№2. Москвка

$\cos(3z) = -2i$

$$\frac{e^{3zi} + e^{-3zi}}{2} = -2i \iff \left(e^{3zi}\right)^2 + 1 + 4i\,e^{3zi} = 0$$

$$e^{3zi} = \frac{-4i \pm \sqrt{-16+4}}{2} = \frac{-4i \pm \sqrt{-12}}{2} = \frac{-4i \pm 2i\sqrt{3}}{2} =$$

$$= -2i \pm \sqrt{3}i = i(-2 \pm \sqrt{3})$$

$$\left[\begin{array}{l} 3zi = \ln(i(-2+\sqrt{3})) \\ 3zi = \ln(i(-2-\sqrt{3})) \end{array}\right. \Rightarrow \left[\begin{array}{l} 3zi = \ln(2-\sqrt{3}) + i\left(-\frac{\pi}{2} + 2\pi k\right) \;/\; -\frac{i}{3} \\ 3zi = \ln(2+\sqrt{3}) + i\,\dfrac{\left(-\frac{\pi}{2} + 2\pi k\right)}{-\frac{i}{3}} \end{array}\right.$$

$$\left[\begin{array}{l} z = \dfrac{-\frac{\pi}{2} + 2\pi k}{3} - i \cdot \dfrac{\ln(2-\sqrt{3})}{3} \\ z = \dfrac{-\frac{\pi}{2} + 2\pi k}{3} - i \cdot \dfrac{\ln(2+\sqrt{3})}{3} \end{array}\right.$$

№3. Москвина

$f(z) = z^2 \cdot \bar{z} + Re(7e^{8i}) = (x^2+y^2)\cdot(x+iy) +$

$+ Im(7(\cos 8 + i\sin 8)) = x^3 + x^2 y i + y^2 x + i y^3 + \sin 8$

$$\underbrace{x^2 + y^2 x + \sin 8}_{u(x,y)} + i\underbrace{(x^2 y + y^3)}_{v(x,y)}$$

$\frac{du}{dx} = 3x^2 + y^2$; $\frac{dv}{dy} = x^2 + 3y^2$ => условие не выполн.

=> функция не аналитична

$Im f(-i) = im(-1 \cdot i + \sin 8) = -1$

Ответ: f. −1

№6 Московка

$$\int\limits_{|z+1|=2} \frac{dz}{(z+1)(z-2)} + \int\limits_{|z|=0,5} \frac{dz}{(z+2)(z-1)}$$

$|z+1| = 2$ $\qquad$ $|z| = 0,5$

$-3 < z < 1$ $\qquad$ $-0,5 < z < 0,5$

$z = -1$

$$\int \frac{dz}{(z+1)(z-2)} = 2\pi i \lim_{z \to -1} \frac{1}{z-2} = 2\pi i \cdot \frac{1}{-3} = -\frac{2\pi i}{3}.$$

Ответ: $e. -\frac{2\pi i}{3}$.

№7. Москвина

$$\int\limits_{|z-2|=2} \frac{dz}{(z-3)(z+2)} + \int\limits_{|z-3|=0,5} \frac{dz}{(z-2)(z-3)}$$

$|z-2|=2$ $\qquad$ $|z-3|=0,5$

$0<z<4$ $\qquad$ $2,5<z<3,5$

$$\int \frac{dz}{(z-3)(z+2)} = 2\pi i \cdot \frac{1}{5} = \frac{2\pi i}{5}$$

$$\int \frac{dz}{(z-2)(z-3)} = 2\pi i$$

$$I = \frac{2\pi i}{5} + 2\pi i = \frac{12\pi i}{5} \leftarrow \text{Ответ.}$$

№8.

$$\int\limits_{-\infty}^{+\infty} \frac{x^2\,dx}{(x^2+9)(x^2+16)} \qquad z = 3i;\ 4i$$

$$\underset{z=3i}{\mathrm{res}}(..) = \lim_{z\to 3i} \frac{z^2}{(z^2+9)(z^2+16)} = \frac{-9}{6i\cdot(-9+16)} = \frac{-9}{7\cdot 6i}$$

$$\underset{z=4i}{\mathrm{res}}(\ldots) = \lim_{z\to 4i} \frac{z^2}{(z^2+9)(z^2+4i)} = \frac{-16}{(-16+9)\cdot(8i)} = \frac{\cancel{-16}^{\,2}}{-7\cdot\cancel{8}i} = \frac{2}{7i}$$

$$I = 2\pi i\left(\frac{-72+96}{7\cdot 48i}\right) = 2\pi \cancel{i}\left(\frac{24}{7\cdot 48\cancel{i}}\right) = \frac{\pi}{7}.$$

Ответ: $\frac{\pi}{7}$.