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Summary Sheet

OFF-THE-GRID: A New Choice of Life

Abstract

It is very expensive to connect to the power grid in remote areas. However, with the development and maturity of battery technology, people can use a set of energy storage system to absorb the energy of the sun through the solar panel and convert it into chemical energy in the battery in a sunny day, to supply people with night lighting, kitchen cooking, sanitation, and entertainment. The construction of off-the-grid power storage system and the purchase of batteries are closely related to the power demand. Based on the power demand analysis, we establish a model to select the best configuration scheme of energy storage system.

Through assuming family members (including father, mother, a son, and a daughter), as well as the electrical appliances they use, we establish an optimal configuration scheme model of energy storage system for a 1600 square foot home. Electrical appliances are divided into high-power equipment working for a short time and low-power equipment working for a long time, and different types of batteries are selected for the two types of equipment. Based on the power demand of the whole family, a multi-stage linear integer programming model is established by comprehensively considering the cost, the round-trip efficiency of the battery and the volume of the system. The model adds the power threshold coefficient and considers the influence of cloudy weather on the charging efficiency, to ensure the normal operation of the system under sudden high load power demand. The results show that under our assumption, we need to purchase one Trojan L-16 -SPRE 6V 415 battery, one Discover AES 7.4 kWh battery and two Tesla Powerwall+ batteries. The whole system costs $23,970 and can provide a total capacity of 36.90kWh. The continuous power rating of low-power battery packs can reach 0.19kW, and the continuous power rating of high-power battery packs can reach 20.65kW. The peak power rating of the entire battery pack can reach 34.4kW

Based on the above model, we generalize the establishment and solution process of the model. Anyone can easily build the most suitable system optimal configuration model according to their own needs. The optimization objectives of the model are divided into two types. First, with sufficient budget, we can fix the power supply demand and minimize the construction cost of the system; Second, in the case of limited budget, we can fix the system construction cost and maximize the power supply capacity of the battery pack. Both optimization models are linear integer programming problems, so they can be solved by simplex method. We also give a grid search method when the alternative battery category is not too large.

Finally, we discuss the possibility and application of cement in energy storage and compare the advantages and disadvantages of new cement battery for solar energy storage with the traditional one. We point out that in order to add the cement battery to the optimization model, we also need to know the continuous and instantaneous power, price, volume, and the capacity of the battery.

Keywords: Linear Integer Programming, Simplex Method, Multistage Optimization

Contents

1. Introduction

1.1 Restatement of the Problem

The cost of connecting to the grid is expensive when living in a remote area, thus, we can consider build an energy storage system to provide enough electricity at night and on rainy days through collecting solar energy. When building our off-the-grid energy storage system, there are many problems that need to be carefully considered and optimized. In short, we need to complete the following tasks:

- Through a detailed discussion of the use scenarios and various factors that determine the energy storage system. Establish an optimal model of the system for a 1600 square foot home and figure out the optimal configuration scheme.
- Generalize the model above to make it applicable to any individual.
- Consider how the new battery (cement battery) affects the optimal configuration.

1.2 Our Work

When considering the construction of an off-the-grid-home power system, there are too many uncertain factors, especially the use of electrical appliances and the charging of the battery in rainy weather, which will affect the configuration of the system. To this end, we collected a lot of data and made many assumptions for actual scenarios to simplify the complex calculations of the model.

In Section 4, we gave a detailed description of the composition of members in a 1,600-square-foot home, including each person's age, occupation, electricity usage habits, and electricity usage time. Regarding the electrical and electronic equipment at home, we gave the rated power and working hours of each piece of equipment when it is working. We discussed the power consumption scenario in detail and established a multi-step linear integer programming problem with the power supply capacity of the system as the constraint condition and the cost of the system as the objective function. The solution of these optimization problems can help us determine the optimal configuration scheme that comprehensively considers the construction cost, battery's Round-Trip Efficiency and battery's volume.

In Section 5, we generalized the discussion, establishment, and solution process of the model, and gave a method to extend the model to any individual. The promoted model has two optimization directions. On one hand, we can consider the fixed power supply demand to minimize the construction cost of the power system; on the other, considering that in real life, the budget of the power storage system is limited, we can fix the maximum construction cost to maximize the power supply ability of the system. Based on various assumptions and model's characteristics, we discussed the advantages and disadvantages of the model.

In Section 6, we discussed the possibility of adding new cement batteries to the off-the-grid power system. According to the principles and characteristics of cement batteries, some interesting ideas for building energy storage systems are given. At the same time, we pointed out the information needed to add the new cement to the optimization model.

In Section 7, we summarize the discussion and solution results of the model as a non-technical news article. Sections 8 and Section 9 are references and appendices, respectively.

2. Model Assumptions

1. When building an energy storage system for a 1,600-square-foot home, it is assumed that we have sufficient budget to ensure that the energy storage system meets any power demand.
2. Assuming that under good sunlight conditions, the battery pack can be quickly charged in a short time, and the power consumed by each electrical appliance during the charging process is negligible.
3. To simplify the calculation, the case that the battery pack is charging and discharging at the same time is not considered.
4. It is assumed that the attenuation ratio of the solar energy absorbed by the battery pack is fixed and quantifiable when it is charged under rainy weather.

(**Note**: More assumptions about specific issues will be given in the text when discussed)

3. Notations

Symbol	Description
$P_{max}^{(low/high)}$	Maximum power rating needed by low/high power appliance while working
$P_i^{(low/high)}$	Rated power of the low/high power appliance while working
$N_i^{(low/high)}$	Number of the low/high power appliance
$T_i^{(low/high)}$	Single working time of low/high power appliance
ε	Threshold ratio of maximum power rating
$C^{(low/high)}$	Capacity of low/high power battery pack
$UC^{(low/high)}$	Battery capacity required for low/high power appliance to work for one night
$w_i^{(low/high)}$	Unite price for low/high power batteries
$n_i^{(low/high)}$	Number of low/high power batteries
$p_i^{(low/high)}$	Continuous power rating of low/high power battery
$c_i^{(low/high)}$	Battery capacity of low/high power battery
$v_i^{(low/high)}$	The volume of a single battery
$r_i^{(low/high)}$	The round-trip efficiency of a single battery
$g_i^{(low/high)}$	The mass of a single battery
r_1	Attenuation ratio of battery charging efficiency in cloudy or rainy days
r_2	The reduce proportion of power consumption in cloudy or rainy days
RTE	Battery round-trip efficiency

4. Planning the Off-the-grid Home

4.1 Factors We Consider

We will consider the power system design of off-the-grid home from practical demands. We divide the factors under consideration into four levels: How many and what kind of people will live in this home? What are appliances needed to use? How to plan the cost of this off-the-grid-home power system? What are other factors that may influence the optimization goals? All the above will be discussed in turn below. The modeling process of Section 4.1 is as follows:

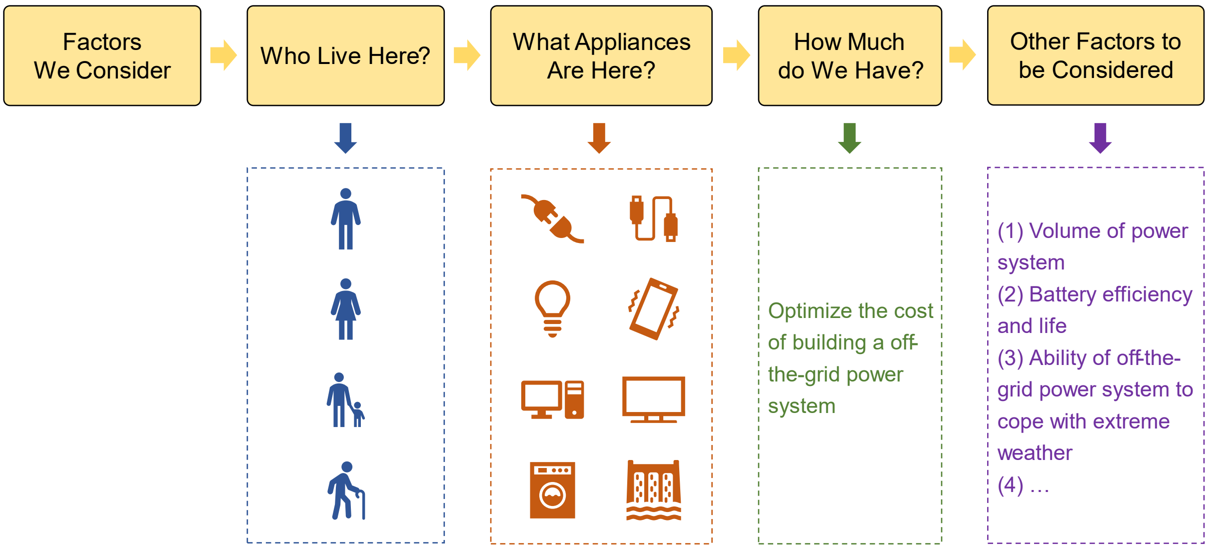

Figure 1： Modeling process in Section 4.1

4.1.1 Who Lives Here?

For the considered 1,600 square foot off-the-grid family, we assume that there are four people living in this home, including a 13-year-old junior high school girl, a 16-year-old high school boy, and their parents. Table 1 gives a description of each person's characteristics, and the electrical or electronic equipment that each person will use. Table 2 shows some electrical appliances shared by four people at home. All our subsequent analyses will be based on the assumptions and data listed in the table.

Table 1: Family member characteristic description

Role	Characteristics & Job Description	Electrical & Electronic Equipment
Son	16 years old, high school student. He goes to school from Monday to Friday, 8 a.m. to 6 p.m., and stays at home in the evenings. He may stay at home at weekends or go out for activities.	One mobile phone, one laptop, one desk lamp and other household appliances

Daughter	13 years old, junior high school student. She goes to school from Monday to Friday, 8 a.m. to 6 p.m., stays at home in the evenings. She may stay at home at weekends or go out for activities.	One laptop, one desk lamp and other household appliances
Father	45 years old. He goes to work every day from 9 a.m. to 6 p.m., stays at home in the evenings and most time of the weekend.	One mobile phone, one laptop and other household appliances
Mother	40 years old. She goes to work every day from 9 a.m. to 5 p.m., and then prepares dinner for the family. She stays at home in the evenings and most time of the weekend.	One mobile phone, kitchen appliances and other household appliances

Table 2: Household Appliances

Type	
Kitchen Appliances	Refrigerator, dishwasher, induction cooker, electric oven, microwave oven, electric water heater, range hood, toaster, dishwasher, electric kettle, rice cooker
Clean Appliances	Washer, dryer, vacuum cleaner, hair dryer, electric iron
Temperature Control	Central air conditioner, electric heater, electric fan
Lighting	Incandescent bulb, energy-saving bulb
Entertainment	Television
Electronic Equipment	Mobile phone, laptop, tablet computer, desktop, radio

4.1.2 What Appliances Are Here?

Table 2 lists the electrical and electronic products required according to the electricity demand of each family member in Table 1. When designing the off-the-grid power system, we also need to understand the rated power of each electrical appliance when working. Only when the provided power is greater than the rated power can the electrical appliance work normally. Table 3 shows the rated power of each electrical appliance in Table 2 we collected from the Internet [1], the number of electrical appliances required according to the demand, and the working time of each electrical appliance when it is used.

Table 3: Power Rating and quantity of electrical appliances

Appliances	Power Rating (kW)	Quantity	Working hours per use
Refrigerator	80-120	1	Long working time
Washer	200-500	1	1
Dryer	500-2000	1	0.5
Central A/C	2200-2600	1	Long working time
Induction Cooker	1000-2000	1	1-2
Electric Oven	1000-2000	1	1-2
Microwave Oven	1000-1500	1	1
Water Heater	1200-2000	1	1
Toaster	750	1	0.5
Rice Cooker	1200	1	1
Range Hood	100-200	1	1-2
Dish Washer	800-1500	1	0.5
Television	80-150	1	Long working time
Incandescent Bulb	50	2	Long working time
Energy-saving Bulb	10	4	Long working time
Desk Lamp	10	3	Long working time
Electric Iron	500-800	1	0.5
Electric Fan	40	2	Long working time
Vacuum Cleaner	400-800	1	1
Electric Heater	800-2000	1	Long working time
Mobile Phone	5-50	3	Long working time
Tablet Computer	20-50	1	Long working time
Laptop	60-100	1	Long working time
Desktop	150-300	1	2-6

4.1.3 How Much do We Have?

The cost of building an off-the-grid power system is not cheap, and the electricity demand of different appliances is flexible rather than fixed. We assume that there are enough funds to construct a power system to meet any power demand, and the cost of the power system will be discussed further in Section 4.2 as the objective of the optimization problem. We hope to meet the power demand and minimize the cost of our power system at the same time.

4.1.4 Other Factors to be Considered

Despite meeting the power demand and minimizing the construction cost of the power system, there is still room for improvement of the construction plan for this system. Some other issues may worthy of our attention are as follows.

(1) Efficiency and lifespan of the battery

The use of a battery with a higher Round-Trip Efficiency can improve the energy efficiency of the battery when it is discharging. In addition, frequent charging and discharging will shorten the battery's life and maximum battery capacity. Choosing a high-quality battery with a longer life can optimize subsequent maintenance costs

(2) The size of the power system

The construction of the power system needs to open additional space for the battery pack. In the case of ensuring the power supply and the cost of the battery pack, the total volume of the battery pack must also be considered. Reducing the volume ratio of the battery pack can reduce space occupation, reduce the construction cost of supporting facilities (such as a bracket specially built for the battery pack), reduce subsequent maintenance costs, and make the entire power system more beautiful.

(3) How to deal with extreme weather

The designed power system must not only be able to support a whole night of household electricity, but we also need to consider the battery's abilities of energy storage and power supply in consecutive extreme weather conditions during which the solar energy received is rather limited. In the discussion in Section 4.2, we assume the maximum number of consecutive cloudy and rainy days and give the assumption of the attenuation ratio of solar energy absorbed when the battery pack is charged under such weather.

4.2 Model to Choose the Best Battery Storage System

After determining the rated power, the number of equipment, and the operating time of each electrical appliance, we can determine the maximum peak power output that the off-the-grid power system needs to carry, and the maximum battery capacity required for various usage scenarios. We will optimize the design of the power system from the aspects of the battery pack construction cost, battery cycle efficiency, and battery volume. The model adopts a step-by-step optimization strategy, that is, each step of optimization is solved in the feasible region of the previous optimization problem.

The modeling process of Section 4.2 is shown in Figure 2.

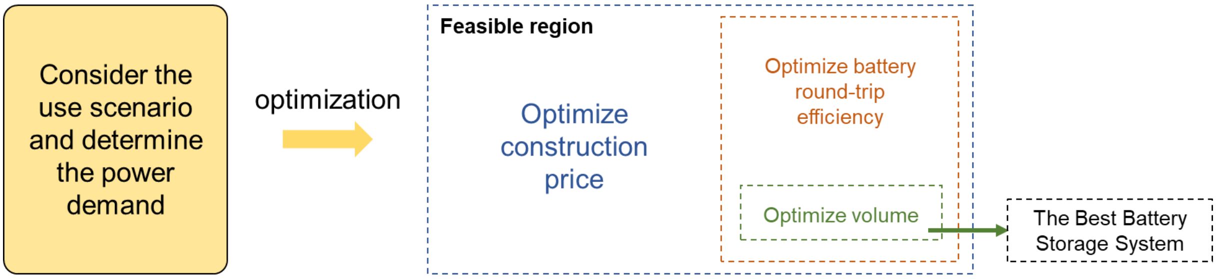

Figure 2: Modeling process in Section 4.2

4.2.1 Power Demand

Considering the maximum load scenario, the simultaneous working status and use time of all appliances, the off-the-grid power system needs to meet the requirement that the output power is greater than the sum of the rated power of the appliances, and the battery capacity needs to be greater than the sum of the power required by the appliances in the working time. For the convenience of calculation, according to the assumption, we do not consider charging and discharging the battery at the same time but assume that the battery pack can be quickly charged in a short time under the condition of good sunlight, and the power that the appliances consume during the charging process is ignored.

We first don't consider the impact of rainy weather on the charging efficiency of the battery pack. Since there is no one at home during the working day, and all four of the family members return home at about 6 p.m., we only need to consider whether the battery pack can support the family's electricity demand from 6 p.m. to the next morning.

Different battery should provide support for different power needs in the home, and their maximum output power and battery capacity vary greatly. Therefore, we will discuss short-term high-power electrical equipment (such as induction cookers, dryers, electric ovens, etc.) and long-term low-power electrical equipment (such as refrigerators, lighting systems, and mobile devices).

(1) High-power electrical equipment for short-term work

For high-power electrical equipment that works for a short time, the largest power consumption scenario is that the following appliances work at the same time: washers, dryers, A/Cs, induction cookers, electric ovens, microwave ovens, electric water heaters, toasters, rice cookers, range hoods, dishwasher, electric iron, vacuum cleaner, electric heater, and desktops. When the rated power is among some range, we take the average value to represent its rated power. Calculate the maximum output power required by the battery pack $P_{max}^{(high)}$

$$P_{max}^{(high)} = 15100\text{W} = 15.1\text{kW} \tag{1}$$

Therefore, when the output power of the battery pack reaches 15.1kW, it can satisfy the demands of all high-power electrical appliances. But from the perspective of practical use and cost optimization, we do not necessarily need to satisfy all high-power appliances to work together. Living in an off-the-grid home, we can design the order of using high-power electrical appliances. By peak-shift use, the maximum output power required by the battery pack can be greatly reduced.

We consider a variety of scenarios and believe the scenario with the largest electricity demand is "when a family of four arrive home, mom and dad are cooking in the kitchen", currently, the induction cooker, range hood, microwave oven, rice cooker, air conditioner and desktop computer work at the same time. Other electrical appliances, such as washers, dishwashers, dryers, vacuum cleaners, electric kettles, etc., can be used later. After reducing the scale of electricity consumption, the maximum output power required by the battery pack $P_{max}^{(high)}$ is calculated.

$$P_{max}^{(high)} = 6725\text{W} = 6.725\text{kW} \tag{2}$$

The required output power is reduced by 55% compared to the original value 15.1kW. In addition, in order to ensure the normal operation of all electrical appliances, we need to set a safety threshold based on the maximum output power, which means given $\varepsilon > 0$, the maximum output power of the high rated power battery pack $P_{max}^{(high)}$ is set as:

$$P_{max}^{(high)} = P_{max}^{(high)} \times (1+\varepsilon) \tag{3}$$

In this way, on one hand we guarantee the basic electricity demand, even if there are additional electrical appliances that need electricity in an emergency, the normal operation of the entire power system can be ensured. In this problem, we take $\varepsilon = 20\%$ and get $P_{max}^{(high)} = 8.07\text{kW}$.

Below we consider the electricity consumed by these high-power appliances in one night. We notice that in addition to air conditioners and electric heaters, all other high-power appliances can complete their work in a short time. If we exclude the two appliances for now, and assume that all other high-power electrical appliances need to be used in one night, the battery capacity that the battery pack needs to provide $C^{(high)}$ is:

$$C^{(high)} = \sum_{i=1} P_i^{(high)} T_i^{(high)} N_i^{(high)} = 14550\text{Wh} = 14.55\text{kWh} \tag{4}$$

Now consider the air conditioner and electric heater. In hot summer or cold winter, the central air conditioner needs to work continuously for about 12 hours (from 7 pm to 7 pm the next day). Therefore, the electricity required for the central air conditioner for one night is:

$$C = 28800\text{Wh} = 28.8\text{kWh} \tag{5}$$

If an electric heater is used, the amount of electricity C required for the electric heater to work for one night is:

$$C = 16800\text{Wh} = 16.8\text{kWh} \tag{6}$$

From formula (5) and (6), we can find that whether it is an air conditioner or an electric heater, the amount of electricity required for long-term work is huge. Moreover, if the battery capacity of the entire battery pack is increased for the air conditioner alone, when the air conditioner is not working, the excess battery capacity will become a great waste of resources. Considering the saving of energy and cost, we assume that the use of air conditioners or electric heaters can be intelligently managed, and the effective working time ratio at night is 50%. Therefore, the required battery capacity C is:

$$C = 28.8\text{kWh} \times 50\% = 14.4\text{kWh} \tag{7}$$

Integrated with other high-power electrical appliances, the entire battery capacity $C^{(high)}$ that provided by the off-the-grid power system for high-power electrical appliances in one night is:

$$C^{(high)} = 14.55\text{kWh} + 14.4\text{kWh} = 28.95\text{kWh} \tag{8}$$

Next, we need to consider the impact of rainy weather on the working status of the battery pack. Assuming that in the daytime of rainy weather, due to lack of sufficient sunlight, the available power of the battery pack can only reach 50% of the maximum battery capacity. Under the extreme weather, as users of off-the-grid power systems, we should also save electricity appropriately to ensure basic living electricity. Wait until the weather is clear, and then resume normal electricity use.

Assume that in the rainy weather, the daily electricity consumption is reduced to 60% of the single-day maximum electricity consumption. To enable the battery pack to cope with the electricity demand under the above extreme weather, the maximum battery capacity designed for the battery pack should be raised by $\tau > 0$ based on the theoretical capacity. To be specific, in average, the designed capacity of the battery pack $UC^{(high)}$ should be:

$$UC^{(high)} = C^{(high)} \times (1+\tau) = \frac{C^{(high)} \times 60\%}{50\%} \tag{9}$$

Solved from the formula above, we have $\tau = 20\%$, which means that we can have an extra 20% space for capacity based on the maximum electricity demand $C^{(high)}$ to cope with the rainy weather. Calculated from the formula (9), we get:

$$UC^{(high)} = C^{(high)} \times (1+20\%) = 34.74\text{kWh} \tag{10}$$

In all, combined with formula (3) and (11), the capacity of the battery pack which provide power for the high-power electrical appliances should be no less than 34.74kWh, and the output power rate should be no less than 8.07kW.

(2) Low-power electrical equipment that works for a long time

For electronic equipment such as refrigerators, televisions, lighting systems, and mobile phones, they need a stable (guaranteed uninterrupted power) power supply with low output power. The total output power required by multiple low-power devices to work together is also low. Similar to the previous processing method, for the case where the rated power is within a range, we take the average value to represent its rated power. For low-power electrical equipment, calculate the maximum output power required by the battery pack $P_{max}{}^{(low)}$:

$$P_{max}{}^{(low)} = \sum_i P_i^{(low)} N_i{}^{(low)} = 655\text{W} = 0.655\text{kW} \tag{11}$$

In fact, in low-power power supply equipment, we can have further division. Except for the refrigerator, which requires 24 hours of uninterrupted power, the long-term work of other equipment generally does not exceed 6 hours (from 6 pm to 12 pm).

The total electricity C and output power P required by rechargeable devices such as mobile phones, tablets and laptops, as well as televisions and electric fans are:

$$C = 1655\text{Wh} = 1.655\text{kWh} \tag{12}$$

$$P = 385\text{W} = 0.385\text{kW} \tag{13}$$

This part of electricity and power only account for about 5% of those of the high-power electrical appliance ($C^{(high)}$ and $P_{max}{}^{(high)}$), we can completely let high-power battery pack take this part of power burden.

Consider that all lighting systems use energy-saving lamps to further reduce power consumption, the entire low-power battery pack only needs to be responsible for basic lighting and refrigerator power supply. Recalculate the required output power of the low-power battery pack $P_{max}{}^{(low)}$ at this time:

$$P_{max}{}^{(low)} = \sum_{i} P_i^{(low)} N_i^{(low)} = 190\text{W} = 0.19\text{kW} \tag{14}$$

Now consider the total power that the low-power battery pack needs to provide. The refrigerator needs to be powered throughout the night, so the working time is 12 hours. Assuming that the lighting system has a working time of 6 hours (from 6 p.m. to 12 p.m.), the battery capacity $C^{(low)}$ that the low-power battery pack needs to provide for one night is:

$$C^{(low)} = \sum_{i=1} P_i^{(low)} T_i^{(low)} N_i^{(low)} = 1740\text{Wh} = 1.74\text{kWh} \tag{15}$$

Also considering the impact of rainy weather on battery charging, we assume that the available power of the battery pack can only reach 50% of the maximum battery capacity. In such condition, we cannot reduce the power consumption of the refrigerator, but we can reduce the power consumption of the lighting system. Set the daily power consumption to 80% of the maximum power consumption in a single day. Therefore, similar to formula (9), the maximum battery capacity designed for the battery pack should be raised by $\tau > 0$ on the basis of the theoretical capacity and the designed capacity of the battery pack $UC^{(low)}$ should be:

$$UC^{(low)} = C^{(low)} \times (1+\tau) = \frac{C^{(low)} \times 80\%}{50\%} \tag{16}$$

Solved from the formula above, we have $\tau = 60\%$, which means that we can have an extra 60% space for capacity on the basis of the maximum electricity demand $C^{(low)}$ to cope with the rainy weather. Calculated from the formula (16), we get:

$$UC^{(low)} = C^{(low)} \times (1+60\%) = 2784\text{W} = 2.784\text{kWh} \tag{17}$$

In all, combined with formula (14) and (17), the capacity of the battery pack which provide power for the low-power electrical appliances should be no less than 2.784kWh, and the output power rate should be no less than 0.19kW.

4.2.2 Establishment of Optimization Model

Now we can construct an optimal model based on the constraints on power demand in Section 4.2.1. First consider the cost of the off-the-grid power system, here we only consider the battery price. Suppose the unit prices of low-power/high-power battery packs are respectively $w_i^{(low/high)}$, the numbers are respectively $n_i^{(low/high)}$, the continuous power rating are respectively $p_i^{(low/high)}$ and the battery capacities are respectively $c_i^{(low/high)}$. Based on the discussion in Section 4.2.1, that optimal problem about the construction cost can be written as:

$$\begin{aligned}
&\underset{n_i^{(low)},n_i^{(high)}}{\arg\min} && \sum_i w_i^{(low)} n_i^{(low)} + \sum_i w_i^{(high)} n_i^{(high)} \\
& && \sum_i c_i^{(low)} n_i^{(low)} \geq 2.784 \\
& && \sum_i c_i^{(high)} n_i^{(high)} \geq 34.74 \\
&\text{subject to} && \sum_i p_i^{(low)} n_i^{(low)} \geq 0.19 \\
& && \sum_i p_i^{(high)} n_i^{(high)} \geq 8.07 \\
& && n_i^{(low)}, n_i^{(high)} \in \mathbb{N}
\end{aligned} \tag{18}$$

Formula (18) is a linear integer programming problem in respect to $n_i^{(low)}, n_i^{(high)}$, which can be solved retrogradely using the simplex method [2]. Since every parameter $c_i^{(low/high)}$ and $w_i^{(low/high)}$ is non-negative, the problem above is due to have a solution.

The constraints divide the solution area into a feasible region, and the optimization goal will find the best one or more points in the feasible region. For subsequent optimization, here we do not limit the solution to only the lowest cost solution but remove it from the feasible region every time a solution is found, and solve the optimal solution in the remaining feasible region until the top 5 battery system schemes are obtained $S = \{s_1, s_2, s_3, s_4, s_5\}$.

Next, based on this optimal problem, we take into consideration the Round-Trip Efficiency (RTE) of the battery. For each scheme of battery system $s_i = (n_i^{(low)}, n_i^{(high)})$, set the RTEs for each battery as $r_i^{(low)}, r_i^{(high)}$, and then we can set up the optimal problem as follows:

$$\begin{aligned}
&\underset{s_i}{\arg\max} && \frac{1}{\sum_i n_i^{(low)} + \sum_i n_i^{(high)}} \sum_i r_i^{(low)} n_i^{(low)} + \sum_i r_i^{(high)} n_i^{(high)} \\
&\text{subject to} && s_i \in S
\end{aligned} \tag{19}$$

Similarly, we take the top 3 battery system schemes of this problem $\overline{S} = \{\overline{s_1}, \overline{s_2}, \overline{s_3}\}$.

In the third step, we consider the optimization of the battery's volume. For each scheme of the battery system $\overline{s_i} = (n_i^{(low)}, n_i^{(high)})$, we set the volumes of each battery as $v_i^{(low)}, v_i^{(high)}$, and set up the following optimal problem:

$$\begin{aligned}
&\underset{\overline{s_i}}{\arg\min} && \sum_i v_i^{(low)} n_i^{(low)} + \sum_i v_i^{(high)} n_i^{(high)} \\
&\text{subject to} && \overline{s_i} \in \overline{S}
\end{aligned} \tag{20}$$

Formulas (18), (19), (20) together constitute the optimization model of the optimal power storage system. In general, the best model can be confirmed only through (18), because the consideration is an integer programming problem, and the high battery price is likely to make the sub-optimal scheme increase the number of batteries based on the optimal scheme. But the two optimization processes (19), (20) can help us determine the best scheme when we get the schemes whose costs are close or the same.

In addition, (18) is an integer programming problem. To increase the range of the feasible region and prevent missing the actual optimal scheme due to integer rounding, we can consider appropriately relaxing the constraints, that is, adding a not too large positive number $\epsilon > 0$, and turn the optimal problem into:

$$\begin{aligned} &\underset{n_i^{(low)},n_i^{(high)}}{\arg\min} && \sum_i w_i^{(low)} n_i^{(low)} + \sum_i w_i^{(high)} n_i^{(high)} \\ &\text{subject to} && \sum_i c_i^{(low)} n_i^{(low)} \geq 2.784 - \epsilon \\ & && \sum_i c_i^{(high)} n_i^{(high)} \geq 34.74 - \epsilon \\ & && \sum_i p_i^{(low)} n_i^{(low)} \geq 0.19 \\ & && \sum_i p_i^{(high)} n_i^{(high)} \geq 8.07 \\ & && n_i^{(low)}, n_i^{(high)} \in \mathbb{N} \end{aligned} \tag{21}$$

4.2.3 Solution of The Best Battery Storage System

We only consider the five batteries provided in the appendix which include: Deka Solar 8GCC2 6V 198, Trojan L-16 -SPRE 6V 415, Discover AES 7.4 kWh, Electriq PowerPod 2 and Tesla Powerwall+.

First, we use formula (21), and set construction cost as the optimization object. We set $\epsilon = 0.5$ and figure out the top 5 battery system schemes as shown in Table 4.

Table 4: Optimal scheme with price as optimization objective

Scheme	Configuration Scheme					Total Cost
	Deka Solar 8GCC2 6V 198	Trojan L-16 - SPRE 6V 415	Discover AES 7.4 kWh	Electriq PowerPod 2	Tesla Powerwall+	
1	0	1	1	0	2	\$23970
2	1	1	1	0	2	\$24338
3	0	2	1	0	2	\$24462
4	2	1	1	0	2	\$24706
5	1	2	1	0	2	\$24830

The above five schemes can meet our various assumptions about electricity demand, and the price difference is not big. Next, the feasible region is reduced to the above five schemes, and the optimization problems of battery's Round-Trip Efficiency (RTE) and battery's volume are solved according to (19) and (20) respectively. Table 5 shows the results of the three optimization problems.

Table 5: Optimization results of price, battery round-trip efficiency and volume

Scheme	Configuration Scheme					Total Cost	Round-Trip Efficiency	Total Volume (inch^3)
	Deka Solar 8GCC2 6V 198	Trojan L-16 - SPRE 6V 415	Discover AES 7.4 kWh	Electriq PowerPod 2	Tesla Powerwall+			
1	0	1	1	0	2	\$23970	89.37%	28539
2	1	1	1	0	2	\$24338	88.00%	29332
3	0	2	1	0	2	\$24462	88.00%	29960
4	2	1	1	0	2	\$24706	87.08%	30125
5	1	2	1	0	2	\$24830	87.08%	30753

From Table 5, we can see that for the 1,600 square foot off-the-grid home, the optimal configuration of the power storage system is: purchase 1 Trojan L-16 -SPRE 6V 415 battery, 1 Discover AES 7.4 kWh battery and Two Tesla Powerwall+ batteries. While ensuring the power supply demand, this configuration scheme requires the lowest construction cost, the highest average Round-Trip Efficiency (RTE) and the smallest volume.

The construction cost of the optimal configuration scheme is \$23,970, the average battery RTE is about 89.37%, and the volume is about 28539 cubic inches. This complete system can provide a total battery capacity of 36.90kWh. The continuous power rating of low-power battery packs can reach 0.19kW, and the continuous power rating of high-power battery packs can reach 20.65kW. The peak power rating of the entire battery pack can reach 34.4kW.

5. Model Generalization

This section will discuss how to abstract the model we established in Section 4, so that the model can be applied to any individual who wants to build an off-the-grid power storage system. We make the expression of optimization problem (21) more general and define a set of standardized model establishment and solution processes for everyone to help people determine the best configuration scheme of the power storage system. In Section 5.2, we discuss the advantages and weakness of the model.

The modeling process of Section 5 is shown in Figure 3:

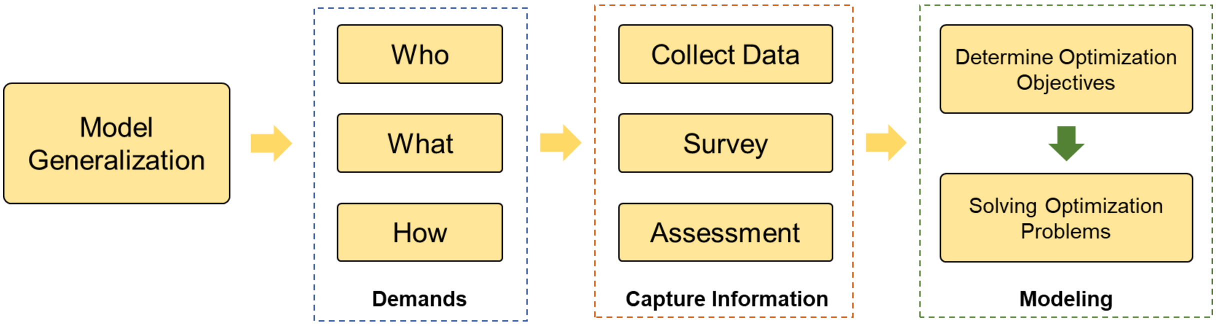

Figure 3: Modeling process of Section 5

5.1 Custom Model on Demand

5.1.1 Determine Requirements

To extend the model to any individual, we need to identify the users of off-the-grid power storage system, and then locate the electrical equipment required by each user. According to Table 3, collect a record of the rated power, quantity, and single working hours of each electrical appliance. The table records the most basic power demand of the whole off-the-grid home.

From this demand table, we need to divide electrical appliances into high-power equipment working for a short time and low-power equipment working for a long time. The following four key variables are calculated:

- Maximum rated power $P_{max}^{(low)}$ required for operation of low power equipment.
- Battery capacity $C^{(low)}$ required for low-power equipment to work one night.
- Maximum rated power $P_{max}^{(high)}$ required for operation of high-power equipment.
- Battery capacity $C^{(high)}$ required for high-power equipment to work one night.

5.1.2 Consider Other Factors

To ensure the normal operation of off-the-grid power storage system, we need to appropriately increase the output power and battery capacity based on the basic needs, to deal with sudden high load demand or poor power storage in extreme weather.

Let $\varepsilon > 0$ be the correction factor for the output power. Appropriately increase ε to improve the bearing capacity of the system when dealing with high load power consumption. Correct $P_{max}^{(low)}$ and $P_{max}^{(high)}$ as follows:

$$
\begin{aligned}
P_{max}^{(low)} &= P_{max}^{(low)} \times (1+\varepsilon) \\
P_{max}^{(high)} &= P_{max}^{(high)} \times (1+\varepsilon)
\end{aligned}
\tag{22}
$$

The influence of cloudy and rainy weather on power storage system is considered below. Before purchasing batteries, we should investigate the historical weather data of the area to estimate the climate characteristics. Collect relevant data or conduct experiments to evaluate the attenuation ratio r_1 of battery charge in cloudy and rainy days.

In addition, we suggest to appropriately reduce unnecessary power consumption in continuous cloudy and rainy weather to ensure the normal power supply of important appliances (such as refrigerator, lighting, etc.), and evaluate the proportion r_2 of power consumption of household appliances that can be reduced.

Using formula (9), the design capacity UC of the battery pack shall meet:

$$\begin{aligned} UC^{(low)} &= C^{(low)} \times (1+\tau) = \frac{C^{(low)} \times r_2}{r_1} \\ UC^{(high)} &= C^{(high)} \times (1+\tau) = \frac{C^{(high)} \times r_2}{r_1} \end{aligned} \tag{23}$$

Restrictions on the number, category, round-trip efficiency, weight or volume of batteries can also be considered. All these conditions are expressed as constraints in (24):

$$\begin{aligned} & n_i^{(l)} \le n_i^{(low)} \le n_i^{(u)}, n_i^{(l)} \le n_i^{(high)} \le n_i^{(u)}, \quad n_i^{(low)}, n_i^{(high)} \in \mathbb{N} \\ & \frac{1}{\sum_i n_i^{(low)} + \sum_i n_i^{(high)}} \sum_i r_i^{(low)} n_i^{(low)} + \sum_i r_i^{(high)} n_i^{(high)} \ge RTE \\ & \sum_i v_i^{(low)} n_i^{(low)} + \sum_i v_i^{(high)} n_i^{(high)} \le V \\ & \sum_i g_i^{(low)} n_i^{(low)} + \sum_i g_i^{(high)} n_i^{(high)} \le G \end{aligned} \tag{24}$$

Where $n_i^{(low/high)}$ represents the purchase quantity of the i-th battery, $r_i^{(low/high)}$ represents the round-trip efficiency of a single battery, and RTE represents the lowest average round-trip efficiency that the battery pack is expected to achieve. $v_i^{(low/high)}$ represents the volume of a single battery and V represents the maximum volume of the power storage system. $g_i^{(low/high)}$ represents the mass of a single battery and G represents the maximum mass of the power storage system. Any constraint condition to be considered can be added to formula (23) by using the relevant information expression of the battery.

5.1.3 Establish Optimization Model

After obtaining a series of information about power storage system capability requirements, we can establish the optimization model from two aspects. First, fix the power demand and minimize the cost of building off-the-grid power storage system. Second, fix the cost of building the system and maximize the power supply capacity that the system can provide.

(1) Fix power demand

Under the ideal condition of sufficient funds, this strategy can be considered to ensure the power supply capacity of the system. The optimization problem is basically the same as problem (21) in Section 4.2. The optimization problem (25) is established as follows:

$$\begin{aligned}
&\underset{n_i^{(low)},n_i^{(high)}}{\arg\min} && \sum_i w_i^{(low)} n_i^{(low)} + \sum_i w_i^{(high)} n_i^{(high)} \\
& && \sum_i c_i^{(low)} n_i^{(low)} \geq UC^{(low)} - \epsilon \\
& && \sum_i c_i^{(high)} n_i^{(high)} \geq UC^{(high)} - \epsilon \\
&\text{subject to} && \sum_i p_i^{(low)} n_i^{(low)} \geq P_{\max}{}^{(low)} \\
& && \sum_i p_i^{(high)} n_i^{(high)} \geq P_{\max}{}^{(high)} \\
& && \text{conditions (23)}
\end{aligned} \tag{25}$$

In which, the normal number $\epsilon > 0$ is used to enhance the solvability of the integer programming problem.

(2) Fix the cost of building the system

In most cases, the construction cost we can use is limited. Therefore, we can consider maximizing the power supply capacity of the power storage system as the optimization goal:

$$\begin{aligned}
&\underset{n_i^{(low)},n_i^{(high)}}{\arg\max} && \sum_i p_i^{(high)} n_i^{(high)}, \sum_i c_i^{(high)} n_i^{(high)} \\
& && \sum_i w_i^{(low)} n_i^{(low)} + \sum_i w_i^{(high)} n_i^{(high)} \leq C \\
& && \sum_i c_i^{(low)} n_i^{(low)} \geq UC^{(low)} - \epsilon_1 \\
&\text{subject to} && \sum_i c_i^{(high)} n_i^{(high)} \geq UC^{(high)} - \epsilon_2 \\
& && \sum_i p_i^{(low)} n_i^{(low)} \geq P_{\max}{}^{(low)} - \epsilon_3 \\
& && \sum_i p_i^{(high)} n_i^{(high)} \geq P_{\max}{}^{(high)} - \epsilon_4 \\
& && \text{conditions (23)}
\end{aligned} \tag{26}$$

After adding the constraints of total cost C, the integer programming problem (25) is likely to be unsolvable because it cannot meet the four constraints related to power and battery capacity. To increase the solvability of problem (25), we add adjustment parameter $\epsilon_1, \cdots, \epsilon_4$ to the constraints on power and capacity. Appropriately increasing the values of these parameters can expand the feasible region of the problem, to ensure that the optimization goal can find the integer solution in the feasible region.

5.1.4 Model Solving

Problems (24) and (25) are linear integer programming problems about battery purchase quantity $n_i^{(low)}, n_i^{(high)}$. these two kinds of optimization problems can be solved directly by simplex method.

In addition, there is a faster and simpler solution method. Generally, the number of batteries purchased $n_i^{(low)}, n_i^{(high)}$ will not be too large (because we either want to minimize the battery cost or the budget is limited, both of which will limit the optimal solution). We can take a small positive integer N_0 (e.g., $N_0 = 4$) and directly search for the optimal solution satisfying the conditions in all possible combinations of $0 \le n_i^{(low)}, n_i^{(high)} \le N_0$. Assuming that there are d types of batteries considered, the time complexity of this method is $\mathcal{O}\left((N_0+1)^d\right)$ [3]. When d is not too large (for example, we have $d = 5$ in Section 4.2), it is feasible to use this method.

5.2 Model Evaluation

5.2.1 Advantages

Aiming at the configuration scheme of off-the-grid power storage system, we establish an optimization model for optimizing expenditure and optimizing power supply capacity. From the perspective of data collection and analysis, model establishment and solution, our method has the following advantages.

- Our model is clear and simple, and each module is highly customized, which is very easy to expand. Anyone can change the constraints according to their actual needs to obtain the best model that is most suitable for them.
- Optimization problems are linear integer programming problems about the number of batteries purchased. The solvability of the problem can be guaranteed. We can use the simplex method to obtain the solution of the problem.
- You can decide whether to optimize the system construction cost or the power supply capacity of the system according to the budget you have.
- We consider the impact of cloudy and rainy weather on battery charging efficiency and set a threshold for the maximum power supply capacity of the power storage system to ensure the normal operation of electrical appliances under sudden high load.
- In addition to optimizing the cost and power supply capacity of the system, the model also considers the battery round-trip efficiency and volume as the objectives for multi-stage optimization.

5.2.2 Weakness

For the convenience of calculation, we have done a lot of assumptions in the process of modeling, and some assumptions will introduce errors into the model. The main weakness of the model are as follows.

- We assume that the battery can be charged in a very short time when the sun is full, so we don't consider the situation of charging and discharging at the same time. In practice, the charging power of the battery module is limited. Therefore, we need a more accurate model to calculate the power supply capacity of the battery pack.
- Battery charging and power supply in cloudy and rainy weather is a complex process. We greatly simplified the modeling process and only considered the charge and discharge requirements of each day. In case of continuous cloudy and rainy weather, it is more practical to consider the battery charge process and discharge for many consecutive days.

- When considering the power supply capacity of the system, the model is based on the power required by household appliances under full load. In real life, the power consumption of electrical appliances is complex and changeable, and most of the time will be far less than the full load we consider, which is a certain waste of resources for the configuration of power storage system. The configuration scheme of off-the-grid power storage system can be further optimized by using more complex and accurate models.

6. Discussion on Cement Battery

In this part, we'll discuss the application of cement in aspect of energy storage. Based on the structure and working principles of the latest rechargeable cement-based battery [4], we first identify the advantages and disadvantages of using cement batteries to store solar power.

6.1 How about Cement Battery

6.1.1 Advantages

- Due to the huge amount of concrete in building materials all over the world, if this kind of new energy storage method could be realized in large scale, it makes possible an innovative functional concrete which may provide solutions to the increasing energy problems.
- Combined with this new technology, the functional concrete can also be used in places where current electricity system may have trouble reaching, say, providing power for monitors and detectors, even signal stations in distant areas.
- The average energy density of the rechargeable cement-based battery has witnessed a great leap (over ten times that of the former concrete batteries) since the concept was proposed. Also, with proper modification of the cement-based electrolytes and electrodes, their conductivity and workability process a promising potential.
- The structure and materials of the cement batteries is commercially available, compared with the large amount of solar energy it can save, and the construction cost may not even be more than that of the current power grid.

6.1.2 Disadvantages:

- Despite the improvement in the average energy density, the energy capacity per unit volume is still quite low, compared with the commercial batteries.
- The electronic resistivity in the cement-based electrolyte may add up to a big number when this technology is used in a large scale.
- Before the technology turns into reality, how to extend the life of the batteries and recycle the old ones would be critical technical problems.
- The technique of adding fabric in the cement electrodes may face difficulties in implementation and maintenance when used in large scale of applications.

When incorporate cement as a battery for our off-the-grid home or any other home, we mainly discuss from the aspect of the cement battery's implementation and maintenance.

First, different from the current commercial batteries, we don't need to especially spare a space to put the batteries since cement is the most frequently used material in construction of buildings. We can just install the cement batteries when building the walls, and connect the batteries with the solar panels, thus making our home a "solar energy station" with sufficient capacity to store the solar energy.

Secondly, we can design the batteries' layout in the walls. For example, we can combine batteries in several large walls to provide power support for high-power appliances. Also, we can make use of a whole wall in one bedroom to generate electricity for the room's power demand, reducing the power loss in transmission, which may be a tricky problem when using current batteries.

Finally, we need to pay attention to the follow-up maintenance of the cement batteries. Considering that the batteries are directly installed in the walls, the cement may expand and contract with heat, we should prioritize the walls that have fewer time directly facing the sunlight and prepare ahead for extreme cold weather, like adding special heat insulation layer to resist low temperature and disperse the heat generated when charging and discharging.

6.2 More Information We Want

To compare the use of cement batteries to currently available batteries for solar power storage, we need to take into several factors including cement battery's dimension, cost, continuous and instantaneous power rating etc. We will discuss the in the following aspects.

- Continuous and Instantaneous power rating

The priority of the batteries is always satisfying the power demand of the off-grid-home, so we need to know the battery's continuous and instantaneous power rating values to determine the number or volume of the cement batteries that are needed. We can use the method that is similar to that we used in designing the power system supported by current batteries. Then we can compare the energy supply efficiency between the two kinds of batteries. For example, we can calculate the number of nights that the system can support the appliances to work with a single full charge or consider the average charging time of the batteries in a whole day's power demand (since we assume that the charging of the batteries is completed in a quite short period).

- Price and Space

After determining the number of cement batteries that are needed, we are able to calculate the total price of the batteries and the space needed to contain the batteries, since we also need to optimize the design of our power system both commercially and spatially. We may compare the ratio of the energy that each power system can supply within their working life to the total cost, or the ratio of the energy to the extra space each system occupy.

- Usable capacity

Even though the unit power capacity of the cement battery is low, considering the relatively low unit price and large amount, its usable capacity may still be satisfying. Therefore, we can calculate the ratio of the power discharged to the power charged in a long period of time to evaluate the usable power capacity in actual use. Also, we can combine this factor with the battery's cost to balance the demand in cost and capacity.

Overall, the above three factors can be considered simultaneously in our model, and the optimal option depends on to what extent we pay our focus on each factor.

7. One-page Non-technical News Article

OFF-THE-GRID: A New Choice of Life

Electricity power is always a critical and precious resource in the development of mankind society. Due to different terrain conditions and the unbalanced commercial situation, the cost of connecting to the grid (i.e., a power company) may be quite expensive. Therefore, off-the-grid becomes a choice. Recently, a research team from 2021 HiMCM designed a mathematical model to decide the optimal design of such a solar power battery storage system.

The model is based on a 1600 square-foot off-the-grid home with a family of 4: father, mother, a son, and a daughter. The team considers the main appliances at home and the use scenario to determine the total power demand. They divide the demands into two parts:

(1) High-power electrical equipment that works for a short time.

(2) Low-power electrical equipment that works for a long time.

The power demands are 34.74kWh and 2.784kWh, respectively. They design the energy system with current available batteries: Deka Solar 8GCC2 6V 198, Trojan L-16-SPRE 6V 145, Discover AES 7.4 kWh, Electriq PowerPod 2 and Tesla Powerwall+.

In the model's main body, the team tries to find the optimal battery system scheme under the consideration of minimizing the cost and volume of batteries. To be specific, they choose the schemes with low batteries' cost from all that satisfy the power demands. The qualified options of schemes go into the next round of selection. The 2rd round of selection is based on one of the battery's significant performance indicators: Round-Trip Efficiency (RTE). They choose from the candidates the ones that have relatively high round-trip efficiency. In the final round, volume of the battery is considered since extra space for the battery system may generate more construction cost of supporting facilities.

From all the schemes that enters the final round, the optimal configuration of the power storage system is: one Trojan L-16 -SPRE 6V 415 battery, one Discover AES 7.4 kWh battery and two Tesla Powerwall+ batteries.

The decision model can easily be generalized to any other home with different power demands. The optimization process can be adjusted by fixing one of the main factors and optimizing the others step by step. It's easy for users to add other factors into the model and it's quite flexible since this process allows them to focus on factors or conditions they care about.

In the final part of the team's report, they propose the future possibility of using cement battery in this system. They give reasons for this new technology's promising future:

(1) Wide use of cement in buildings and bridges provides conditions for this proposal to come true, making the cement battery a possible solution to the problem of energy storage.

(2) Compared with the expensive cost of grid in remote areas, cement battery would be more commercially friendly.

(3) Recent research on cement battery witnessed a great leap in its conductivity and workability which shows the battery's great potential.

It's worth waiting that this off-the-grid energy storage system may come into reality someday and the development of cement battery will undoubtedly ignite people's thinking of the problem of energy storage and the imagination of the life in the future.

8. References

[1]
[2] Rosen, K. . (2003). Discrete Mathematics and Its Applications. McGraw-Hill.
[3] Cormen, T. H. , Leiserson, C. E. , Rivest, R. L. , & Stein, C. . (2001). Introduction to algorithms 2nd ed. Journal of the Operational Research Society, 1(9), 14-24.
[4] Zhang, Emma Q., and Luping Tang. 2021. "Rechargeable Concrete Battery" Buildings 11, no. 3: 103. https://doi.org/10.3390/buildings11030103.

9. Appendix

Python code for our model

```python
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt

data = pd.read_excel("electrical appliances.xlsx")
data.head()

# grid search
# generate schema list
schema = []
for n1 in range(0,5):
    for n2 in range(0,5):
        for n3 in range(0,4):
            for n4 in range(0,4):
                for n5 in range(0,4):
                    schema.append([n1,n2,n3,n4,n5])
schema = np.array(schema)

# multistage optimization
# optimize the cost
def OptimizeCost(eps,C_low,C_high,P_low,P_hight):
    result = {}
    index = 1
    for s in schema:
        c_low = s[0:2]@Battery['UC'][0:2]
        c_high = s[2:]@Battery['UC'][2:]
        p_low = s[0:2]@Battery['CPR'][0:2]
        p_high = s[2:]@Battery['CPR'][2:]
        if  c_low >= C_low - eps and c_high >= C_high - eps and p_low >= P_low and p_high >= P_hight:
                cost = s@Battery['Cost']
                uc = s@Battery['UC']
                low_cpr = s[0:2]@Battery['CPR'][0:2]
                high_cpr = s[2:]@Battery['CPR'][2:]
```

```
                ipr = s[2:]@Battery['IPR'][2:]
                result[index] = list(s) + [cost,uc,low_cpr,high_cpr,ipr]
                index += 1
    result = pd.DataFrame(result).T
    result.columns = list(Battery['Battery']) + ['Cost','UC','low CPR','High CPR','IPR']
    result = result.sort_values('Cost').reset_index(drop=True)
    return result.iloc[0:5,:]
CostOpt = OptimizeCost(eps=0.5,C_low=2.784,C_high=34.74,P_low=0.19,P_hight=8.07)

# optimize the Round-Trip Efficiency
def OptimizeRTE(S):
    S = S.copy()
    S['RTE'] = None
    for i,s in enumerate(S.values):
        S.loc[i,'RTE'] = s[0:5]@Battery['RTE'] / sum(s[0:5]) * 100
    S - S.sort_values('RTE',ascending=False).reset_index(drop=True)
    return S
rteOpt = OptimizeRTE(CostOpt)

# optimize the volume of the system
def OptimizeVolume(S):
    S = S.copy()
    S['Volume'] = None
    for i,s in enumerate(S.values):
        S.loc[i,'Volume'] = s[0:5]@Battery['Volume']
    S - S.sort_values('Volume',ascending=False).reset_index(drop=True)
    return S
VolumeOpt = OptimizeVolume(rteOpt)
```