## User's Manual

Software Version: 1.9.0b Date: 2005/02/09 09:25:00



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UCRL-MA-137155 DR

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# Introduction and Mailing List Information

hypre is a software library for solving large, sparse linear systems of equations on massively parallel computers. The library was created with the primary goal of providing users with advanced parallel preconditioners. Issues of robustness, ease of use, flexibility, and interoperability also play an important role.

There are three hypre Mailing Lists that can be subscribed to through the hypre web page, which is located at http://www.llnl.gov/CASC/hypre.

- hypre-announce (hypre-announce@lists.llnl.gov)
  The development team uses this list to announce new general releases of hypre. It cannot be posted to by users.
- hypre-beta-announce (hypre-beta-announce@lists.llnl.gov)
  The development team uses this list to announce new beta releases of hypre. It cannot be posted to by users.
- hypre-users (hypre-users@lists.llnl.gov)
  This list is for hypre users to communicate with the development team and each other.
  It can be posted to by any member of the list.

### 1.1 Features

• Scalable preconditioners provide efficient solution on today's and tomorrow's systems: hypre contains several families of preconditioner algorithms focused on the scalable solution of very large sparse linear systems. hypre includes "grey-box" algorithms that use more than just the matrix to solve certain classes of problems more efficiently than general-purpose libraries. This includes algorithms such as structured multigrid.

- Suite of common iterative methods provides options for a spectrum of problems: hypre provides several of the most commonly used Krylov-based iterative methods to be used in conjunction with its scalable preconditioners. This includes methods for nonsymmetric systems such as GMRES and methods for symmetric matrices such as Conjugate Gradient.
- Intuitive grid-centric interfaces obviate need for complicated data structures and provide access to advanced solvers: hypre has made a major step forward in usability from earlier generations of sparse linear solver libraries in that users do not have to learn complicated sparse matrix data structures. Instead, hypre does the work of building these data structures for the user through a variety of interfaces, each appropriate to different classes of users. These include stencil-based structured/semi-structured interfaces most appropriate for finite-difference applications; a finite-element based unstructured interface; and a linear-algebra based interface. Each interface provides access to several solvers without the need to write new interface code.
- User options accommodate beginners through experts: hypre allows a spectrum of expertise to be applied by users. The beginning user can get up and running with a minimal amount of effort. More expert users can take further control of the solution process through various parameters.
- Configuration options to suit your computing system: hypre utilizes the GNU Autoconf package to allow simple and flexible installation on a wide variety of computing systems. Users can tailor the installation to match their computing system. Options include debug and optimized modes, the ability to change required libraries such as MPI and BLAS, a sequential mode, and modes enabling threads for certain solvers. On most systems, however, hypre can be built by simply typing configure followed by make.
- Interfaces in multiple languages provide greater flexibility for applications: hypre contains interfaces for both Fortran and C users.

### 1.2 Assumptions and Limitations

- hypre is designed for large, sparse, linear systems on parallel computers. Small linear systems, systems that are solvable on a sequential computer, and dense systems are all better addressed by other libraries that are designed specifically for them.
- To run in parallel, hypre requires an installation of MPI.
- Configuration of hypre with threads requires an implementation of OpenMP. Currently, only a subset of hypre is threaded.
- hypre currently does not support complex-valued systems.

# **Getting Started**

Before writing any code:

- 1. Choose a conceptual interface (see Sections 2.2 and 2.3). Generally, the choice is fairly obvious. A structured-grid interface is clearly inappropriate for an unstructured-grid application. It is desirable to use a more specific interface if appropriate, e.g., the linear-algebraic interface is usable from any type of grid but will involve much more user work and prevent access to some grid-type-specific preconditioners.
- 2. Choose your desired solver strategy. For the typical user, this will mean a single Krylov method and a single preconditioner.
- 3. Look up matrix requirements for each solver and preconditioner. Each specific solver and preconditioner has requirements from the input matrix. This information is provided in several places: Chapter 7, the hypre Reference Manual, and the hypre header files.
- 4. Choose a matrix class that is compatible with your solvers and preconditioners and your conceptual interface. Note that some of the interfaces currently only support one matrix class choice.

Once the previous decisions have been made, it is time to code your application to call hypre:

- 1. Build any necessary auxiliary structures for your chosen conceptual interface. This includes, e.g., the grid and stencil structures for the structured-grid interface.
- 2. Build the matrix, solution vector, and right-hand-side vector through your chosen conceptual interface. Each conceptual interface provides a series of calls for entering information about your problem into hypre.
- 3. Build solvers and preconditioners and set solver parameters (optional). Some parameters like convergence tolerance are the same across solvers, while others are solver specific.

- 4. Call the solve function for the solver.
- 5. Retrieve desired information from solver. Depending on your application, there may be different things you may want to do with the solution vector. Also, performance information such as number of iterations is typically available, though it may differ from solver to solver.

### 2.1 A Simple Example

The following code serves as a simple example of the usage of hypre. In this example, the structured-grid interface (discussed in Chapter 3) is used to enter the problem into hypre, and the PFMG Multigrid solver is used to solve the system. Since the structured-grid interface currently only supports one underlying matrix class, there are no choices to make here. If we were using the semi-structured grid interface instead, then we would have to choose between the SStruct and ParCSR matrix classes, depending on the solver we want to use.

This example and all other examples in this manual are written in C, but *hypre* also supports Fortran. See Section 8.2 for details.

```
/*----
* Set up the grid and stencil
*----*/
HYPRE_StructGridCreate(MPI_COMM_WORLD, dim, &grid);
HYPRE_StructGridSetExtents(grid, ilower, iupper);
HYPRE_StructGridAssemble(grid);
HYPRE_StructStencilCreate(dim, stencil_size, &stencil);
HYPRE_StructStencilSetElement(stencil, 0, offset0);
/*----
* Set up the matrix, right-hand side, and initial guess
 *-----*/
HYPRE_StructMatrixCreate(MPI_COMM_WORLD, grid, stencil, &A);
HYPRE_StructMatrixInitialize(A);
HYPRE_StructMatrixSetBoxValues(A, ilower, iupper, nelts, elts, Avalues);
HYPRE_StructMatrixAssemble(A);
HYPRE_StructVectorCreate(MPI_COMM_WORLD, grid, &b);
HYPRE_StructVectorInitialize(b);
```

```
HYPRE_StructVectorSetBoxValues(b, ilower, iupper, bvalues);
HYPRE_StructVectorAssemble(b);
HYPRE_StructVectorCreate(MPI_COMM_WORLD, grid, &x);
HYPRE_StructVectorInitialize(x);
HYPRE_StructVectorSetBoxValues(x, ilower, iupper, xvalues);
HYPRE_StructVectorAssemble(x);
/*----
* Set up the solver
*----*/
HYPRE_StructPFMGCreate(MPI_COMM_WORLD, &solver);
HYPRE_StructPFMGSetMaxIter(solver, 50);
                                 /* optional */
HYPRE_StructPFMGSetTol(solver, 1.0e-06); /* optional */
HYPRE_StructPFMGSetup(solver, A, b, x);
/*----
* Solve the linear system
*----*/
HYPRE_StructPFMGSolve(solver, A, b, x);
/*----
* Get solution info and free up memory
*----*/
HYPRE_StructVectorGetBoxValues(x, ilower, iupper, xvalues);
. . .
HYPRE_StructPFMGDestroy(solver);
HYPRE_StructGridDestroy(grid);
HYPRE_StructStencilDestroy(stencil);
HYPRE_StructMatrixDestroy(A);
HYPRE_StructVectorDestroy(b);
HYPRE_StructVectorDestroy(x);
```

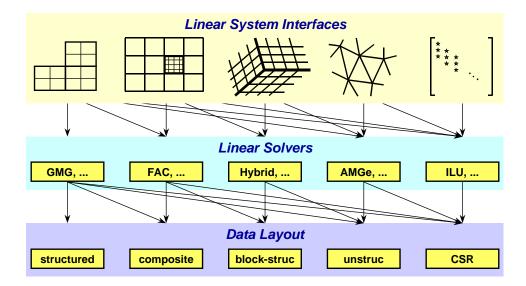


Figure 2.1: Graphic illustrating the notion of conceptual interfaces. All of these elements are not necessarily in *hypre*.

### 2.2 What are conceptual interfaces?

The top row of Figure 2.1 illustrates a number of conceptual interfaces. Generally, the conceptual interfaces are denoted by different types of computational grids, but other application features might also be used, such as geometrical information. These conceptual interfaces are intended to represent the way that applications developers naturally think of their linear problem, and provide natural interfaces for them to pass the data that defines their linear system into hypre. Essentially, these conceptual interfaces can be considered convenient utilities for helping a user build a matrix data structure for hypre solvers and preconditioners. For example, applications that use structured grids (such as in the left-most interface in the Figure 2.1) typically view their linear problems in terms of stencils and grids. On the other hand, applications that use unstructured grids and finite elements typically view their linear problems in terms of elements and element stiffness matrices. Finally, the right-most interface is the standard linear-algebraic (matrix rows/columns) way of viewing the linear problem.

The second row of Figure 2.1 is a set of linear solver algorithms. Each linear solver group requires different information from the user through the conceptual interfaces. So, the geometric multigrid algorithm (GMG) listed in the left-most box, for example, can only be used with the left-most conceptual interface. On the other hand, the ILU algorithm in the right-most box may be used with any conceptual interface.

The third row of Figure 2.1 is a list of data layouts or matrix/vector storage schemes. The relationship between linear solver and storage scheme is similar to that of interface and linear solver.

### 2.3 Which conceptual interface should I use?

hypre currently supports four conceptual interfaces:

- Structured-Grid System Interface (Struct): This interface is appropriate for applications whose grids consist of unions of logically rectangular grids with a fixed stencil pattern of nonzeros at each grid point. This interface supports only a single unknown per grid point. See Chapter 3 for details.
- Semi-Structured-Grid System Interface (SStruct): This interface is appropriate for applications whose grids are mostly structured, but with some unstructured features. Examples include block-structured grids, composite grids in structured adaptive mesh refinement (AMR) applications, and overset grids. This interface supports multiple unknowns per cell. See Chapter 4 for details.
- Finite Element Interface (FEI): This is appropriate for users who form their linear systems from a finite element discretization. The interface mirrors typical finite element data structures, including element stiffness matrices. Though this interface is provided in *hypre*, its definition was determined elsewhere (www.z.ca.sandia.gov/fei). See Chapter 5 for details.
- Linear-Algebraic System Interface (IJ): This is the traditional linear-algebraic interface. It can be used as a last resort by users for whom the other grid-based interfaces are not appropriate. It requires more work on the user's part, though still less than building parallel sparse data structures. General solvers and preconditioners are available through this interface, but not specialized solvers which need more information. Our experience is that users with legacy codes, in which they already have code for building matrices in particular formats, find the IJ interface relatively easy to use. See Chapter 6 for details.

Generally, a user should choose the most specific interface that matches their application, because this will allow them to use specialized and more efficient solvers and preconditioners without losing access to more general solvers.

# Structured-Grid System Interface (Struct)

In order to get access to the most efficient and scalable solvers for scalar structured-grid applications, users should use the Struct interface described in this chapter. This interface will also provide access (this is not yet supported) to solvers in hypre that were designed for unstructured-grid applications and sparse linear systems in general. These additional solvers are usually provided via the unstructured-grid interface (FEI) or the linear-algebraic interface (IJ) described in Chapters 5 and 6.

Figure 3.1 gives an example of the type of grid currently supported by the Struct interface. The interface uses a finite-difference or finite-volume style, and currently supports only scalar PDEs (i.e., one unknown per gridpoint). There are four basic steps involved in setting up the linear system to be solved:

- 1. set up the grid,
- 2. set up the stencil,
- 3. set up the matrix,
- 4. set up the right-hand-side vector.

To describe each of these steps in more detail, consider solving the 2D Laplacian problem

$$\begin{cases} \nabla^2 u = f, & \text{in the domain,} \\ u = 0, & \text{on the boundary.} \end{cases}$$
 (3.1)

Assume (3.1) is discretized using standard 5-pt finite-volumes on the uniform grid pictured in 3.1, and assume that the problem data is distributed across two processes as depicted.

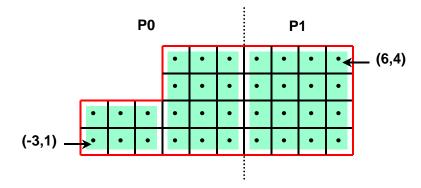


Figure 3.1: An example 2D structured grid, distributed across two processors.

### 3.1 Setting Up the Struct Grid

The grid is described via a global *index space*, i.e., via integer tuples (triples in 3D). The integers may have any value, negative or positive. The global indexes allow *hypre* to discern how data is related spatially, and how it is distributed across the parallel machine. Each process describes that portion of the grid that it "owns", one *box* at a time. For example, in the figure, the global grid can be described in terms of three boxes, two owned by process 0, and one owned by process 1. A box is described in terms of a lower and upper index.

On process 0, the following code will set up the grid shown in the figure (the code for process 1 is similar).

The Create() routine creates an empty 2D grid object that lives on the MPI\_COMM\_WORLD communicator. The SetExtents() routine adds a new box to the grid. The Assemble() routine is a collective call (i.e., must be called on all processes from a common synchronization point), and finalizes the grid assembly, making the grid "ready to use".

### 3.2 Setting Up the Struct Stencil

The geometry of the discretization stencil is described by an array of integer tuples in 2D (triples in 3D), each representing a relative offset (in index space) from some gridpoint on the grid. For example, the geometry of the 5-pt stencil for the example problem being considered can be represented in the following way:

$$\begin{bmatrix}
(-1,0) & (0,1) \\
(0,0) & (1,0) \\
(0,-1)
\end{bmatrix} \equiv \begin{bmatrix}
S_4 \\
S_1 & S_0 & S_2 \\
S_3
\end{bmatrix}.$$
(3.2)

In (3.2), the (0,0) entry represents the "center" coefficient, and is the 0th entry in the array  $(S_0)$ . The (0,-1) entry represents the "south" coefficient, and is the 3rd entry in the array  $(S_3)$ . And so on.

On process 0 or 1, the following code will set up the stencil in (3.2). The stencil must be the same on all processes.

The Create() routine creates an empty 2D, 5-pt stencil object. The SetElement() routine defines the geometry of the stencil and assigns the array numbers for each of the stencil entries. None of the calls are collective calls.

### 3.3 Setting Up the Struct Matrix

The matrix is set up in terms of the grid and stencil objects described in Sections 3.1 and 3.2. The coefficients associated with each stencil entry will typically vary from gridpoint to gridpoint, but in the example problem being considered, they are as follows over the entire grid (except at boundaries; see below):

$$\begin{bmatrix} -1 \\ -1 & 4 & -1 \\ -1 & \end{bmatrix}. \tag{3.3}$$

On process 0, the following code will set up matrix values associated with the center  $(S_0)$  and south  $(S_3)$  stencil entries in (3.2) / (3.3) (boundaries are ignored here temporarily).

```
HYPRE_StructMatrix A;
double
                    values[36];
                    stencil_indices[2] = {0,3};
int
int
                    i;
HYPRE_StructMatrixCreate(MPI_COMM_WORLD, grid, stencil, &A);
HYPRE_StructMatrixInitialize(A);
for (i = 0; i < 36; i += 2)
{
   values[i]
               = 4.0;
   values[i+1] = -1.0;
}
HYPRE_StructMatrixSetBoxValues(A, ilower[0], iupper[0], 2,
                                stencil_indices, values);
HYPRE_StructMatrixSetBoxValues(A, ilower[1], iupper[1], 2,
                                stencil_indices, values);
/* set boundary conditions */
HYPRE_StructMatrixAssemble(A);
```

The Create() routine creates an empty matrix object. The Initialize() routine indicates that the matrix coefficients (or values) are ready to be set. This routine may or may not involve the allocation of memory for the coefficient data, depending on the implementation. The optional Set routines mentioned later in this chapter and in the Reference Manual, should be called before this step. The SetBoxValues() routine sets the matrix coefficients for some set of stencil entries over the gridpoints in some box. Note that the box need not correspond to any of the boxes used to create the grid, but values should be set for all gridpoints that this process "owns". The Assemble() routine is a collective call, and finalizes the matrix assembly, making the matrix "ready to use".

Matrix coefficients that reach outside of the boundary should be set to zero. For efficiency reasons, hypre does not do this automatically. The most natural time to insure this is when the boundary conditions are being set, and this is most naturally done after the coefficients on the grid's interior have been set. For example, during the implementation of the Dirichlet boundary condition on the lower boundary of the grid in Figure 3.1, the "south" coefficient must be set to zero. To do this on process 0, the following code could be used:

```
int ilower[2] = {-3, 1};
int iupper[2] = { 2, 1};

/* create matrix and set interior coefficients */
...

/* implement boundary conditions */
...

for (i = 0; i < 12; i++)
{
    values[i] = 0.0;
}

i = 3;
HYPRE_StructMatrixSetBoxValues(A, ilower, iupper, 1, &i, values);

/* complete implementation of boundary conditions */
...</pre>
```

### 3.4 Setting Up the Struct Right-Hand-Side Vector

The right-hand-side vector is set up similarly to the matrix set up described in Section 3.3 above. The main difference is that there is no stencil (note that a stencil currently does appear in the interface, but this will eventually be removed).

On process 0, the following code will set up the right-hand-side vector values.

```
HYPRE_StructVectorSetBoxValues(b, ilower[1], iupper[1], values);
HYPRE_StructVectorAssemble(b);
```

The Create() routine creates an empty vector object. The Initialize() routine indicates that the vector coefficients (or values) are ready to be set. This routine follows the same rules as its corresponding Matrix routine. The SetBoxValues() routine sets the vector coefficients over the gridpoints in some box, and again, follows the same rules as its corresponding Matrix routine. The Assemble() routine is a collective call, and finalizes the vector assembly, making the vector "ready to use".

### 3.5 Symmetric Matrices

Some solvers and matrix storage schemes provide capabilities for significantly reducing memory usage when the coefficient matrix is symmetric. In this situation, each off-diagonal coefficient appears twice in the matrix, but only one copy needs to be stored. The Struct interface provides support for matrix and solver implementations that use symmetric storage via the SetSymmetric() routine.

To describe this in more detail, consider again the 5-pt finite-volume discretization of (3.1) on the grid pictured in Figure 3.1. Because the discretization is symmetric, only half of the off-diagonal coefficients need to be stored. To turn symmetric storage on, the following line of code needs to be inserted somewhere between the Create() and Initialize() calls.

```
HYPRE_StructMatrixSetSymmetric(A, 1);
```

Note that symmetric storage may or may not actually be used, depending on the underlying storage scheme. Currently in *hypre*, symmetric storage is always used when indicated.

To most efficiently utilize the Struct interface for symmetric matrices, notice that only half of the off-diagonal coefficients need to be set. To do this for the example being considered, we simply need to redefine the 5-pt stencil of Section 3.2 to an "appropriate" 3-pt stencil, then set matrix coefficients (as in Section 3.3) for these three stencil elements *only*. For example, we could use the following stencil

$$\begin{bmatrix} (0,1) \\ (0,0) & (1,0) \end{bmatrix} \equiv \begin{bmatrix} S_2 \\ S_0 & S_1 \end{bmatrix}. \tag{3.4}$$

This 3-pt stencil provides enough information to recover the full 5-pt stencil geometry and associated matrix coefficients.

# Semi-Structured-Grid System Interface (SStruct)

In order to get access to the most efficient and scalable solvers for semi-structured-grid applications (applications with grids that are mostly structured, but with some unstructured features), users should use the SStruct interface described in this chapter. This interface also provides access to solvers in hypre that were designed for unstructured-grid applications and sparse linear systems in general. These additional solvers are usually provided via the unstructured-grid interface (FEI) or the linear-algebraic interface (IJ) described in Chapters 5 and 6.

Figure 4.1 gives an example of the type of grid currently supported by the SStruct interface. The grid is composed of three "parts". There is a single cell-centered variable across the entire grid, except at the grid cell marked with a square, which has one additional cell-centered variable. There are five basic steps involved in setting up the linear system to be solved:

- 1. set up the grid,
- 2. set up the stencils,
- 3. set up the graph,
- 4. set up the matrix.
- 5. set up the right-hand-side vector.

To describe each of these steps in more detail, consider solving the 2D Laplacian problem

$$\begin{cases} \nabla^2 u = f, & \text{in the domain,} \\ u = 0, & \text{on the boundary.} \end{cases}$$
 (4.1)

Assume (3.1) is discretized using standard 9-pt finite-volumes on the grid pictured in 4.1, and assume that the problem data is distributed across three processes as depicted. In the figure, each grid part is distributed onto a different process, but this need not be the case; each grid part may also be distributed across several processes. Assume for simplicity and illustration that there is a

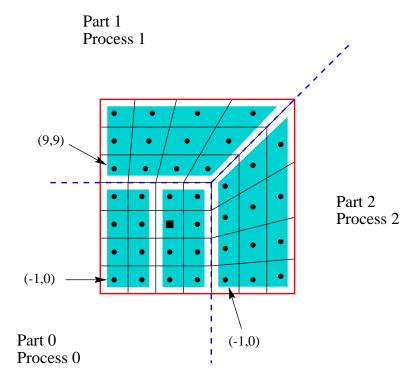


Figure 4.1: An example 2D block-structured grid, distributed across three processes.

single coupling between the first and second variables in grid part 0 at index (1,2). In general, this variable may also be coupled to variables at neighboring indices.

### 4.1 Setting Up the SStruct Grid

On process 0, the following code will set up the grid shown in the figure (the code for processes 1 and 2 is similar).

```
HYPRE_SStructGridSetExtents(grid, 0, ilower[1], iupper[1]);
HYPRE_SStructGridSetVariables(grid, 0, 1, vars);
HYPRE_SStructGridAddVariables(grid, 0, addindex, 1, addvars);
HYPRE_SStructGridAssemble(grid);
```

The Create() routine creates an empty 2D grid object that lives on the MPI\_COMM\_WORLD communicator. The SetExtents() routine adds a new box to the grid. The SetVariables() routine sets the variables on a grid part, and the AddVariables() routine adds variables to a grid part index (in the above example, there are two cell-centered variables at index (1,2)). The Assemble() routine is a collective call (i.e., must be called on all processes from a common synchronization point), and finalizes the grid assembly, making the grid "ready to use".

### 4.2 Setting Up the SStruct Stencil

The geometry of the discretization stencil is described by an array of integer tuples in 2D (triples in 3D), each representing a relative offset (in index space) from some gridpoint on the grid. For example, the geometry of the 9-pt stencil for the example problem being considered can be represented in the following way:

$$\begin{bmatrix} (-1,1) & (0,1) & (1,1) \\ (-1,0) & (0,0) & (1,0) \\ (-1,-1) & (0,-1) & (1,-1) \end{bmatrix} \equiv \begin{bmatrix} S_7 & S_4 & S_8 \\ S_1 & S_0 & S_2 \\ S_5 & S_3 & S_6 \end{bmatrix}.$$
(4.2)

In (3.2), the (0,0) entry represents the "center" coefficient, and is the 0th entry in the array  $(S_0)$ . The (0,-1) entry represents the "south" coefficient, and is the 3rd entry in the array  $(S_3)$ . And so on

On process 0, 1, or 2, the following code will set up the stencil in (3.2).

The Create() routine creates an empty 2D, 9-pt stencil object. The SetEntry() routine defines the geometry of the stencil, and assigns the array numbers for each of the stencil entries. None of the calls are collective calls.

### 4.3 Setting Up the SStruct Graph

The graph will represent the nonzero structure of the matrix in Section 4.4 below. It is defined in terms of the grid and stencil objects described in Sections 4.1 and 4.2 above.

On process 0, the following code will set up the graph for the example problem.

```
HYPRE_SStructGraph graph;
                                     = \{\{2,0\},\{2,1\},\{2,2\},\{2,3\},
                    addindex[9][2]
int
                                        {-1,3},{0,3},{1,3},{2,3},
                                        { 1,2}};
int
                   addentries[9][2] = \{\{-1,0\},\{-1,1\},\{-1,2\},\{-1,3\},
                                        { 9,9},{10,9},{11,9},{12,9},
                                        { 1,2}};
HYPRE_SStructGraphCreate(MPI_COMM_WORLD, grid, &graph);
HYPRE_SStructGraphSetStencil(graph, 0, 0, stencil);
/* Add graph edge entries at grid part boundaries */
HYPRE_SStructGraphAddEntries(graph, 0, addindex[0], 0, 2, addentries[0], 0);
HYPRE_SStructGraphAddEntries(graph, 0, addindex[0], 0, 2, addentries[1], 0);
HYPRE_SStructGraphAddEntries(graph, 0, addindex[1], 0, 2, addentries[0], 0);
. . .
/* Add graph edge entries at index (1,2) */
HYPRE_SStructGraphAddEntries(graph, 0, addindex[8], 0, 0, addentries[8], 1);
HYPRE_SStructGraphAddEntries(graph, 0, addindex[8], 1, 0, addentries[8], 0);
```

### 4.4 Setting Up the SStruct Matrix

The matrix is set up in terms of the graph object described in Section 4.3 above. The coefficients associated with each graph entry will typically vary from gridpoint to gridpoint, but in the example problem being considered, they are as follows over the entire grid (except at boundaries; see below):

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}. \tag{4.3}$$

On process 0, the following code will set up matrix values associated with the center  $(S_0)$  and south  $(S_3)$  stencil entries in (3.2) / (3.3). Matrix values associated with the non-stencil graph entries are also set up.

```
HYPRE_SStructMatrix A;
double
                     values[32];
                     sindices[2] = \{0,3\};
int
int
                     gindices[4] = \{9,10,11,0\};
int
                     i;
HYPRE_SStructMatrixCreate(MPI_COMM_WORLD, graph, &A);
HYPRE_SStructMatrixInitialize(A);
for (i = 0; i < 32; i += 2)
   values[i] = 8.0;
   values[i+1] = -1.0;
}
HYPRE_SStructMatrixSetBoxValues(A, 0, ilower[0], iupper[0], 0,
                                 2, sindices, values);
/* set values at non-stencil graph entries */
for (i = 0; i < 3; i++)
{
   values[i] = -1.0;
}
HYPRE_SStructMatrixSetValues(A, 0, addindex[0], 0, 2, gindices, values);
HYPRE_SStructMatrixSetValues(A, 0, addindex[1], 0, 3, gindices, values);
HYPRE_SStructMatrixSetValues(A, 0, addindex[2], 0, 3, gindices, values);
HYPRE_SStructMatrixSetValues(A, 0, addindex[7], 0, 1, gindices, values);
HYPRE_SStructMatrixSetValues(A, 0, addindex[7], 1, 1, &gindices[3], values);
/* zero out coefficients that reach outside of the domain or grid part */
. . .
/* set boundary conditions at domain boundaries, */
. . .
HYPRE_SStructMatrixAssemble(A);
```

## 4.5 Setting Up the SStruct Right-Hand-Side Vector

The right-hand-side vector is set up similarly to the matrix set up described in Section 4.4 above. The main difference is that there is no graph.

# Finite Element Interface (FEI)

The finite element interface (FEI) defines a linear solver interface for finite element applications. For information on how to use this interface, see [4]. This chapter describes the iterative methods and preconditioners in the hypre implementation of this interface.

Solving a linear system from a finite element problem consists of four steps in the FEI:

- 1. Initialize the structure of the finite-element data, including loading the element connectivity data
- 2. Load the element or super-element stiffness matrices and forcing terms
- 3. Set solver parameters and solve the linear system
- 4. Retrieve the solution to the linear system

Parameters to the hypre solvers are specified by calling

```
void FEI_parameters(int sysHandle, int numParams, char **paramStrings);
```

where sysHandle is an identifier for the linear system being solved, numParams is the number of parameter strings, and paramStrings is an array of null-terminated strings with the format: "parameter\_name value". For example, setting the preconditioner can be accomplished by:

```
char **paramStrings[1];
paramStrings[0] = (char *) malloc(64*sizeof(char));
strcpy(paramStrings[0], "preconditioner parasails");
FEI_parameters(sysHandle, 1, paramStrings);
```

All possible parameters are listed in Table 5.1.

A linear system is then solved by calling

```
void FEI_iterateToSolve(int sysHandle);
```

### 5.1 Iterative methods and preconditioners available

### 5.1.1 Iterative methods

- 1. Krylov solvers (conjugate gradient, GMRES, TFQMR, BiCGSTAB)
- 2. BoomerAMG (a parallel algebraic multigrid solver)
- 3. SuperLU direct solver (sequential)
- 4. SuperLU direct solver with iterative refinement (sequential)

### 5.1.2 Preconditioners

- 1. diagonal
- 2. parallel incomplete LU with threshold (PILUT)
- 3. another parallel incomplete LU (Euclid)
- 4. parallel algebraic multigrid (BoomerAMG)
- 5. parallel sparse approximate inverse (ParaSails)

Parameter Name	Parameter Values
solver	cg, gmres $(default)$ , bicgstab, tfqmr, boomeramg, superlu, superlux
preconditioner	diagonal (default), pilut, parasails, boomeramg, euclid
gmresDim	an integer specifying the value of m in restarted GMRES(m). The default value is 50.
maxIterations	an integer specifying the maximum number of iterations permitted for CG or GMRES. The default value is 1000.
tolerance	a floating point number specifying the termination criterion for CG or GMRES. The default value is 1.0E-10.
pilutFillin	an integer specifying the maximum number of nonzeros kept in the formation of incomplete L and U. If this is not called, a value will be selected based on the sparsity of the matrix.
pilutDropTol	a floating point number specifying the threshold to drop small entries in L and U. The default value is 0.0.
euclidNlevels	a non-negative integer specifying the desired sparsity of the incomplete factors. The default value is 0.
${ m euclidThreshold}$	a floating point number specifying the threshold used to sparsify the incomplete factors. The default value is 0.0.
superluOrdering	natural (default) or mmd (minimum degree ordering). This ordering is used to minimize the number of nonzeros generated in the LU decomposition. The default is natural ordering.
superluScale	y (yes; perform row and column scalings before decomposition) or n (no; default).
${ m amgCoarsenType}$	falgout, ruge, or default (CLJP) coarsening for BoomerAMG.
${ m amgNumSweeps}$	an integer specifying the number of pre- and post-smoothing at each level of BoomerAMG. The default is one pre- and one post-smoothings.
amgRelaxType	jacobi (Damped Jacobi), gs-slow (sequential Gauss-Seidel), gs-fast (Gauss-Seidel on interior nodes), hybrid, or direct. The default is hybrid.
${ m amgRelaxWeight}$	a floating point number between 0 and 1 specifying the damping factor for BoomerAMG's damped Jacobi smoother. The default value is 0.5.
${ m amgStrongThreshold}$	a floating point number between 0 and 1 specifying the threshold used to determine strong coupling in BoomerAMG's coasening. The default value is 0.25.
parasailsThreshold	a floating point number between 0 and 1 specifying the threshold used to prune small entries in setting up the sparse approximate inverse. The default value is 0.0.
parasailsNlevels	an integer larger than 0 specifying the desired sparsity of the approximate inverse. The default value is 1.
parasailsFilter	a floating point number between 0 and 1 defining the threshold used to prune small entries in A. The default is 0.0.
parasailsLoadbal	a floating point number between 0 and 1 specifying how load balancing has to be done. The default is 0.0.
parasailsSymmetric	set ParaSails to take A as symmetric.
parasailsUnSymmetric	set ParaSails to take A as nonsymmetric (default).
parasailsReuse	set ParaSails to reuse the sparsity pattern of A.

Table 5.1: Parameters.

# Linear-Algebraic System Interface (IJ)

The IJ interface described in this chapter is the lowest common denominator for specifying linear systems in *hypre*. This interface provides access to general sparse-matrix solvers in *hypre*, not to the specialized solvers that require more problem information.

### 6.1 IJ Matrix Interface

As with the other interfaces in *hypre*, the IJ interface expects to get data in distributed form because this is the only scalable approach for assembling matrices on thousands of processes. Matrices are assumed to be distributed by blocks of rows as follows:

$$\begin{bmatrix}
A_0 \\
A_1 \\
\vdots \\
A_{P-1}
\end{bmatrix}$$
(6.1)

In the above example, the matrix is distributed across the P processes, 0, 1, ..., P-1 by blocks of rows. Each submatrix  $A_p$  is "owned" by a single process and its first and last row numbers are given by the global indices ilower and iupper in the Create() call below.

The following example code illustrates the basic usage of the IJ interface for building matrices:

```
MPI_Comm comm;
HYPRE_IJMatrix ij_matrix;
HYPRE_ParCSRMatrix parcsr_matrix;
int ilower, iupper;
int jlower, jupper;
int nrows;
```

```
int
                   *ncols;
int
                   *rows;
int
                   *cols;
double
                   *values;
HYPRE_IJMatrixCreate(comm, ilower, iupper, jlower, jupper, &ij_matrix);
HYPRE_IJMatrixSetObjectType(ij_matrix, HYPRE_PARCSR);
HYPRE_IJMatrixInitialize(ij_matrix);
/* set matrix coefficients */
HYPRE_IJMatrixSetValues(ij_matrix, nrows, ncols, rows, cols, values);
/* add-to matrix cofficients, if desired */
HYPRE_IJMatrixAddToValues(ij_matrix, nrows, ncols, rows, cols, values);
HYPRE_IJMatrixAssemble(ij_matrix);
HYPRE_IJMatrixGetObject(ij_matrix, (void **) &parcsr_matrix);
```

The Create() routine creates an empty matrix object that lives on the comm communicator. This is a collective call (i.e., must be called on all processes from a common synchronization point), with each process passing its own row extents, ilower and iupper. The row partitioning must be contiguous, i.e., iupper for process i must equal ilower—1 for process i+1. Note that this allows matrices to have 0- or 1-based indexing. The parameters jlower and jupper define a column partitioning, and should match ilower and iupper when solving square linear systems. See the Reference Manual for more information.

The SetObjectType() routine sets the underlying matrix object type to HYPRE\_PARCSR (this is the only object type currently supported). The Initialize() routine indicates that the matrix coefficients (or values) are ready to be set. This routine may or may not involve the allocation of memory for the coefficient data, depending on the implementation. The optional SetRowSizes() and SetDiagOffdSizes() routines mentioned later in this chapter and in the Reference Manual, should be called before this step.

The SetValues() routine sets matrix values for some number of rows (nrows) and some number of columns in each row (ncols). The actual row and column numbers of the matrix values to be set are given by rows and cols. After the coefficients are set, they can be added to with an AddTo() routine. Each process should set only those matrix values that it "owns" in the data distribution.

The Assemble() routine is a collective call, and finalizes the matrix assembly, making the matrix "ready to use". The GetObject() routine retrieves the built matrix object so that it can be passed on to hypre solvers that use the ParCSR internal storage format. Note that this is not an expensive routine; the matrix already exists in ParCSR storage format, and the routine simply returns a "handle" or pointer to it. Although we currently only support one underlying data storage format, in the future several different formats may be supported.

One can preset the row sizes of the matrix in order to reduce the execution time for the matrix specification. One can specify the total number of coefficients for each row, the number of coefficients in the row that couple the diagonal unknown to (Diag) unknowns in the same processor domain, and the number of coefficients in the row that couple the diagonal unknown to (Offd) unknowns in other processor domains:

```
HYPRE_IJMatrixSetRowSizes(ij_matrix, sizes);
HYPRE_IJMatrixSetDiagOffdSizes(matrix, diag_sizes, offdiag_sizes);
```

Once the matrix has been assembled, the sparsity pattern cannot be altered without completely destroying the matrix object and starting from scratch. However, one can modify the matrix values of an already assembled matrix. To do this, first call the Initialize() routine to re-initialize the matrix, then set or add-to values as before, and call the Assemble() routine to re-assemble before using the matrix. Re-initialization and re-assembly are very cheap, essentially a no-op in the current implementation of the code.

### 6.2 IJ Vector Interface

The following example code illustrates the basic usage of the IJ interface for building vectors:

```
MPI_Comm
                 comm;
HYPRE_IJVector
                 ij_vector;
HYPRE_ParVector par_vector;
                 jlower, jupper;
int
                 nvalues;
int
                *indices;
double
                *values;
HYPRE_IJVectorCreate(comm, jlower, jupper, &ij_vector);
HYPRE_IJVectorSetObjectType(ij_vector, HYPRE_PARCSR);
HYPRE_IJVectorInitialize(ij_vector);
/* set vector values */
HYPRE_IJVectorSetValues(ij_vector, nvalues, indices, values);
HYPRE_IJVectorAssemble(ij_vector);
HYPRE_IJVectorGetObject(ij_vector, (void **) &par_vector);
```

The Create() routine creates an empty vector object that lives on the comm communicator. This is a collective call, with each process passing its own index extents, jlower and jupper. The names

of these extent parameters begin with a j because we typically think of matrix-vector multiplies as the fundamental operation involving both matrices and vectors. For matrix-vector multiplies, the vector partitioning should match the column partitioning of the matrix (which also uses the j notation). For linear system solves, these extents will typically match the row partitioning of the matrix as well.

The SetObjectType() routine sets the underlying vector storage type to HYPRE\_PARCSR (this is the only storage type currently supported). The Initialize() routine indicates that the vector coefficients (or values) are ready to be set. This routine may or may not involve the allocation of memory for the coefficient data, depending on the implementation.

The SetValues() routine sets the vector values for some number (nvalues) of indices. Each process should set only those vector values that it "owns" in the data distribution.

The Assemble() routine is a trivial collective call, and finalizes the vector assembly, making the vector "ready to use". The GetObject() routine retrieves the built vector object so that it can be passed on to hypre solvers that use the ParVector internal storage format.

Vector values can be modified in much the same way as with matrices by first re-initializing the vector with the Initialize() routine.

# Solvers and Preconditioners

There are several solvers available in *hypre* via different conceptual interfaces (see Table 7.1). The procedure for setup and use of solvers and preconditioners is largely the same. We will refer to them both as solvers in the sequel except when noted. In normal usage, the preconditioner is chosen and constructed before the solver, and then handed to the solver as part of the solver's setup. In the following, we assume the most common usage pattern in which a single linear system is set up and then solved with a single righthand side. We comment later on considerations for other usage patterns.

	System Interfaces			
Solvers	Struct	SStruct	FEI	IJ
Jacobi	X			
SMG	X			
PFMG	X			
BoomerAMG		X	X	X
ParaSails		X	X	X
Euclid		X	X	X
PILUT		X	X	X
PCG	X	X	X	X
GMRES	X	X	X	X

Table 7.1: Current solver availability via hypre conceptual interfaces.

### Setup:

1. Pass to the solver the information defining the problem. In the typical user cycle, the user has passed this information into a matrix through one of the conceptual interfaces prior to setting up the solver. In this situation, the problem definition information is then passed to the solver by passing the constructed matrix into the solver. As described before, the

matrix and solver must be compatible, in that the matrix must provide the services needed by the solver. Krylov solvers, for example, need only a matrix-vector multiplication. Most preconditioners, on the other hand, have additional requirements such as access to the matrix coefficients.

- 2. Choose parameters for the preconditioner and/or solver. Parameters are chosen through the Set() calls provided by the solver. As is true throughout *hypre*, all parameters have reasonable defaults if not chosen. Note that in *hypre*, convergence criteria can be chosen after the preconditioner/solver has been setup.
- 3. Pass the preconditioner to the solver. For solvers that are not preconditioned, this step is omitted. The preconditioner is passed through the SetPreconditioner() call.
- 4. Set up the solver. This is just the Setup() routine.

At this point, the solver/preconditioner is fully constructed and ready for use.

### Use:

- 1. **Set convergence criteria.** Convergence can be controlled by the number of iterations, as well as various tolerances such as relative residual, preconditioned residual, etc. Like all parameters, reasonable defaults are used. Users are free to change these, though care must be taken. For example, if an iterative method is used as a preconditioner for a Krylov method, a constant number of iterations is usually required.
- 2. Solve the system. This is just the Solve() routine.

### 7.1 SMG

SMG is a parallel semicoarsening multigrid solver for the linear systems arising from finite difference, finite volume, or finite element discretizations of the diffusion equation,

$$\nabla \cdot (D\nabla u) + \sigma u = f \tag{7.1}$$

on logically rectangular grids. The code solves both 2D and 3D problems with discretization stencils of up to 9-point in 2D and up to 27-point in 3D. See [10, 2, 5] for details on the algorithm and its parallel implementation/performance.

SMG is a particularly robust method. The algorithm semicoarsens in the z-direction and uses plane smoothing. The xy plane-solves are effected by one V-cycle of the 2D SMG algorithm, which semicoarsens in the y-direction and uses line smoothing.

 $7.2. \ PFMG$  31

### **7.2 PFMG**

PFMG is a parallel semicoarsening multigrid solver similar to SMG. See [1, 5] for details on the algorithm and its parallel implementation/performance.

The main difference between the two methods is in the smoother: PFMG uses simple pointwise smoothing. As a result, PFMG is not as robust as SMG, but is much more efficient per V-cycle.

### 7.3 BoomerAMG

BoomerAMG is a parallel implementation of algebraic multigrid. It can be used both as a solver or as a preconditioner. The user can choose between various different parallel coarsening techniques and relaxation schemes. See [6] for a detailed description of the coarsening algorithms, the interpolation and nueemerical results. The following coarsening techniques are available:

- the Cleary-Luby-Jones-Plassman (CLJP) coarsening,
- various variants of the classical Ruge-Stueben (RS) coarsening algorithm, and
- the Falgout coarsening which is a combination of CLJP and the classical RS coarsening algorithm.

The following relaxation techniques are available:

- weighted Jacobi relaxation,
- a hybrid Gauss-Seidel / Jacobi relaxation scheme,
- a symmetric hybrid Gauss-Seidel / Jacobi relaxation scheme, and
- sequential Gauss-Seidel relaxation.

### 7.3.1 Synopsis

The solver is set up and run using the following routines, where A is the matrix, b the right hand side and x the solution vector of the linear system to be solved:

### 7.3.2 Interface functions

Parameters for setting up the code are specified using the following routines:

#### HYPRE\_BoomerAMGSetMaxLevels

```
int HYPRE_BoomerAMGSetMaxLevels( HYPRE_Solver solver, int max_levels);
```

max\_levels defines the maximal number of multigrid levels allowed. The default is 25.

#### HYPRE\_BoomerAMGSetMaxIter

```
int HYPRE_BoomerAMGSetMaxIter( HYPRE_Solver solver, int max_iter);
```

max\_iter defines the maximal number of iterations allowed. The default is 20.

#### HYPRE\_BoomerAMGSetTol

```
int HYPRE_BoomerAMGSetTol( HYPRE_Solver solver, double tol);
```

tol defines the tolerance needed for the stopping criterion  $||b - Ax^{(n)}||_2/||b||_2 \le \text{tol.}$  The default for tol is  $10^{-7}$ .

#### HYPRE\_BoomerAMGSetStrongThreshold

```
int HYPRE_BoomerAMGSetStrongThreshold( HYPRE_Solver solver,
double strong_threshold);
```

A point i is strongly connected to j, if  $-a_{i,j} \geq \theta \max_{j \neq i} a_{ij}$ , where the strong threshold  $\theta$  is a value between 0 and 1. Weak connections are usually ignored when determining the next lower level. Consequently, choosing a larger strong threshold leads in general to smaller coarse grids, but worse convergence rates. The default value for  $\theta$  is 0.25, which appears to be a good choice for 2-dimensional problems. A better choice for 3-dimensional problems appears to be 0.5. However, the choice of the strength threshold is problem dependent and therefore there could be better choices than the two suggested ones.

### HYPRE\_BoomerAMGSetMaxRowSum

```
int HYPRE_BoomerAMGSetMaxRowSum( HYPRE_Solver solver, double max_row_sum);
```

This feature leads to a more efficient treatment of very diagonally dominant portions of the matrix. If the absolute row sum of row i weighted by the diagonal is greater than max\_row\_sum all dependencies of variable i are set to be weak. This feature can be switched off by setting max\_row\_sum to 1.0. The default is 0.9.

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### HYPRE\_BoomerAMGSetCoarsenType

int HYPRE\_BoomerAMGSetCoarsenType(HYPRE\_Solver solver, int coarsen\_type); coarsen\_type defines the coarsening used. The following options are possible:

- 0 CLJP-coarsening
- 1 Ruge-Stueben coarsening without boundary treatment
- 3 Ruge-Stueben coarsening with a 3rd 'second' pass on the boundaries
- 6 Falgout coarsening (default)

### HYPRE\_BoomerAMGSetMeasureType

```
int HYPRE_BoomerAMGSetMeasureType( HYPRE_Solver solver, int measure_type );
```

measure\_type defines whether local (measure\_type = 0, default) or global measures (measure\_type = 1) are used within the coarsening algorithm. This feature is ignored for the CLJP and the Falgout coarsening.

### HYPRE\_BoomerAMGSetNumGridSweeps

```
int HYPRE_BoomerAMGSetNumGridSweeps( HYPRE_Solver solver, int* num_grid_sweeps );
```

num\_grid\_sweeps[k] defines the number of sweeps over the grid on the fine grid (k=0), the down cycle (k=1), the up cycle (k=2) and the coarse grid (k=3).

### HYPRE\_BoomerAMGSetGridRelaxType

```
int HYPRE_BoomerAMGSetGridRelaxType( HYPRE_Solver solver, int* grid_relax_type );
```

grid\_relax\_type[k] defines the relaxation used on the fine grid (k=0), the down cycle (k=1), the up cycle (k=2) and the coarse grid (k=3). The following options are possible for grid\_relax\_type[k]:

- 0 weighted Jacobi
- 1 sequential Gauss-Seidel (very slow!)
- 3 Gauss-Seidel / Jacobi hybrid method (default)
- 6 symmetric Gauss-Seidel / Jacobi hybrid method
- 9 Gaussian elimination (only for the coarsest level (k=3), not recommended if the system on the coarsest level is large)

#### HYPRE\_BoomerAMGSetGridRelaxPoints

```
int HYPRE_BoomerAMGSetGridRelaxPoints( HYPRE_Solver solver,
int** grid_relax_points);
```

grid\_relax\_points[i][j] defines which points are to be relaxed during the (j+1)-th sweep on the fine grid (i=0), the down cycle (i=1), the up cycle (i=2) and the coarse grid (i=3), e.g. if

grid\_relax\_points[1][0] is -1, all points marked -1 (which are in general fine points) are relaxed on the first sweep of the down cycle. Note: grid\_relax\_points[3][j] needs to be 0 always, since the concept of coarse and fine points does not exist on the coarsest grid. If the user sets it to another value, it will be automatically set to 0 and a warning printed, unless the direct solver is used.

### ${\bf HYPRE\_BoomerAMGS} et Relax Weight$

```
int HYPRE_BoomerAMGSetRelaxWeight( HYPRE_Solver solver, double* relax_weight);
```

defines the relaxation weights used on each level, if weighted Jacobi is used as relaxation method. The default relaxation weight is 1.0 on each level.

### HYPRE\_BoomerAMGSetPrintLevel

```
int HYPRE_BoomerAMGSetPrintLevel( HYPRE_Solver solver, int print_level);
```

where ioutdat determines whether statistics information is generated and printed. The information is printed to standard output. The following options are possible:

- 0 no output (default)
- 1 matrix statistics (includes information on interpolation operators and matrices generated on each level)
- 2 cycle statistics (includes residuals generated during solve phase)
- 3 matrix and cycle statistics

### 7.4 ParaSails

ParaSails is a parallel implementation of a sparse approximate inverse preconditioner, using a priori sparsity patterns and least-squares (Frobenius norm) minimization. Symmetric positive definite (SPD) problems are handled using a factored SPD sparse approximate inverse. General (nonsymmetric and/or indefinite) problems are handled with an unfactored sparse approximate inverse. It is also possible to precondition nonsymmetric but definite matrices with a factored, SPD preconditioner.

ParaSails uses a priori sparsity patterns that are patterns of powers of sparsified matrices. ParaSails also uses a post-filtering technique to reduce the cost of applying the preconditioner. In advanced usage not described here, the pattern of the preconditioner can also be reused to generate preconditioners for different matrices in a sequence of linear solves.

For more details about the ParaSails algorithm, see [3].

### 7.4.1 Synopsis

```
#include "HYPRE_parcsr_ls.h"
int HYPRE_ParaSailsCreate(MPI_Comm comm, HYPRE_Solver *solver,
  int symmetry);
```

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```
int HYPRE_ParaSailsSetParams(HYPRE_Solver solver,
  double thresh, int nlevel, double filter);
int HYPRE_ParaSailsSetup(HYPRE_Solver solver, HYPRE_ParCSRMatrix A,
  HYPRE_ParVector b, HYPRE_ParVector x);
int HYPRE_ParaSailsSolve(HYPRE_Solver solver, HYPRE_ParCSRMatrix A,
  HYPRE_ParVector b, HYPRE_ParVector x);
int HYPRE_ParaSailsStats(HYPRE_Solver solver);
int HYPRE_ParaSailsDestroy(HYPRE_Solver solver);
```

The accuracy and cost of ParaSails are parameterized by the real thresh and integer nlevels parameters,  $0 \le thresh \le 1$ ,  $0 \le nlevels$ . Lower values of thresh and higher values of nlevels lead to more accurate, but more expensive preconditioners. More accurate preconditioners are also more expensive per iteration. The default values are thresh = 0.1 and nlevels = 1. The parameters are set using HYPRE\_ParaSailsSetParams.

Mathematically, given a symmetric matrix A, the pattern of the approximate inverse is the pattern of  $\tilde{A}^m$  where  $\tilde{A}$  is a matrix that has been sparsified from A. The sparsification is performed by dropping all entries in a symmetrically diagonally scaled A whose values are less than thresh in magnitude. The parameter nlevel is equivalent to m+1. Filtering is a post-thresholding procedure. For more details about the algorithm, see [3].

The storage required for the ParaSails preconditioner depends on the parameters thresh and nlevels. The default parameters often produce a preconditioner that can be stored in less than the space required to store the original matrix. ParaSails does not need a large amount of intermediate storage in order to construct the preconditioner.

### 7.4.2 Interface functions

A ParaSails solver solver is returned with

```
int HYPRE_ParaSailsCreate(MPI_Comm comm, HYPRE_Solver *solver,
  int symmetry);
```

where comm is the MPI communicator.

The value of symmetry has the following meanings, to indicate the symmetry and definiteness of the problem, and to specify the type of preconditioner to construct:

value	meaning
0	nonsymmetric and/or indefinite problem, and nonsymmetric preconditioner
1	SPD problem, and SPD (factored) preconditioner
2	nonsymmetric, definite problem, and SPD (factored) preconditioner

For more information about the final case, see section 7.4.3.

Parameters for setting up the preconditioner are specified using

```
int HYPRE_ParaSailsSetParams(HYPRE_Solver solver,
  double thresh, int nlevel, double filter);
```

The parameters are used to specify the sparsity pattern and filtering value (see above), and are described with suggested values as follows:

parameter	type	range	sug. values	default	meaning
nlevel	integer	${\tt nlevel} \geq 0$	0, 1, 2	1	$m = \mathtt{nlevel} + 1$
thresh	$\operatorname{real}$	$\mathtt{thresh} \geq 0$	0, 0.1, 0.01	0.1	$\mathit{thresh} = \mathtt{thresh}$
		$\mathtt{thresh} < 0$	-0.75, -0.90		thresh selected automatically
filter	$\operatorname{real}$	$filter \ge 0$	0, 0.05, 0.001	0.05	filter value = filter
		filter < 0	-0.90		filter value selected automatically

When thresh < 0, then a threshold is selected such that -thresh represents the fraction of the nonzero elements that are dropped. For example, if thresh = -0.9 then  $\tilde{A}$  will contain approximately ten percent of the nonzeros in A.

When filter < 0, then a filter value is selected such that -filter represents the fraction of the nonzero elements that are dropped. For example, if filter = -0.9 then approximately 90 percent of the entries in the computed approximate inverse are dropped.

### 7.4.3 Preconditioning nearly symmetric matrices

A nonsymmetric, but definite and nearly symmetric matrix A may be preconditioned with a symmetric preconditioner M. Using a symmetric preconditioner has a few advantages, such as guaranteeing positive definiteness of the preconditioner, as well as being less expensive to construct.

The nonsymmetric matrix A must be definite, i.e.,  $(A+A^T)/2$  is SPD, and the a priori sparsity pattern to be used must be symmetric. The latter may be guaranteed by 1) constructing the sparsity pattern with a symmetric matrix, or 2) if the matrix is structurally symmetric (has symmetric pattern), then thresholding to construct the pattern is not used (i.e., zero value of the thresh parameter is used).

### 7.5 Euclid

The Euclid library is a scalable implementation of the Parallel ILU algorithm that was presented at SC99 [7], and published in expanded form in the SIAM Journal on Scientific Computing [8]. By scalable we mean that the factorization (setup) and application (triangular solve) timings remain nearly constant when the global problem size is scaled in proportion to the number of processors. As with all ILU preconditioning methods, the number of iterations is expected to increase with global problem size.

Experimental results have shown that PILU preconditioning is in general more effective than Block Jacobi preconditioning for minimizing total solution time. For scaled problems, the relative advantage appears to increase as the number of processors is scaled upwards. Euclid may also be used to good advantage as a smoother within multigrid methods.

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### 7.5.1 Synopsis

Euclid is best thought of as an "extensible ILU preconditioning framework." Extensible means that Euclid can (and eventually will, time and contributing agencies permitting) support many variants of ILU(k) and ILUT preconditioning. (The current release includes Block Jacobi ILU(k) and Parallel ILU(k) methods.) Due to this extensibility, and also because Euclid was developed independently of the hypre project, the methods by which one passes runtime parameters to Euclid preconditioners differ in some respects from the hypre norm. While users can directly set options within their code, options can also be passed to Euclid preconditioners via command line switches and/or small text-based configuration files. The latter strategies have the advantage that users will not need to alter their codes as Euclid's capabilities are extended.

The following fragment illustrates the minimum coding required to invoke Euclid preconditioning within *hypre* application contexts. The next subsection provides examples of the various ways in which Euclid's options can be set. The final subsection lists the options, and provides guidance as to the settings that (in our experience) will likely prove effective for minimizing execution time.

```
#include "HYPRE_parcsr_ls.h"
HYPRE_Solver eu;
HYPRE_Solver pcg_solver;
HYPRE_ParVector b, x;
HYPRE_ParCSRMatrix A;
//Instantiate the preconditioner.
HYPRE_EuclidCreate(comm, &eu);
//Optionally use the following two calls to set runtime options.
// 1. pass options from command line or string array.
HYPRE_EuclidSetParams(eu, argc, argv);
// 2. pass options from a configuration file.
HYPRE_EuclidSetParamsFromFile(eu, "filename");
//Set Euclid as the preconditioning method for some
//other solver, using the function calls HYPRE_EuclidSetup
//and HYPRE_EuclidSolve. We assume that the pcg_solver
//has been properly initialized.
HYPRE_PCGSetPrecond(pcg_solver,
                    (HYPRE_PtrToSolverFcn) HYPRE_EuclidSolve,
                    (HYPRE_PtrToSolverFcn) HYPRE_EuclidSetup,
                    eu);
//Solve the system by calling the Setup and Solve methods for,
```

```
//in this case, the HYPRE_PCG solver. We assume that A, b, and x
//have been properly initialized.
HYPRE_PCGSetup(pcg_solver, (HYPRE_Matrix)A, (HYPRE_Vector)b, (HYPRE_Vector)x);
HYPRE_PCGSolve(pcg_solver, (HYPRE_Matrix)parcsr_A, (HYPRE_Vector)b, (HYPRE_Vector)x);
//Destroy the Euclid preconditioning object.
HYPRE_EuclidDestroy(eu);
```

### 7.5.2 Setting options: examples

For expositional purposes, assume you wish to set the ILU(k) factorization level to the value k=3. There are several methods of accomplishing this. Internal to Euclid, options are stored in a simple database that contains (name, value) pairs. Various of Euclid's internal (private) functions query this database to determine, at runtime, what action the user has requested. If you enter the option "-eu\_stats 1", a report will be printed when Euclid's destructor is called; this report lists (among other statistics) the options that were in effect during the factorization phase.

**Method 1.** By default, Euclid always looks for a file titled "database" in the working directory. If it finds such a file, it opens it and attempts to parse it as a configuration file. Configuration files should be formatted as follows.

```
>cat database
#this is an optional comment
-level 3
```

Any line in a configuration file that contains a "#" character in the first column is ignored. All other lines should begin with an option name, followed by one or more blanks, followed by the option value. Note that option names always begin with a "-" character. If you include an option name that is not recognized by Euclid, no harm should ensue.

Method 2. To pass options on the command line, call

```
HYPRE_EuclidSetParams(HYPRE_Solver solver, int argc, char *argv[]);
```

where argc and argv carry the usual connotation: main(int argc, char \*argv[]). If your hypre application is called phoo, you can then pass options on the command line per the following example.

```
mpirun -np 2 phoo -level 3
```

Since Euclid looks for the "database" file when HYPRE\_EuclidCreate is called, and parses the command line when HYPRE\_EuclidSetParams is called, option values passed on the command line will override any similar settings that may be contained in the "database" file. Also, if same option name appears more than once on the command line, the final appearance determines the setting.

Some options, such as "-bj" (see next subsection) are boolean. Euclid always treats these options as the value "1" (true) or "0" (false). When passing boolean options from the command

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line the value may be committed, in which case it assumed to be "1." Note, however, that when boolean options are contained in a configuration file, either the "1" or "0" must stated explicitly.

Method 3. There are two ways in which you can read in options from a file whose name is other than "database." First, you can call HYPRE\_EuclidSetParamsFromFile to specify a configuration filename. Second, if you have passed the command line arguments as described above in Method 2, you can then specify the configuration filename on the command line using the -db\_filename filename option, e.g.,

mpirun -np 2 phoo -db\_filename ../myConfigFile

### 7.5.3 Options summary

- -level  $\langle int \rangle$  Factorization level for ILU(k). Default: 1. Guidance: for 2D convection-diffusion and similar problems, fastest solution time is typically obtained with levels 4 through 8. For 3D problems fastest solution time is typically obtained with level 1.
- -bj Use Block Jacobi ILU preconditioning instead of PILU. Default: 0 (false). Guidance: if subdomains contain relatively few nodes (less than 1,000), or the problem is not well partitioned, Block Jacobi ILU may give faster solution time than PILU.
- -eu\_stats When Euclid's destructor is called a summary of runtime settings and timing information is printed to stdout. Default: 0 (false). The timing marks in the report are the maximum over all processors in the MPI communicator.
- -eu\_mem When Euclid's destructor is called a summary of Euclid's memory usage is printed to std-out. Default: 0 (false). The statistics are for the processor whose rank in MPI\_COMM\_WORLD is 0.
- -printTestData This option is used in our autotest procedures, and should not normally be invoked by users.

The following options are partially implemented, but not yet fully functional (i.e, don't use them until further notice).

- -sparseA  $\langle float \rangle$  Drop-tolerance for ILU(k) factorization. Default: 0 (no dropping). Entries are treated as zero if their absolute value is less than (sparseA \* max), where "max" is the largest absolute value of any entry in the row. Guidance: try this in conjunction with -rowScale. CAUTION: If the coefficient matrix A is symmetric, this setting is likely to cause the filled matrix, F = L + U I, to be unsymmetric. This setting has no effect when ILUT factorization is selected.
- -rowScale Scale values prior to factorization such that the largest value in any row is +1 or -1. Default: 0 (false). CAUTION: If the coefficient matrix A is symmetric, this setting is likely to cause the filled matrix, F = L + U I, to be unsymmetric. Guidance: if the matrix is poorly scaled, turning on row scaling may help convergence.

- -ilut  $\langle float \rangle$  Use ILUT factorization instead of the default, ILU(k). Here,  $\langle float \rangle$  is the drop tolerance, which is relative to the largest absolute value of any entry in the row being factored. CAUTION: If the coefficient matrix A is symmetric, this setting is likely to cause the filled matrix, F = L + U I, to be unsymmetric.
- -maxNzPerRow  $\langle int \rangle$  This sets the maximum number of nonzeros that is permitted in any row of F, in addition to the number that would result from an ILU(0) factorization. A negative value indicates infinity (no limit). This setting is effective for both ILU(k) and ILUT factorization methods. Default: infinity, for ILU(k); 5, for ILUT.

## 7.6 PILUT: Parallel Incomplete Factorization

PILUT is a parallel preconditioner based on Saad's dual-threshold incomplete factorization algorithm. The original version of PILUT was done by Karypis and Kumar [9] in terms of the Cray SHMEM library. The code was subsequently modified by the hypre team: SHMEM was replaced by MPI; some algorithmic changes were made; and it was software engineered to be interoperable with several matrix implementations, including hypre's ParCSR format, PETSc's matrices, and ISIS++RowMatrix. The algorithm produces an approximate factorization LU, with the preconditioner M defined by M = LU.

**Note:** *PILUT* produces a nonsymmetric preconditioner even when the original matrix is symmetric. Thus, it is generally inappropriate for preconditioning symmetric methods such as Conjugate Gradient.

### Parameters:

- SetMaxNonzerosPerRow( int LFIL ); (Default: 20) Set the maximum number of nonzeros to be retained in each row of L and U. This parameter can be used to control the amount of memory that L and U occupy. Generally, the larger the value of LFIL, the longer it takes to calculate the preconditioner and to apply the preconditioner and the larger the storage requirements, but this trades off versus a higher quality preconditioner that reduces the number of iterations.
- SetDropTolerance( double tol ); (Default: 0.0001) Set the tolerance (relative to the 2-norm of the row) below which entries in L and U are automatically dropped. PILUT first drops entries based on the drop tolerance, and then retains the largest LFIL elements in each row that remain. Smaller values of tol lead to more accurate preconditioners, but can also lead to increases in the time to calculate the preconditioner.

## Chapter 8

## Additional Information

### 8.1 Building the Library

Usually, hypre can be built by simply typing ./configure followed by make in the top-level source directory. hypre uses GNU Autoconf, Automake, and libtool so the building of the hypre library should be familiar to you if you've built GNU based packages before.

### 8.1.1 Getting the Library

The hypre distribution available through the Lawrence Livermore National Laboratory hypre web page at http://www.llnl.gov/CASC/hypre/ from the Software distribution page. The hypre Software distribution page will contain the tarballs for the 2 latest general, beta distributions, as well as documentation of hypre. You may also subscribe to the hypre software release announcement mailing lists form the hypre web page.

### 8.1.2 Library configuration

After unpacking the HYPRE tarball, the distribution will be in a subdirectory of the form, hypre-VERSION (e.g., hypre-1.8.4, with a "b" appended if the release is a beta).

```
$ gzip -cd hypre-1.8.4b.tar.gz | tar -xf -
$ cd hypre-1.8.4b
$ ls
```

```
CHANGELOG INSTALL docs
COPYRIGHT_and_DISCLAIMER README src
```

\$ cd src
\$ ls

FEI\_mv blas matrix\_matrix struct\_ls

HYPRE.h	config	nopoe	struct_mv
IJ_mv	configure	parcsr_es	tarch
Makefile.am	distributed_ls	parcsr_ls	test
Makefile.in	distributed_matrix	parcsr_mv	utilities
aclocal.m4	krylov	seq_mv	
babel	lapack	sstruct_ls	
babel-runtime	lib	sstruct_mv	

The *hypre* distribution subdirectory will contain files and directories similar to the above directory listing.

To automatically generate machine specific makefiles, type ./configure in the top level 'src' directory. The configure script is a portable script generated by GNU Autoconf. It runs a series of tests to determine characteristics of the machine on which it is running, and it uses the results of the these tests to produce the machine specific makefiles, called 'Makefile', from template files called 'Makefile.in' in each directory. Once the makefiles are produced you can run make as you would with any other makefile.

The configure script primarily does the following things:

- selects a compiler
- provides either optimization or debugging options for the compiler
- finds the headers and libraries for MPI

The configure script makes these decisions based on a hierarchical check. First, it attempts to identify the machine on which it is running as a specific supported machine. Next it will try to identify the architecture as a supported architecture. If both of these fail, generic default decisions are made by the script. However, the script does have some command-line options that can give you control over the choices it will make. You can type configure —help to see the list of all of the command-line options to configure. This is the best resource for information on configure options. The 'INSTALL' file in the top-level directory contains a little more information on installing software using configure.

Configure automatically generates a file HYPRE\_config.h that includes all the header files found to be necessary by configure. This file may be used to see how a compiled version of the library was configured and may also be included by the user in his/her own code.

hypre libraries are built from the top-level source directory by simply typing:

- \$ ./configure
- \$ make install

Note that by default hypre libraries will be installed in a subdirectory hypre/lib, and the include files in hypre/include (i.e., -prefix='pwd'/hypre). This differs from the standard GNU install which uses /usr/local/lib, and /usr/local/include. After testing (below) configure can be rerun with the -prefix=/usr/local option, followed by make install, to get the GNU behavior.

### 8.1.3 Testing the Library

After the make install step above, a few sample drivers can be used to test the build. cd test make The test drivers ij, ij\_es, ij\_mv, new\_ij, struct, and sstruct will be built, and should be runable from the command line.

```
$ ./configure
```

- \$ make test
- \$ cd test
- \$ ./ij
- \$ ./ij\_es
- \$ ./ij\_mv
- \$ ./sstruct
- \$ ./struct

To build the (stand-alone) fei driver the following steps are needed, starting from the top-level source directory:

```
$ make nofei
```

- \$ cd test
- \$ make fei++
- \$ ./fei

### 8.1.4 Linking to the Library

A program linking with hypre must be compiled with -I\$PREFIX/include and linked with -L\$PREFIX/lib -lhypre library name... -lhypre library name..., where \$PREFIX is the directory where hypre is installed, specified by the configure option --prefix=PREFIX. Additionally, any other libraries to which hypre is linked must also be linked to by the users application. For example, the BLAS library or PETSc library are often (but not always) linked in by hypre and would also need to be linked in by the users application.

It may be useful to reference the Makefile in the test subdirectory. This makefile is designed to build test applications that link with and call hypre. All include and linking flags that are used by hypre and needed by these test applications get exported to this file by the configure script.

### 8.1.5 Configure Options

Configure does it best to try and determine the compile options needed to build HYPRE. However the defaults that configure chooses can not meet everyones needs. Configure will display all of its available options by typing ./configure –help. A few of the more useful options include:

- \$ ./configure --enable-shared=yes, will build both shared and static libraries.
- \$ ./configure --enable-debug --without-MPI, will compile a serial version and debugging.
- \$ ./configure --with-babel, will build the babel interface libraries

### $\$ ./configure --prefix=\$HOME, will install hypre in to \$HOME

To build 64-bit static libraries on AIX (Note: configure will need to execute compiled code for its test, so configure needs to be run so that the default AIX poe scheduling is disabled), use the following:

```
$ ./nopoe ./configure CC=mpcc CXX=mpCC F77=mpxlf \
CFLAGS="-q64 -qmaxmem=8192 -DHYPRE_COMM_SIMPLE -03 -qstrict"\
CXXFLAGS="-q64 -qmaxmem=8192 -DHYPRE_COMM_SIMPLE -03 -qstrict"\
F77FLAGS="-q64 -03 -qstrict"\
LDFLAGS="-q64" AR="ar -X64"
make -i install
To support a read-only source tree, and build everything in a directory, build (VPATH):
$ mkdir build
$ cd build
$ cd build
$ ../configure
$ make test
```

### 8.1.6 Make Targets

all Make all the top-level targets, the default.

clean Undo what ever make does.

mostlyclean Like clean, but leaves libraries.

mostlyclean Like clean, but leaves libraries.

Maintainer-clean Delete everything than can be rebuilt.

distclean Undo what ever configure does.

install Copy (install) executables/libraries.

install-strip Strip the installed files.

uninstall Undo what every make install does.

TAGS Update the tags table.

test Build, install libraries, then build test drivers. dist Create a distribution file of the sources files.

check Perform self-test (if any).
installcheck Build and test an install.
distcheck Build and test a distribution.

### 8.1.7 Hints

Use MPI wrappers for the C, C++, and Fortran compilers (e.g., mpicc, mpicxx, mpif77). This will cause the least amount of confusion for configure and allow you to conveniently tune the compiler flags and libraries to your particular system. If MPI is enabled, mpicc will be the preferred compiler for C, while mpiCC or mpicxx will be used for C++, and mpif77 for Fortran.

If when running ./configure on AIX you get the error:

checking for C compiler default output file name... a.out

```
checking whether the C compiler works... configure: error: cannot run C compiled programs. If you meant to cross compile, use '--host'. See 'config.log' for more details.
```

## 8.2 Calling from Fortran

Rerun configure using ./nopoe ./configure.

Although *hypre* is written in C, a Fortran interface is provided. The Fortran interface is very similar to the C interface, and can be determined from the C interface by a few simple conversion rules. These conversion rules are described below.

Let us start out with a simple example. Consider the following hypre prototype:

The corresponding Fortran code for calling this routine is as follows:

The Fortran subroutine name is the same, unless the name is longer than 31 characters. In these situations, the name is condensed to 31 characters, usually by simple truncation. For now, users should look at the Fortran drivers in the test directory for the correct condensed names. In the future, this aspect of the interface conversion will be made consistent and straightforward.

The Fortran subroutine argument list is always the same as the corresponding C routine, except that the error return code ierr is always last. Conversion from C parameter types to Fortran argument type is summarized in Table 8.1. Arrays arguments in hypre are always of type (int \*) or (double \*), and the corresponding Fortran types are simply integer or double precision arrays. Note that the Fortran arrays may be indexed in any manner. For example, an integer array of length N may be declared in fortran as either of the following:

```
integer array(N)
integer array(0:N-1)
```

hypre objects can usually be declared as in the table because integer\*8 usually corresponds to the length of a pointer. However, there may be some machines where this is not the case (although we are not aware of any at this time). On such machines, the Fortran type for a hypre object should be an integer of the appropriate length.

C parameter	Fortran argument		
int i	integer i		
double d	double precision d		
int *array	integer array(*)		
double *array	<pre>double precision array(*)</pre>		
char *string	character string(*)		
HYPRE_Type object	integer*8 object		
HYPRE_Type *object	integer*8 object		

Table 8.1: Conversion from C parameters to Fortran arguments

## 8.3 Bug Reporting

hypre has an automated bug reporting mechanism in place that may be used as a resource for submitting bugs, desired features, and documentation problems, as well as querying the status of previous reports. Access http://www-casc.llnl.gov/bugs for full bug tracking details or to submit or query a bug report. When using the CASC bug reporting site for the first time, click on "Open a new Bugzilla account" under the "User login account management" heading.

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