№2. Московка

$\cos(3iz) = 2i$

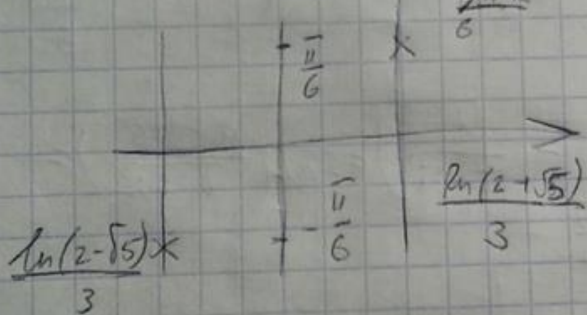
$$\frac{e^{i(3iz)} + e^{-i(3iz)}}{2} = 2i \Leftrightarrow e^{3z} + e^{-3z} - 4i = 0 \Leftrightarrow$$

$$(e^{3z})^2 - 4i(e^{3z}) + 1 = 0$$

$$e^{3z} = 2i \pm i\sqrt{5}$$

$$\left[\begin{array}{l} 3z = \ln(i(2+\sqrt{5})) \\ 3z = \ln(i(2-\sqrt{5})) \end{array}\right. \Rightarrow \left[\begin{array}{l} 3z = \ln(2+\sqrt{5}) + \frac{\pi}{2}i + 2\pi i k \\ 3z = \ln(2-\sqrt{5}) + \frac{\pi}{2}i + \pi i + 2\pi i k \end{array}\right.$$

$$z = \pm \frac{\ln(2 \pm \sqrt{5})}{3} \pm i\underbrace{\left(\frac{\pi}{6} + \frac{2\pi k}{3}\right)}_{\frac{5\pi k}{6}} \pm \frac{\pi}{6}i + \frac{2\pi i k}{3}$$



Ответ: a.

№3 Московка

$f(z) = z^2 \cdot \bar{z} + Re(7e^{8i}) = (x^2+y^2)\cdot(x+iy) +$

$+ Im(7(\cos 8 + i\sin 8) = x^3 + x^2 yi + y^2 x + iy^3 + \sin 8$

$$\underbrace{x^2 + y^2 x + \sin 8}_{u(x,y)} + i\underbrace{(x^2 y + y^3)}_{v(x,y)}$$

$\frac{du}{dx} = 3x^2 + y^2$; $\frac{dv}{dy} = x^2 + 3y^2$ => условие не выполн.

=> функция не аналитична

$Im f(-i) = im(-1 \cdot i + \sin 8) = -1$

Ответ: d. -1

№6

$$\int\limits_{|z-4|=1} \frac{dz}{(z+1)(z-2)} + \int\limits_{|z+2|=1} \frac{dz}{(z+2)(z-1)}$$

$|z-4| = 1$ $\qquad |z+2| = 1$

$3 < z < 5$ $\qquad -3 < z < -1$

$z = -2$

$$\int \frac{dz}{(z+2)(z-1)} = 2\pi i \, res f(-2) = 2\pi i \lim_{z \to -2} \frac{1}{z-1} = 2\pi i \frac{1}{-3}$$

Ответ: $-\frac{2\pi i}{3}$.

№7. Московка

$$\int\limits_{|z-1|=1,5}\frac{dz}{(z-3)(z-2)}+\int\limits_{|z-2|=2}\frac{dz}{(z-2)(z+3)}$$

$|z-1|=1,5$ $\quad$ $|z-2|=2$

$-0,5<z<2,5$ $\quad$ $0<z<4$

$z=2$ $\quad$ $z=2$

$$\int\frac{dz}{(z-3)(z-2)}=2\pi i\operatorname{res}_{z=2}f(z)=2\pi i\lim_{z\to 2}\frac{1}{z-3}=-2\pi i$$

$$\int\frac{dz}{(z-2)(z+3)}=2\pi i\operatorname{res}_{z=2}f(z)=2\pi i\lim_{z\to 2}\frac{1}{z+3}=\frac{2\pi i}{5}.$$

Ответ: $-\frac{10\pi i}{5}+\frac{2\pi i}{5}=-\frac{8i\pi}{5}$.

№8.

$$\int\limits_{-\infty}^{0}\frac{x^2dx}{(x^2+16)(x^2+1)}$$ $\quad$ $z_{1,2}=\pm 4i;\ z_{3,4}=\pm i$

$$I=\frac{1}{2}\int\limits_{-\infty}^{+\infty}\frac{x^2dx}{(x^2+16)(x^2+1)}$$

$$F(z)=\frac{z^2}{(z^2+16)(z^2+1)}$$

В верхней полуплоскости: $4i;\ i$.

$$\operatorname{res}_{z=4i}\frac{z^2}{(z^2+16)(z^2+1)}=\lim_{z\to 4i}\frac{z^2}{(z^2+16)(z^2+1)}=\frac{-16}{8i\cdot(-16+1)}=\frac{2}{15i}$$

$$\operatorname{res}_{z=i}=\ldots=\lim_{z\to i}\frac{z^2}{(z^2+16)(z^2+1)}=\frac{-1}{(-1+16)(i+i)}=\frac{-1}{15\cdot 2i}=\frac{-1}{30i}$$

Ответ: $\frac{1}{2}\cdot 2\pi i\left(\frac{2}{15i}-\frac{1}{30i}\right)=\pi i\left(\frac{3}{30i}\right)=\frac{\pi}{10}.$