

№2. Макробна АА.

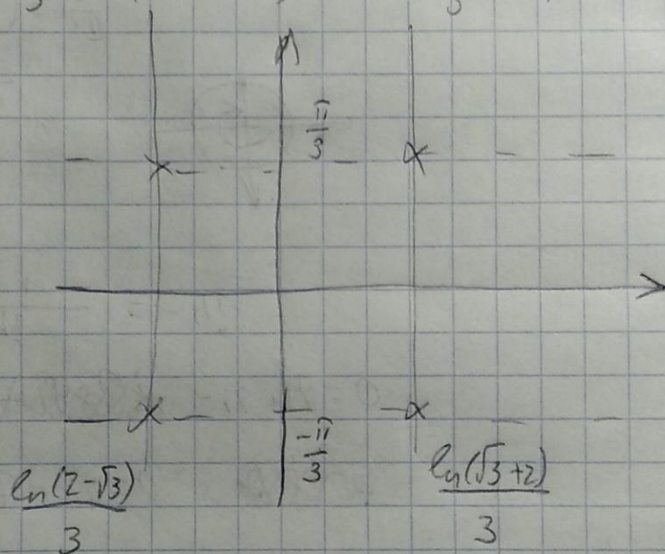
$$\cos(3iz) = -2$$

$$(e^{3z})^2 + 4e^{3z} + 1 = 0$$

$$e^{3iz} = -2 \pm \sqrt{3}$$

$$3z = \ln(-2 \pm \sqrt{3}) + \pi k + 2\pi ki$$

$$z = \pm \frac{1}{3} \ln(2 \pm \sqrt{3}) + \frac{(1 + 2ki)\pi}{3}, k \in \mathbb{Z}$$



Ответ: c.

№3. Мокшова А.А.

$$f(z) = z^2 \cdot \bar{z} + \operatorname{Re}(5e^z \cdot i)$$

$$\begin{aligned} f'(z) &= (x+iy)^2 \cdot (-i) + 5e^z = x^3 + x^2yi + y^3x + iy^3 = \\ &= \underbrace{x^3 + yx^2}_{u(x,y)} + i \underbrace{(x^2y + y^3)}_{v(x,y)} \end{aligned}$$

$$\frac{du}{dx} = 3x^2 + y^2; \quad \frac{dv}{dy} = x^2 + 3y^2 \Rightarrow \text{Кроков не вынуждены} \Rightarrow$$

$$\operatorname{Im} f(-i) = \operatorname{Im}(-i \cdot f(1)) = -1$$

Ответ: -1.

№6.

$$\int_{|z-2|=2} \frac{dz}{(z+1)(z-2)} + \int_{|z-4|=2} \frac{dz}{(z+2)(z-1)}$$

$$|z-2|=2$$

$$|z-4|=2$$

$$0 < z < 4$$

$$2 < z < 6$$

$$z = -1, 2.$$

$$z = 1, -2$$

$$z = z - 0.7.$$

$$\int_{(z+1)(z-2)} \frac{dz}{(z+1)(z-2)} = 2\pi i \operatorname{Res}_{z=2} f(z) = 2\pi i \lim_{z \rightarrow 2} \frac{dz}{z+1} = 2\pi i \frac{1}{3} = \frac{2\pi i}{3}$$

Ответ: а) $\frac{2\pi i}{3}$.

№7. Найдите А.А.

$$\int_{|z+2|=\frac{1}{2}} \frac{dz}{(z+2)(z+3)} + \int_{|z+1|=2} \frac{dz}{(z+2)(z-3)}$$

$$|z+2|=\frac{1}{2}$$

$$-\frac{5}{2} < z < -\frac{3}{2}$$

$$z = -2$$

$$|z+1|=2$$

$$-3 < z < 1$$

$$z = -2$$

$$\int \frac{dz}{(z+3)(z+2)} = 2\pi i \operatorname{res} f(z)_{z=-2} = 2\pi i \lim_{z \rightarrow -2} \frac{1}{z+3} = 2\pi i$$

$$\int \frac{dz}{(z+2)(z-3)} = 2\pi i \operatorname{res} f(z)_{z=-2} = 2\pi i \lim_{z \rightarrow -2} \frac{1}{z-3} = -\frac{2\pi i}{5}$$

$$2\pi i - \frac{2\pi i}{5} = \frac{10\pi i - 2\pi i}{5} = \frac{8\pi i}{5}$$

Ответ: $\frac{8\pi i}{5}$.

~ 8. Мокноба д.д.

$$\int_{-\infty}^{+\infty} \frac{x^2 dx}{(x^2+9)(x^2+1)} \quad z = i; 3i$$

$$\operatorname{res}_{z=3i} \int_{-\infty}^{+\infty} \frac{x^2 dx}{(x^2+9)(x^2+1)} = \lim_{z \rightarrow 3i} \frac{z^2}{(z+3i)(z^2+1)} =$$

$$= \frac{-9}{(6i)(-9+1)} = \frac{-9}{-8 \cdot 6i} = \frac{9}{6 \cdot 8i}$$

$$\operatorname{res}_{z=i} \int_{-\infty}^{+\infty} \frac{x^2 dx}{(x^2+9)(x^2+1)} = \lim_{z \rightarrow i} \frac{z^2}{(z^2+9)(z+i)} = \frac{-1}{(-1+9)(2i)} =$$

$$= \frac{-1}{8 \cdot 2i}$$

$$I = 2\pi i \left(\frac{9}{6 \cdot 8i} - \frac{1}{2 \cdot 8i} \right) = 2\pi i \left(\frac{9-3}{8 \cdot 6i} \right) =$$

$$= 2\pi i \cdot \frac{1}{8i} = \frac{\pi}{4}$$

Омбем: д. $\frac{\pi}{4}$.