribution and wall thickness. To keep the indoor temperature at a comfortable $22℃$, the results show that the optimal wall thickness should be set at 149cm. The wall thickness will be used for the calculation of space station construction scheme.

When considering the foundation excavation plan and the geometry of the space station, we abstract each astronaut and the required workspace as a fixed cylinder with a bottom radius of 1m and a height of 4m. To minimize the excavation volume of foundation, the optimization process is divided into two steps. In the first step, we consider making the cylinder represented by the astronaut fill the space station tightly to minimize the bottom area of the foundation; in the second step, we minimize the depth of foundation excavation. The optimal construction plan of the space station is as follows: the geometry of the space station is a cylinder with a height of 4m and a bottom radius of 5.122m. A cylinder foundation with a height of 3.0897m and a bottom radius of 5.122m is excavated, and all the excavated lunar soil is made into the cylinder shell of the space station. The height of the above part of the station is 0.9103m, the thickness of the wall is 1.49m, and the excavation volume of the foundation is 254.65m^3.

Basalt used for shelter construction has very high compressive strength (300MPa). Through simple calculation, we prove that pressure the space station needs to take is much less than 300MPa, which ensures the structural stability of the space station.

Cosmos is full of unknowns, and in order to simplify the model, we make a lot of reasonable assumptions. In model evaluation, we discuss the errors caused by these assumptions, and propose the improvement direction of the model. Finally, we extend the model to the general optimization problem with fixed volume demand.

Keywords: heat conduction, forward difference scheme, convective heat transfer, optimization

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1. Introduction

1.1 Restatement of the Problem

Backgrounds: Exploring the universe and realizing interstellar navigation has always been a human dream. The successful sampling of lunar soil promotes our further exploration of it, and the establishment of a space station on the moon is the first step for our interstellar space exploration. With the help of 3D printing technology and intelligent machinery, we have great hope to complete the construction of the space station through remote control. Considering the complex outer space environment, to ensure the feasibility of the scheme, we need to complete the following tasks:

- Figure out the wall thickness needed to build the space station to cope with the drastic change of temperature on the moon.
- Optimize the construction plan (including both the foundation excavation part and the building part on ground) of space station to meet the demands.
- Stability analysis of space station structure.

1.2 Our Work

Through consulting the data, we know that the diurnal temperature range of the moon can reach -180℃ to 160℃. We simplify the temperature transfer as a one-dimensional heat transfer model, and divide the heat transfer process into three parts: the heat transfer on the wall of the space station, the convective heat transfer between the inner wall of the station and the indoor air, and the heat transfer of the air inside the station. By solving the heat conduction equation through numerical methods, we determine the optimal wall thickness for the construction of the space station to ensure that the indoor temperature is stable at about 22℃, The wall thickness obtained will be used for the optimization of the construction scheme of the space station. The analysis of heat conduction is discussed in Section 4.

In Section 5, when we consider the geometry of the space station, we abstract each astronaut as a cylinder of fixed size and give them enough space to work. We considered several regular geometries and made the station as full as possible of the space occupied by astronauts without wasting any resources. For each geometry, we first optimize the size of the bottom area of the foundation excavation, and then take the excavation depth as the independent variable. On the one hand, we ensure that the volume of the station, the wall thickness meets the requirements, on the other hand, we minimize the excavation depth, so as to determine the design scheme for the construction of the shelter.

In Section 6, the stability of the space station structure is discussed in view of the special gravity environment on the moon, to make ensure that the structure conforms to the basic principles of physics and mechanics. In Seventh 7, we evaluate the model, analyze the source of model error and the direction of improvement. Finally, we show how to extend the model to the general optimization problem with fixed volume demand.

We have made many reasonable simplifications and assumptions in building the model to solve the key problems, all the errors caused by them will be discussed in Section 7.

2. Model Assumptions

1. It is assumed that the construction material of the space station (i.e. lunar soil) considered in the model is basalt [1].
2. It is assumed that the transfer of temperature in all directions of the medium is uniform.
3. It is assumed that convection does not occur when the heat transfer of air is considered in the model.
4. It is assumed that the influence of the day night change of the moon on the temperature transfer is not considered.
5. It is assumed that the heat transfer between all objects in the space station, including astronauts, and the air is not considered.
6. It is assumed that the structure of the building materials obtained by 3D printing is compact without pores, that is, the heat transfer of porous media is not considered.
7. It is assumed that each astronaut and the space he needs can be abstracted as a cylinder of fixed size.
8. According to Reference [1], it is assumed that the material generated by 3D printing has no mass loss compared with the raw material.

3. Notations

Symbol	Description
ρ_w	Density of basalt
c_w	Specific heat capacity of basalt
λ_w	Thermal conductivity of basalt
a_w	The coefficient of heat conductivity of basalt
ρ_a	Density of air
c_a	Specific heat capacity of air
λ_a	Thermal conductivity of air
a_a	The coefficient of heat conductivity of air
Φ	Heat flux in the process of heat conduction
T_0	Temperature on the surface of the moon
T_C	Temperature of convective heat transfer layer
$T(x,t)$	Temperature at position x at time t
Δx	Spatial step size of forward difference schemes
Δt	Time step size of forward difference schemes

(Continued)

Symbol	Description
d	Wall thickness of space station
d_a	Thickness of convective heat transfer layer
T_s	Steady state temperature of indoor air
h	Convective heat transfer coefficient of air
d_h	Wall thickness required to withstand high temperature
d_l	Wall thickness required to withstand low temperature
$h_{astronaut}$	The height of a small cylinder occupied by astronauts
R	The radius of the bottom of a small cylinder occupied by astronauts
V_{base}	Volume of foundation to be excavated
V_{wall}	Wall volume of space station
H	Foundation excavation depth
$a\ (f(R))$	The length of cuboid in cuboid model
$b\ (g(R))$	The width of cuboid in cuboid model
h	The height of the above ground part of the space station
r	Excavation radius of foundation in cylinder model
P_0	The compressive strength of basalt
g_m	The acceleration of gravity on the moon
V_{roof}	Volume of space station roof
G_{roof}	Gravity of space station roof
S	The area of the stress surface
P	The pressure on the wall of the space station

Note: The coefficient of heat conductivity a has the following relationship with the density ρ, specific heat capacity c and thermal conductivity λ of heat transfer medium:

$$a = \frac{\lambda}{\rho c} \tag{3.1}$$

The parameter a measures the ability of the medium to conduct heat [2]. The larger the value of a is, the faster the temperature transfer speed is, and the stronger the thermal conductivity of the medium is.

4. Heat Transfer Model of Space Station

The moon alternates day and night for about a month, so the space station will be exposed to extremely hot (about 160℃) sun, or extremely cold (about −180℃) shadow for a long time. There is no atmosphere on the surface of the moon. According to Assumption 4, we can take the temperature of the surface of the moon as the outermost temperature of the space station. The temperature is transferred along the wall of the space station. With the increase of the wall thickness, the steady temperature of the innermost wall tends to be flat. The inner surface of the station will have convective heat transfer with a layer of air nearby (tropospheric air), and then the tropospheric air will transfer the temperature to other places in the house along the gradient direction. According to Assumption 5, we don't consider the heat transfer between other objects in the station and the air.

We will use the numerical method to solve the heat conduction equation and determine the optimal wall thickness to make the temperature inside the station is stabilized at a comfortable level about 22 ℃.

4.1 Heat Transfer on the Wall of Space Station

Using the energy conservation equation, we can get the general differential equation of heat conduction as follows [2]:

$$\rho c\frac{\partial T}{\partial t}=\left[\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)\right]+\Phi \tag{4.1}$$

For the heat transfer on the wall of the space station, it is assumed that the temperature of the outermost layer of the space station is constant (constant temperature heat source) as the temperature of the moon surface, and the schematic diagram of heat transfer from the high temperature (low temperature) heat source in the wall is as follows:

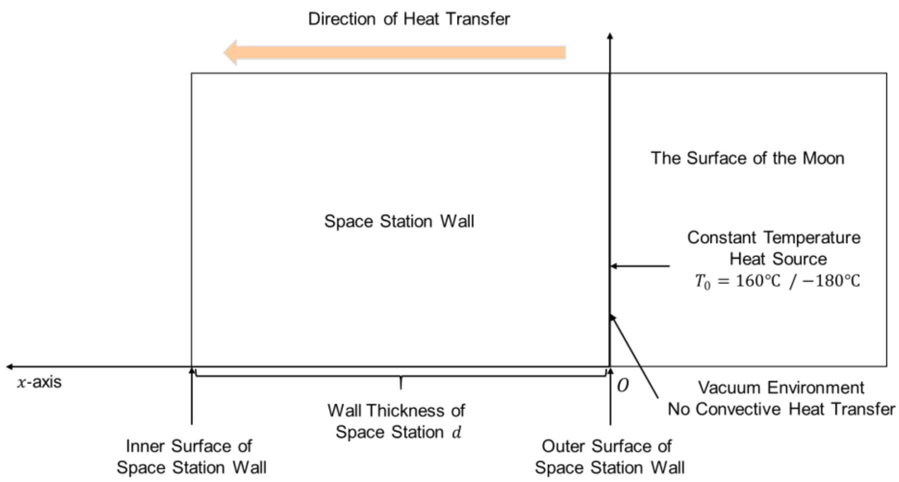

Figure 1: Heat transfer diagram of space station wall

Because of the assumption of uniform heat conduction (Assumption 2), the heat transfer process can be abstracted as one-dimensional heat transfer, and there is no spontaneous heat source inside the wall of the space station. Taking the outer surface of the space station wall as the coordinate origin, the coordinate axis is established along the heat transfer direction, and the positive direction points to the interior of the space station. Therefore, the differential equation of the heat transfer system can be simplified as:

$$\frac{\partial T}{\partial t} = \frac{\lambda_w}{\rho_w c_w}\frac{\partial^2 T}{\partial x^2} = a_w \frac{\partial^2 T}{\partial x^2} \tag{4.2}$$

On the left side of the above equation is the partial derivative of temperature with respect to time variable t, on the right side of the equation, the first term coefficient a_w is the thermal conductivity coefficient of material, which characterizes the ability of material to transfer temperature. The second term is the second-order partial derivative of temperature with respect to wall thickness variable x. In addition, the equation has the following important initial and boundary value conditions (constant temperature heat source condition):

$$\begin{aligned} &T(0,t) = T_0,\ \forall t \geq 0 \\ &T(x,0) = 0,\ x \in (0,d] \end{aligned} \tag{4.3}$$

By discretizing equation (4.2) in time and space [3], the difference equation is obtained：

$$\frac{T_n^{(i+1)} - T_n^{(i)}}{\Delta t} = a\frac{T_{n+1}^{(i)} - 2T_n^{(i)} + T_{n-1}^{(i)}}{\Delta x^2} \tag{4.4}$$

The upper corner of temperature T represents the time corner, and the lower corner is the space corner. For the convenience of iterative calculation, the above formula can be rewritten as forward difference scheme [3]：

$$T_n^{(i+1)} = \frac{a\Delta t}{\Delta x^2}\left(T_{n+1}^{(i)} + T_{n-1}^{(i)}\right) + \left(1 - 2\frac{a\Delta t}{\Delta x^2}\right)T_n^{(i)} \tag{4.5}$$

Δx and Δt represent length step and time step respectively. The value range of thickness x is $x \in [0, d]$, which is equally divided into N parts; similarly, a long enough time range is taken to ensure the system to reach steady state (here $t \in [0, 3600 * 24 * 14]$), and the time is equally divided into M parts. Then the value range of length step variable n and time step variable i is:

$$0 \leq n \leq N, 0 \leq i \leq M \tag{4.6}$$

In the calculation process of the above iterative program, in addition to the reasonable setting of the initial value of the temperature field, we also need to pay attention to the selection of the step size when constructing the difference equation. In formula (4.5), in the process of temperature transfer from time i to $i+1$, from a practical point of view, the higher the temperature at time i, the higher the temperature should be at time $i+1$. Therefore, we should guarantee that the coefficient before $T_n^{(i)}$ is not less than 0:

$$\frac{a\Delta t}{\Delta x^2} \leq \frac{1}{2} \tag{4.7}$$

Formula (4.7) is an important criterion to determine whether the thickness step and time step are reasonable when constructing the difference equation. It is a key guarantee for the convergence of the numerical solution.

According to Assumption 1, the material used to build the space station is basalt. The parameters to measure the thermal conductivity of basalt are density $\rho_w \approx 3050\ \mathrm{kg/m^3}$, specific heat capacity $c_w \approx 854\ \mathrm{J/(kg \cdot ℃)}$ and thermal conductivity $\lambda_w \approx 2.177\ \mathrm{W/(m \cdot ℃)}$. Substituting the data into the heat conduction equation, the conduction process of temperature in the space station wall is obtained as shown in the figure below:

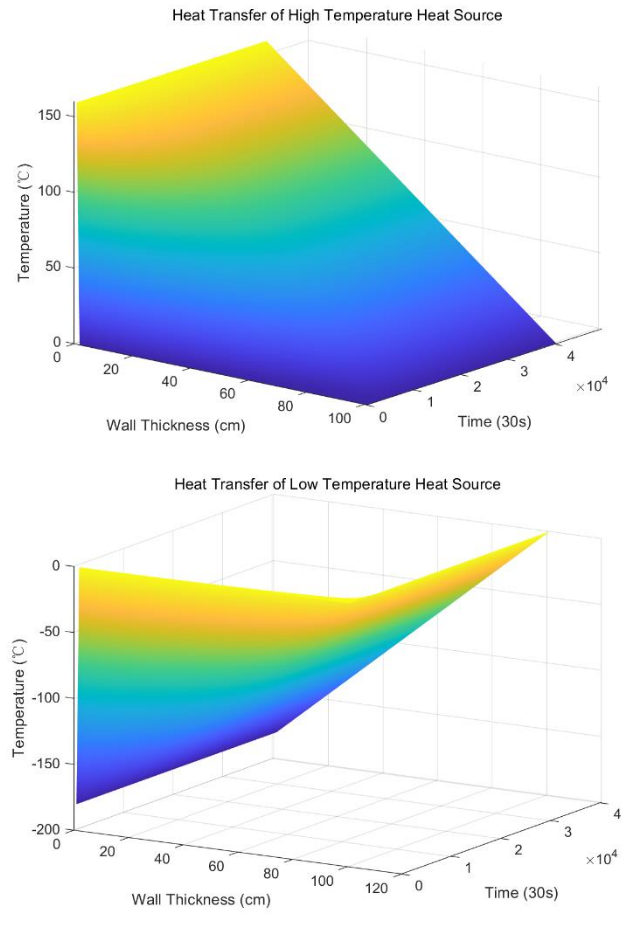

Figure 2: Temperature transfer of space station wall under high and low temperature heat sources

Taking the space station wall thickness of 10 cm, 30 cm, 50 cm, 70 cm and 90 cm as examples, the relationship between temperature and time is drawn as follows:

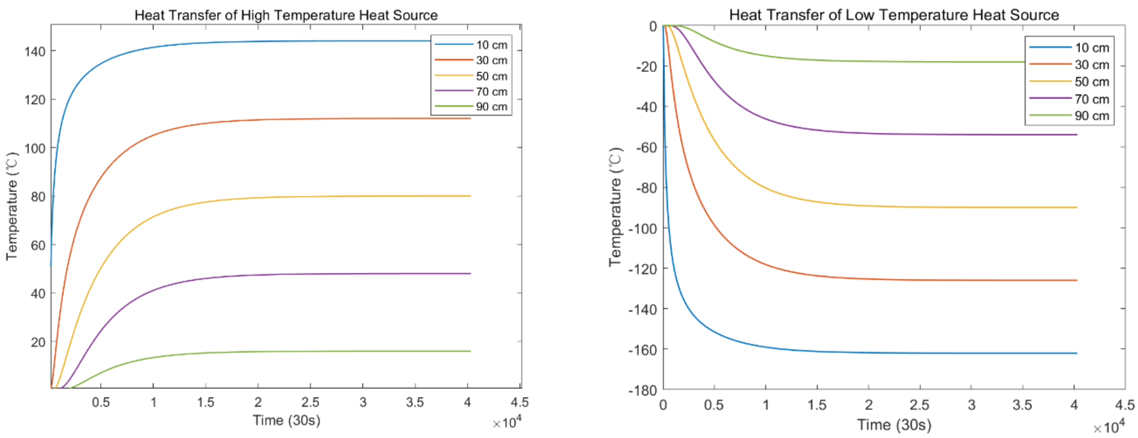

Figure 3: Conduction of temperature in space station wall of different thickness

The following figure shows the temperature distribution of the space station wall when the heat conduction reaches a steady state:

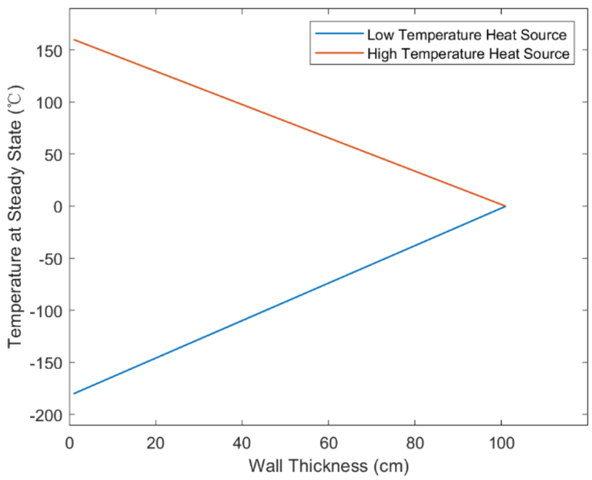

Figure 4: temperature distribution of the space station wall in steady state

As can be seen from the above figure, as the thickness of the wall increases, the steady-state temperature becomes “mild”. The inner surface of the wall will continue to transfer heat to the indoor air. In order to maintain the indoor temperature at about 22 ℃, the wall of the space station must be thick enough.

To calculate the required wall thickness, the latter two heat transfer models are needed, that is, the convective heat transfer between the inner surface of the space station and the indoor air, and the heat transfer of the indoor air.

4.2 Convective Heat Transfer between the Wall and the Indoor Air

We now consider the convective heat transfer between the wall and the air on the surface of the space station, and the heat transfer diagram is as follows:

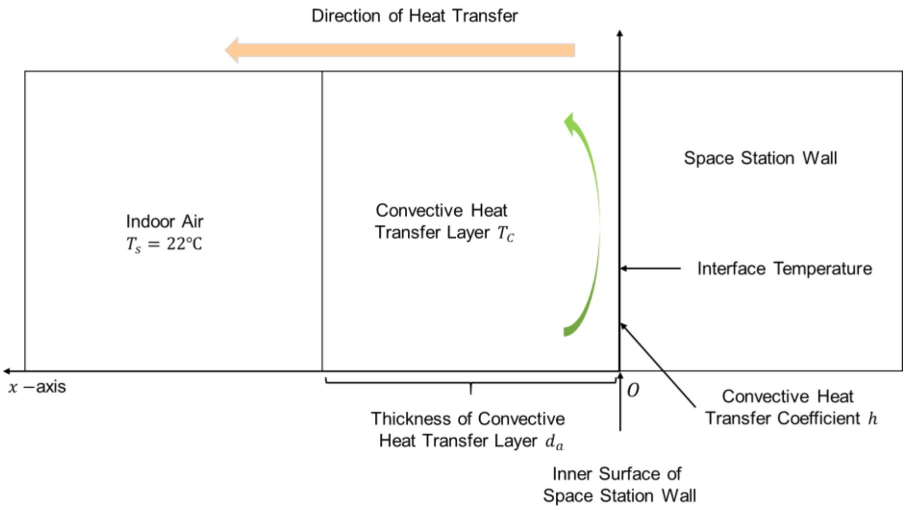

Figure 5: Heat transfer diagram of convective heat transfer

The thermal conductivity of air a_a is calculated from the density $\rho_a \approx 1.18\,\mathrm{kg/m^3}$, specific heat capacity $c_a \approx 1005\,\mathrm{J/(kg \cdot ℃)}$ and thermal conductivity $\lambda_a \approx 0.023\,\mathrm{J/(kg \cdot ℃)}$ of air. The thermal conductivity coefficient of air a_a is much larger than that of other substances, so its thermal conductivity is excellent. Therefore, we can assume that the temperature field of air is uniform, and the temperature is constant in the thinner air layer.

It is assumed that there is a 50 mm thick layer of air near the inner surface of the wall and convective heat transfer occurs between the wall and the air in the troposphere, and the temperature of the air in the troposphere is constant T_C. In a very short range of 50 mm, tropospheric air and indoor air at $T_s = 22℃$ achieve heat balance through heat transfer.

At the contact node between the inner surface of the station and the indoor air, the element with small thickness is taken for analysis. According to the law of conservation of energy [6]:

$$\lambda_w \frac{T_{N-1}^{(i)}}{\Delta x} + h\left(T_s - T_N^{(i)}\right) = \rho_w c_w \frac{\Delta x}{2} \frac{T_N^{(i+1)} - T_N^{(i)}}{\Delta t} \tag{4.8}$$

Using forward difference scheme, we can get:

$$T_N^{(i+1)} = T_N^{(i)}\left(1 - \frac{2h\Delta t}{\rho_w c_w \Delta x} - \frac{2a_w \Delta t}{\Delta x^2}\right) + \frac{2a_w \Delta t}{\Delta x^2} T_{N-1}^{(i)} + \frac{2h\Delta t}{\rho_w c_w \Delta x} T_s \tag{4.9}$$

Formula (4.9) can be used to calculate the temperature of contact surface between air and wall, which will be used to calculate the T_C.

The heat inflow from the outer troposphere minus the heat outflow from the inner troposphere is equal to the heat increment of the troposphere itself [6]:

$$h(T_{N-1}^{(i)} - T_C^{(i)}) - h_q(T_C^{(i)} - T_s) = \rho_a c_a d_a \frac{T_C^{(i+1)} - T_C^{(i)}}{\Delta t} \tag{4.10}$$

After simplification, the formula of T_C is obtained as follows:

$$T_C^{(i+1)} = \frac{h(T_{N-1}^{(i)} - 2T_C^{(i)} + T_s)\Delta t}{\rho_a c_a d_a} + T_C^{(i)} \tag{4.11}$$

We assume that the troposphere is naturally convective, the convective heat transfer coefficient $h \approx 15\ \mathrm{W/(m^2 \cdot K)}$. The temperature changes of the interface between wall and air, and convective heat transfer layer are as follows:

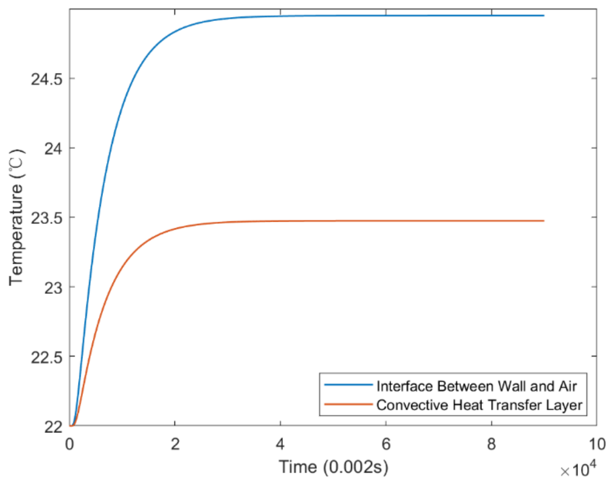

Figure 6: Temperature of interface and convective layer

4.3 Heat Transfer of the Indoor Air

According to Assumption 3, since the heat transfer of the internal air does not consider the air convection, we can use the model in Section 4.1 to calculate the temperature distribution. The only difference is that the density, specific heat capacity, thermal conductivity, and geometric thickness of the medium have changed.

Considering the heat transfer of the inner air can helps us to determine the range of temperature T_C in the convective heat transfer layer. If T_C is too high (or too low) than 22℃, the convective heat transfer layer will continue to transfer heat to the internal air, making the indoor temperature unable to stabilize at 22℃. Therefore, T_C should change around 22℃, and the critical value of T_C determines whether the convective layer can reach heat balance after heat exchange with indoor air in a very short distance (50 mm).

The heat transfer diagram of indoor air is as follows:

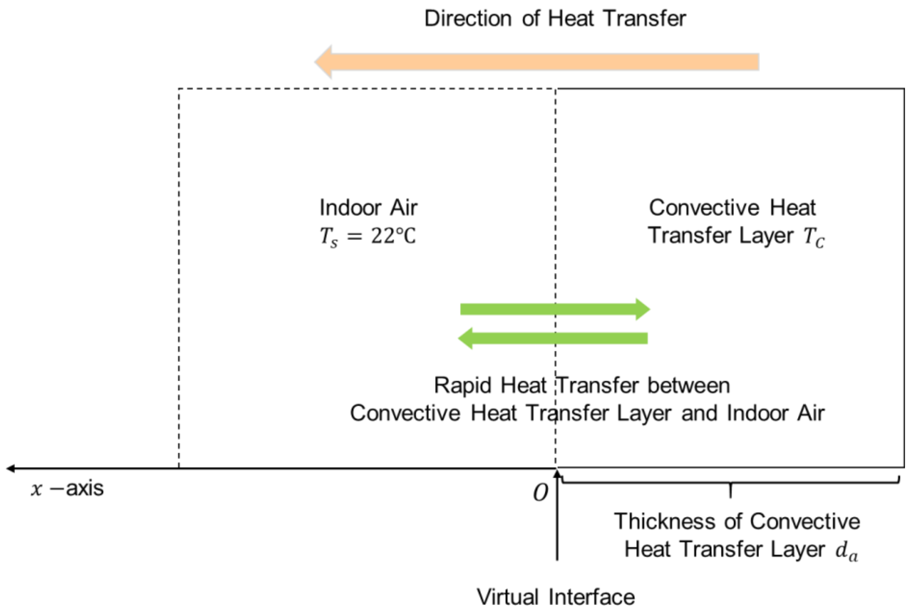

Figure 7: Heat transfer diagram of indoor air

4.4 Wall Thickness of Space Station

From the model in Section 4.3, to make the temperature T_C of convective heat transfer layer quickly exchanges heat in the range of 50 mm to recover the temperature indoor to about 22 ℃. We set an allowable error of 70% (the maximum difference between the indoor air temperature and the wall temperature). The allowable range of temperature T_C is obtained:

$$6.6℃ \le T_C \le 37.4℃ \tag{4.12}$$

Combined with three heat transfer models, the external ambient temperature was set at 160℃ and −180℃ respectively. By changing the wall thickness d of the station continuously, after enough heat exchange time (Half of the day night cycle on the moon, about 14 days), then check whether the value of T_C meets the requirements of formula (4.12). If not, increase the value of d until it reaches the critical value of T_C in (4.12). The minimum wall thickness d_h and d_l of the space station at high (160℃) and low (−180℃) temperature are calculated by numerical experiments:

$$d_h = 102 \text{ cm}, \quad d_l = 149 \text{ cm} \tag{4.13}$$

Take the maximum of the two above. Therefore, to resist extreme temperature changes on the moon, the wall thickness required for the space station is $d = 149$ cm.

5. Optimization of Space Station Construction Plan

We assume that the average height of the astronaut at the lunar workstation is 1.7m and the body shape is relatively uniform. The volume of the astronaut's body and its basic range of motion are approximated as a small cylinder with a smooth surface, a bottom radius of 1m and a height of 4m. We also assume that there is no mutual embedding and overlap between the cylinders. The schematic diagram of astronauts and their working space is shown below:

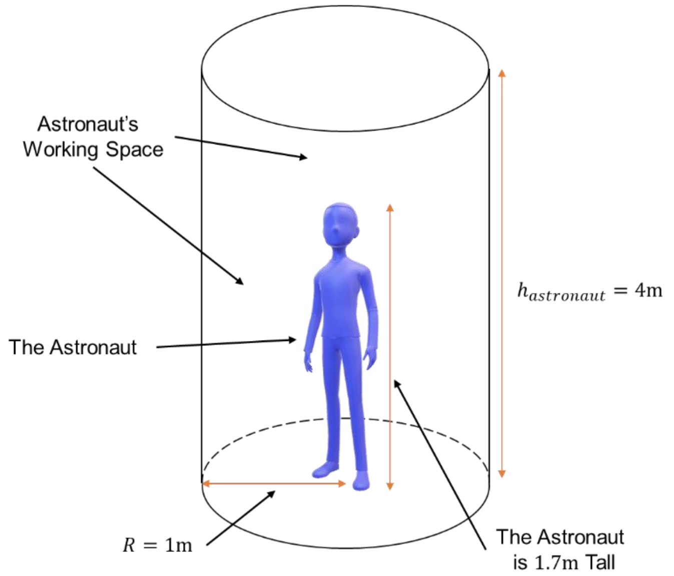

Figure 8: The schematic diagram of astronaut and his working space

Assuming that the excavation space and the walls of the house are geometric bodies with a smooth surface and uniform thickness, the small cylinder that represents the astronaut can be tangent to the inside of the wall, but cannot be embedded or contained in the wall.

Considering that the astronauts and their moving spaces are approximated as small cylinders, based on information about most existing space stations, to maximize the space utilization in the lunar station, we discuss the cuboid and cylinder geometry models separately.

5.1 Cuboid Model

Imagine the underground excavation space and the space above the ground which is enclosed by the walls as two cuboids spliced up and down. The wall of uniform thickness wraps the above-ground part into a larger cube (as shown in Figure 9). We choose this kind of model in that if the bottom area of the underground part is larger than the bottom area of the above-ground part, it will cause partial waste of the excavation space; if the bottom area of the underground part is smaller than the bottom area of the above-ground part, then the excavation must dig deeper to obtain enough materials for the walls' building and it will increase the difficulty of excavation.

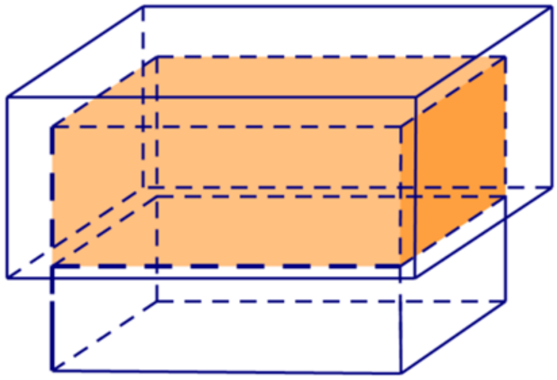

Figure 9: Cuboid model diagram

Suppose the foundation depth (excavation depth) is H meters, the length and width of the inner bottom surface of the workstation are a meters and b meters, respectively, the height of the above-ground part inside the workstation is h meters, and the thickness of the outer wall is d meters. According to the background of the question, the materials of the underground excavation will be used for the construction of the wall above the ground, so we have:

$$\begin{aligned} V_{base} &= abH \\ V_{wall} &= (a+2d)(b+2d)(h+d) - abh = V_{base} \end{aligned} \tag{5.1}$$

Besides, $h + H = h_{astronaut} = 4\text{m}$, so we have the formula:

$$h = \frac{4ab}{(a+2d)(b+2d)} - d \tag{5.2}$$

$$H = 4 - h = 4 + d - \frac{4ab}{(a+2d)(b+2d)} \tag{5.3}$$

The volume of the foundation can be calculated:

$$V_{base} = abH = (4+d)ab - \frac{4a^2b^2}{(a+2d)(b+2d)} \tag{5.4}$$

5.2 Cylinder Model

Like the cuboid model, we imagine the underground excavation space and the space above the ground which is enclosed by the walls as two vertically spliced cylinders. The walls of uniform thickness wrap the above-ground part into a larger cylinder (as shown in Figure 10).

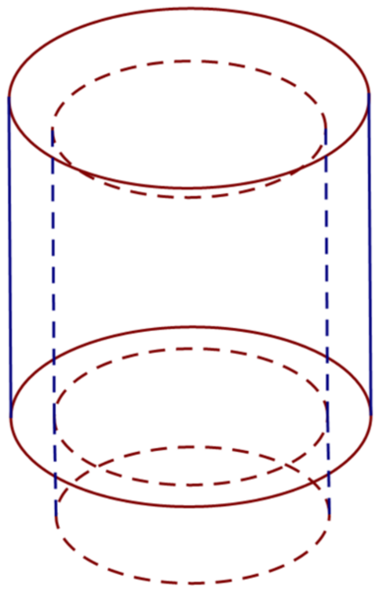

Figure 10: Cylinder model diagram

Similarly, suppose the depth of the foundation (excavation depth) is H, the radius of the inner bottom surface of the workstation is r, the height of the part above the ground inside the workstation is h, and the thickness of the outer walls is d. According to the background of the question, the materials of the underground excavation will be used for the construction of the walls, so we have:

$$\begin{aligned} V_{base} &= \pi r^2 H \\ V_{wall} &= \pi(r+d)^2(h+d) - \pi r^2 h = V_{base} \end{aligned} \tag{5.5}$$

Besides, $h + H = h_{astronaut} = 4\text{m}$, so we have the formula:

$$h = \frac{4r^2}{(r+d)^2} - d \tag{5.6}$$

$$H = 4 - h = 4 + d - \frac{4r^2}{(r+d)^2} \tag{5.7}$$

The volume of the foundation can be calculated:

$$V_{base} = \pi r^2 H = (4+d)\pi r^2 - \frac{4\pi r^4}{(r+d)^2} \tag{5.8}$$

The above two schemes (the cuboid model and the cylinder model) will be used to optimize the subsequent construction scheme.

5.3 Optimization of the Space Station Construction

According to the geometry of the workstation, we discuss the space optimization in the station under two circumstances: cuboid model and cylinder model. The problem turns into finding the rectangle or circle with the smallest area so that it can accommodate 20 small circles with a radius of 1m. We will discuss the calculation of the two models.

5.3.1 Optimization of Cuboid Model

- **Simple Accumulation:**

Since the number of astronauts in the workstation is 20, which is not a square number, so if we use simple accumulated packing (a small circle is tangent to the surrounding four small circles), the integer 20 can be decomposed into 4×5 and 2×10 (closest to square decomposition), the resulting rectangles and squares are as follows:

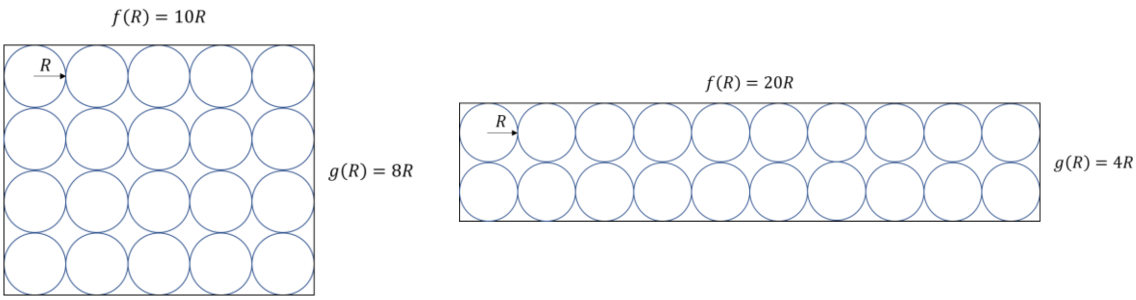

Figure 11: Simple accumulation diagram

Take $a = 10$ m,$b = 8\ m$, $d = 1.49$ m into the formula (5.2), then we get $h \approx 0.7553$ m, furthermore, we have the volume of the excavation base:

$$V_{base} = abH = 259.58\ \mathrm{m}^3 \tag{5.9}$$

As comparison, take $= 20$ m, $b = 4\ m$, $d = 1.49$ m into (5.2), then we get $h \approx 0.505$ m， furthermore, we have the volume of the excavation base.

$$V_{base} = abH = 279.6\ \mathrm{m}^3 \tag{5.10}$$

- Dense Accumulation:

Now we consider the dense accumulated packing where a small circle is tangent to the surrounding six small circles. It can be proved that this kind of packing has the highest area utilization when the number of small circles is large enough (or the area of a single small circle is small enough). Pack the small circles in this way to get the following two arrangements: (2×10 and 1×20 decompositions are not discussed here because they do not meet the dense accumulated packing requirement).

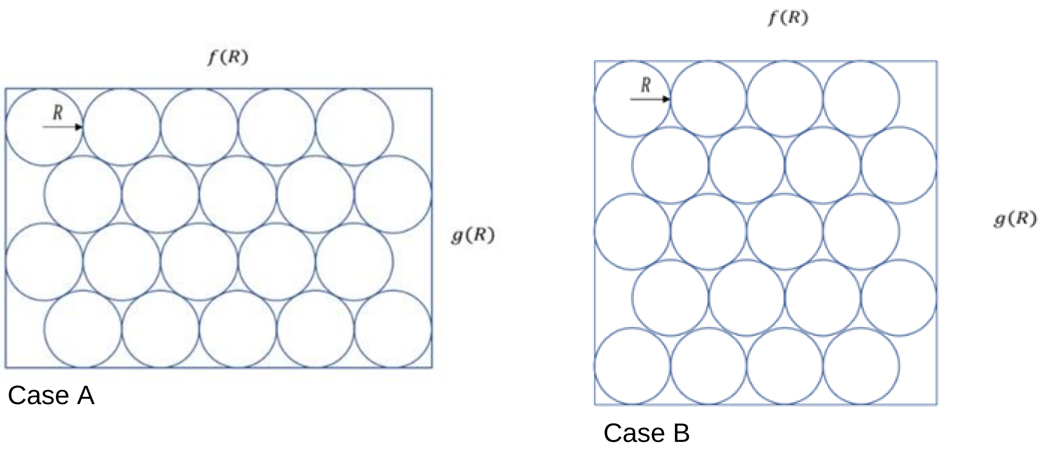

Figure 12: Dense accumulation diagram

(1) Case A:

The small circles in the rectangle are arranged by 5×4 (see Figure 13 below), and the length and width are calculated as $a = 11$ m and $b = \left(3\sqrt{3} + 2\right)$ m. We take a, b, d into the formula (5.2), then we get $h \approx 0.7357$ m. We have the volume of the excavation base:

$$V_{base} = abH = 258.39 \text{ m}^3 \tag{5.11}$$

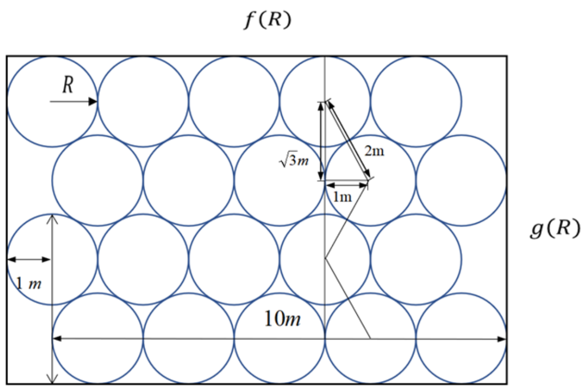

Figure 13: Geometric parameters of Case A

(2) Case B:

The small circles in the rectangle are arranged by 4×5 (see Figure 14 below), and the length and width are calculated as $a = 9$ m and $b = \left(4\sqrt{3} + 2\right)$ m. We take a, b, d into the formula (5.2), then we get $h \approx 0.763$ m. We have the volume of the excavation base:

$$V_{base} = abH = 260.10 \text{ m}^3 \tag{5.12}$$

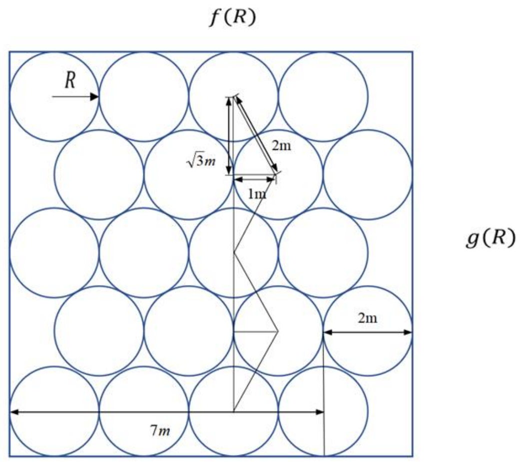

Figure 14: Geometric parameters of Case B

5.3.2 Optimization of Cylinder Model

In the end we discuss the case of the cylinder. In our model, the problem of studying the arrangement of small circles on the bottom of a cylinder can be transformed into using a large circle to include 20 small circles with a radius of 1m that do not coincide with each other, and find the minimum radius of this large circle.

- **Simple Accumulation:**

It is easy to prove that the area utilization rate of this packing method is low, so we skip this part and jump directly to other methods.

- **Optimal Packing Method [6]:**

Based on computer simulation of controlled circle packing, we have the conclusion that the smallest circle to include 20 small circles with radius of 1 has a radius of about 5.122, [6].

Take $r = 5.122$ m and $d = 1.49$ m into the formula (5.6), and then we get $h \approx 0.9103m$, furthermore, we have the volume of the excavated base:

$$V_{base} = \pi r^2 H = 254.65 \text{ m}^3 \tag{5.13}$$

5.4 The Best Construction Plan

Based on all the above results, the plan with the smallest volume of excavated foundation is selected. We decided to use the cylinder model. The excavation radius of the foundation is $r = 5.122$m, the excavation depth is $H = 3.0897$m, and the wall thickness of the space station is $d = 1.49$m. At this time, the height of the ground part of the space station is $h = 0.9103$m, and the foundation volume to be developed is $V_{base} = 254.65 \text{ m}^3$.

6. Structural Stability of Space Station

There is no atmosphere on the moon, so weathering effects are not considered. In this section, we mainly discuss the load-bearing capacity of the side walls of our chosen cylinder model. The data show that the compressive strength of basalt can reach $P_0 = 300$ MPa, and the acceleration of gravity on the moon $g_m \approx 1.63\ \mathrm{m/s^2}$ is about one sixth of that on earth.

We divide the cylinder model of the space station into two parts: the roof and the side wall. The wall needs to bear the gravity of the roof. We assume that the gravity of the roof uniformly acts on the ring area of the wall, the force diagram is as follows:

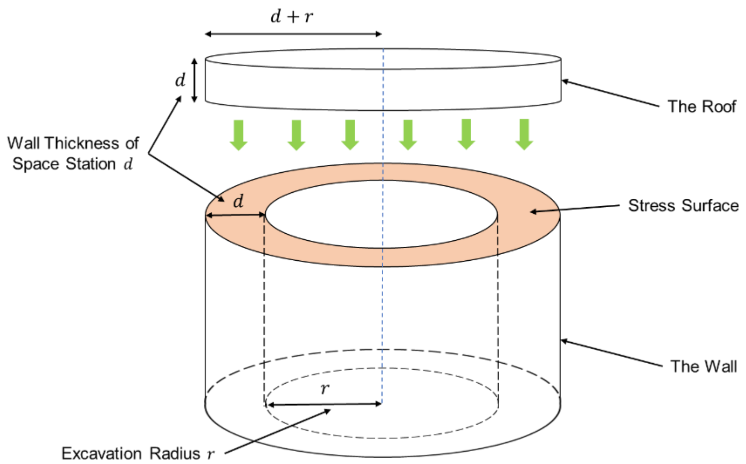

Figure 15: Force diagram of wall

Using volume, density ρ_w and acceleration of gravity g_m, the gravity of the space station roof G_{roof} is calculated:

$$G_{roof} = \rho_w V_{roof} g_m = \rho_w \left[\pi (r+d)^2 d \right] g_m \approx 1017394\ \mathrm{N} \tag{6.1}$$

Calculate the area of the stress surface:

$$S = \pi (r+d)^2 - \pi r^2 = 54.9265\ \mathrm{m}^2 \tag{6.2}$$

Calculate the pressure on the wall of the space station:

$$P = \frac{G_{roof}}{S} \approx 18523\ \mathrm{Pa} \ll 300\ \mathrm{MPa} = P_0 \tag{6.3}$$

It can be seen from the above calculation that the basalt used for the construction of the space station has extremely high compressive strength ($P_0 = 300$ MPa), and the pressure on its stress surface is far less than the compressive strength, so the structure of the cylinder model space station is stable and reliable.

7. Model Evaluation

7.1 Model Merits

To give the most suitable construction plan to meet the requirements of heat insulation and working space, we designed heat transfer and space optimization models and they perform better than others in following reasons:

- The numerical method used to solve the differential equation guarantees a rational and explainable output as the ideal thickness of the wall, and the discussion of the iteration step makes sure the accuracy of the equation's solution.
- To make the plan closer to reality, we take the heat transfer between wall and indoor air into consideration as well so that the actual temperature indoor and thickness of wall will be much more reasonable.
- The geometry model of human body and excavation base makes the calculation and of volume easier and helps to cope with irregular shape of subjects in the station, further, to obtain the optimization of the volume.

7.2 Error Analysis and Model Improvements

The construction plan of the lunar station in former sections is based on some assumptions which may lead to errors in actual operations as below:

- The one-dimensional heat transfer is set based on the assumption that the heat deduction in all medium is uniform and the heat source is constant, however, these two conditions may change because the density of the basalt may be different from different directions (pores or uneven distribution) and the temperature on the moon may fluctuate owing to other factors.
- In the model we assume that the convection does not occur when the heat is transferred in the air while in practical circumstance the convection among the air can't be ignored.
- In the geometric model of astronauts and the station, we only consider common geometries and assume that the surface of the wall is smooth. In fact, the irregular shape of equipment and 3D-print outputs may influence the calculation of the volume needed.
- As the optimal packing of circles is still a big challenge to even the cutting-edge scholars, the prove of the minimum radius of the big circle to include 20 small circles with radius of 1 meter is still in process and we have tried to carry out a dynamic stimulation of circles to figure out the best configuration.

7.3 Generalization

When we consider the general optimization problem with fixed volume demand, we can consider using the optimization strategy in Section 5. Because the volume is equal to the product of the bottom area and the height, and the volume increases monotonically for both. From the perspective of space saving, we can first minimize the bottom area of the geometry, and then minimize the height of the geometry. This strategy can simplify the problem and be extended to optimization problems such as freight transportation.

8. Reference

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[2] Crank, J. , and P. A. Nicolson . "A practical method for numerical evaluation of solutions of partial differential equations of the heat-conduction type." Advances in Computational Mathematics 6.1(1996):207-226.
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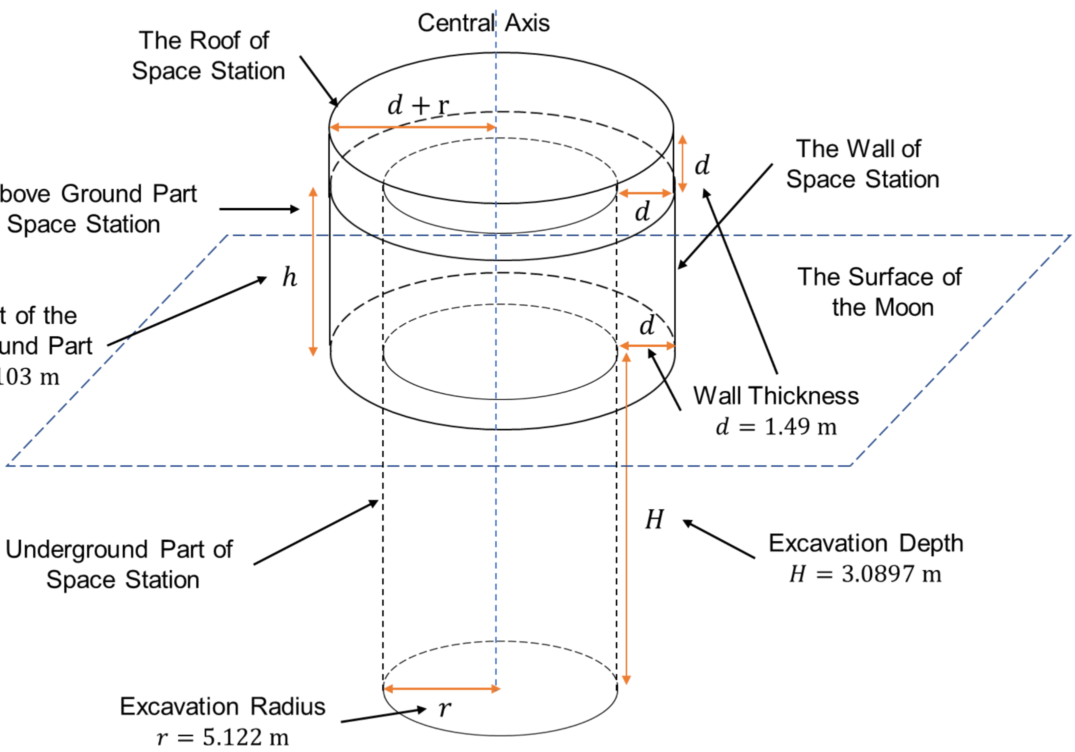
[5] Wenqi, Huang , et al. "Quasi-Physical Algorithm for the Equal Circles Packing Problem" System science and Mathematics, 28.8(2008):993-1001.
[6] Tao Wenquan. Numerical heat transfer. Xi'an Jiaotong University Press, 2001.

9. Attachments

- **Three-dimensional structure of space station**



A 1: Structure diagram of space station

- **Calculation of heat conduction (MATLAB)**

```
clear all;
a = 0;
b = 1;
c = 0;
d = 3600*24*14;
h = 0.01;
tau = 0.2;
Lambda = 2.177;
C = 854;
rho = 3050;
rho2 = 1.293;
c2 = 1005;
hd = 0.05;
q = 15;
alpha = Lambda/(C*rho);
n = (b-a)/h;
m = (d-c)/tau;
T = zeros(m+1,n+2) + 22; % Initial temperature
T(:,1) = zeros(m+1,1) - 180; % Boundary value temperature
r = alpha*tau/h^2;
for i=1:m
    t_next = zeros(1, n);
    % Forward difference scheme
    for j = 2 : n-1
        t_next(j-1) = r*(T(i, j+1) + T(i, j-1)) + (1 - 2*r)*T(i, j);
    end
    % Contact surface temperature
    t_next(n-1) =  T(i,n)*(1 - 2*q*tau/(rho*C*h) - 2*alpha*tau/h^2) + 2*alpha*tau*T(i, n-1)/h^2 + 2*q*tau*T(i, 1 + n)/(rho*C*h);
    % T_C
    t_next(n) = T(i, 1+n) + q*tau*(T(i,n) + T(i, 2+n) - 2*T(i, 1+n))/(hd*rho2*c2);
    T(i+1,2:n+1) = t_next;
end
% surf(T);
% shading interp
```