> #Лабораторная работа 3(Вариант 4)
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> #PART 1

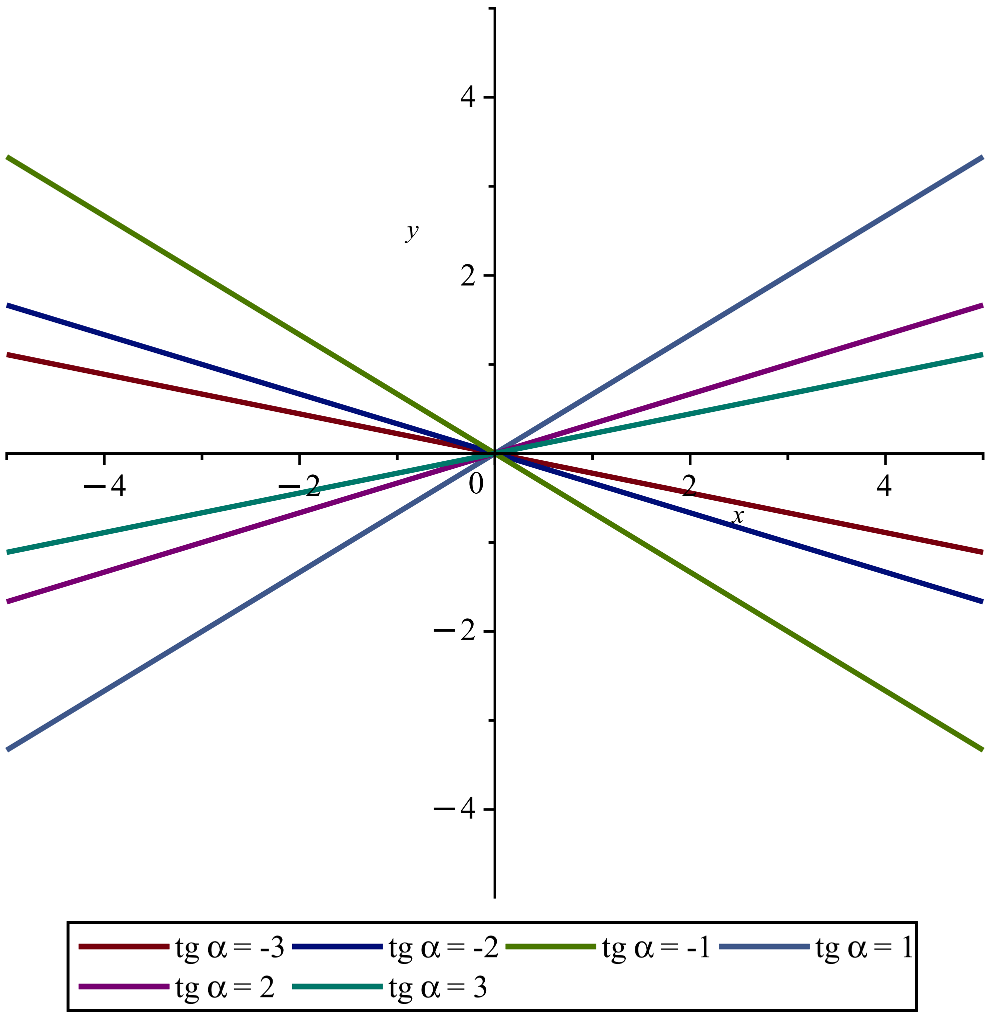
> #Задание 1

> $diff(y(x), x) = \frac{2 \cdot x}{3 \cdot y(x)}$

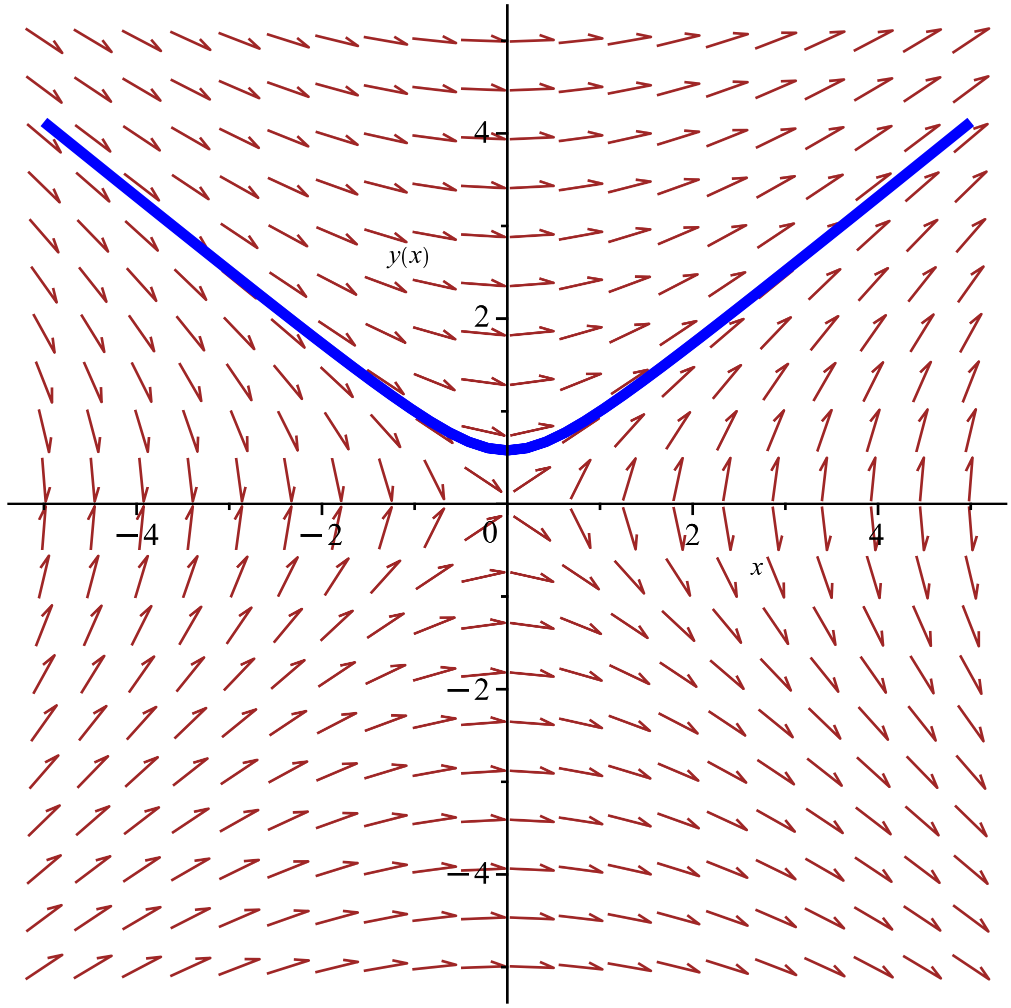
$$\frac{d}{dx}\, y(x) = \frac{2\, x}{3\, y(x)} \qquad (1)$$

> $with(DETools):$

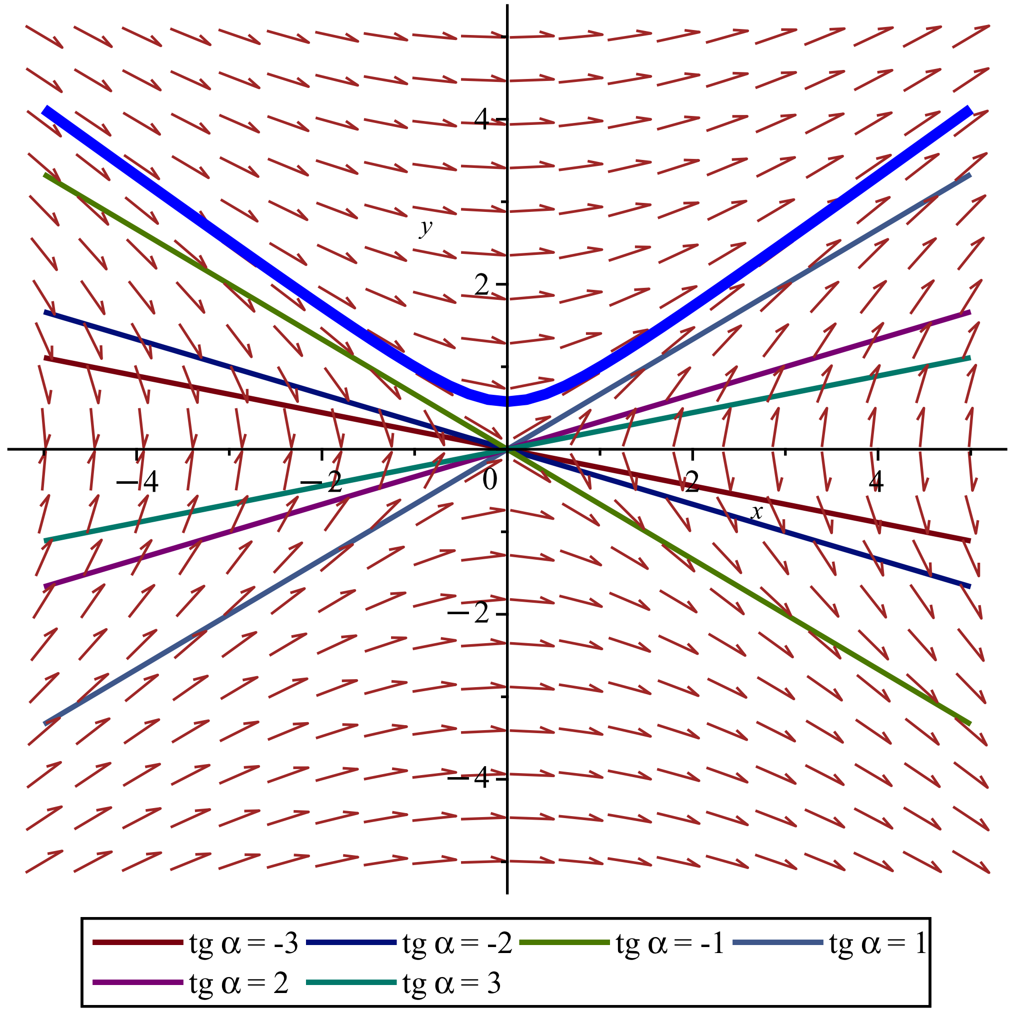
> $isoclines := plot\left(\left[seq\left(\frac{2 \cdot x}{3 \cdot k},\ k = [-3, -2, -1, 1, 2, 3]\right)\right], x = -5 .. 5, y = -5 .. 5, legend = [\text{"tg α = -3"}, \text{"tg α = -2"}, \text{"tg α = −1"}, \text{"tg α = 1"}, \text{"tg α = 2"}, \text{"tg α = 3"}], thickness = 2\right);$



> $dplot := DEplot\left(diff(y(x), x) = \frac{2 \cdot x}{3 \cdot y(x)}, y(x), x = -5 ..5, y = -5 ..5, [\, y(1) = 1], linecolor = blue, thickness = 4\right)$



> $plots[display](isoclines, dplot)$



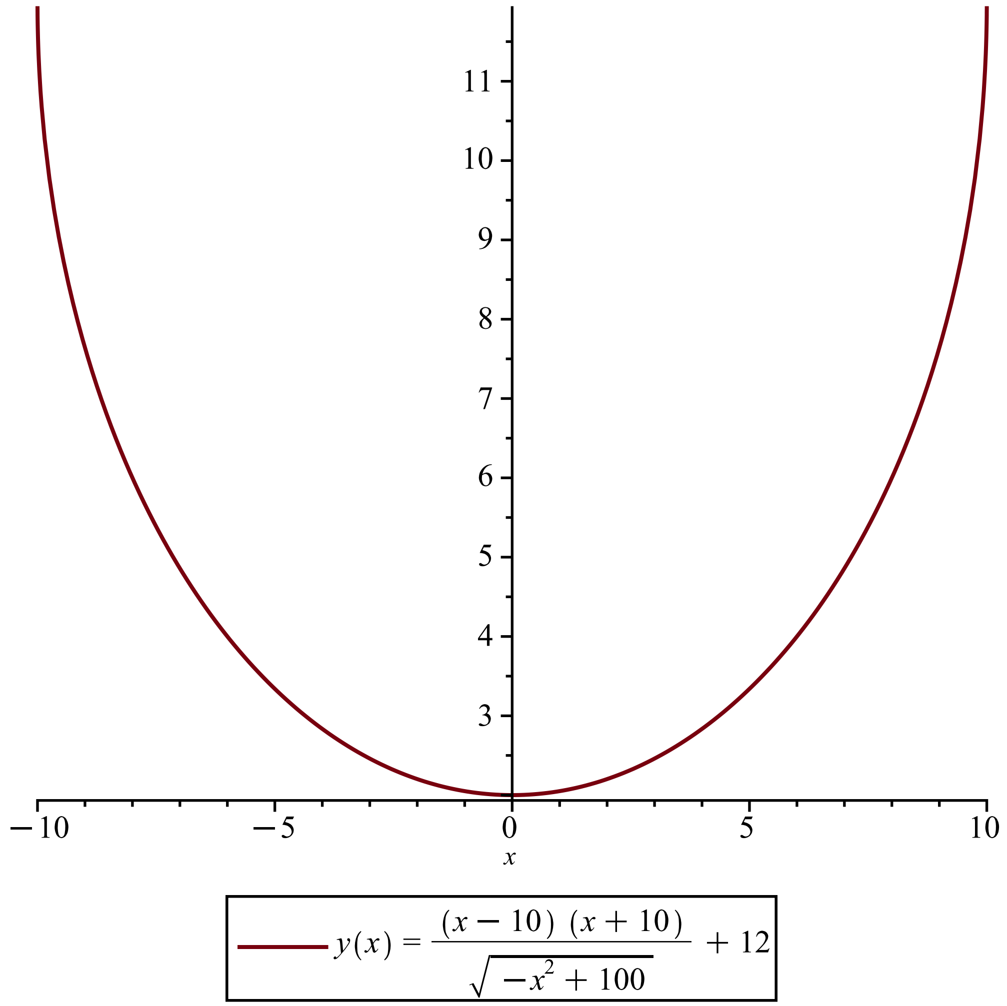
> *restart* :

> ***#Задание 2.1***

> $line := dsolve\left(\left\{diff(y(x), x) = \frac{x}{\mathrm{sqrt}(10^2 - x^2)}, y(6) = 4\right\}\right)$

$$line := y(x) = \frac{(x-10)\ (x+10)}{\sqrt{-x^2+100}} + 12 \quad \textbf{(2)}$$

> $plot(rhs(line), legend = line)$ **# *rhs() извлекает правую часть***



> *restart* :

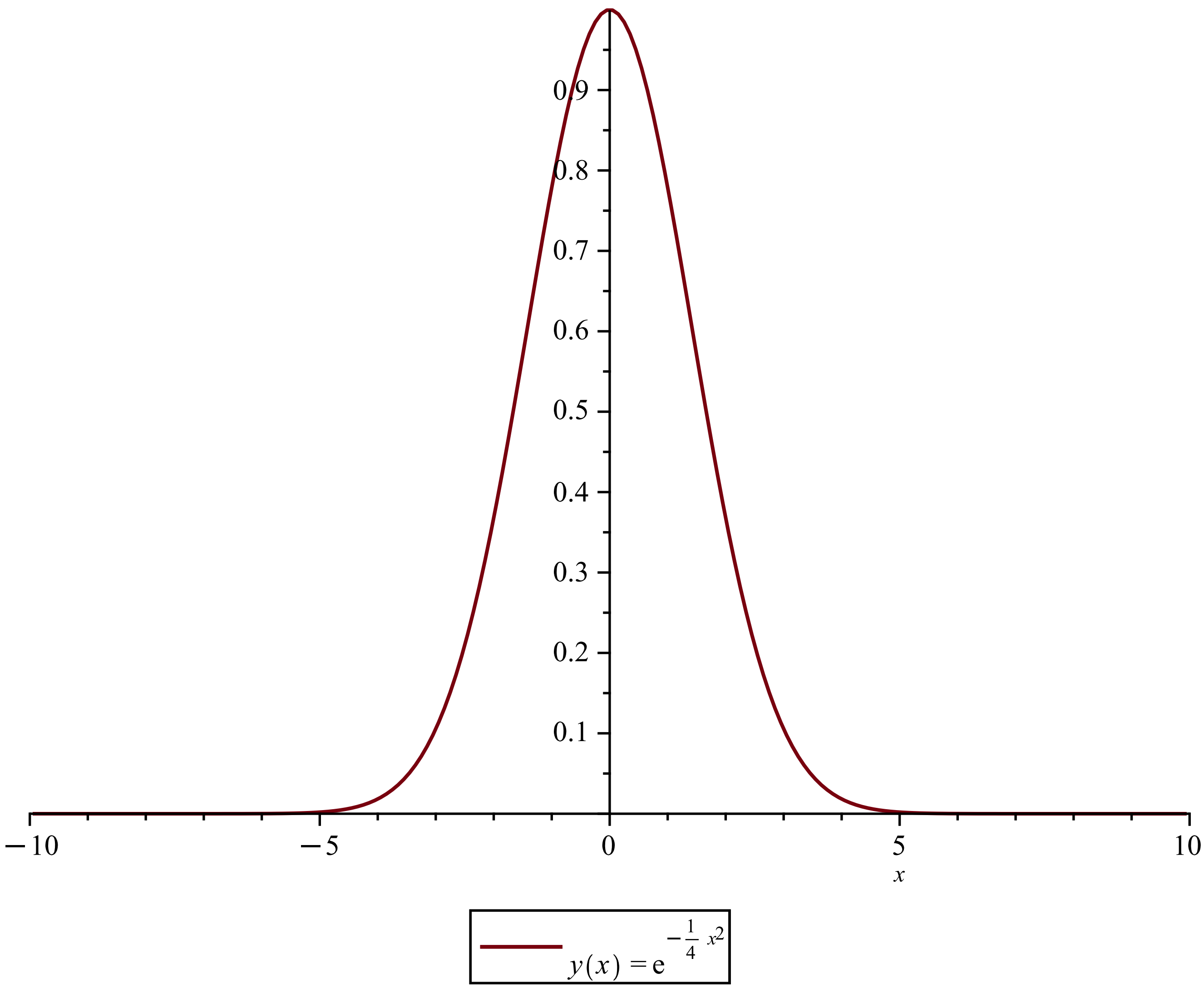
> ***#Задание 2.2***

> $a := 2:$

> $line := simplify\left(dsolve\left(\left\{diff(y(x), x) = -\frac{y(x) \cdot x}{a}, y(2) = \frac{1}{\mathrm{e}}\right\}\right)\right)$

$$line := y(x) = \mathrm{e}^{-\frac{x^2}{4}} \qquad \textbf{(3)}$$

> $plot(rhs(line), legend = line)$



> *restart* :

> ***#Задание 3***

> $func := diff(y(x), x) = \dfrac{-12\cdot x - 5\cdot y(x) + 34}{2\cdot x + y(x) - 6}$

$$func := \frac{d}{dx}\, y(x) = \frac{-12\,x - 5\,y(x) + 34}{2\,x + y(x) - 6} \tag{4}$$

> $func\_solve := dsolve(func, y(x))$

$$func\_solve := y(x) = 2 - \frac{8\,(x-2)\,c_1 + 1 + \sqrt{4\,(x-2)\,c_1 + 1}}{2\,c_1} \tag{5}$$

> $A := Matrix([[-12, -5], [2, 1]])$

$$A := \begin{bmatrix} -12 & -5 \\ 2 & 1 \end{bmatrix} \tag{6}$$

> $linalg[det](A)$

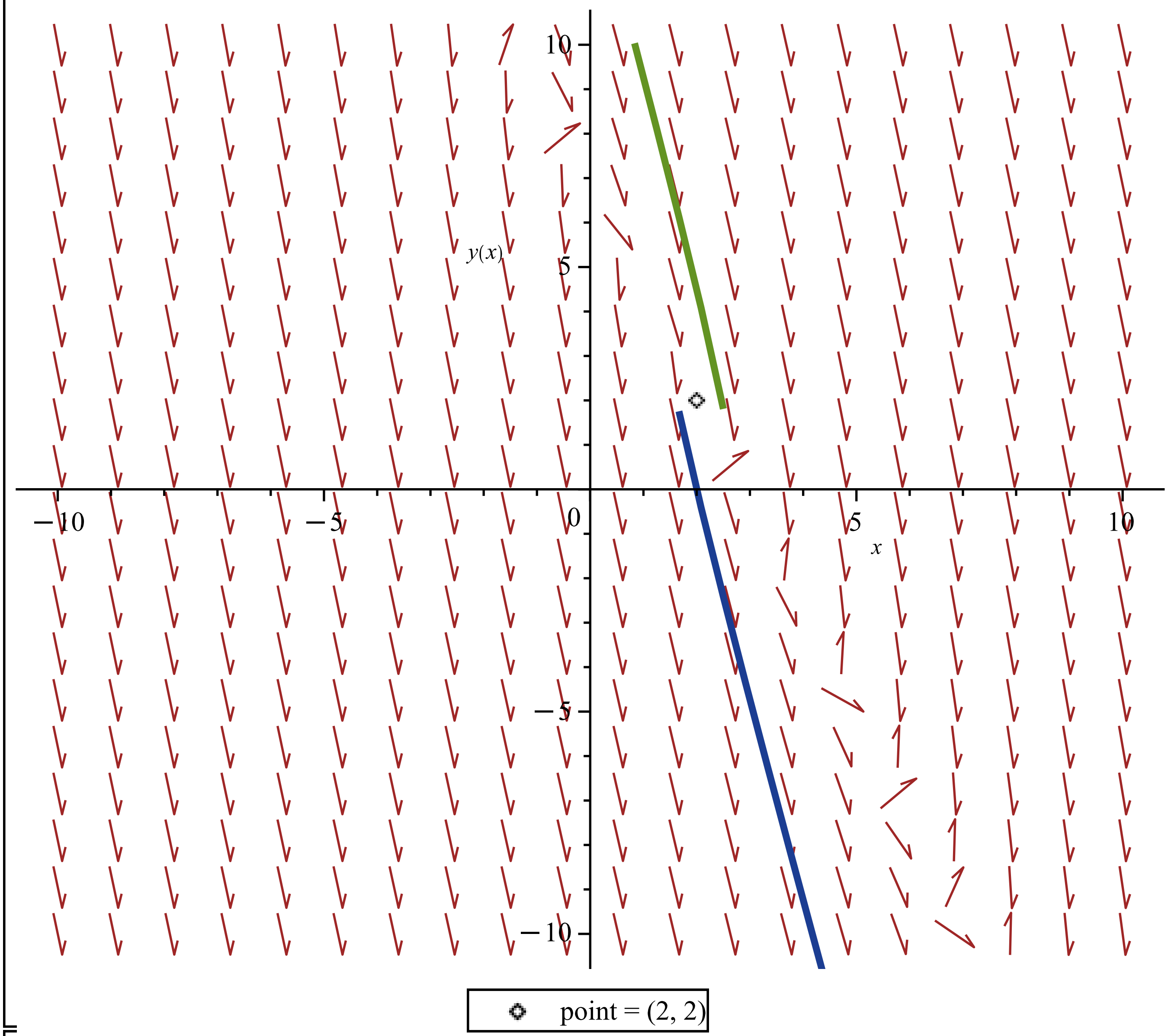
$$-2 \tag{7}$$

> $solve(\{-12\cdot x - 5\cdot y + 34 = 0, 2\cdot x + y - 6 = 0\})$

$$\{x = 2, y = 2\} \qquad (8)$$

```
> plot1 := DEtools[DEplot](func, y(x), x = -10 ..10, y = -10 ..10 , [[y(2) = 0], [y(2)
      = 4.5]]):
  plot2 := plots[pointplot]([2, 2], legend = "point = (2, 2)"):
  plots[display](plot1, plot2)
```

Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further left of 1.4999999, maxfun limit exceeded (see ?dsolve,maxfun for details)
Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further right of 2.6250001, probably a singularity




```
> M := Matrix([[2 − λ, 1], [−12, −5 − λ]]);
  solve(LinearAlgebra[Determinant](M) = 0):
  λ1 = convert(%[1], float);
  λ2 = convert(%%[2], float);
```

$$M := \begin{bmatrix} 2-\lambda & 1 \\ -12 & -5-\lambda \end{bmatrix}$$

$$\lambda 1 = -1.$$

$$\lambda 2 = -2. \tag{9}$$

> $\lambda 1$ − отриц , $\lambda 2$ − отриц ⇒ точка ассимптотически устройчива (устойчивый узел)

$\lambda 1$ − отриц, $\lambda 2$ − отриц ⇒ точка ассимптотически устройчива (устойчивый узел) **(10)**

> *restart* :

> ***#Задание 4***

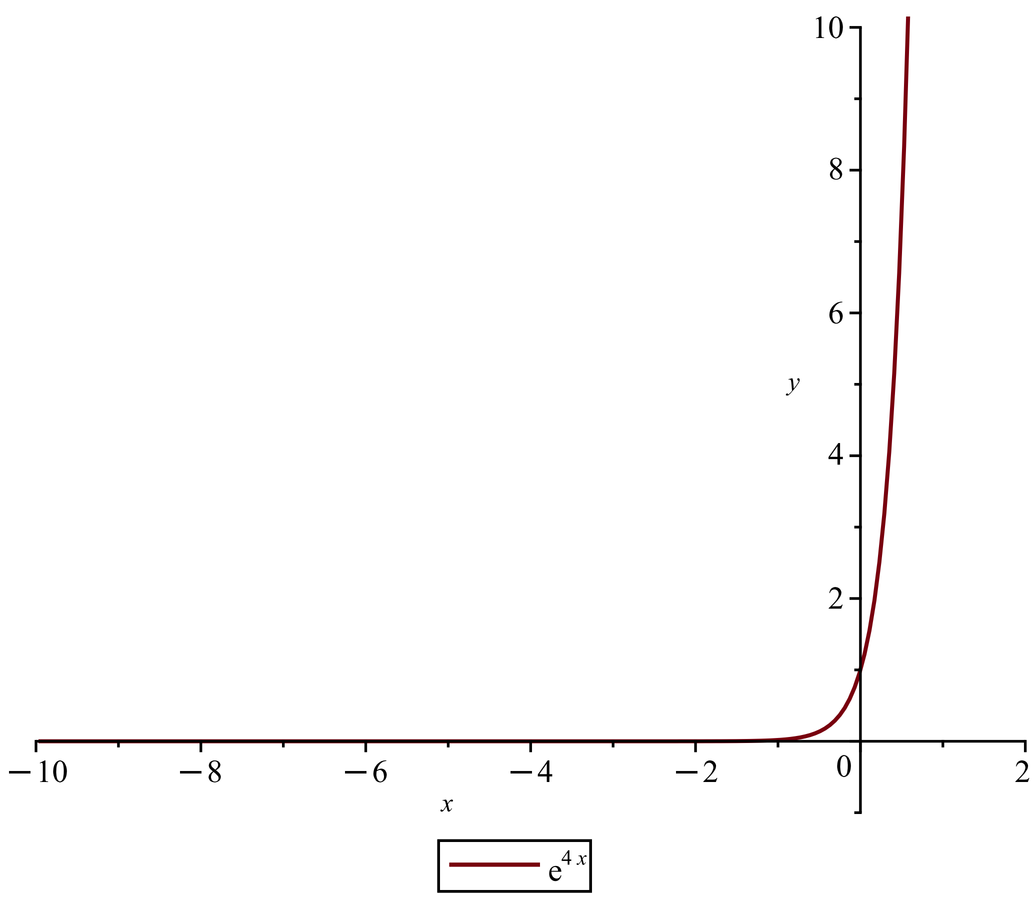
> $func := diff(y(x), x) + 4\,x^3 \cdot y(x) = 4(x^3+1)\cdot \exp(-4\cdot x)\cdot y(x)^2;$

$ans := solve(dsolve(\{func, y(0) = 1\}), y(x));$

$plot(ans, x = -10\,..\,2, y = -1\,..10, legend = ans)$

$$func := \frac{d}{dx}\, y(x) + 4\,x^3\, y(x) = 4\,(x^3+1)\,e^{-4x}\, y(x)^2$$

$$ans := e^{4x}$$



> *restart* :

> ***#Задание 5.1***

> $task := x = diff(y(x), x)\cdot \ln(diff(y(x), x)) - diff(y(x), x);$

$x\_fun := diff(y(t), t) = t\cdot \ln(t) - t;$

$dx\_fun := diff(y(t), t) = \ln(t);$

```
dy_fun := diff(y(t), t) = t·ln(t);
y_solve := dsolve(dy_fun);
```

$$task := x = \left(\frac{d}{dx}\, y(x)\right) \ln\left(\frac{d}{dx}\, y(x)\right) - \frac{d}{dx}\, y(x)$$

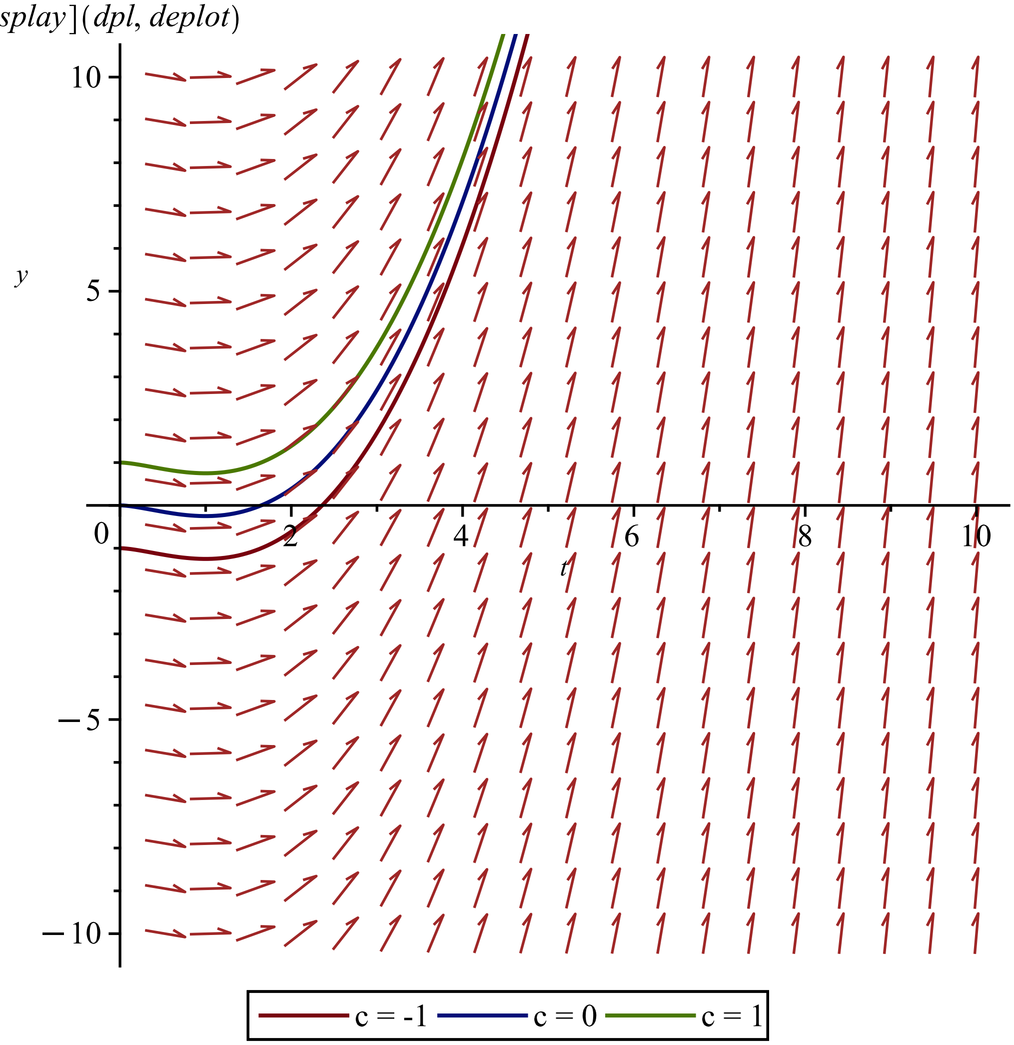
$$x\_fun := \frac{d}{dt}\, y(t) = t \ln(t) - t$$

$$dx\_fun := \frac{d}{dt}\, y(t) = \ln(t)$$

$$dy\_fun := \frac{d}{dt}\, y(t) = t \ln(t)$$

$$y\_solve := y(t) = \frac{t^2 \ln(t)}{2} - \frac{t^2}{4} + c_1 \tag{11}$$

```
> deplot := DETools[DEplot](dy_fun, y(t), t = 0..10, y = -10..10, thickness = 5) :
> dpl := plot([seq(t^2·ln(t)/2 - t^2/4 + c, c = -1..1)], t = 0..10, y = -10..10, legend = ["c = -1", "c = 0", "c = 1"]) :
> plots[display](dpl, deplot)
```



> $restart:$

> ***#Задание 5.2***

> $task := y(x) = \ln(\mathrm{abs}(\cos(diff(y(x), x)))) + diff(y(x), x) \cdot \tan(diff(y(x), x));$
$y\_fun := \ln(\mathrm{abs}(\cos(t))) + t \cdot \tan(t);$
$dy\_fun := diff(x(t), t) = \frac{t}{\cos(t)^2};$
$dx\_fun := diff(x(t), t) = \frac{1}{\cos(t)^2};$
$x\_solve := dsolve(\mathrm{dx\_fun});$

$$task := y(x) = \ln\left(\left|\cos\left(\frac{d}{dx}\, y(x)\right)\right|\right) + \left(\frac{d}{dx}\, y(x)\right) \tan\left(\frac{d}{dx}\, y(x)\right)$$

$$y\_fun := \ln(|\cos(t)|) + t \tan(t)$$

$$dy\_fun := \frac{d}{dt}\, x(t) = \frac{t}{\cos(t)^2}$$

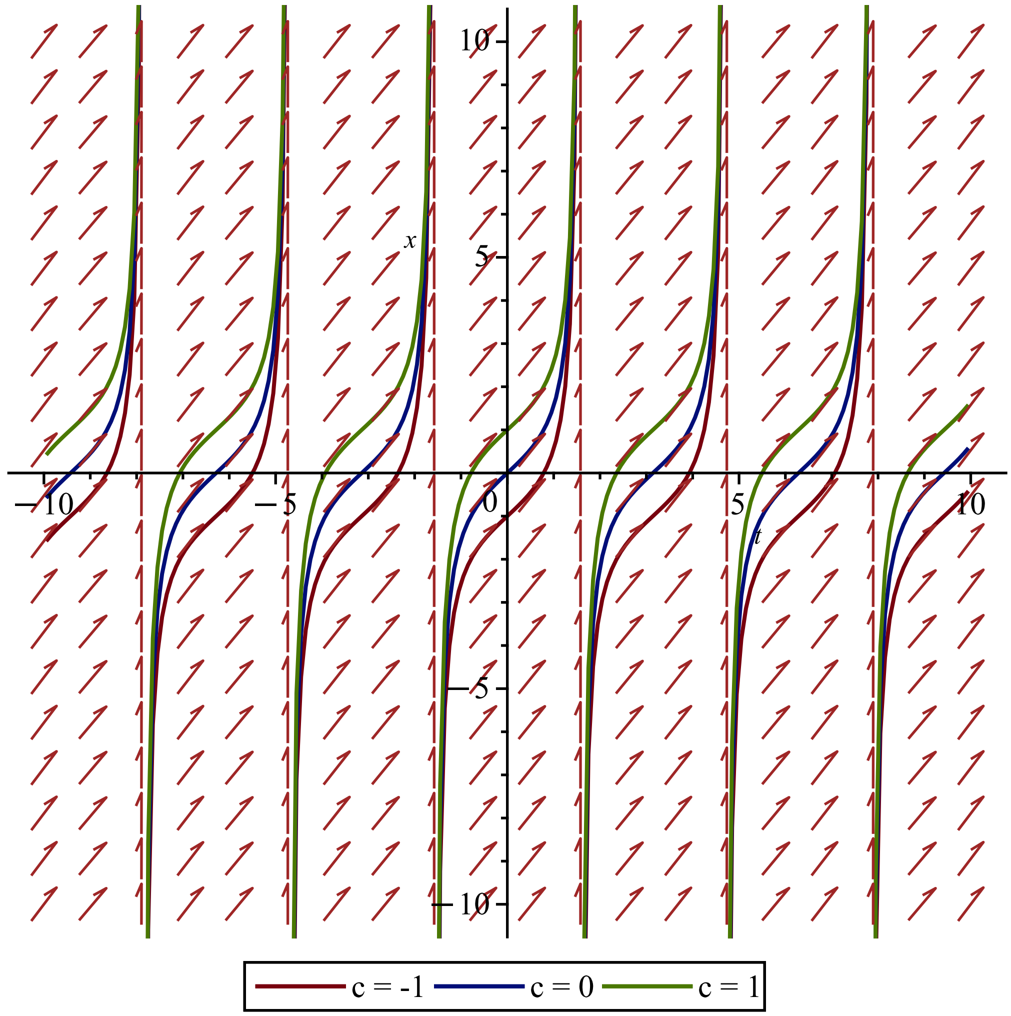
$$dx\_fun := \frac{d}{dt}\, x(t) = \frac{1}{\cos(t)^2}$$

$$x\_solve := x(t) = \tan(t) + c_1 \tag{12}$$

> $deplot := DETools[DEplot](dx\_fun, x(t), t = -10..10, x = -10..10, thickness = 5):$

> $dpl := plot([seq(\tan(t) + c, c = -1..1)], \mathrm{t} = -10..10, \mathrm{x} = -10..10, \mathrm{legend} = [\text{"c = -1"}, \text{"c = 0"}, \text{"c = 1"}]):$

> $plots[display](dpl, deplot)$

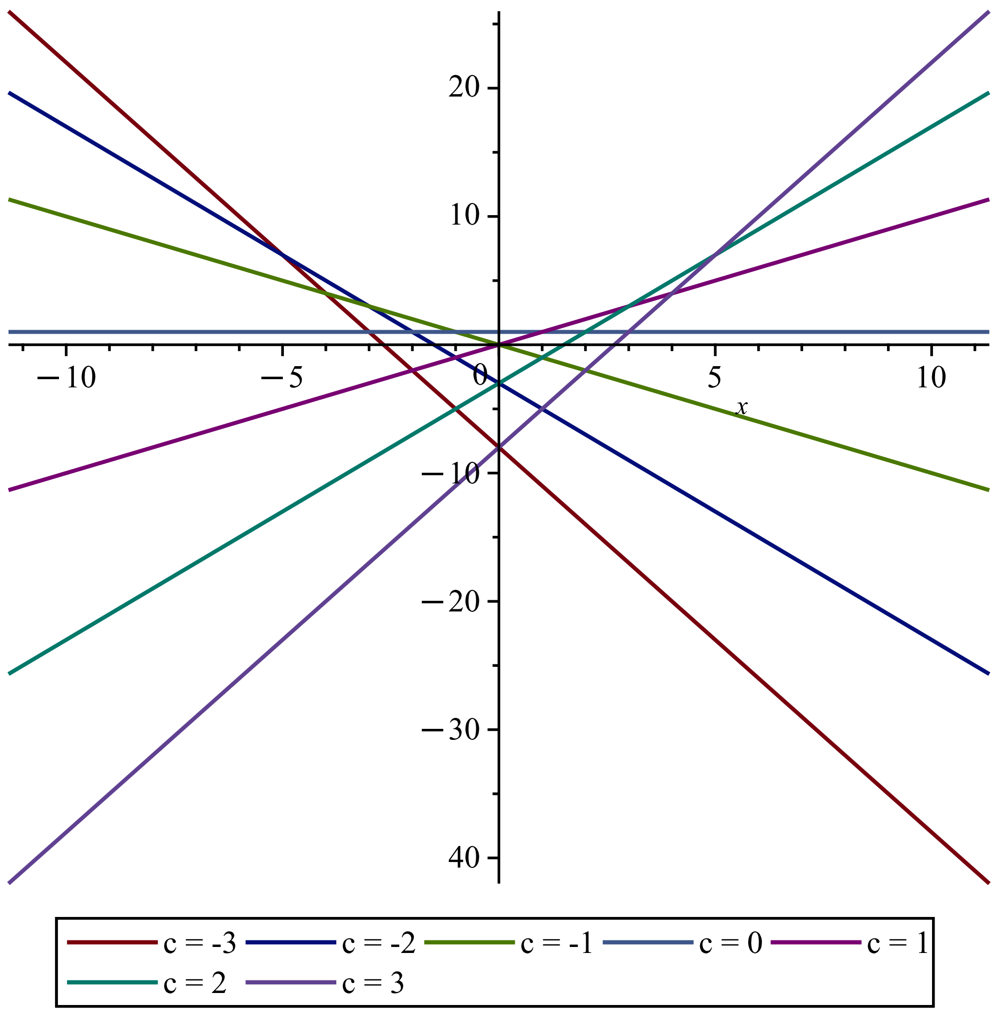



```
> restart :
> #Задание 6
> task := y(x) = x·diff(y(x), x) − diff(y(x), x)^2 + 1;
  ans := dsolve(task, y(x)) :
  ans_1 := solve(ans[1], y(x));
  ans_2 := solve(ans[2], y(x));
  plot([seq(ans_2, _C1 = −3 .. 3)], legend = ["c = -3", "c = -2", "c = -1", "c = 0", "c = 1", "c = 2",
       "c = 3"]);
```

$$task := y(x) = x\left(\frac{d}{dx}\,y(x)\right) - \left(\frac{d}{dx}\,y(x)\right)^2 + 1$$

$$ans\_1 := \frac{x^2}{4} + 1$$

$$ans\_2 := -c_1^{\,2} + x\,c_1 + 1$$



> *restart* :

>

> ***#PART 2***

>

> ***#Задание 1.1***

> $de := x = \left(\frac{d^2}{d\,x^2}(y(x))\right)^2 + \sin\left(\frac{d^2}{d\,x^2}(y(x))\right)$

$$de := x = \left(\frac{d^2}{dx^2}\,y(x)\right)^2 + \sin\left(\frac{d^2}{dx^2}\,y(x)\right) \tag{13}$$

> $x := t + \sin(t)$

$$x := t + \sin(t) \tag{14}$$

> $dx := diff(x, t);$

$$dx := 1 + \cos(t) \tag{15}$$

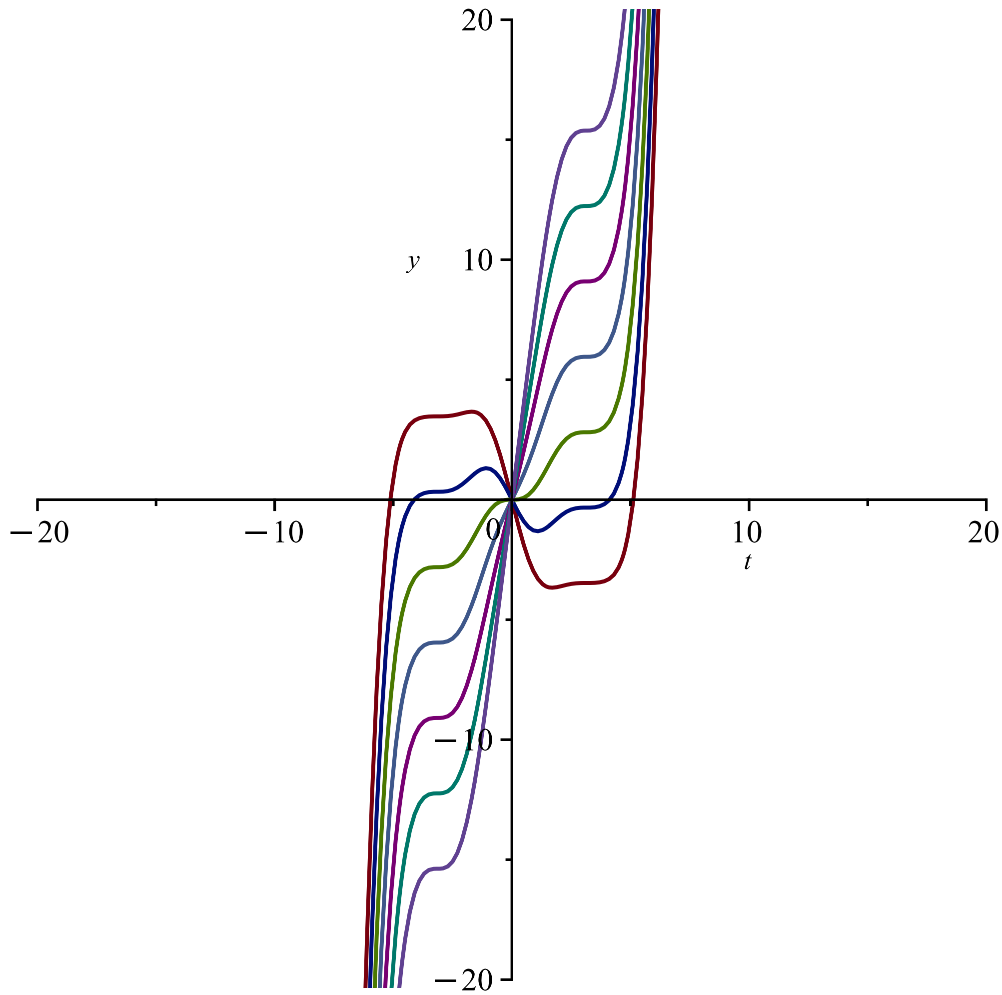
> $y1 := int(t \cdot dx, t);$

$$y1 := \frac{t^2}{2} + \cos(t) + t\sin(t) \tag{16}$$

> $sol := y = int((y1 + C1) \cdot dx, t) + C2;$

$$sol := y = -\frac{t\cos(t)^2}{2} + \frac{3\cos(t)\sin(t)}{4} + \frac{3t}{4} + \frac{t^2\sin(t)}{2} + \sin(t) + C1\sin(t) + \frac{t^3}{6} + C1\,t + C2 \qquad (17)$$

```
> dpl := plot([seq(seq(rhs(sol), C2 = [0]), C1 =-3..3)], t =-20..20, y =-20..20, thickness
    =1) :
  plots[display](dpl);
```



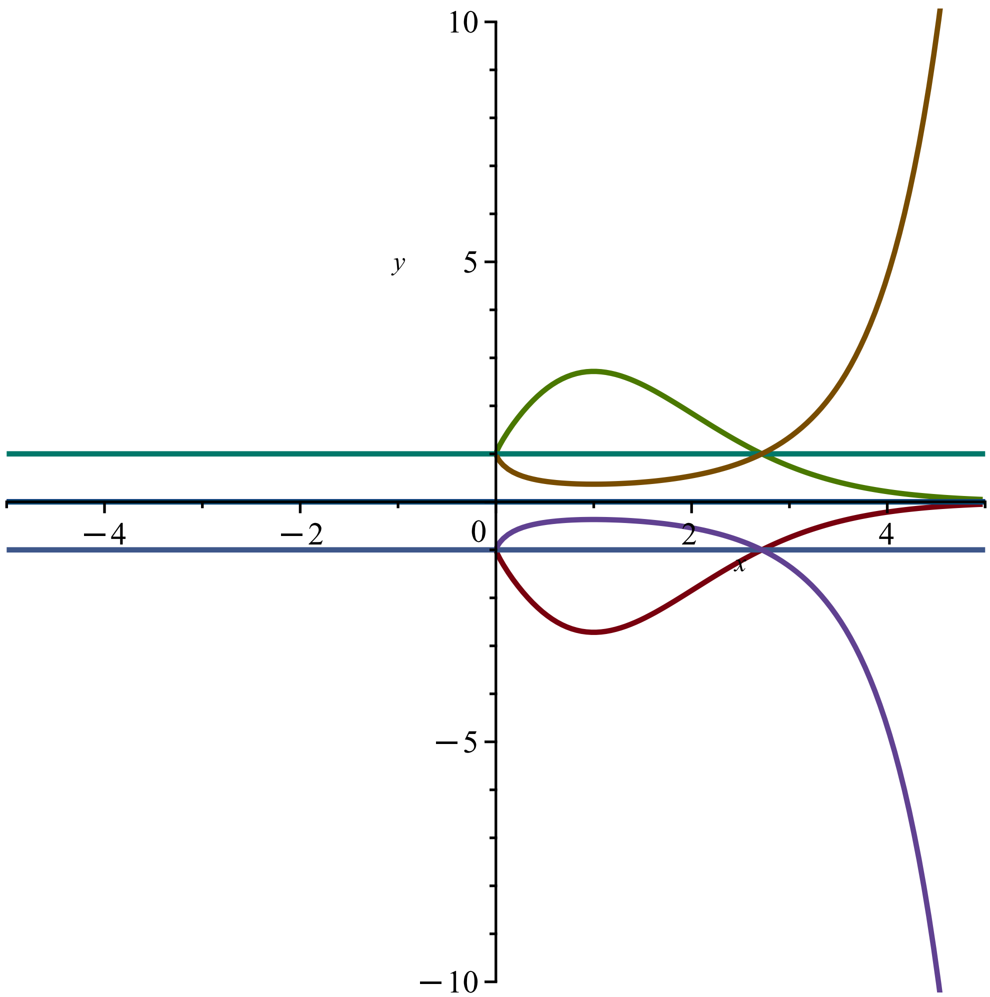

```
> restart :
>
> #Задание 1.2
> de := x·ln(x)·(y(x)·diff(diff(y(x), x), x) − diff(y(x), x)^2) = y(x)·diff(y(x), x);
```

$$de := x\ln(x)\left(y(x)\left(\frac{d^2}{dx^2}\,y(x)\right) - \left(\frac{d}{dx}\,y(x)\right)^2\right) = y(x)\left(\frac{d}{dx}\,y(x)\right) \qquad (18)$$

```
> sol := simplify(dsolve(de))
```

$$sol := y(x) = x^{c_1 x}\,e^{-c_1 x}\,c_2 \qquad (19)$$

```
> dpl := plot([seq(seq(rhs(sol), _C2 =-1..1), _C1 =-1..1)], x =-5..5, y =-10..10,
    thickness=2) :
  plots[display](dpl);
```



> *restart* :

>

> ***#Задание 1.3***

> $de := diff(y(x), x) = x \cdot diff(diff(y(x), x), x) - \frac{(diff(diff(y(x), x), x))^6}{6}$

$$de := \frac{d}{dx}\, y(x) = x \left(\frac{d^2}{dx^2}\, y(x)\right) - \frac{\left(\frac{d^2}{dx^2}\, y(x)\right)^6}{6} \tag{20}$$

> $de := 6 \cdot u = 6 \cdot u' \cdot x - (u')^6$

$$de := 6\, u(x) = 6 \left(\frac{d}{dx}\, u(x)\right) x - \left(\frac{d}{dx}\, u(x)\right)^6 \tag{21}$$

> $dsolve(de)$

$$u(x) = \frac{5\, x^{6/5}}{6},\ u(x) = \frac{5 \left(-\frac{\sqrt{5}}{4} - \frac{1}{4} - \frac{\mathrm{I}\sqrt{2}\,\sqrt{5-\sqrt{5}}}{4}\right) x^{6/5}}{6},\ u(x) \tag{22}$$

$$= \frac{5\left(-\frac{\sqrt{5}}{4}-\frac{1}{4}+\frac{I\sqrt{2}\sqrt{5-\sqrt{5}}}{4}\right)x^{6/5}}{6}, u(x)$$

$$= \frac{5\left(\frac{\sqrt{5}}{4}-\frac{1}{4}-\frac{I\sqrt{2}\sqrt{5+\sqrt{5}}}{4}\right)x^{6/5}}{6}, u(x)$$

$$= \frac{5\left(\frac{\sqrt{5}}{4}-\frac{1}{4}+\frac{I\sqrt{2}\sqrt{5+\sqrt{5}}}{4}\right)x^{6/5}}{6}, u(x) = -\frac{1}{6}c_1^{\,6}+c_1 x$$

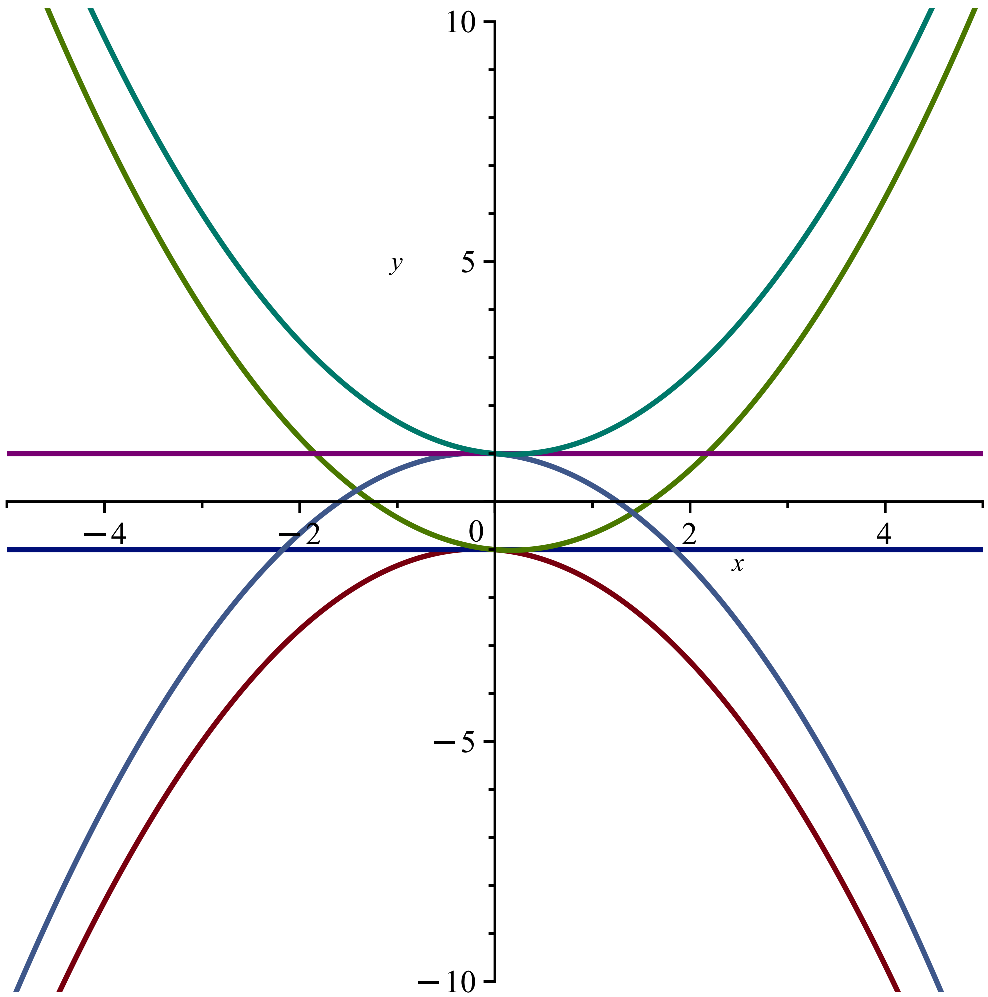
> $sol\_1 := dsolve\left(diff(y(x), x) = C \cdot x - \frac{(C)^6}{6}\right)$

$$sol\_1 := y(x) = \frac{1}{2}C x^2 - \frac{1}{6}C^6 x + c_1 \tag{23}$$

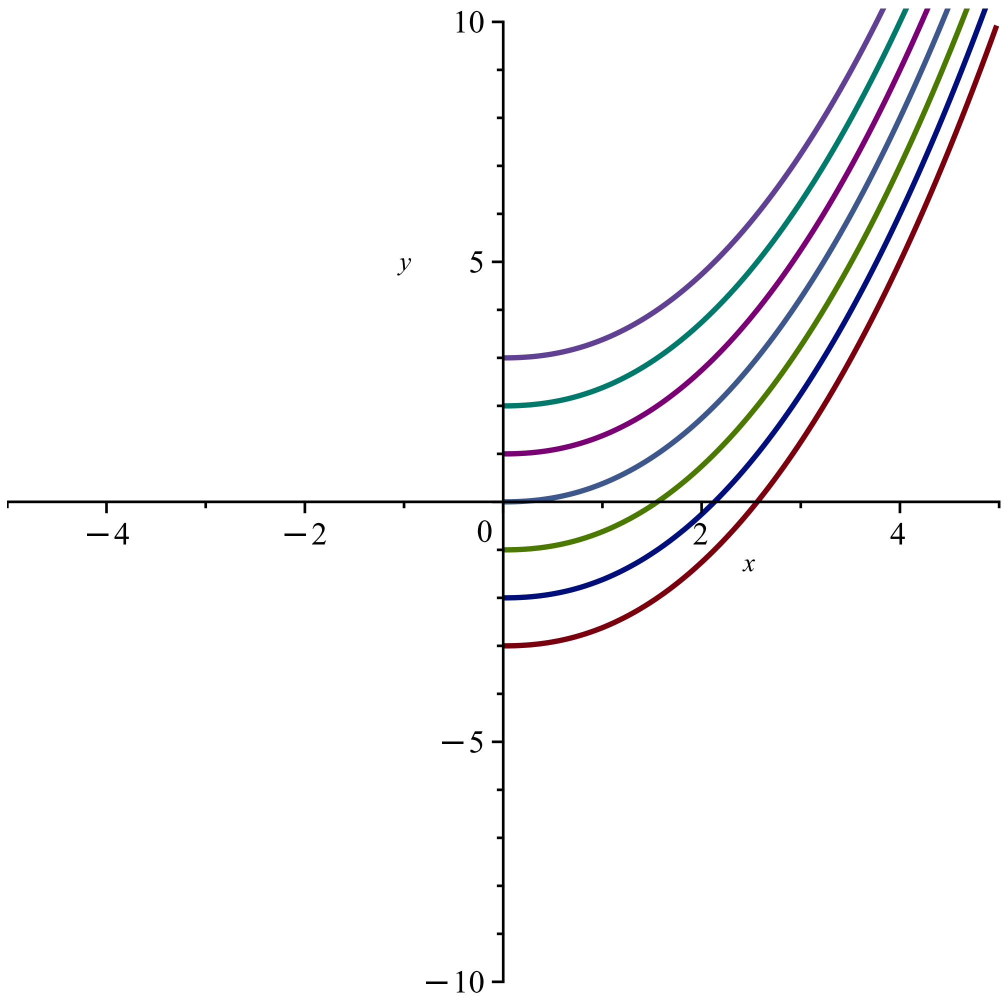
> $sol\_2 := dsolve\left(diff(y(x), x) = \frac{5x^{\frac{6}{5}}}{6}\right)$

$$sol\_2 := y(x) = \frac{25 x^{11/5}}{66} + c_1 \tag{24}$$

> $dpl\_1 := plot([seq(seq(rhs(sol\_1), C = -1..1), \_C1 = [-1, 1])], x = -5..5, y = -10..10,$
> $thickness = 2)$



> $dpl\_2 := plot([seq(rhs(sol\_2), \_C1 = -3..3)], x = -5..5, y = -10..10, thickness = 2)$



> restart :

>

> **#Задание 1.4**

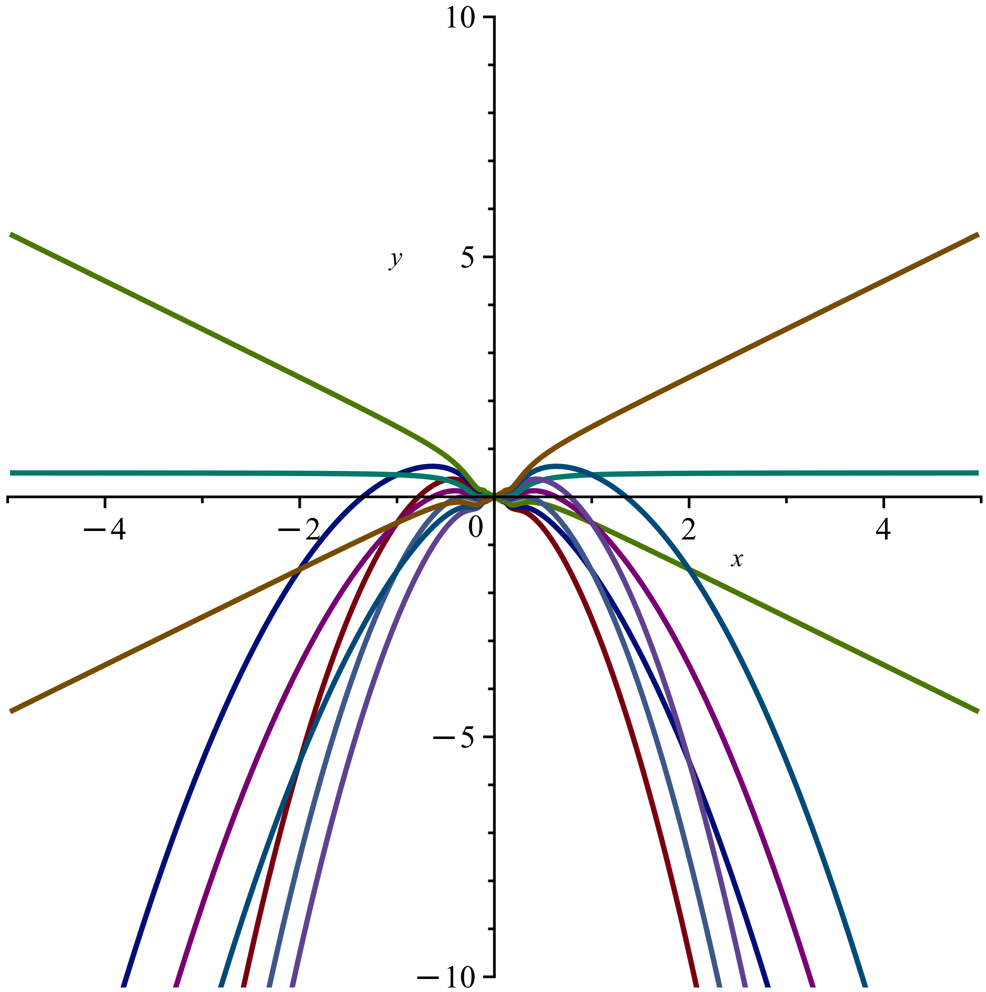
> $de := diff(diff(y(x), x), x) = 2\cdot\left(\frac{diff(y(x), x)}{x} - \frac{y(x)}{x^2}\right) + \frac{1}{x^2}\cdot\cos\left(\frac{1}{x}\right)$

$$de := \frac{d^2}{dx^2}\, y(x) = \frac{2\left(\frac{d}{dx}\, y(x)\right)}{x} - \frac{2\, y(x)}{x^2} + \frac{\cos\left(\frac{1}{x}\right)}{x^2} \tag{25}$$

> $sol := dsolve(de);$

$$sol := y(x) = -\cos\left(\frac{1}{x}\right) x^2 + c_2\, x^2 + c_1\, x \tag{26}$$

> $dpl := plot([seq(seq(rhs(sol), \_C2 = -1..1), \_C1 = -1..1)], x = -5..5, y = -10..10, thickness = 2) :$
$plots[display](dpl);$



> *restart* :

>

> ***#Задание 2***

> $de := x \cdot diff(diff(diff(y(x), x), x), x) + diff(diff(y(x), x), x) = x + 1$

$$de := x\left(\frac{d^3}{dx^3}\,y(x)\right) + \frac{d^2}{dx^2}\,y(x) = x + 1 \quad \textbf{(27)}$$

> *dsolve*(*de*)

$$y(x) = \frac{x^3}{12} + \frac{x^2}{2} + c_1\,(x\ln(x) - x) + c_2\,x + c_3 \quad \textbf{(28)}$$

> *restart* :

>

> ***#Задание 3***

> $de := diff(diff(y(x), x), x) + y(x) = 2\cos(7x) + 3\sin(7x)$

$$de := \frac{d^2}{dx^2}\,y(x) + y(x) = 2\cos(7x) + 3\sin(7x) \quad \textbf{(29)}$$

> *dsolve*(*de*)

$$y(x) = \sin(x)\,c_2 + \cos(x)\,c_1 - \frac{\sin(7x)}{16} - \frac{\cos(7x)}{24} \quad \textbf{(30)}$$

> restart :

>

> **#PART 3**

>

> **#Задание 1**

> $sys\_diff := \left\{ \frac{d}{dx} y\_1(x) = -y\_1(x) + 2 \cdot y\_2(x), \frac{d}{dx} y\_2(x) = 3 \cdot y\_1(x) \right\}:$

sys_diff

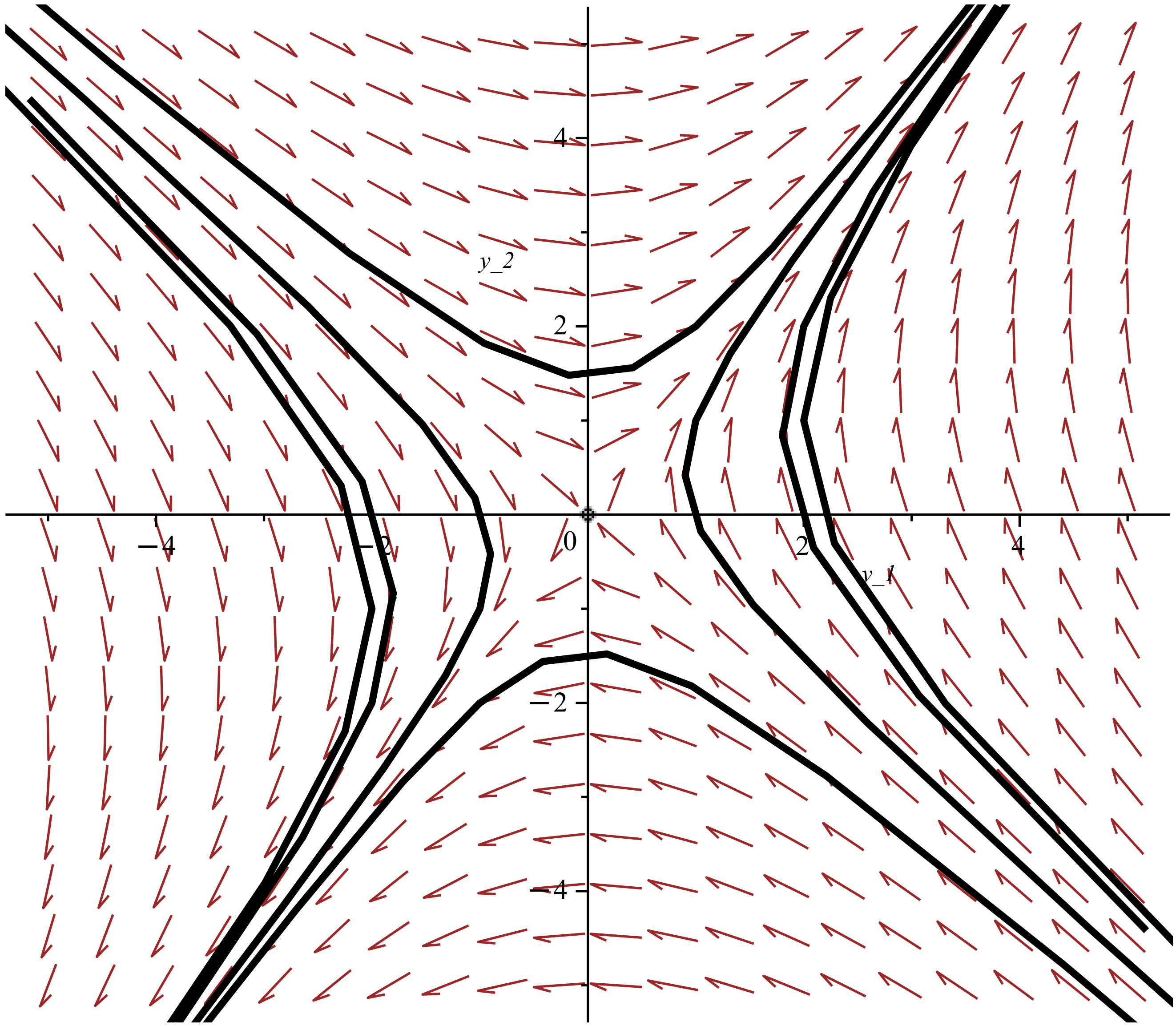
$$\left\{ \frac{d}{dx} y\_1(x) = -y\_1(x) + 2\, y\_2(x), \frac{d}{dx} y\_2(x) = 3\, y\_1(x) \right\} \quad \textbf{(31)}$$

> $res := dsolve(sys\_diff)$

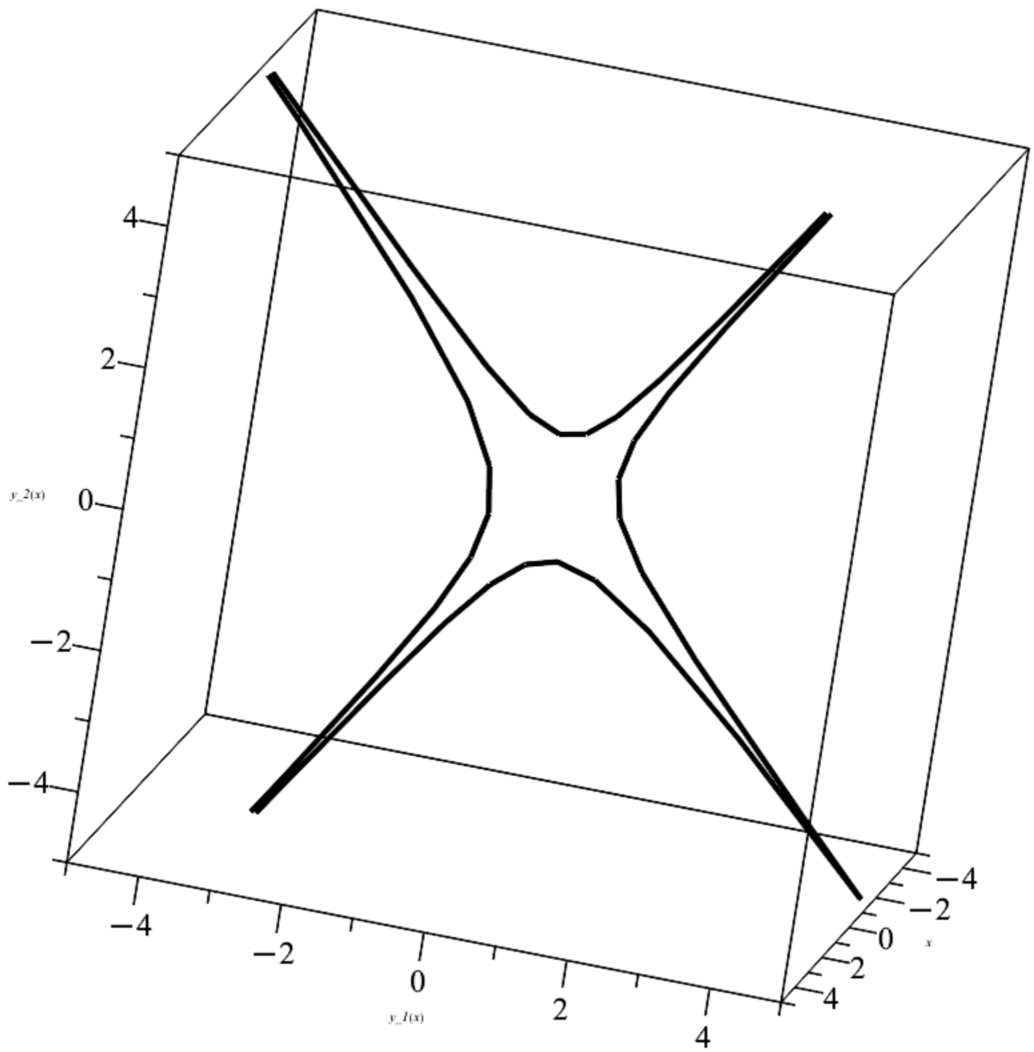
$$res := \left\{ y\_1(x) = \frac{2\, c_1\, e^{2x}}{3} - c_2\, e^{-3x}, y\_2(x) = c_1\, e^{2x} + c_2\, e^{-3x} \right\} \quad \textbf{(32)}$$

> **#Точка покоя (0, 0)**

```
plot([[0, 0]], color = black, style = point, symbolsize = 10) :
DEtools[phaseportrait]([sys_diff[1], sys_diff[2]], [y_1, y_2], x = -5 ..5, [[0, 1, 1], [0, 1, 2],
    [0, 2, 1], [0, -1, -1], [0, 2, 2], [0, -1, -2], [0, -2, -1], [0, -2, -2]], y_1 = -5 ..5, y_2 =
    -5 ..5, linecolor = black, thickness = 3) :
plots[display](%, %%);
```



> *DEtools*[*DEplot3d*]([sys_diff[1], *sys_diff*[2]], [*y_1*, *y_2*], *x* = −5 ..5, [[0, 1, 0], [0, 0, 1], [−1, −1, 0], [−1, 0, −1]], *y_1* = −5 ..5, *y_2* = −5 ..5, *thickness* = 4, *linecolor* = *black*);



> $dfe := diff\left(y\_2(y\_1), y\_1\right) = \dfrac{3\, y\_1}{-y\_1 + 2 \cdot y\_2}$

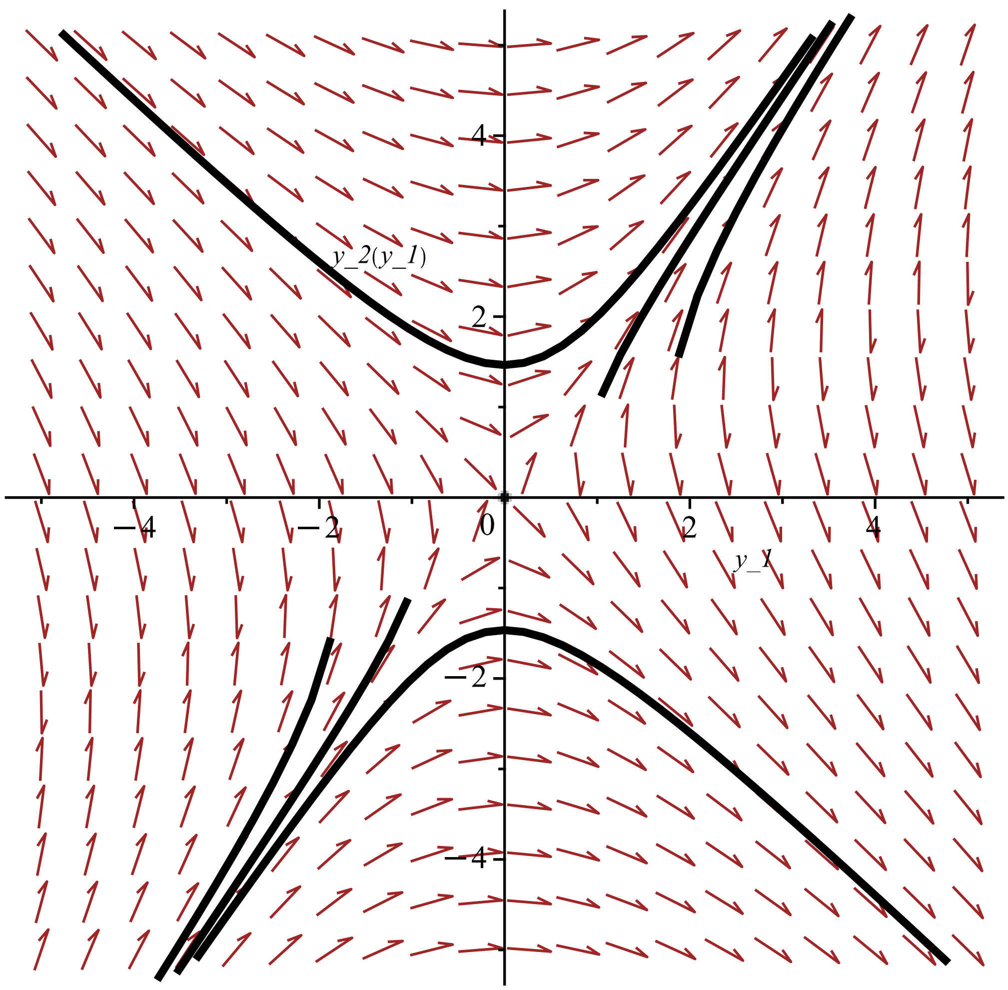
$$dfe := \frac{\mathrm{d}}{\mathrm{d}y\_1}\, y\_2(y\_1) = \frac{3\, y\_1}{-y\_1 + 2\, y\_2} \tag{33}$$

> plot([[0, 0]], color = black, style = point, symbolsize = 10) :
DEtools[DEplot](dfe, y_2(y_1), y_1 = −5 ..5, y_2 = −5 ..5, [[1, 1] , [1, 2], [2, 2], [−1, −1], [−1, −2] , [−2, −2]] , linecolor = black, thickness = 3) :
plots[display](%, %%);

Warning, y_2 is present as both a dependent variable and a name.
Inconsistent specification of the dependent variable is deprecated,

and it is assumed that the name is being used in place of the dependent variable.

```
Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further left of .90064002, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further left of 1.8012801, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further right of -.90064002, probably a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further right of -1.8012801, probably a singularity
```



> *restart* :

> ***#Задание 2***

> $de := \{diff(y\_1(x), x) = 4 \cdot y\_1(x) + y\_2(x), diff(y\_2(x), x) = 11 \cdot y\_1(x) - 6 \cdot y\_2(x)\};$

$$de := \left\{ \frac{d}{dx}\, y\_1(x) = 4\, y\_1(x) + y\_2(x),\ \frac{d}{dx}\, y\_2(x) = 11\, y\_1(x) - 6\, y\_2(x) \right\} \qquad (34)$$

> $dsolve(de)$

$$\left\{ y\_1(x) = c_1\, e^{5x} + c_2\, e^{-7x},\ y\_2(x) = c_1\, e^{5x} - 11\, c_2\, e^{-7x} \right\} \qquad (35)$$

> *restart* :

> ***#Задание 3***

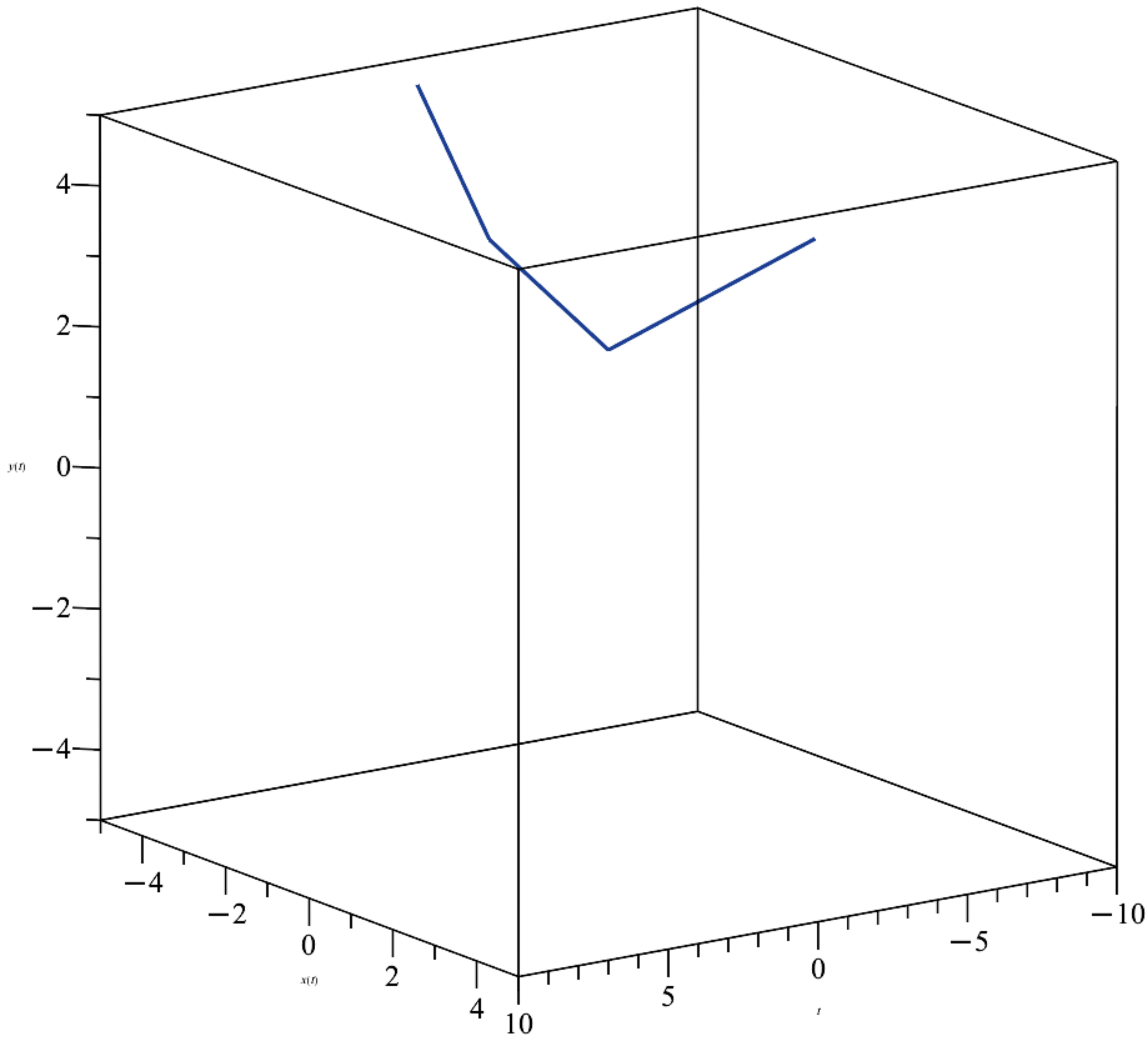
> $de := \{diff(x(t), t) = 3 \cdot x(t) + 5 \cdot y(t) + 2, diff(y(t), t) = 3 \cdot x(t) + y(t) + 1\}$

$$de := \left\{ \frac{\mathrm{d}}{\mathrm{d}t} x(t) = 3\, x(t) + 5\, y(t) + 2, \frac{\mathrm{d}}{\mathrm{d}t} y(t) = 3\, x(t) + y(t) + 1 \right\} \tag{36}$$

> $dsolve(de)$

$$\left\{ x(t) = \mathrm{e}^{6\,t}\, c_2 + \mathrm{e}^{-2\,t}\, c_1 - \frac{1}{4}, y(t) = \frac{3\, \mathrm{e}^{6\,t}\, c_2}{5} - \mathrm{e}^{-2\,t}\, c_1 - \frac{1}{4} \right\} \tag{37}$$

> $DETools[DEplot3d](de, [x, y], t = -10 ..10, [[x(0) = 0, y(0) = 2]], x = -5 ..5, y = -5 ..5)$



> *restart* :