

```
> #Лабораторная работа 3(Вариант 4)
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#гр. 353503
#PART 1
```

```
> #Задание 1
```

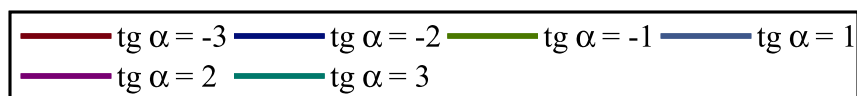
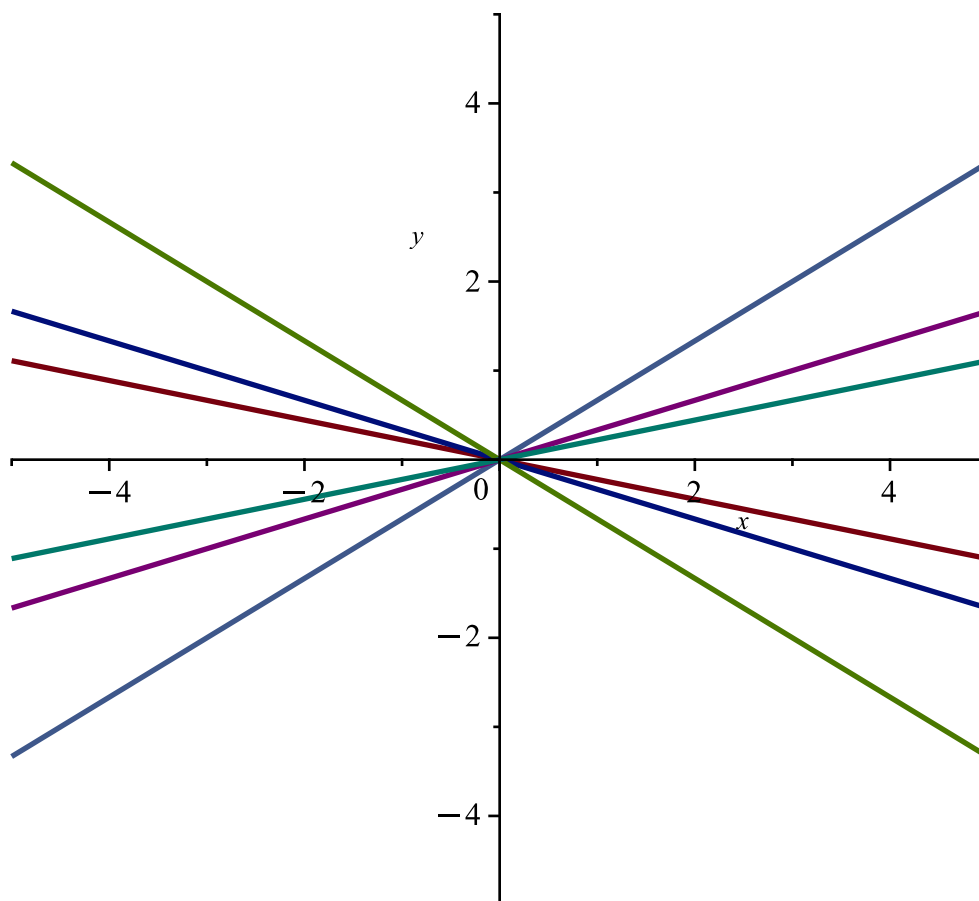
```
> diff(y(x), x) =  $\frac{2 \cdot x}{3 \cdot y(x)}$ 
```

$$\frac{d}{dx} y(x) = \frac{2x}{3y(x)}$$

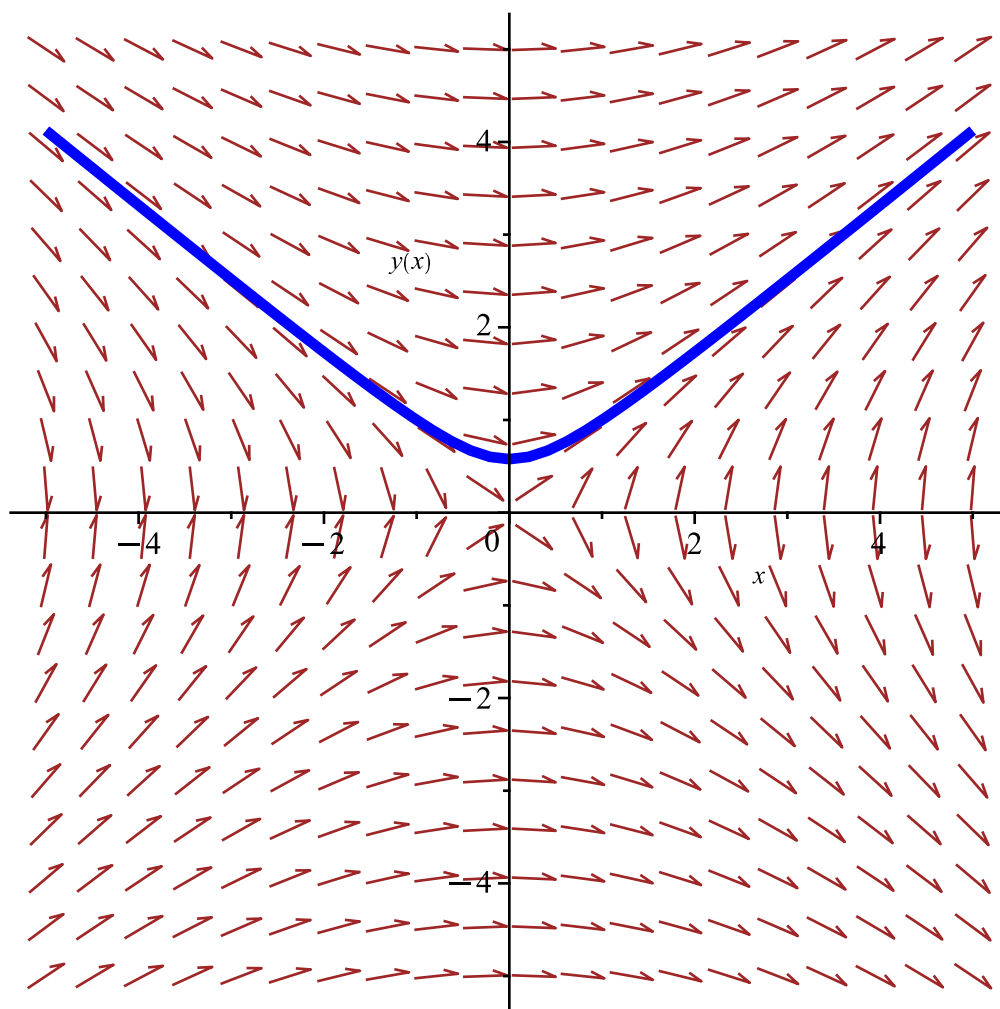
(1)

```
> with(DETools):
```

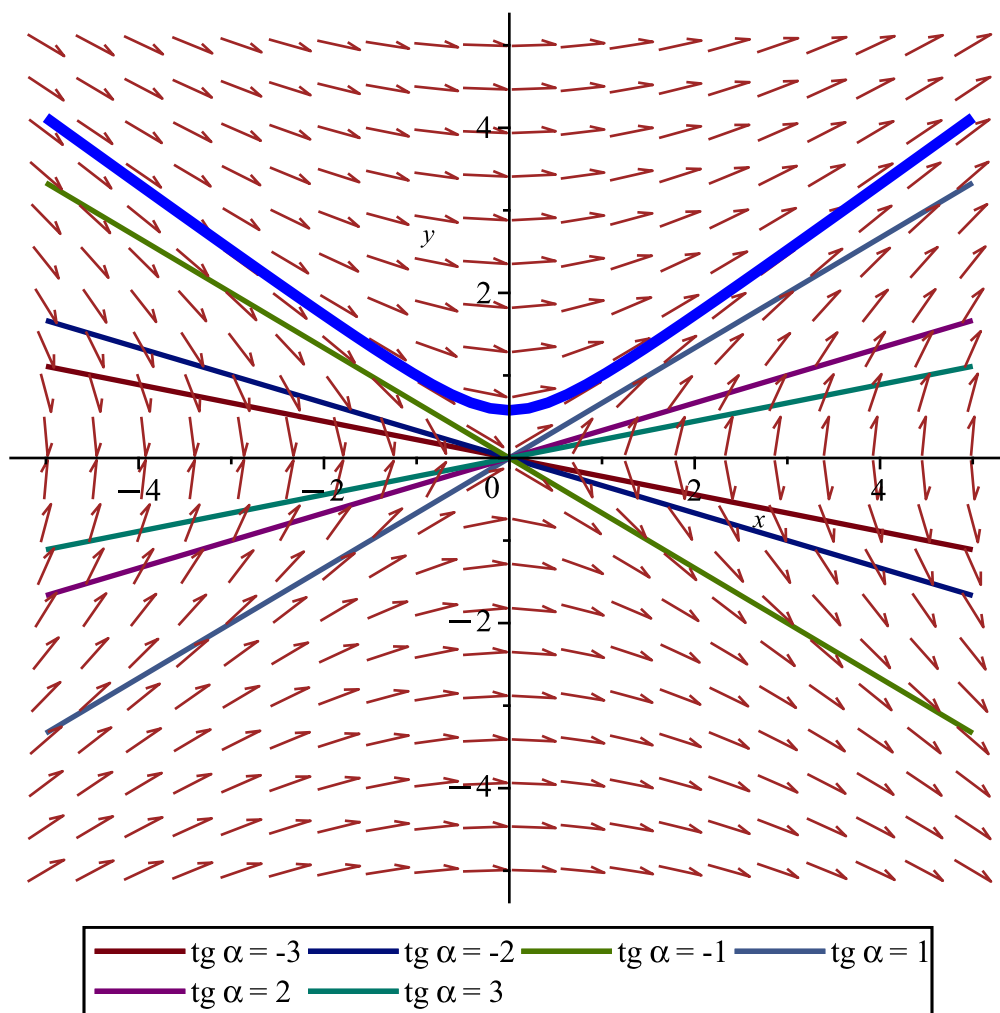
```
> isoclines := plot( [ seq(  $\frac{2 \cdot x}{3 \cdot k}$ , k = [ -3, -2, -1, 1, 2, 3 ] ) ], x = -5 .. 5, y = -5 .. 5, legend
= [ "tg α = -3", "tg α = -2", "tg α = -1", "tg α = 1", "tg α = 2", "tg α = 3"], thickness = 2 );
```



```
> dplot := DEplot( diff(y(x), x) =  $\frac{2 \cdot x}{3 \cdot y(x)}$ , y(x), x = -5 .. 5, y = -5 .. 5, [ y(1) = 1 ], linecolor
= blue, thickness = 4 )
```



```
> plots[display](isoclines, dplot)
```



> restart :

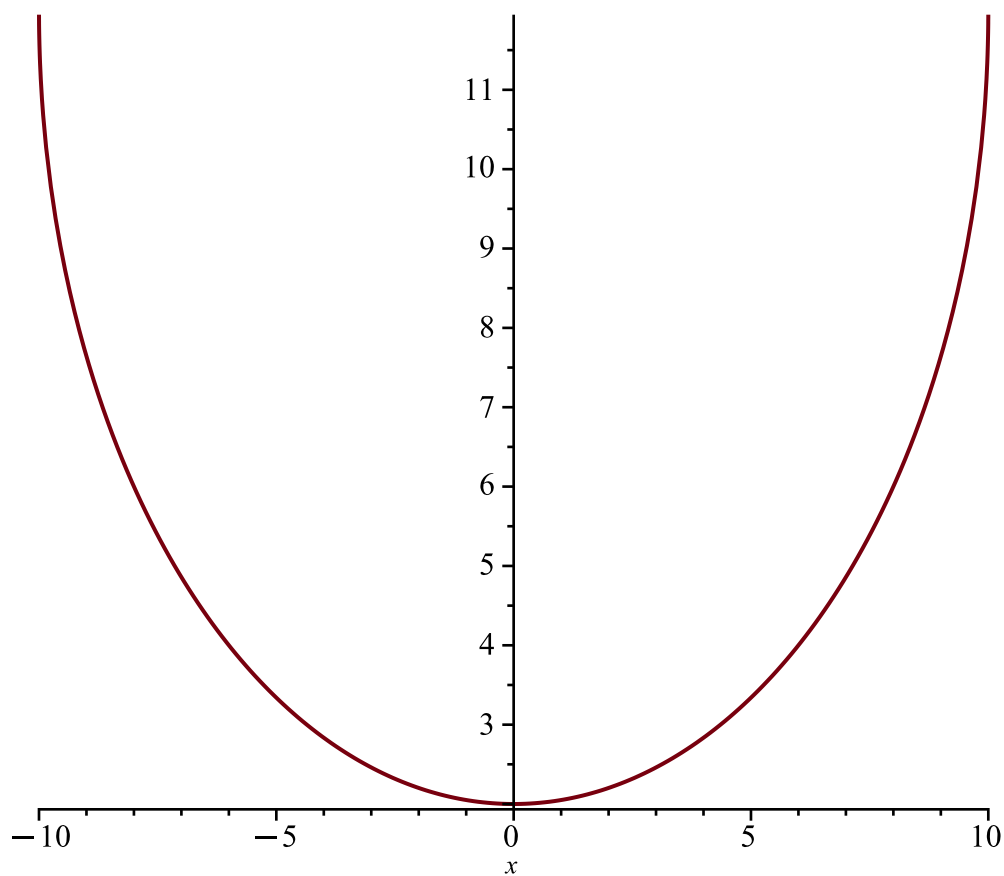
> #Задание 2.1

>  $\text{line} := \text{dsolve}\left(\left\{\text{diff}(y(x), x) = \frac{x}{\sqrt{10^2 - x^2}}, y(6) = 4\right\}\right)$

$\text{line} := y(x) = \frac{(x - 10)(x + 10)}{\sqrt{-x^2 + 100}} + 12$

>  $\text{plot}(\text{rhs}(\text{line}), \text{legend} = \text{line})$  # *rhs()* извлекает правую часть

(2)



$$y(x) = \frac{(x-10)(x+10)}{\sqrt{-x^2+100}} + 12$$

```
> restart :
```

```
> #Задание 2.2
```

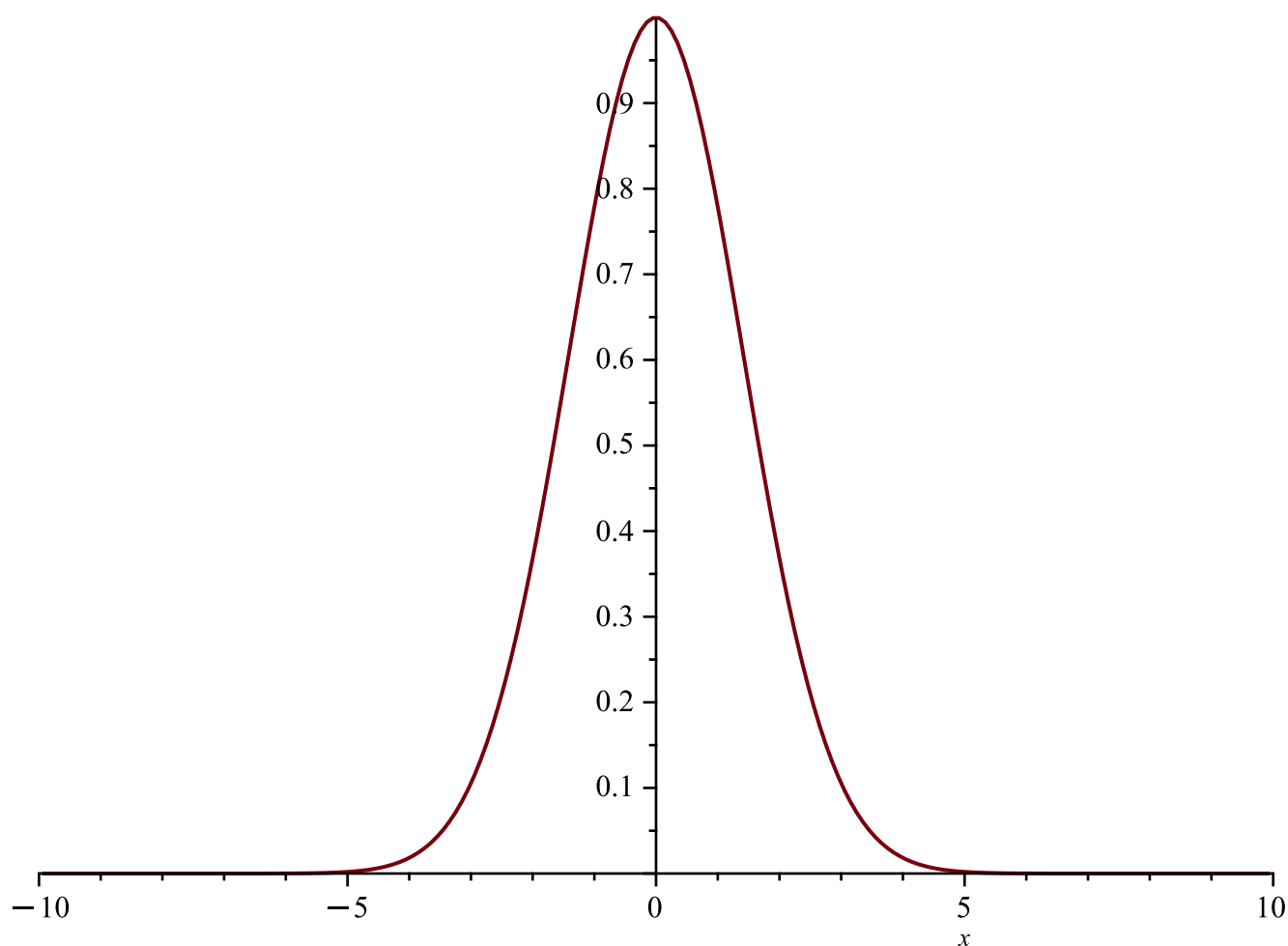
```
> a := 2 :
```

```
> line := simplify( dsolve( { diff(y(x), x) = - (y(x) * x) / a, y(2) = 1/e } ) )
```

$$line := y(x) = e^{-\frac{x^2}{4}}$$

```
> plot(rhs(line), legend=line)
```

(3)



$$y(x) = e^{-\frac{1}{4} x^2}$$

> restart :

> #Задание 3

>  $func := diff(y(x), x) = \frac{-12 \cdot x - 5 \cdot y(x) + 34}{2 \cdot x + y(x) - 6}$

$$func := \frac{d}{dx} y(x) = \frac{-12 x - 5 y(x) + 34}{2 x + y(x) - 6} \quad (4)$$

>  $func\_solve := dsolve(func, y(x))$

$$func\_solve := y(x) = 2 - \frac{8 (x - 2) c_1 + 1 + \sqrt{4 (x - 2) c_1 + 1}}{2 c_1} \quad (5)$$

>  $A := Matrix([[ -12, -5], [2, 1]])$

$$A := \begin{bmatrix} -12 & -5 \\ 2 & 1 \end{bmatrix} \quad (6)$$

>  $linalg[det](A)$

$$-2 \quad (7)$$

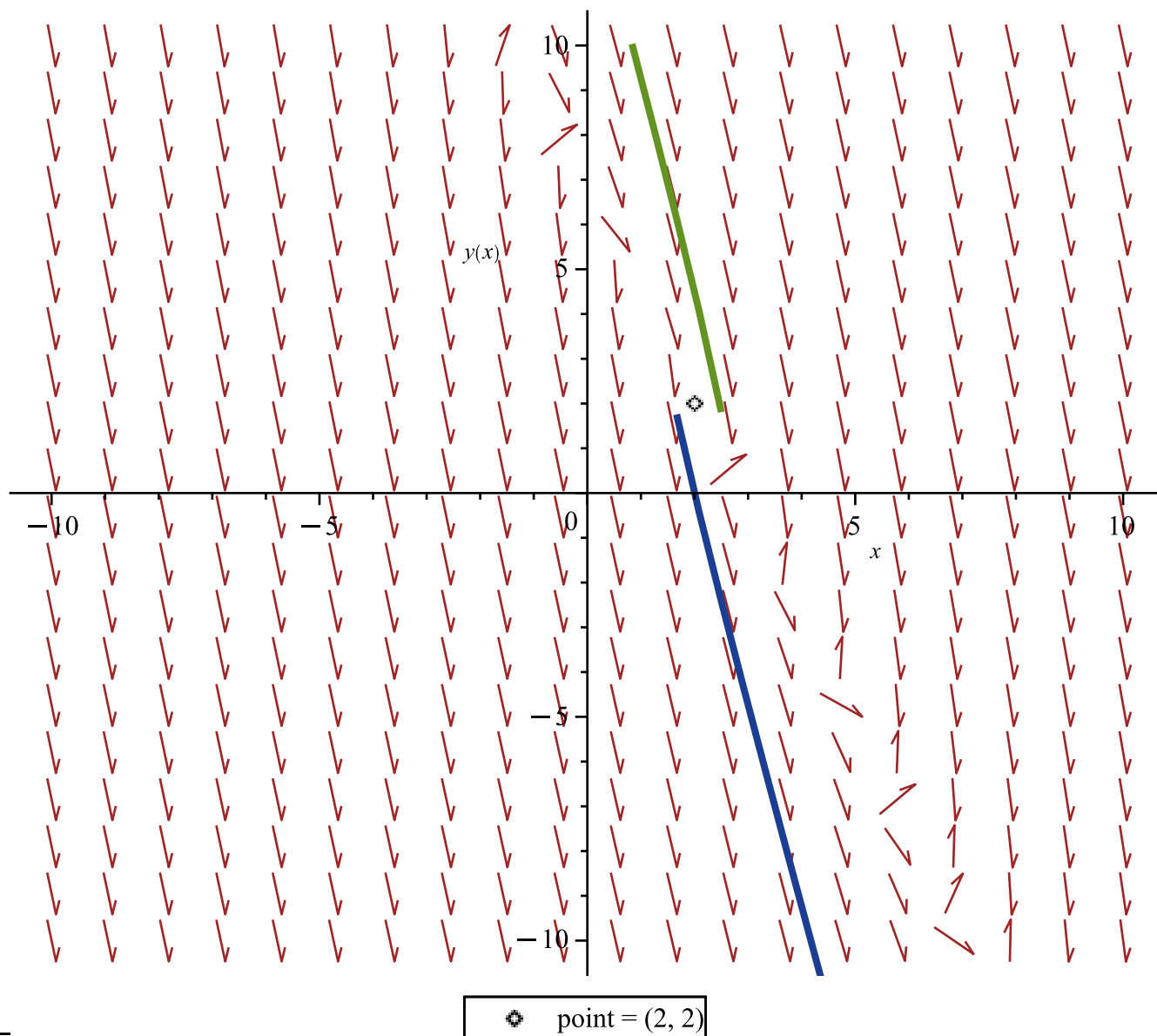
>  $solve(\{-12 \cdot x - 5 \cdot y + 34 = 0, 2 \cdot x + y - 6 = 0\})$

$\{x=2, y=2\}$

(8)

```
> plot1 := DEtools[DEplot](func, y(x), x=-10..10, y=-10..10, [[y(2)=0], [y(2)=4.5]]):  
plot2 := plots[pointplot]([2, 2], legend="point = (2, 2)":  
plots[display](plot1, plot2)
```

Warning, plot may be incomplete, the following errors(s) were issued:  
cannot evaluate the solution further left of 1.4999999, maxfun  
limit exceeded (see ?dsolve, maxfun for details)  
Warning, plot may be incomplete, the following errors(s) were issued:  
cannot evaluate the solution further right of 2.6250001, probably  
a singularity



```
> M := Matrix([[-12 - λ, -5], [2, 1 - λ]]);  
solve(LinearAlgebra[Determinant](M) = 0):  
λ1 = convert(%[1], float);  
λ2 = convert(%%[2], float);
```

$$M := \begin{bmatrix} -12 - \lambda & -5 \\ 2 & 1 - \lambda \end{bmatrix}$$

$$\lambda_1 = 0.178908345$$

$$\lambda_2 = -11.17890834$$

(9)

>  $\lambda_1$  — полож,  $\lambda_2$  — отриц  $\Rightarrow$  точка покоя неустойчива (седло)

$\lambda_1$  — полож,  $\lambda_2$  — отриц  $\Rightarrow$  точка покоя неустойчива (седло)

(10)

> restart :

> #Задание 4

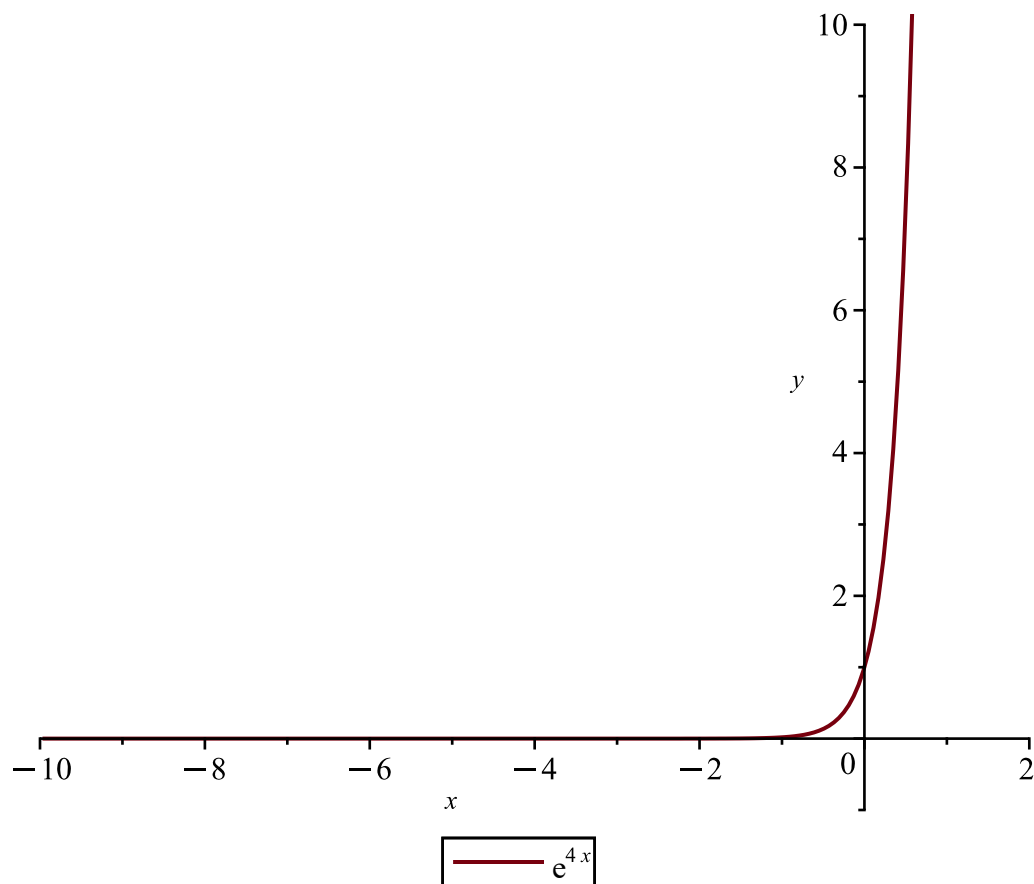
>  $func := diff(y(x), x) + 4x^3 \cdot y(x) = 4(x^3 + 1) \cdot \exp(-4 \cdot x) \cdot y(x)^2;$

$ans := solve(dsolve(\{func, y(0) = 1\}), y(x));$

$plot(ans, x = -10..2, y = -1..10, legend = ans)$

$$func := \frac{d}{dx} y(x) + 4x^3 y(x) = 4(x^3 + 1) e^{-4x} y(x)^2$$

$$ans := e^{4x}$$



> restart :

> #Задание 5.1

>  $task := x = diff(y(x), x) \cdot \ln(diff(y(x), x)) - diff(y(x), x);$

$x\_fun := diff(y(t), t) = t \cdot \ln(t) - t;$

$dx\_fun := diff(y(t), t) = \ln(t);$

```
dy_fun := diff(y(t), t) = t·ln(t);
y_solve := dsolve(dy_fun);
```

$$task := x = \left( \frac{d}{dx} y(x) \right) \ln \left( \frac{d}{dx} y(x) \right) - \frac{d}{dx} y(x)$$

$$x\_fun := \frac{d}{dt} y(t) = t \ln(t) - t$$

$$dx\_fun := \frac{d}{dt} y(t) = \ln(t)$$

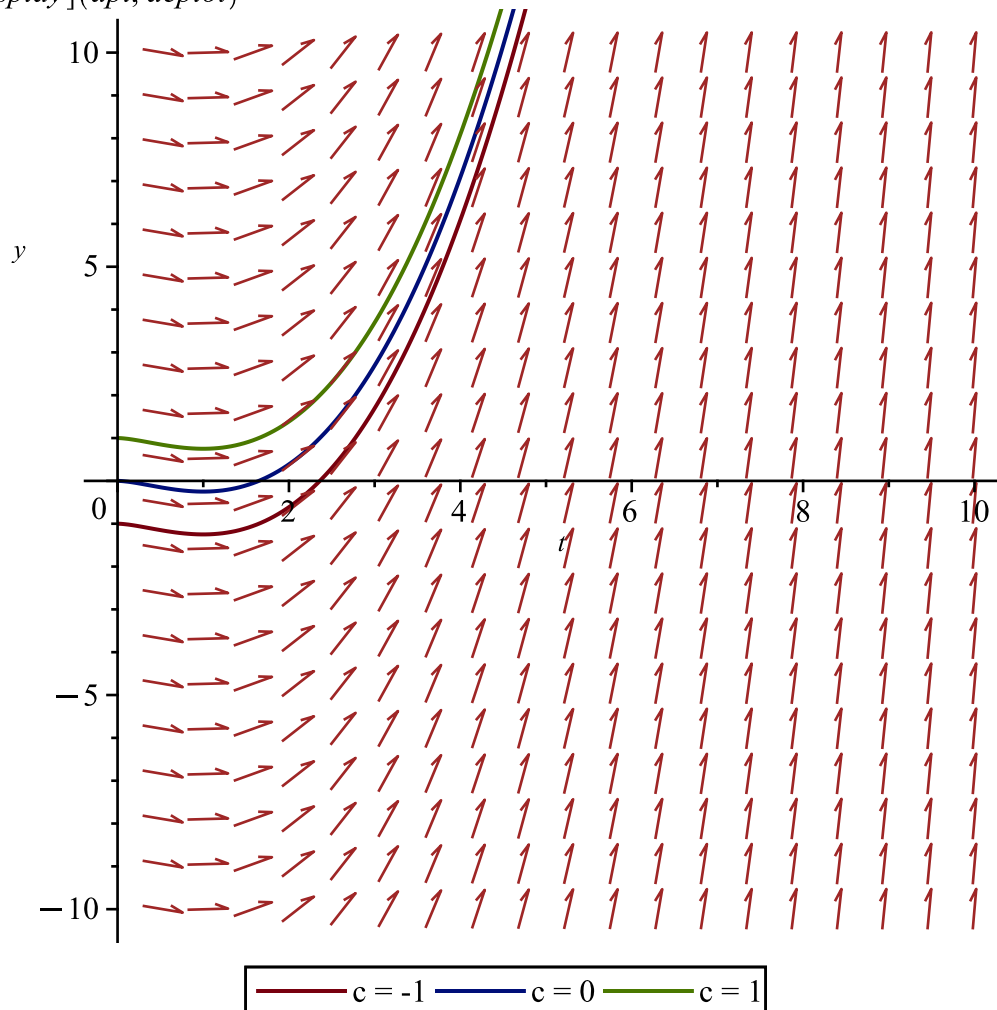
$$dy\_fun := \frac{d}{dt} y(t) = t \ln(t)$$

$$y\_solve := y(t) = \frac{t^2 \ln(t)}{2} - \frac{t^2}{4} + c_1 \quad (11)$$

```
> deplot := DETools[DEplot](dy_fun, y(t), t=0..10, y=-10..10, thickness=5) :
```

```
> dpl := plot( [ seq( [ t^2·ln(t)/2 - t^2/4 + c, c=-1..1 ] ), t=0..10, y=-10..10, legend = [ "c = -1",
    "c = 0", "c = 1" ] ) :
```

```
> plots[display](dpl, deplot)
```





> restart :

> #Задание 5.2

> task := y(x) = ln(abs(cos(diff(y(x), x)))) + diff(y(x), x) · tan(diff(y(x), x));

y\_fun := ln(abs(cos(t))) + t · tan(t);

dy\_fun := diff(x(t), t) =  $\frac{t}{\cos(t)^2}$ ;

dx\_fun := diff(x(t), t) =  $\frac{1}{\cos(t)^2}$ ;

x\_solve := dsolve(dx\_fun);

task := y(x) =  $\ln\left(\left|\cos\left(\frac{d}{dx} y(x)\right)\right|\right) + \left(\frac{d}{dx} y(x)\right) \tan\left(\frac{d}{dx} y(x)\right)$

y\_fun := ln(|cos(t)|) + t tan(t)

dy\_fun :=  $\frac{d}{dt} x(t) = \frac{t}{\cos(t)^2}$

dx\_fun :=  $\frac{d}{dt} x(t) = \frac{1}{\cos(t)^2}$

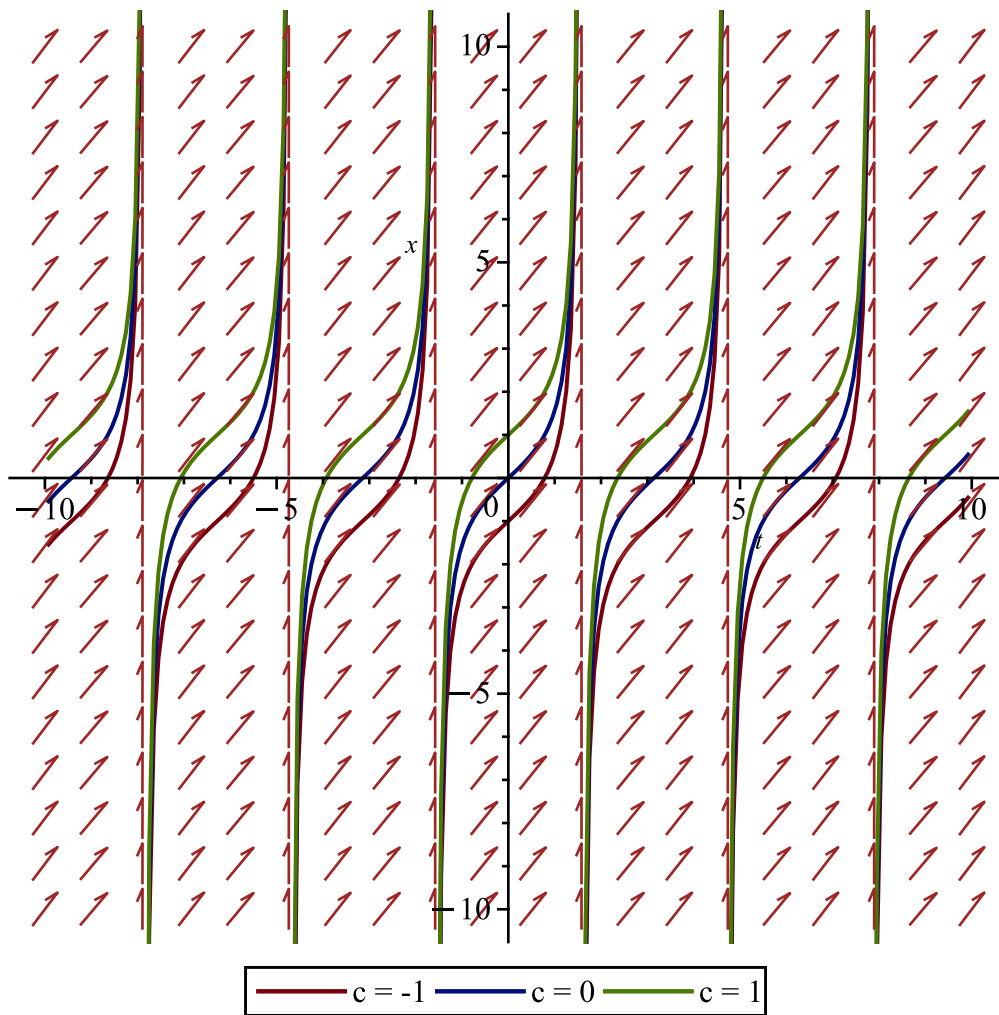
x\_solve := x(t) = tan(t) + c<sub>1</sub>

(12)

> deplot := DETools[DEplot](dx\_fun, x(t), t = -10..10, x = -10..10, thickness = 5) :

> dpl := plot([seq(tan(t) + c, c = -1..1)], t = -10..10, x = -10..10, legend = ["c = -1", "c = 0", "c = 1"]):

> plots[display](dpl, deplot)



> restart :

> #Задание 6

> task := y(x) = x · diff(y(x), x) - diff(y(x), x)<sup>2</sup> + 1;

ans := dsolve(task, y(x)) :

ans\_1 := solve(ans[1], y(x));

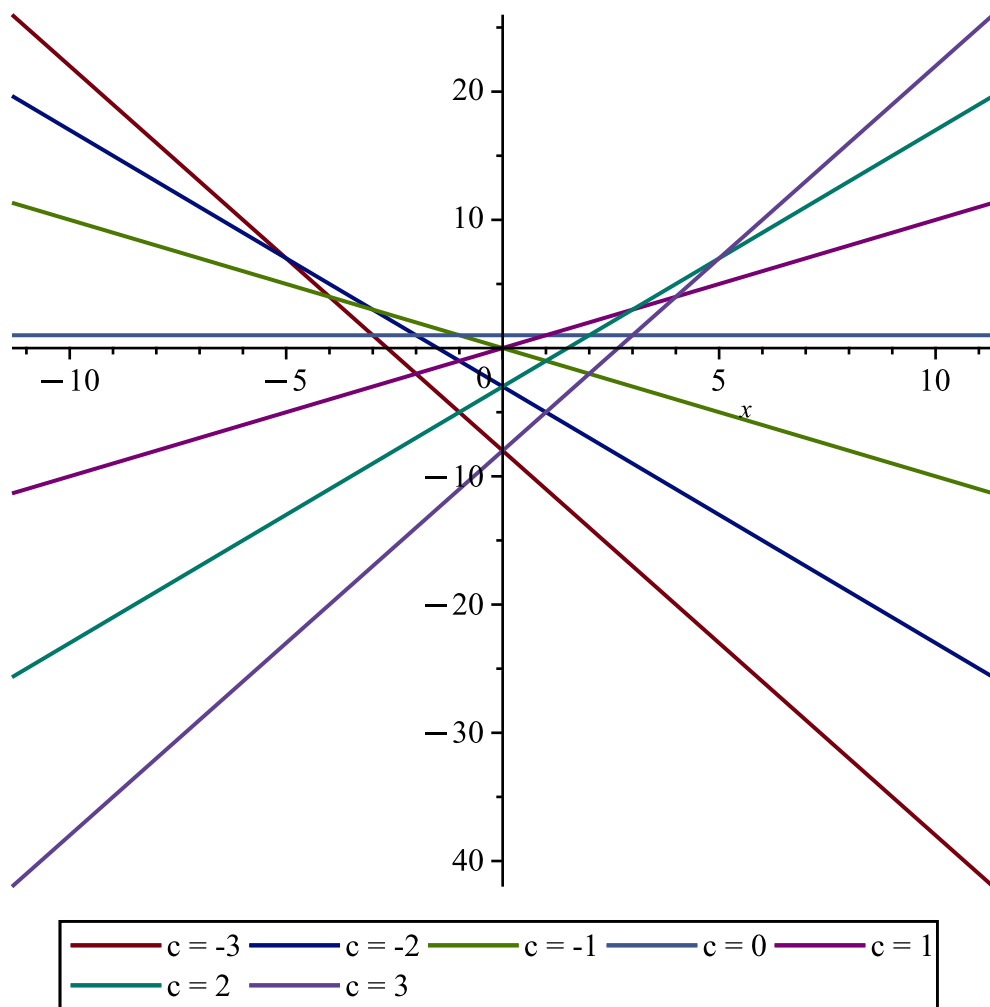
ans\_2 := solve(ans[2], y(x));

plot([seq(ans\_2, \_C1 = -3 .. 3)], legend = ["c = -3", "c = -2", "c = -1", "c = 0", "c = 1", "c = 2",  
"c = 3"]);

$$task := y(x) = x \left( \frac{d}{dx} y(x) \right) - \left( \frac{d}{dx} y(x) \right)^2 + 1$$

$$ans_1 := \frac{x^2}{4} + 1$$

$$ans_2 := -c_1^2 + x c_1 + 1$$



```
> restart :
```

```
> #PART 2
```

```
> #Задание 1.1
```

```
> de := x = \left( \frac{d^2}{dx^2} (y(x)) \right)^2 + \sin \left( \frac{d^2}{dx^2} (y(x)) \right)
```

$$de := x = \left( \frac{d^2}{dx^2} y(x) \right)^2 + \sin \left( \frac{d^2}{dx^2} y(x) \right) \quad (13)$$

```
> x := t + sin(t)
```

$$x := t + \sin(t) \quad (14)$$

```
> dx := diff(x, t);
```

$$dx := 1 + \cos(t) \quad (15)$$

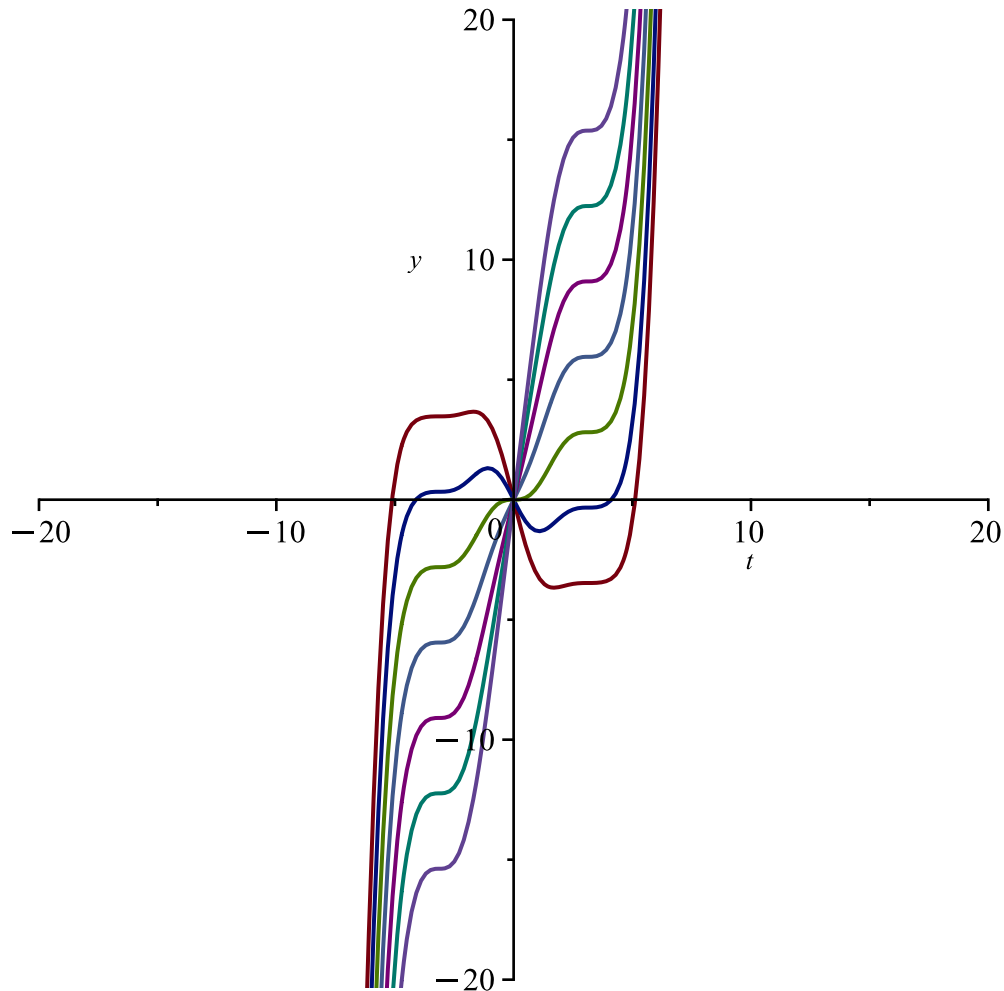
```
> y1 := int(t*dx, t);
```

$$y1 := \frac{t^2}{2} + \cos(t) + t \sin(t) \quad (16)$$

```
> sol := y = int((y1 + C1)*dx, t) + C2;
```

$$sol := y = -\frac{t \cos(t)^2}{2} + \frac{3 \cos(t) \sin(t)}{4} + \frac{3t}{4} + \frac{t^2 \sin(t)}{2} + \sin(t) + C1 \sin(t) + \frac{t^3}{6} + C1 t + C2 \quad (17)$$

```
> dpl := plot([seq(seq(rhs(sol), C2 = [0]), C1 = -3 .. 3)], t = -20 .. 20, y = -20 .. 20, thickness
= 1) :
plots[display](dpl);
```



```
> restart :
```

```
> #Задание 1.2
```

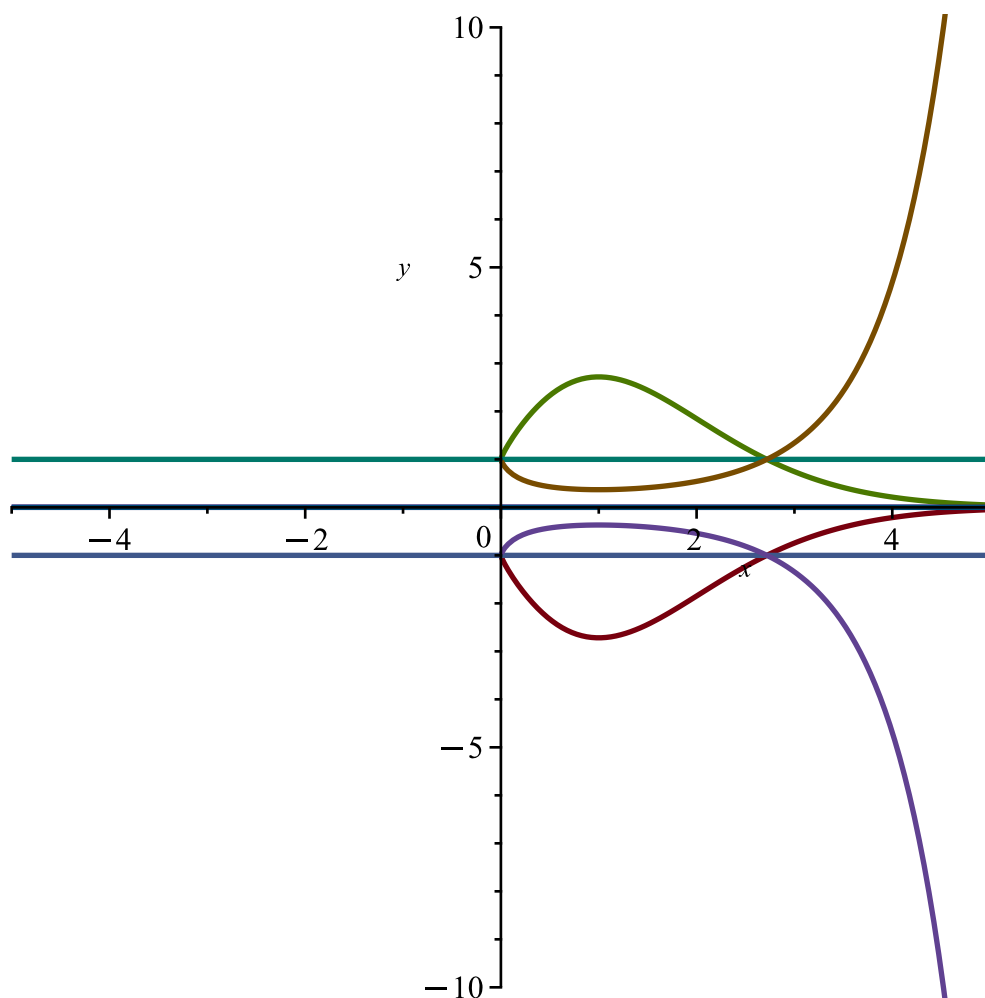
```
> de := x·ln(x)·(y(x)·diff(diff(y(x), x), x) - diff(y(x), x)^2) = y(x)·diff(y(x), x);
```

$$de := x \ln(x) \left( y(x) \left( \frac{d^2}{dx^2} y(x) \right) - \left( \frac{d}{dx} y(x) \right)^2 \right) = y(x) \left( \frac{d}{dx} y(x) \right) \quad (18)$$

```
> sol := simplify(dsolve(de))
```

$$sol := y(x) = x^{c_1 x} e^{-c_1 x} c_2 \quad (19)$$

```
> dpl := plot([seq(seq(rhs(sol), _C2 = -1 .. 1), _C1 = -1 .. 1)], x = -5 .. 5, y = -10 .. 10,
thickness = 2) :
plots[display](dpl);
```



> restart :

>

> #Задание 1.3

>  $de := \text{diff}(y(x), x) = x \cdot \text{diff}(\text{diff}(y(x), x), x) - \frac{(\text{diff}(\text{diff}(y(x), x), x))^6}{6}$

$$de := \frac{d}{dx} y(x) = x \left( \frac{d^2}{dx^2} y(x) \right) - \frac{\left( \frac{d^2}{dx^2} y(x) \right)^6}{6} \quad (20)$$

>  $de := 6 \cdot u = 6 \cdot u' \cdot x - (u')^6$

$$de := 6 u(x) = 6 \left( \frac{d}{dx} u(x) \right) x - \left( \frac{d}{dx} u(x) \right)^6 \quad (21)$$

> dsolve(de)

$$u(x) = \frac{5 x^{6/5}}{6}, u(x) = \frac{5 \left( -\frac{\sqrt{5}}{4} - \frac{1}{4} - \frac{I\sqrt{2}\sqrt{5-\sqrt{5}}}{4} \right) x^{6/5}}{6}, u(x) \quad (22)$$

$$= \frac{5 \left( -\frac{\sqrt{5}}{4} - \frac{1}{4} + \frac{I\sqrt{2}\sqrt{5-\sqrt{5}}}{4} \right) x^{6/5}}{6}, u(x)$$

$$= \frac{5 \left( \frac{\sqrt{5}}{4} - \frac{1}{4} - \frac{I\sqrt{2}\sqrt{5+\sqrt{5}}}{4} \right) x^{6/5}}{6}, u(x)$$

$$= \frac{5 \left( \frac{\sqrt{5}}{4} - \frac{1}{4} + \frac{I\sqrt{2}\sqrt{5+\sqrt{5}}}{4} \right) x^{6/5}}{6}, u(x) = -\frac{1}{6} c_I^6 + c_I x$$

---

>  $sol\_1 := dsolve\left(diff(y(x), x) = C \cdot x - \frac{(C)^6}{6}\right)$

$$sol\_1 := y(x) = \frac{1}{2} C x^2 - \frac{1}{6} C^6 x + c_I \quad (23)$$

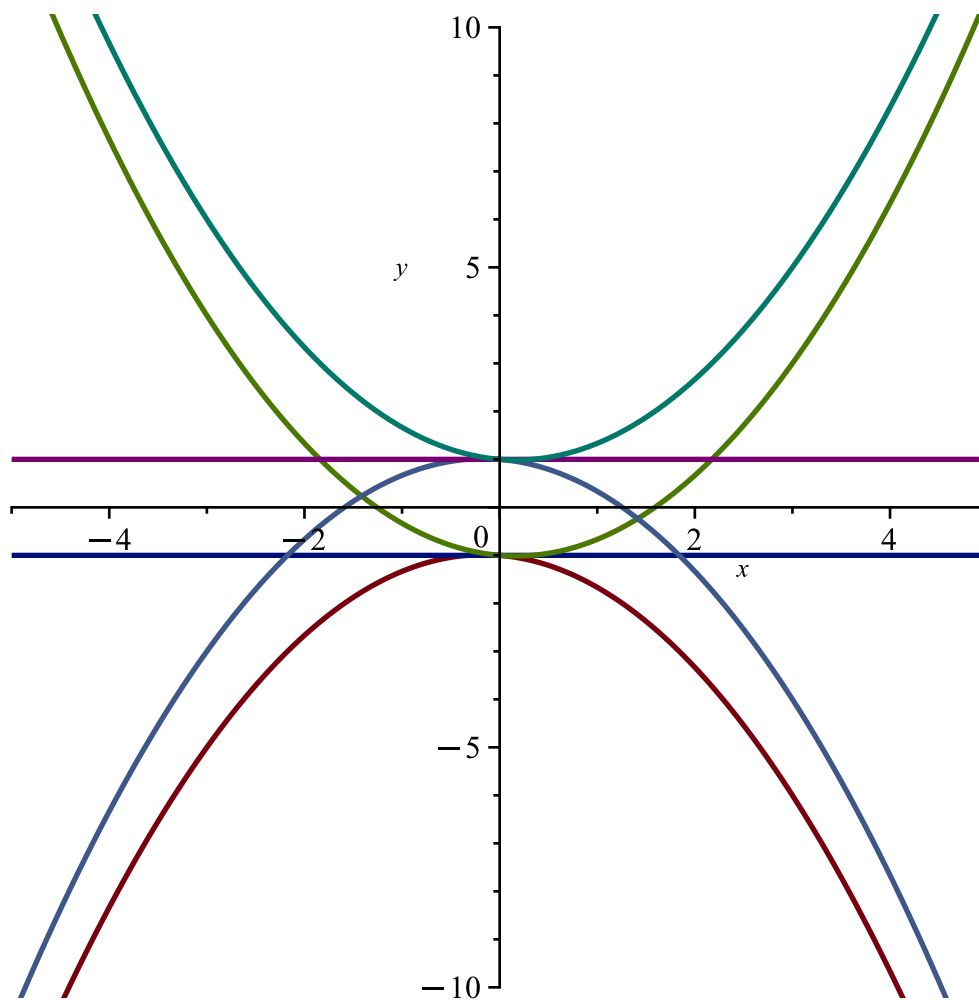

---

>  $sol\_2 := dsolve\left(diff(y(x), x) = \frac{5x^{\frac{6}{5}}}{6}\right)$

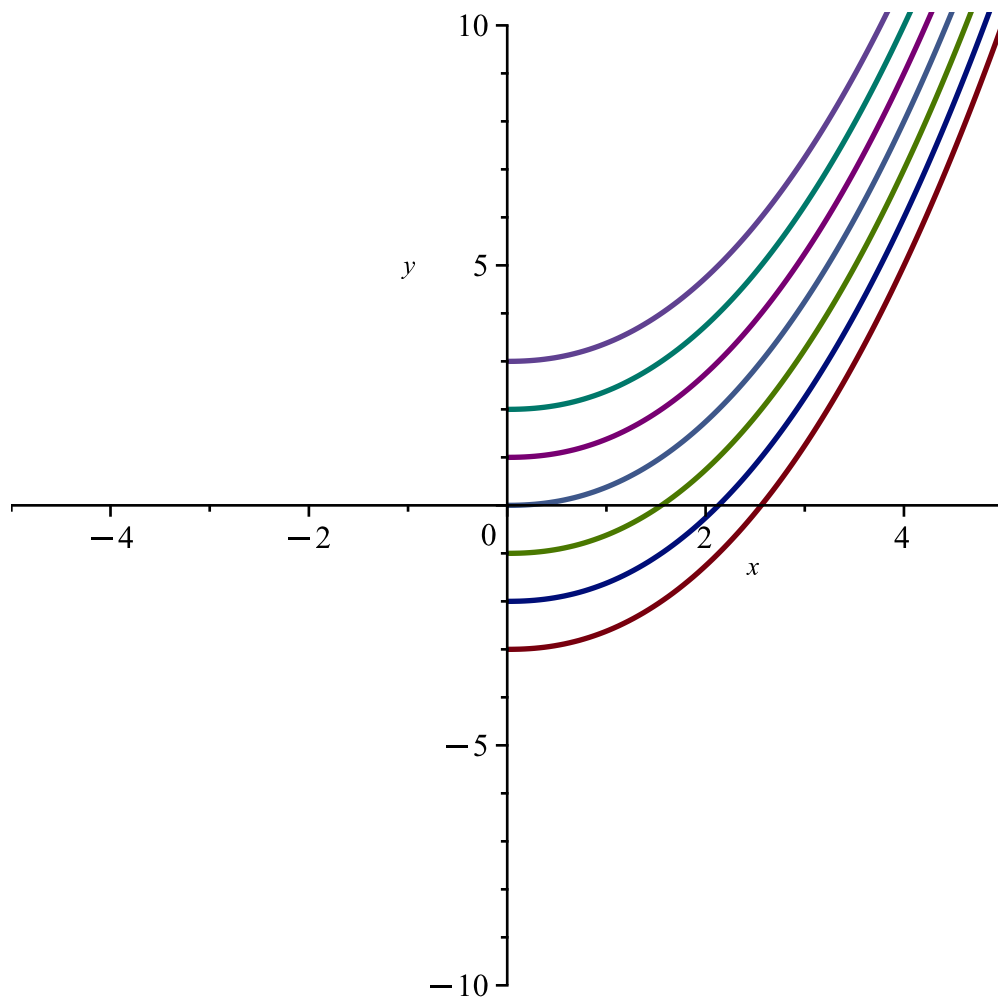
$$sol\_2 := y(x) = \frac{25x^{11/5}}{66} + c_I \quad (24)$$


---

>  $dpl\_1 := plot([seq(seq(rhs(sol\_1), C=-1..1), \_CI=[-1, 1])], x=-5..5, y=-10..10, thickness=2)$



```
> dpl_2 := plot([seq(rhs(sol_2), _C1=-3..3)], x=-5..5, y=-10..10, thickness=2)
```



```
> restart;
```

```
>
```

```
> #Задание 1.4
```

```
> de := diff(diff(y(x), x), x) = 2 * ( diff(y(x), x) / x - y(x) / x^2 ) + 1 / x^2 * cos(1 / x)
```

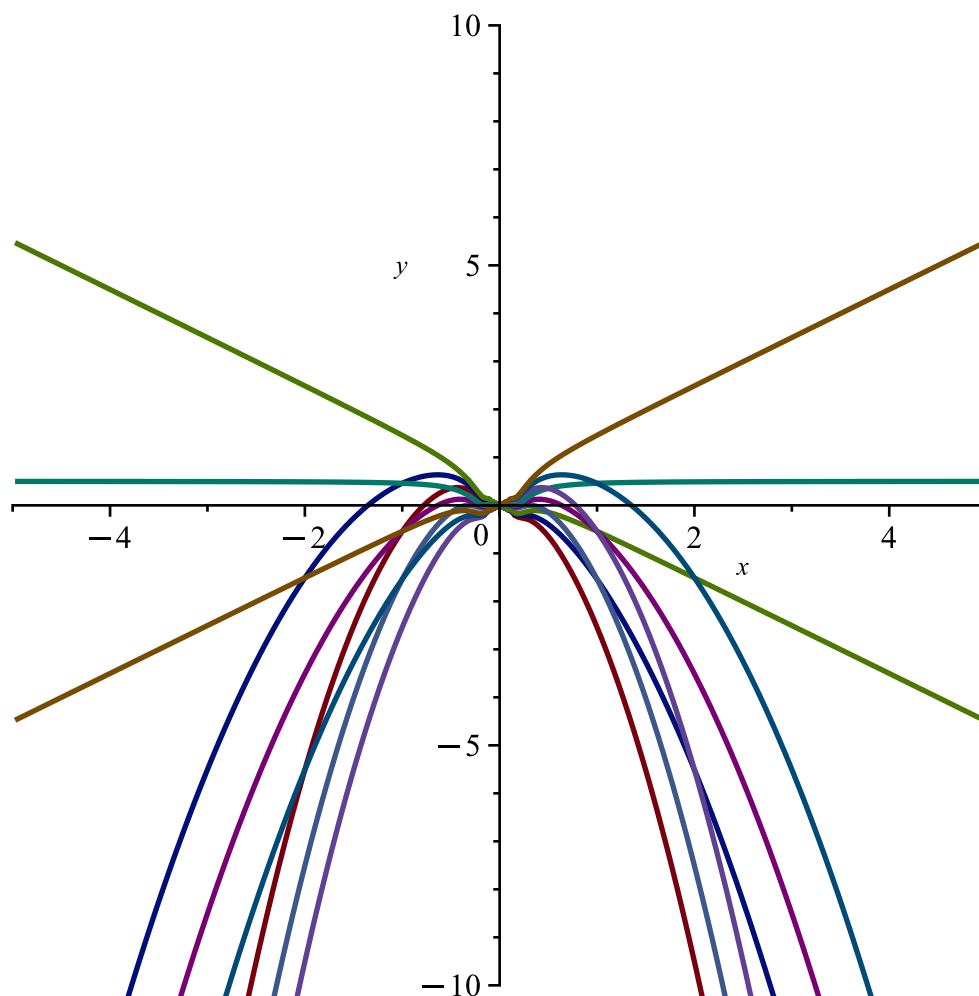
$$de := \frac{d^2}{dx^2} y(x) = \frac{2 \left( \frac{d}{dx} y(x) \right)}{x} - \frac{2 y(x)}{x^2} + \frac{\cos\left(\frac{1}{x}\right)}{x^2} \quad (25)$$

```
> sol := dsolve(de);
```

$$sol := y(x) = -\cos\left(\frac{1}{x}\right) x^2 + c_2 x^2 + c_1 x \quad (26)$$

```
> dpl := plot([seq(seq(rhs(sol), _C2 = -1 .. 1), _C1 = -1 .. 1)], x = -5 .. 5, y = -10 .. 10, thickness = 2);
plots[display](dpl);
```





```
> restart :
```

```
>
```

```
> #Задание 2
```

```
> de := x·diff(diff(diff(y(x), x), x), x) + diff(diff(y(x), x), x) = x + 1
```

$$de := x \left( \frac{d^3}{dx^3} y(x) \right) + \frac{d^2}{dx^2} y(x) = x + 1 \quad (27)$$

```
> dsolve(de)
```

$$y(x) = \frac{x^3}{12} + \frac{x^2}{2} + c_1 (x \ln(x) - x) + c_2 x + c_3 \quad (28)$$

```
> restart :
```

```
>
```

```
> #Задание 3
```

```
> de := diff(diff(y(x), x), x) + y(x) = 2 cos(7 x) + 3 sin(7 x)
```

$$de := \frac{d^2}{dx^2} y(x) + y(x) = 2 \cos(7 x) + 3 \sin(7 x) \quad (29)$$

```
> dsolve(de)
```

$$y(x) = \sin(x) c_2 + \cos(x) c_1 - \frac{\sin(7 x)}{16} - \frac{\cos(7 x)}{24} \quad (30)$$

```
> restart :
```

```
>
```

```
> #PART 3
```

```
>
```

```
> #Задание 1
```

```
> sys_diff := { d/dx y_1(x) = -y_1(x) + 2*y_2(x), d/dx y_2(x) = 3*y_1(x) } :  
sys_diff
```

$$\left\{ \frac{d}{dx} y_1(x) = -y_1(x) + 2 y_2(x), \frac{d}{dx} y_2(x) = 3 y_1(x) \right\} \quad (31)$$

```
> res := dsolve(sys_diff)
```

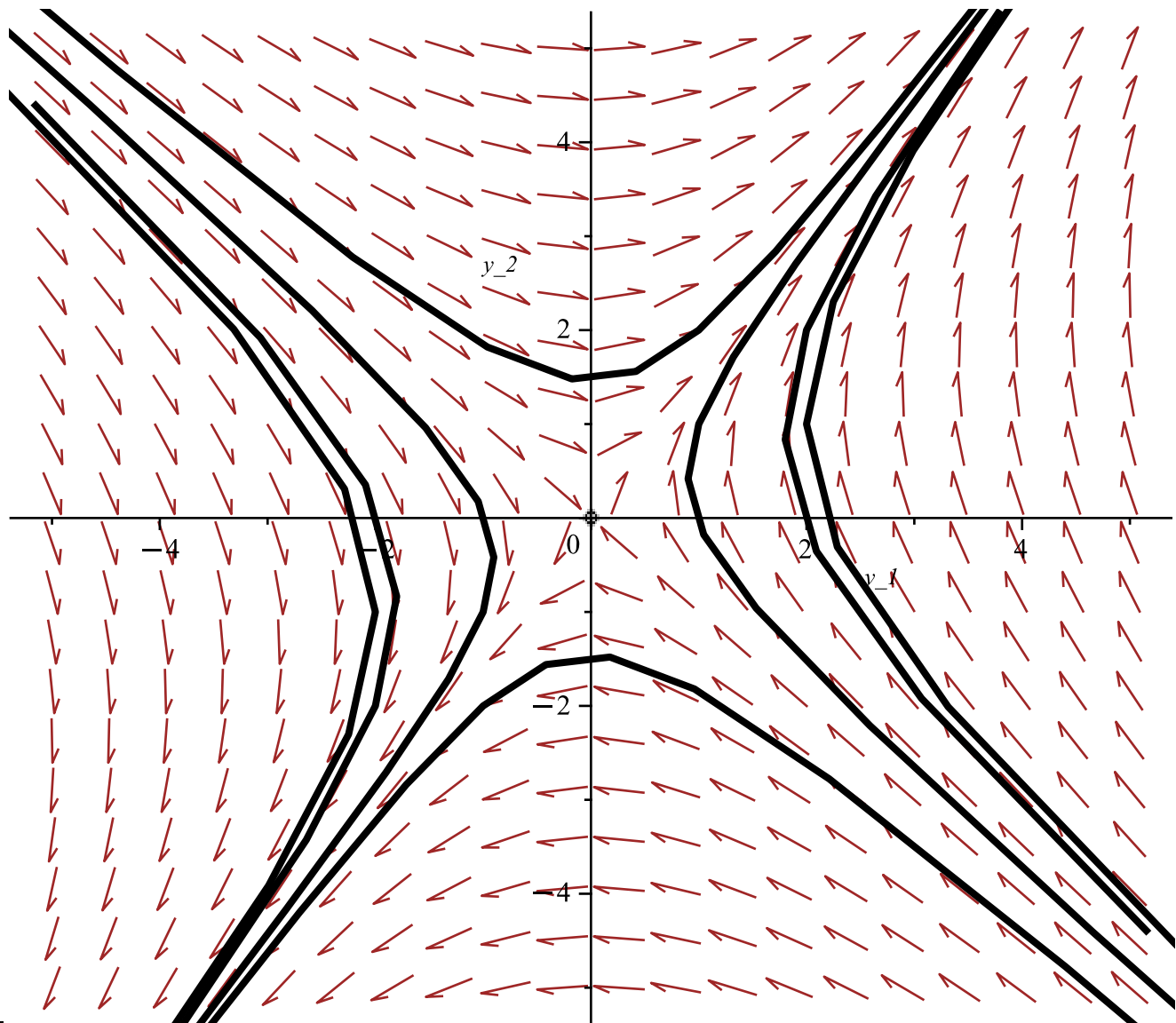
$$res := \left\{ y_1(x) = \frac{2 c_1 e^{2x}}{3} - c_2 e^{-3x}, y_2(x) = c_1 e^{2x} + c_2 e^{-3x} \right\} \quad (32)$$

```
> #Точка покоя (0, 0)
```

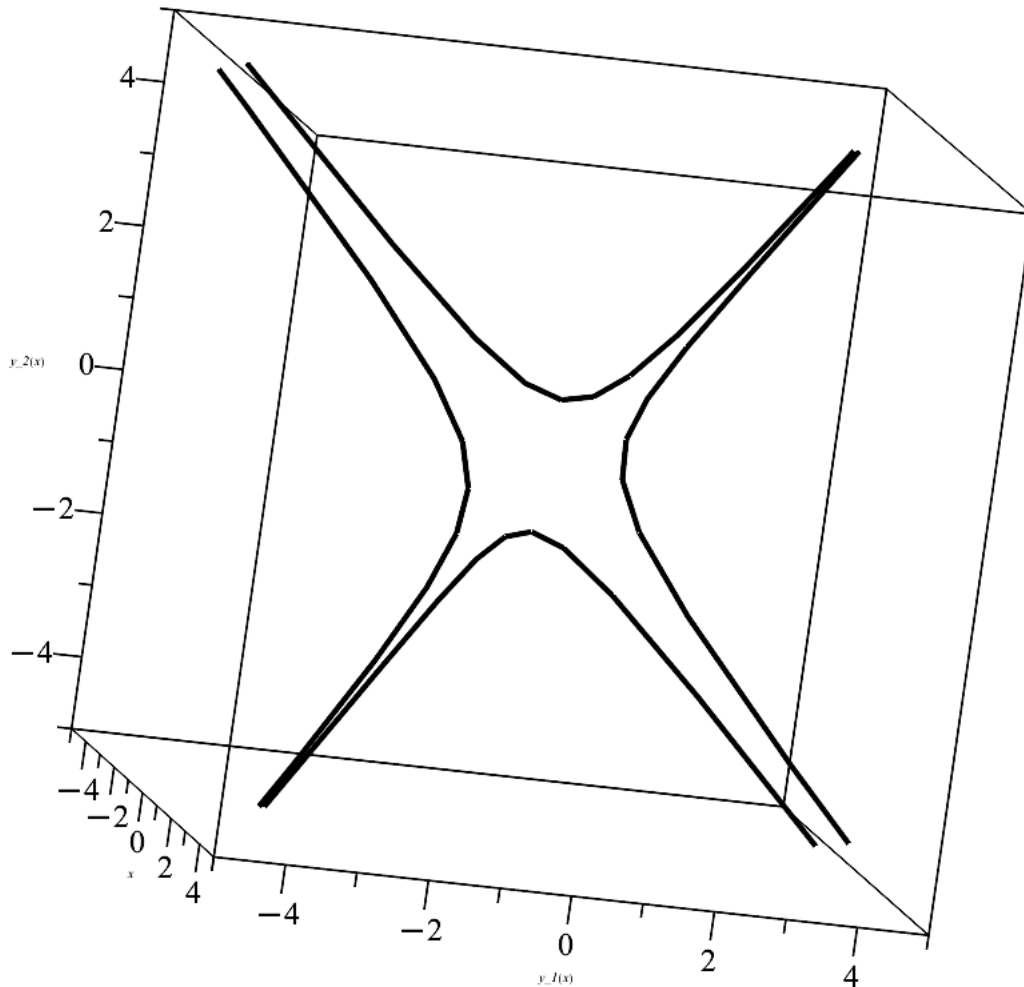
```
plot([ [0, 0]], color = black, style = point, symbolsize = 10) :
```

```
DEtools[phaseportrait]([sys_diff[1], sys_diff[2]], [y_1, y_2], x = -5 .. 5, [[0, 1, 1], [0, 1, 2],  
[0, 2, 1], [0, -1, -1], [0, 2, 2], [0, -1, -2], [0, -2, -1], [0, -2, -2]], y_1 = -5 .. 5, y_2 =  
-5 .. 5, linecolor = black, thickness = 3) :
```

```
plots[display](%, %%);
```



```
> DEtools[DEplot3d]([sys_diff[1], sys_diff[2]], [y_1, y_2], x=-5..5, [[0, 1, 0], [0, 0, 1], [-1, -1, 0], [-1, 0, -1]], y_1=-5..5, y_2=-5..5, thickness=4, linecolor=black);
```



>  $dfe := \text{diff}(y_2(y_1), y_1) = \frac{3 y_1}{-y_1 + 2 \cdot y_2}$

$dfe := \frac{d}{dy_1} y_2(y_1) = \frac{3 y_1}{-y_1 + 2 y_2}$  (33)

> `plot([[0, 0]], color=black, style=point, symbolsize=10) :`  
`DEtools[DEplot](dfe, y_2(y_1), y_1=-5..5, y_2=-5..5, [[1, 1], [1, 2], [2, 2], [-1, -1], [-1, -2], [-2, -2]], linecolor=black, thickness=2) :`  
`plots[display](%, %%);`

Warning, y\_2 is present as both a dependent variable and a name.  
Inconsistent specification of the dependent variable is deprecated.

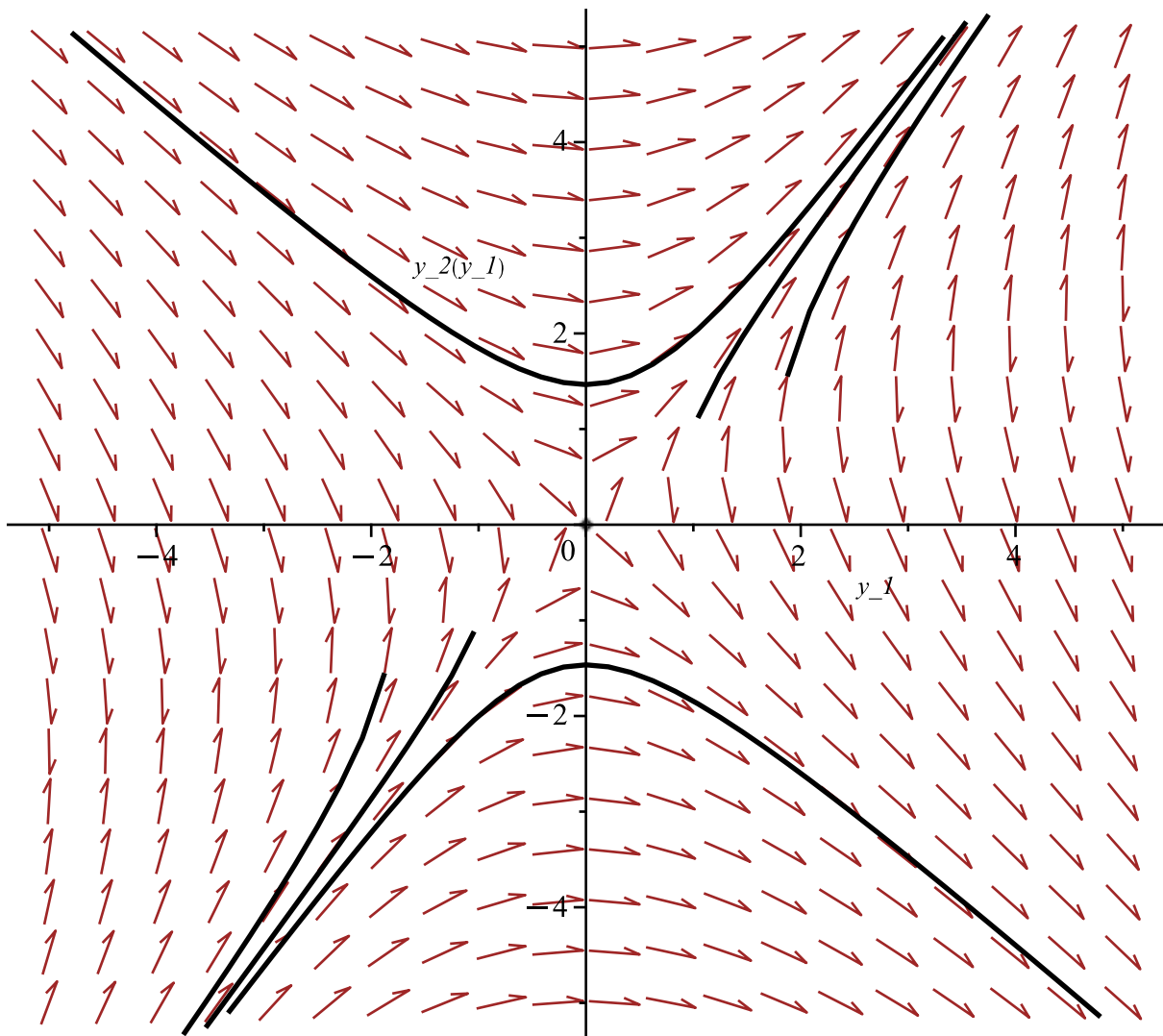
and it is assumed that the name is being used in place of the dependent variable.

Warning, plot may be incomplete, the following errors(s) were issued:  
cannot evaluate the solution further left of .90064002, probably a singularity

Warning, plot may be incomplete, the following errors(s) were issued:  
cannot evaluate the solution further left of 1.8012801, probably a singularity

Warning, plot may be incomplete, the following errors(s) were issued:  
cannot evaluate the solution further right of -.90064002, probably a singularity

Warning, plot may be incomplete, the following errors(s) were issued:  
cannot evaluate the solution further right of -1.8012801, probably a singularity



> restart :

> #Задание 2

> de := {diff(y\_1(x), x) = 4 · y\_1(x) + y\_2(x), diff(y\_2(x), x) = 11 · y\_1(x) - 6 · y\_2(x)};

$$de := \left\{ \frac{d}{dx} y_1(x) = 4 y_1(x) + y_2(x), \frac{d}{dx} y_2(x) = 11 y_1(x) - 6 y_2(x) \right\}$$

(34)

$$\begin{aligned} &> \text{dsolve}(de) \\ &\{y_1(x) = c_1 e^{-7x} + c_2 e^{5x}, y_2(x) = -11 c_1 e^{-7x} + c_2 e^{5x}\} \end{aligned} \quad (35)$$

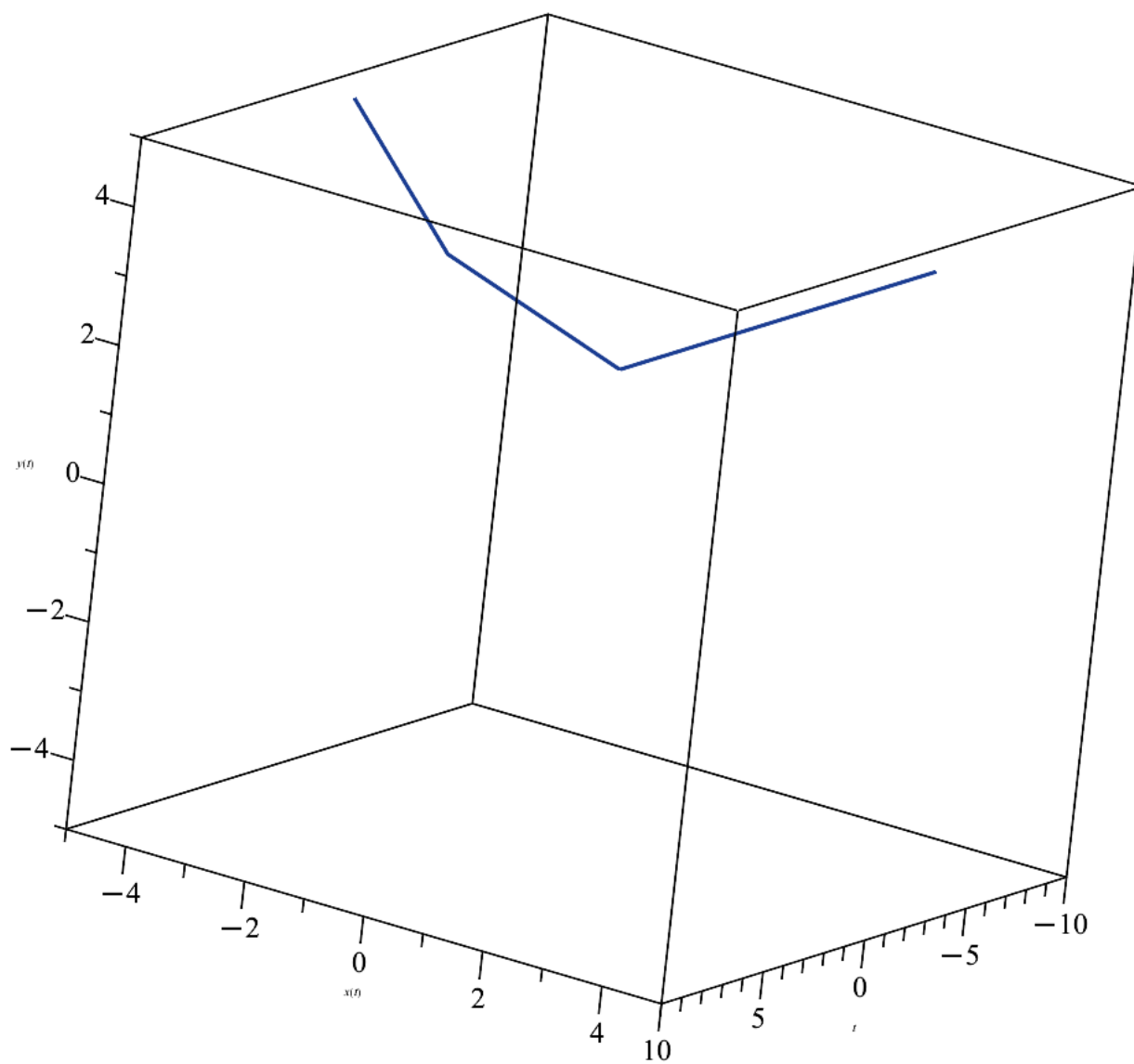
$\text{restart} :$

$\# \text{Задание 3}$

$$\begin{aligned} &> de := \{diff(x(t), t) = 3 \cdot x(t) + 5 \cdot y(t) + 2, diff(y(t), t) = 3 \cdot x(t) + y(t) + 1\} \\ &de := \left\{ \frac{d}{dt} x(t) = 3 x(t) + 5 y(t) + 2, \frac{d}{dt} y(t) = 3 x(t) + y(t) + 1 \right\} \end{aligned} \quad (36)$$

$$\begin{aligned} &> \text{dsolve}(de) \\ &\left\{ x(t) = e^{-2t} c_2 + e^{6t} c_1 - \frac{1}{4}, y(t) = -e^{-2t} c_2 + \frac{3 e^{6t} c_1}{5} - \frac{1}{4} \right\} \end{aligned} \quad (37)$$

$> \text{DETools}[\text{DEplot3d}](de, [x, y], t = -10..10, [[x(0) = 0, y(0) = 2]], x = -5..5, y = -5..5)$



```
[> restart :  
[>
```