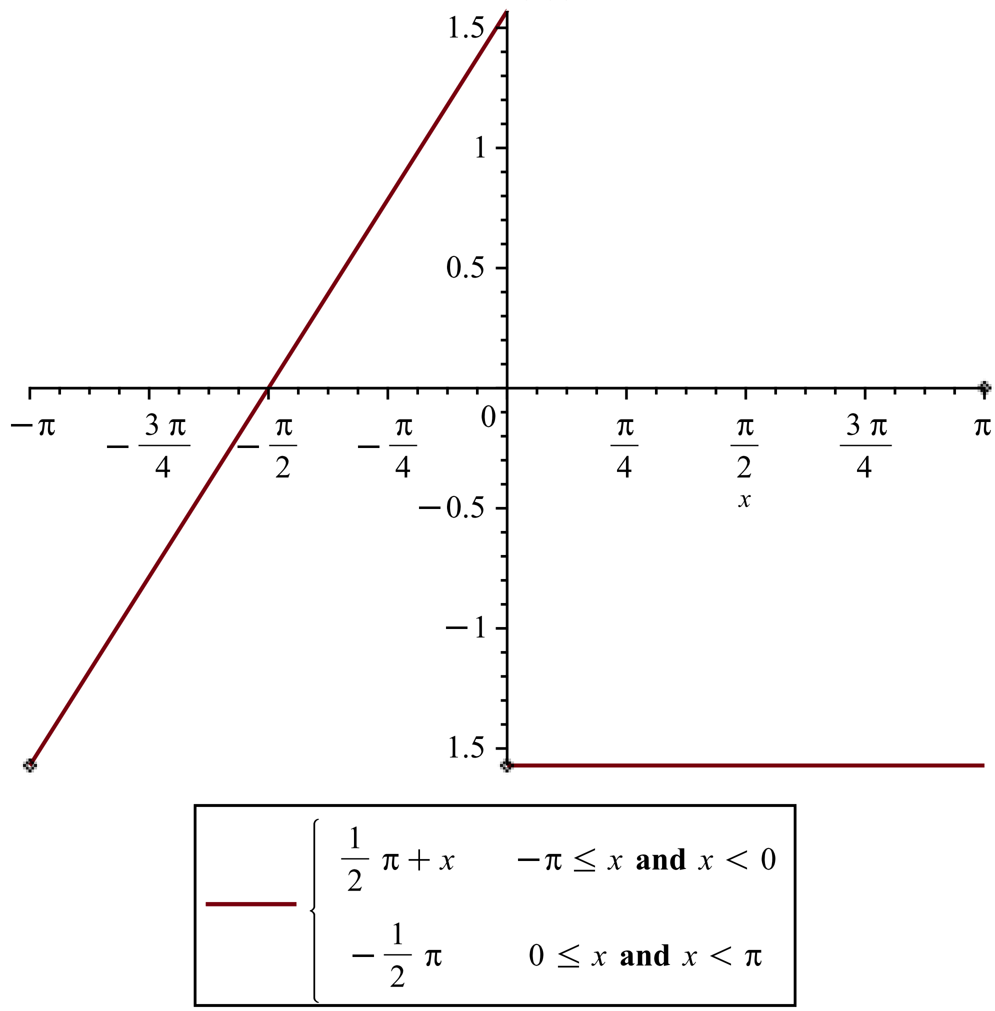
> *#Лабораторная работа 2(Вариант 4)*
#Кохан Артём Игоревич
#гр. 353503

> **#Задание 1. Для 2п-периодической кусочно-непрерывной функции f(x) по ее аналитическому определению на главном периоде**
#получите разложение в тригонометрический ряд Фурье.
#Постройте в одной системе координат на промежутке $[-3\pi, 3\pi]$ графики частичных сумм $S1(x)$, $S3(x)$, $S7(x)$ ряда и его суммы $S(x)$.

> $f := x \to piecewise\left(-\mathrm{Pi} \le x < 0, \frac{\mathrm{Pi}}{2} + x, 0 \le x < \mathrm{Pi}, -\frac{\mathrm{Pi}}{2}\right):$
$plot(f(x), x = -Pi..Pi, discont = true, legend = f(x))$



> $a0 := simplify\left(\frac{1}{\mathrm{Pi}} \cdot Int(f(x), x = -\mathrm{Pi}..\mathrm{Pi})\right) = simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x), x = -\mathrm{Pi}..\mathrm{Pi})\right);$

$$a0 := \frac{\int_{-\pi}^{\pi} \left(\begin{cases} \pi + 2x & x < 0 \\ -\pi & 0 \le x \end{cases}\right) \mathrm{d}x}{2\pi} = -\frac{\pi}{2} \qquad \textbf{(1)}$$

> $an := simplify\left(\frac{1}{\mathrm{Pi}} \cdot Int(f(x) \cdot \cos(n \cdot x), x = -\mathrm{Pi}..\mathrm{Pi})\right) = simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x) \cdot \cos(n \cdot x), x = -\mathrm{Pi}..\mathrm{Pi})\right)$ assuming $n :: posint$

$$an := \frac{\int_{-\pi}^{\pi} \cos(n\,x)\left(\begin{cases} \pi + 2\,x & x < 0 \\ -\pi & 0 \le x \end{cases}\right) \mathrm{d}x}{2\,\pi} = \frac{-(-1)^n + 1}{\pi\, n^2} \quad (2)$$

> $bn := simplify\left(\frac{1}{\mathrm{Pi}} \cdot Int(f(x) \cdot \sin(n \cdot x), x = -\mathrm{Pi}..\mathrm{Pi})\right) = simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x) \cdot \sin(n \cdot x), x = -\mathrm{Pi}..\mathrm{Pi})\right)$ assuming $n :: posint$;

$$bn := \frac{\int_{-\pi}^{\pi} \sin(n\,x)\left(\begin{cases} \pi + 2\,x & x < 0 \\ -\pi & 0 \le x \end{cases}\right) \mathrm{d}x}{2\,\pi} = -\frac{1}{n} \quad (3)$$

> $FourierTrigSum :=$ **proc**(f, k, a, b)
    local a_0, a_n, b_n, n, l;
    $l := \frac{(b-a)}{2}$;
    $assume(n::posint)$;
    $a\_0 := simplify(int(f(x), x = a..b) / l)$;
    $a\_n := simplify\left(int\left(f(x) \cdot \cos\left(\frac{\mathrm{Pi} \cdot n}{l} \cdot x\right), x = a..b\right) / l\right)$;
    $b\_n := simplify\left(int\left(f(x) \cdot \sin\left(\frac{\mathrm{Pi} \cdot n}{l} \cdot x\right), x = a..b\right) / l\right)$;
    return $\frac{a\_0}{2} + sum\left(a\_n \cdot \cos\left(\frac{\mathrm{Pi} \cdot n}{l} \cdot x\right) + b\_n \cdot \sin\left(\frac{\mathrm{Pi} \cdot n}{l} \cdot x\right), n = 1..k\right)$
end proc:

> $S1 := FourierTrigSum(f, 1, -\mathrm{Pi}, \mathrm{Pi})$;
$S3 := FourierTrigSum(f, 3, -\mathrm{Pi}, \mathrm{Pi})$;
$S7 := FourierTrigSum(f, 7, -\mathrm{Pi}, \mathrm{Pi})$;
$S := FourierTrigSum(f, \text{infinity}, -\mathrm{Pi}, \mathrm{Pi})$;
$S50000 := FourierTrigSum(f, 50000, -\mathrm{Pi}, \mathrm{Pi})$:

$$S1 := -\frac{\pi}{4} + \frac{2\cos(x)}{\pi} - \sin(x)$$

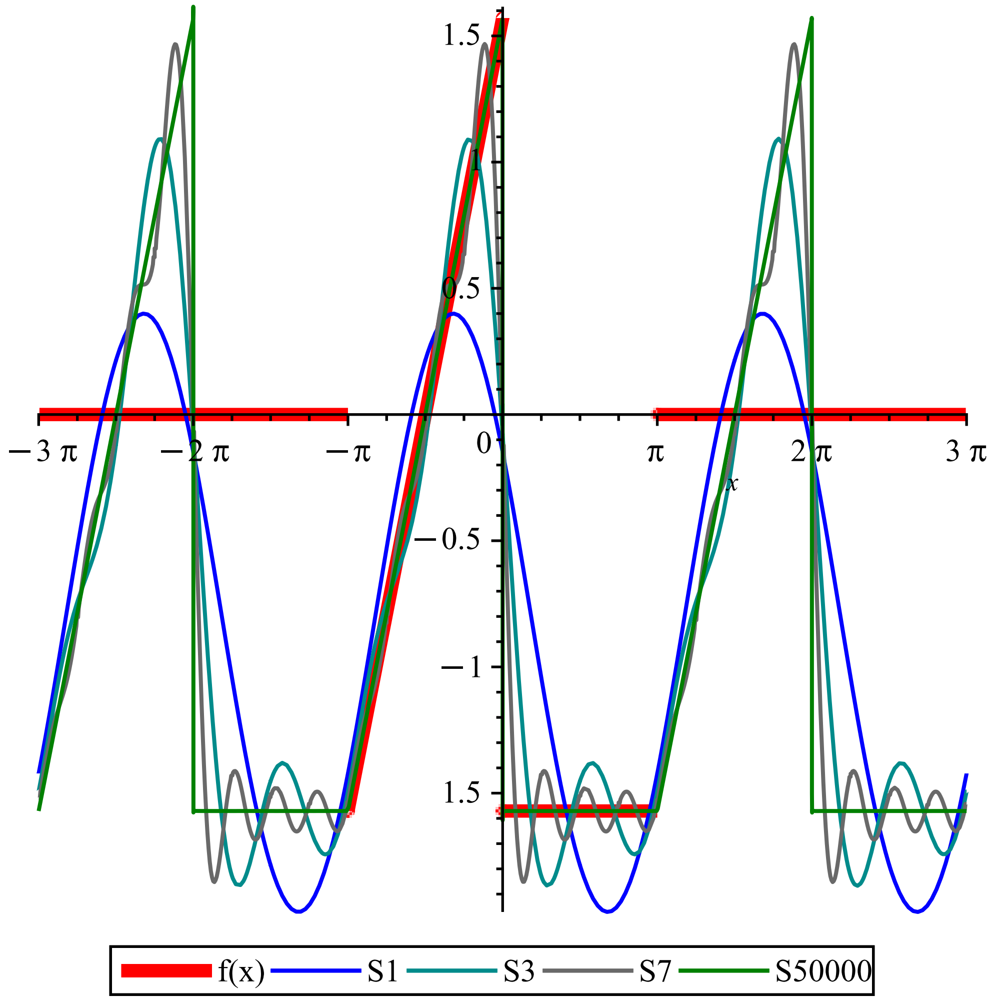
$$S3 := -\frac{\pi}{4} + \frac{2\cos(x)}{\pi} - \sin(x) - \frac{\sin(2\,x)}{2} + \frac{2\cos(3\,x)}{9\,\pi} - \frac{\sin(3\,x)}{3}$$

$$S7 := -\frac{\pi}{4} + \frac{2\cos(x)}{\pi} - \sin(x) - \frac{\sin(2\,x)}{2} + \frac{2\cos(3\,x)}{9\,\pi} - \frac{\sin(3\,x)}{3} - \frac{\sin(4\,x)}{4}$$

$$+\frac{2\cos(5x)}{25\pi}-\frac{\sin(5x)}{5}-\frac{\sin(6x)}{6}+\frac{2\cos(7x)}{49\pi}-\frac{\sin(7x)}{7}$$

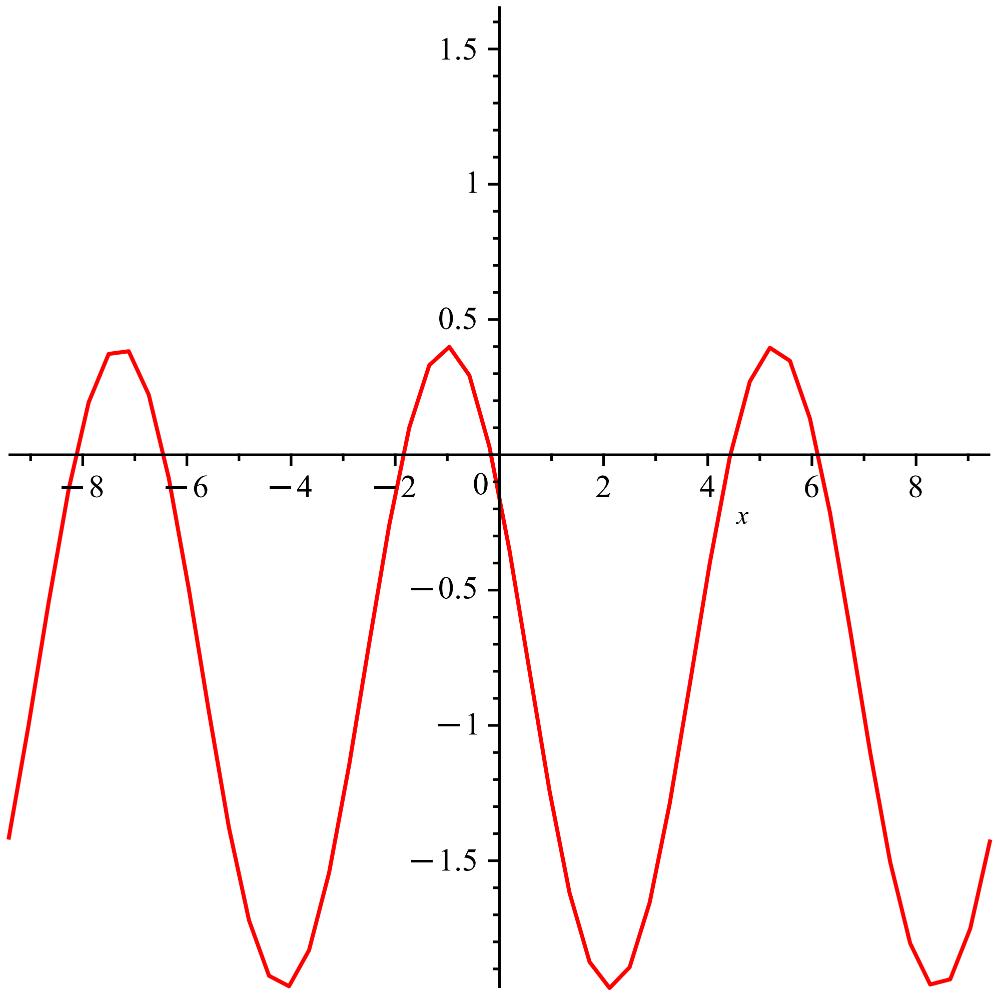
$$S := -\frac{\pi}{4}+\sum_{n\sim=1}^{\infty}\left(\frac{\left(-(-1)^{n\sim}+1\right)\cos(n\sim x)}{n\sim^{2}\,\pi}-\frac{\sin(n\sim x)}{n\sim}\right) \quad \textbf{(4)}$$

> *plot*([*S1*, *S3*, *S7*, *S50000*], *x* = −3 ·Pi ..3·Pi, *legend* = ["S1", "S3", "S7", "S50000"], *color* = ["Blue", "DarkCyan", "DimGray", "Green"]) :
plot(*f*(*x*), *x* = −3 ·Pi ..3·Pi, *legend* = "f(x)", *discont* = *true*, *color* = *red*, *thickness* = 5) :
plots[*display*](%, %%)



> ***#Анимация***

> *plots*[*animate*](*FourierTrigSum*(*f*, *k*, −Pi, Pi) , *x* = −3·Pi ..3·Pi, *k* = 1 ..16, *numpoints* = 50);



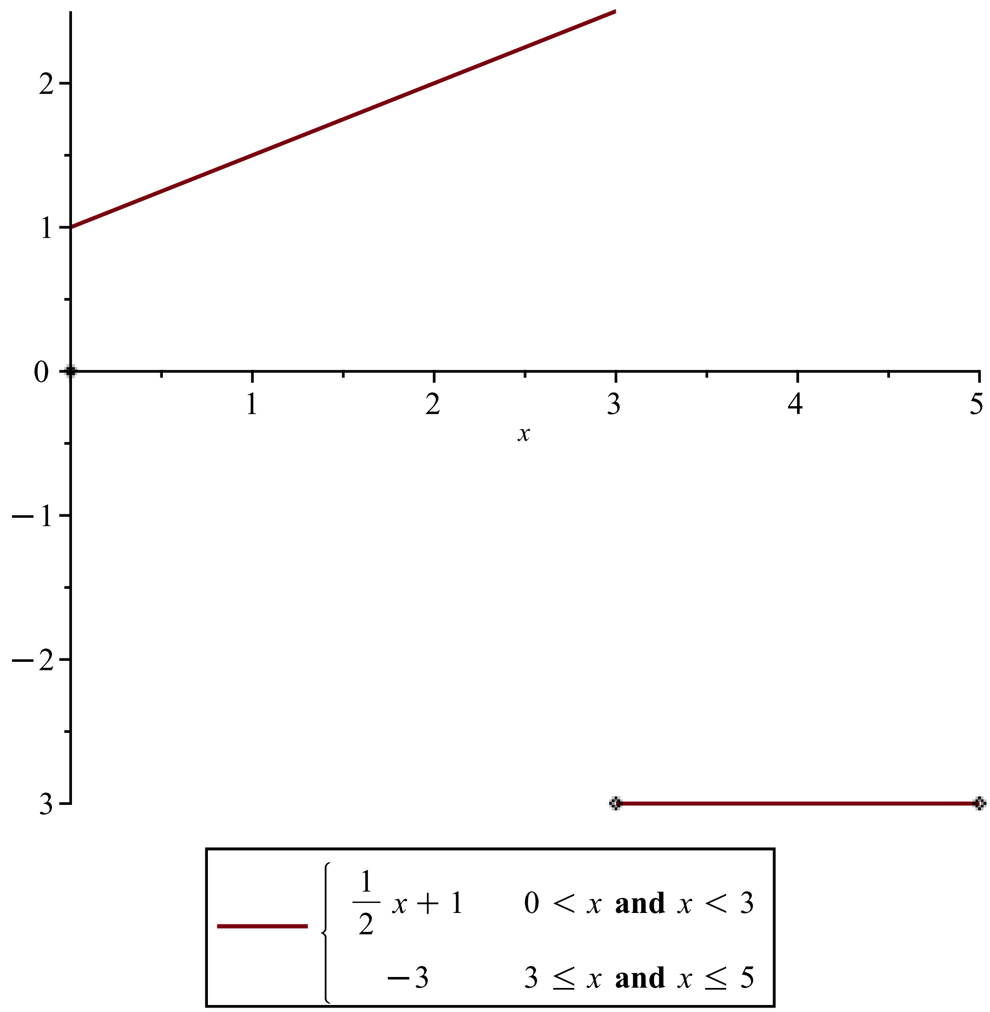
> *restart* :

>

> **#Задание 2. Разложите в ряд Фурье x2 - периодическую функцию y = f(x) , заданную на промежутке (0, x1) формулой**
> **#y = ax + b, а на [x1, x2] y = c.**
> **#Постройте в одной системе координат на промежутке[−2 x2, 2 x2], графики частичных сумм S1(x) , S3(x) , S7(x) ряда и его суммы S(x)**

> $f := x \to piecewise\left(0 < x < 3,\ \frac{1}{2} \cdot x + 1,\ 3 \le x \le 5, -3\right)$:

> $plot(f(x), x = 0\,..5, discont = true, legend = f(x))$;



> $a0 := simplify\left(\frac{2}{5}\cdot Int(f(x), x=0..5)\right) = simplify\left(\frac{2}{5}\cdot int(f(x), x=0..5)\right);$

$$a0 := \frac{2\left(\displaystyle\int_0^5 \left(\begin{cases} \frac{x}{2}+1 & x < 3 \\ -3 & 3 \le x \end{cases}\right) \mathrm{d}x\right)}{5} = -\frac{3}{10} \qquad \textbf{(5)}$$

> $an := simplify\left(\frac{2}{5}\cdot Int\left(f(x)\cdot\cos\left(\frac{2\cdot n\cdot \mathrm{Pi}\cdot x}{5}\right), x=0..5\right)\right) = simplify\left(\frac{2}{5}\cdot int\left(f(x)\cdot\cos\left(\frac{2\cdot n\cdot \mathrm{Pi}\cdot x}{5}\right), x=0..5\right)\right)$ assuming $n :: posint;$

$$an := \frac{2\left(\displaystyle\int_0^5 \left(\begin{cases} \frac{x}{2}+1 & x < 3 \\ -3 & 3 \le x \end{cases}\right) \cos\left(\frac{2\,n\,\pi\,x}{5}\right) \mathrm{d}x\right)}{5} \qquad \textbf{(6)}$$

$$= \frac{22\,n\,\pi\sin\left(\frac{6\,n\,\pi}{5}\right) + 5\cos\left(\frac{6\,n\,\pi}{5}\right) - 5}{4\,n^2\,\pi^2}$$

> $bn := simplify\left(\frac{2}{5}\cdot Int\left(f(x)\cdot\sin\left(\frac{2\cdot n\cdot \mathrm{Pi}\cdot x}{5}\right), x=0..5\right)\right) = simplify\left(\frac{2}{5}\cdot int\left(f(x)\cdot\sin\left(\frac{2\cdot n\cdot \mathrm{Pi}\cdot x}{5}\right), x=0..5\right)\right)$ assuming n :: *posint*;

$$bn := \frac{2\left(\int_0^5 \left(\begin{cases}\frac{x}{2}+1 & x<3\\ -3 & 3\le x\end{cases}\right)\sin\left(\frac{2\,n\,\pi\,x}{5}\right)\mathrm{d}x\right)}{5} \tag{7}$$

$$= \frac{-\frac{11\,n\,\pi\cos\left(\frac{6\,n\,\pi}{5}\right)}{2} + 4\,n\,\pi + \frac{5\sin\left(\frac{6\,n\,\pi}{5}\right)}{4}}{n^2\,\pi^2}$$

> $FourierTrigSum := \mathbf{proc}(f, k, a, b)$
  local a_0, a_n, b_n, n, l;
  $l := \frac{(b-a)}{2}$;
  $assume(n::posint)$;
  $a\_0 := simplify(int(f(x), x=a..b)/l)$;
  $a\_n := simplify\left(int\left(f(x)\cdot\cos\left(\frac{\mathrm{Pi}\cdot n}{l}\cdot x\right), x=a..b\right)/l\right)$;
  $b\_n := simplify\left(int\left(f(x)\cdot\sin\left(\frac{\mathrm{Pi}\cdot n}{l}\cdot x\right), x=a..b\right)/l\right)$;
  return $\frac{a\_0}{2} + sum\left(a\_n\cdot\cos\left(\frac{\mathrm{Pi}\cdot n}{l}\cdot x\right) + b\_n\cdot\sin\left(\frac{\mathrm{Pi}\cdot n}{l}\cdot x\right), n=1..k\right)$
end proc:

> $S1 := FourierTrigSum(f, 1, 0, 5)$;
$S3 := FourierTrigSum(f, 3, 0, 5)$;
$S7 := FourierTrigSum(f, 7, 0, 5)$;
$S := FourierTrigSum(f, \infty, 0, 5)$;
$S50000 := FourierTrigSum(f, 50000, 0, 5)$:

$$S1 := -\frac{3}{20} + \frac{\left(-22\,\pi\sin\left(\frac{\pi}{5}\right) - 5\cos\left(\frac{\pi}{5}\right) - 5\right)\cos\left(\frac{2\,\pi\,x}{5}\right)}{4\,\pi^2}$$
$$+ \frac{\left(\frac{11\,\pi\cos\left(\frac{\pi}{5}\right)}{2} + 4\,\pi - \frac{5\sin\left(\frac{\pi}{5}\right)}{4}\right)\sin\left(\frac{2\,\pi\,x}{5}\right)}{\pi^2}$$

$$S3 := -\frac{3}{20} + \frac{\left(-22\,\pi \sin\left(\frac{\pi}{5}\right) - 5\cos\left(\frac{\pi}{5}\right) - 5\right)\cos\left(\frac{2\,\pi\,x}{5}\right)}{4\,\pi^2} + \frac{\left(\frac{11\,\pi\cos\left(\frac{\pi}{5}\right)}{2} + 4\,\pi - \frac{5\sin\left(\frac{\pi}{5}\right)}{4}\right)\sin\left(\frac{2\,\pi\,x}{5}\right)}{\pi^2} + \frac{\left(44\,\pi\sin\left(\frac{2\,\pi}{5}\right) + 5\cos\left(\frac{2\,\pi}{5}\right) - 5\right)\cos\left(\frac{4\,\pi\,x}{5}\right)}{16\,\pi^2} + \frac{\left(-11\,\pi\cos\left(\frac{2\,\pi}{5}\right) + 8\,\pi + \frac{5\sin\left(\frac{2\,\pi}{5}\right)}{4}\right)\sin\left(\frac{4\,\pi\,x}{5}\right)}{4\,\pi^2} + \frac{\left(-66\,\pi\sin\left(\frac{2\,\pi}{5}\right) + 5\cos\left(\frac{2\,\pi}{5}\right) - 5\right)\cos\left(\frac{6\,\pi\,x}{5}\right)}{36\,\pi^2} + \frac{\left(-\frac{33\,\pi\cos\left(\frac{2\,\pi}{5}\right)}{2} + 12\,\pi - \frac{5\sin\left(\frac{2\,\pi}{5}\right)}{4}\right)\sin\left(\frac{6\,\pi\,x}{5}\right)}{9\,\pi^2}$$

$$S7 := -\frac{3}{20} + \frac{\left(-22\,\pi \sin\left(\frac{\pi}{5}\right) - 5\cos\left(\frac{\pi}{5}\right) - 5\right)\cos\left(\frac{2\,\pi\,x}{5}\right)}{4\,\pi^2} + \frac{\left(\frac{11\,\pi\cos\left(\frac{\pi}{5}\right)}{2} + 4\,\pi - \frac{5\sin\left(\frac{\pi}{5}\right)}{4}\right)\sin\left(\frac{2\,\pi\,x}{5}\right)}{\pi^2} + \frac{\left(44\,\pi\sin\left(\frac{2\,\pi}{5}\right) + 5\cos\left(\frac{2\,\pi}{5}\right) - 5\right)\cos\left(\frac{4\,\pi\,x}{5}\right)}{16\,\pi^2} + \frac{\left(-11\,\pi\cos\left(\frac{2\,\pi}{5}\right) + 8\,\pi + \frac{5\sin\left(\frac{2\,\pi}{5}\right)}{4}\right)\sin\left(\frac{4\,\pi\,x}{5}\right)}{4\,\pi^2}$$

$$+\frac{\left(-66\,\pi\sin\left(\frac{2\,\pi}{5}\right)+5\cos\left(\frac{2\,\pi}{5}\right)-5\right)\cos\left(\frac{6\,\pi\,x}{5}\right)}{36\,\pi^{2}}$$

$$+\frac{\left(-\frac{33\,\pi\cos\left(\frac{2\,\pi}{5}\right)}{2}+12\,\pi-\frac{5\sin\left(\frac{2\,\pi}{5}\right)}{4}\right)\sin\left(\frac{6\,\pi\,x}{5}\right)}{9\,\pi^{2}}$$

$$+\frac{\left(88\,\pi\sin\left(\frac{\pi}{5}\right)-5\cos\left(\frac{\pi}{5}\right)-5\right)\cos\left(\frac{8\,\pi\,x}{5}\right)}{64\,\pi^{2}}$$

$$+\frac{\left(22\,\pi\cos\left(\frac{\pi}{5}\right)+16\,\pi+\frac{5\sin\left(\frac{\pi}{5}\right)}{4}\right)\sin\left(\frac{8\,\pi\,x}{5}\right)}{16\,\pi^{2}}-\frac{3\sin(2\,\pi\,x)}{10\,\pi}$$

$$+\frac{\left(-132\,\pi\sin\left(\frac{\pi}{5}\right)-5\cos\left(\frac{\pi}{5}\right)-5\right)\cos\left(\frac{12\,\pi\,x}{5}\right)}{144\,\pi^{2}}$$

$$+\frac{\left(33\,\pi\cos\left(\frac{\pi}{5}\right)+24\,\pi-\frac{5\sin\left(\frac{\pi}{5}\right)}{4}\right)\sin\left(\frac{12\,\pi\,x}{5}\right)}{36\,\pi^{2}}$$

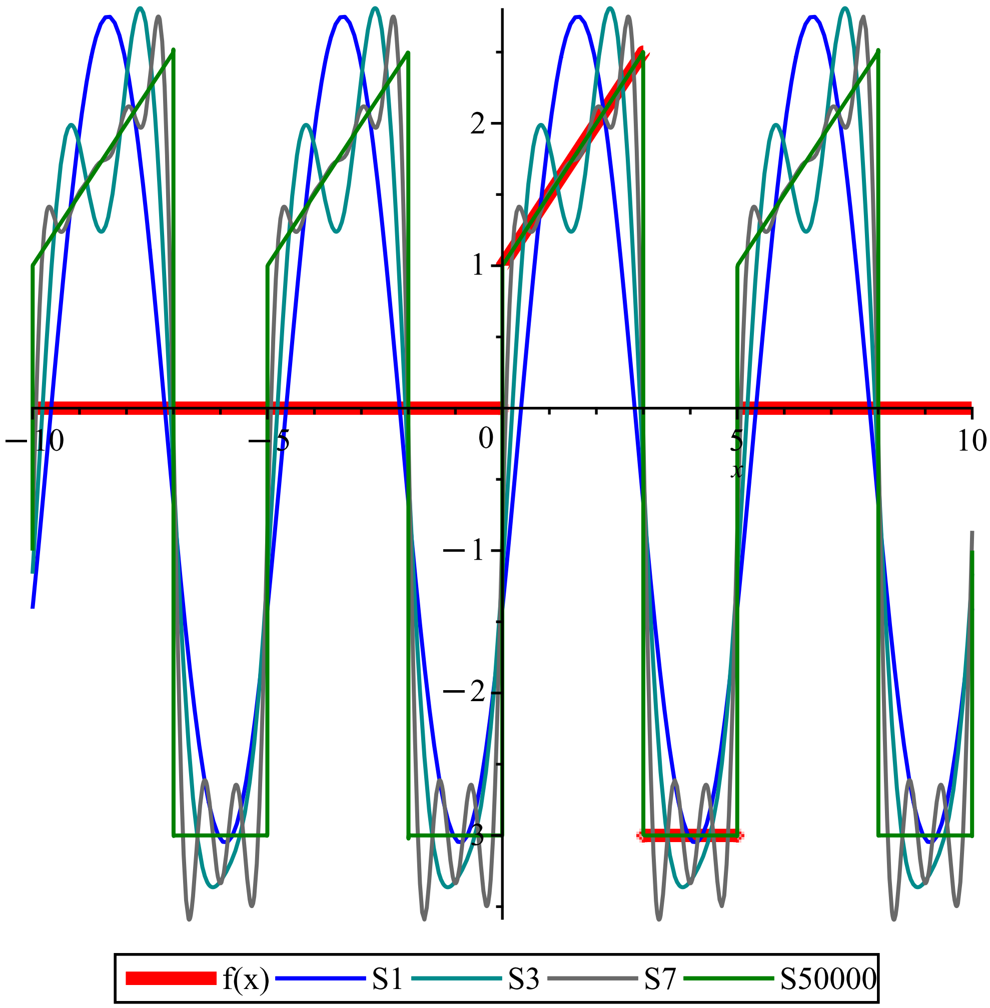
$$+\frac{\left(154\,\pi\sin\left(\frac{2\,\pi}{5}\right)+5\cos\left(\frac{2\,\pi}{5}\right)-5\right)\cos\left(\frac{14\,\pi\,x}{5}\right)}{196\,\pi^{2}}$$

$$+\frac{\left(-\frac{77\,\pi\cos\left(\frac{2\,\pi}{5}\right)}{2}+28\,\pi+\frac{5\sin\left(\frac{2\,\pi}{5}\right)}{4}\right)\sin\left(\frac{14\,\pi\,x}{5}\right)}{49\,\pi^{2}}$$

$$S:=-\frac{3}{20}+\sum_{n\sim=1}^{\infty}\left(\frac{\left(22\,\pi\,n\sim\sin\left(\frac{6\,\pi\,n\sim}{5}\right)+5\cos\left(\frac{6\,\pi\,n\sim}{5}\right)-5\right)\cos\left(\frac{2\,\pi\,n\sim x}{5}\right)}{4\,n\sim^{2}\,\pi^{2}}+\frac{\left(-\frac{11\,\pi\,n\sim\cos\left(\frac{6\,\pi\,n\sim}{5}\right)}{2}+4\,\pi\,n\sim+\frac{5\sin\left(\frac{6\,\pi\,n\sim}{5}\right)}{4}\right)\sin\left(\frac{2\,\pi\,n\sim x}{5}\right)}{n\sim^{2}\,\pi^{2}}\right) \quad \textbf{(8)}$$

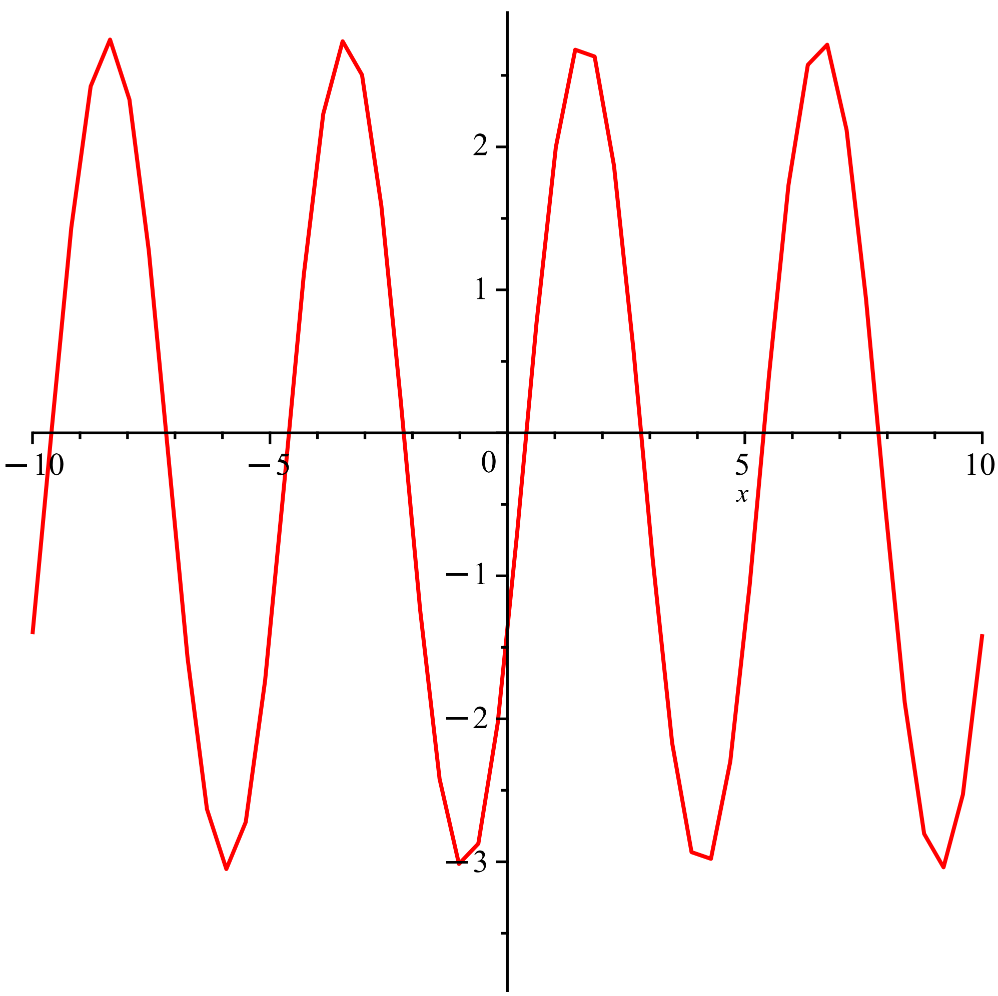
> *plot*([*S1*, *S3*, *S7*, *S50000*], *x* = −10 ..10, *legend* = ["S1", "S3", "S7", "S50000"], *color* = ["Blue",

"DarkCyan", "DimGray", "Green"]) :
plot(*f*(*x*), *x* = −10 ..10, *legend* = "f(x)", *discont* = *true*, *color* = *red*, *thickness* = 5) :
plots[*display*](%, %%)



> ***#Анимация***

> *plots*[*animate*](*FourierTrigSum*(*f*, *k*, 0, 5), *x* = −10 ..10, *k* = 1 ..16, *numpoints* = 50);



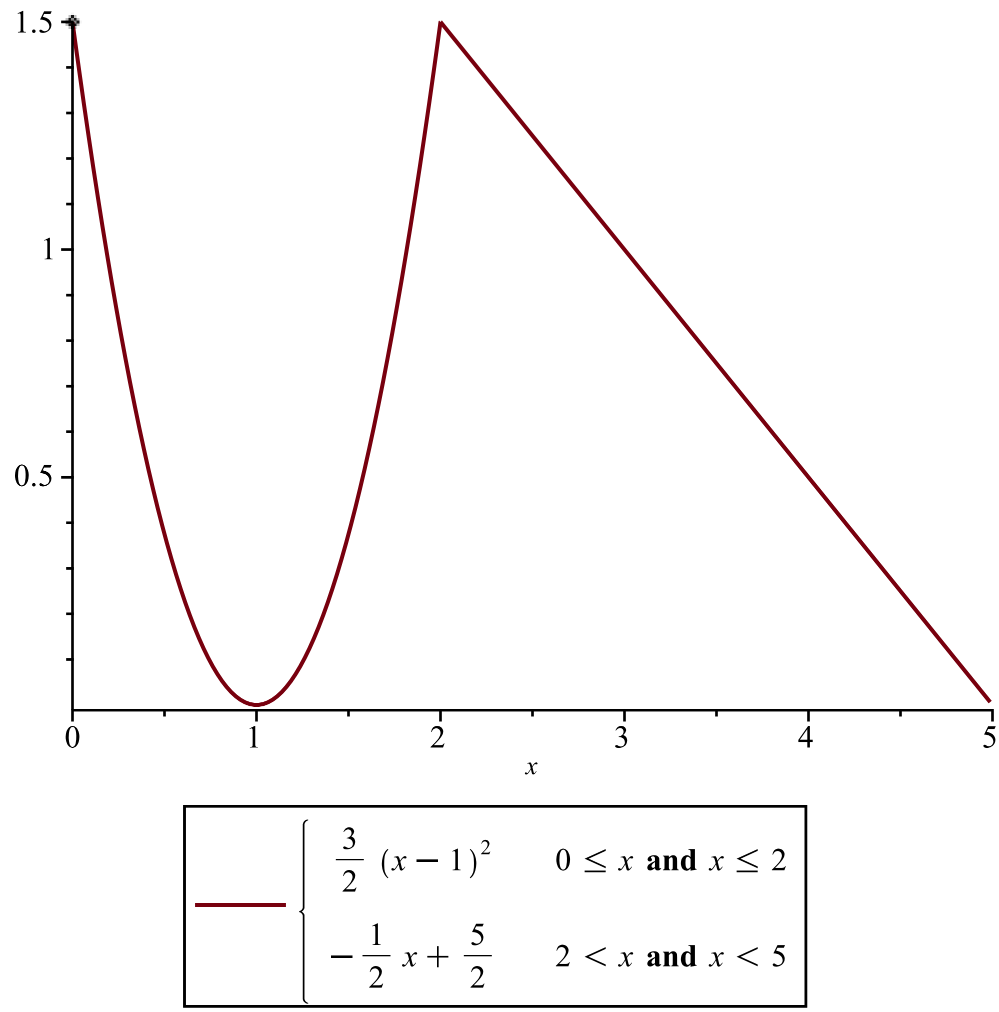
> *restart* :

>

> **#Задание 3. Для графически заданной функции построить три разложения в тригонометрический ряд Фурье.**
> **#Построить графики сумм рядов на промежутке превышающем длину заданного в три раза.**

> $f := x \to piecewise\left(0 \le x \le 2, \frac{3}{2} \cdot (x-1)^2, 2 < x < 5, -\frac{1}{2} \cdot x + \frac{5}{2}\right)$:

> $plot(f(x), x = 0..5, legend = f(x), discont = true);$



> $a0 := simplify\left(\frac{2}{5} \cdot int(f(x), x = 0..5)\right)$

$$a0 := \frac{13}{10} \tag{9}$$

> $an := simplify\left(\frac{2}{5} \cdot int\left(f(x) \cdot \cos\left(\frac{2 \cdot \mathrm{Pi} \cdot n \cdot x}{5}\right), x = 0..5\right)\right)$ assuming $n :: posint$

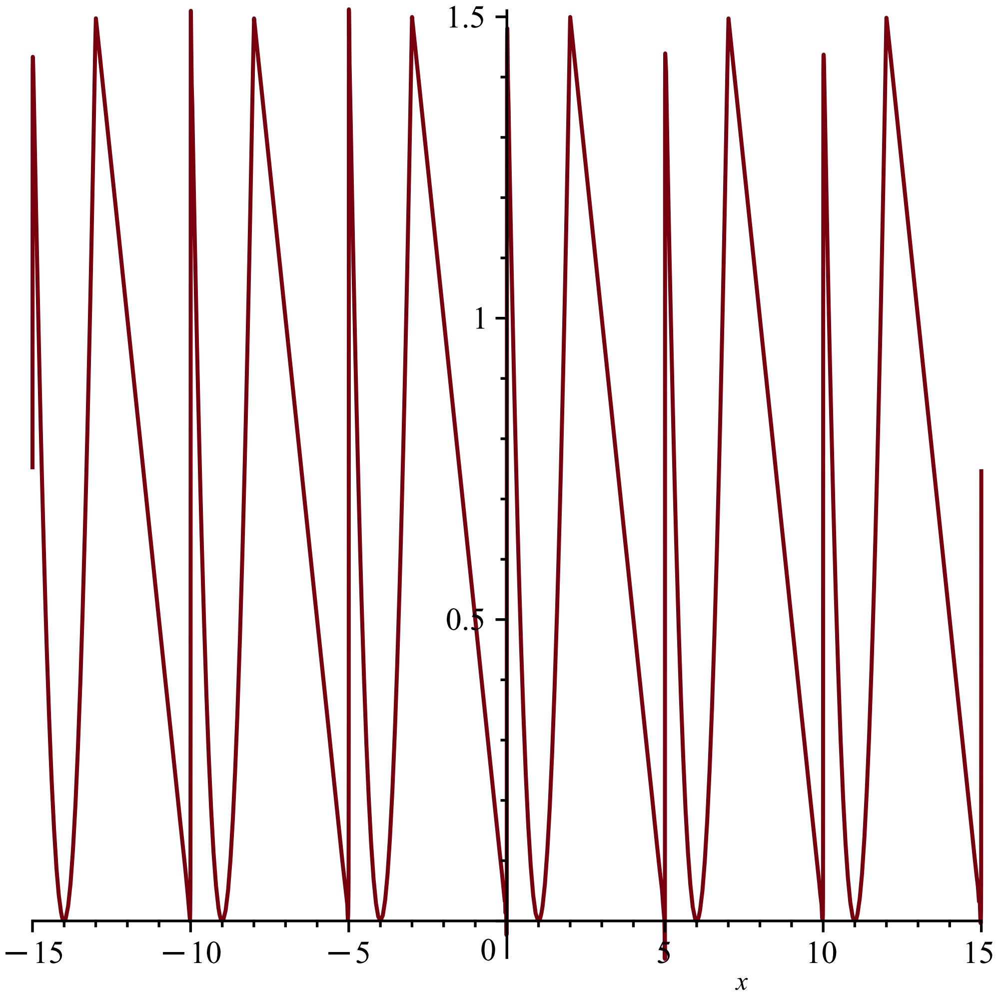
$$an := \frac{5\left(7\pi n \cos\left(\frac{4\pi n}{5}\right) + 5\pi n - 15\sin\left(\frac{4\pi n}{5}\right)\right)}{4\pi^3 n^3} \tag{10}$$

> $bn := simplify\left(\frac{2}{5} \cdot int\left(f(x) \cdot \sin\left(\frac{2 \cdot \mathrm{Pi} \cdot n \cdot x}{5}\right), x = 0..5\right)\right)$ assuming $n :: posint$

$$bn := \frac{6\pi^2 n^2 + 35\pi n \sin\left(\frac{4\pi n}{5}\right) + 75\cos\left(\frac{4\pi n}{5}\right) - 75}{4 n^3 \pi^3} \tag{11}$$

> $S := k \rightarrow \frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{2 \cdot \mathrm{Pi} \cdot n \cdot x}{5}\right) + bn \cdot \sin\left(\frac{2 \cdot \mathrm{Pi} \cdot n \cdot x}{5}\right), n = 1..k\right):$

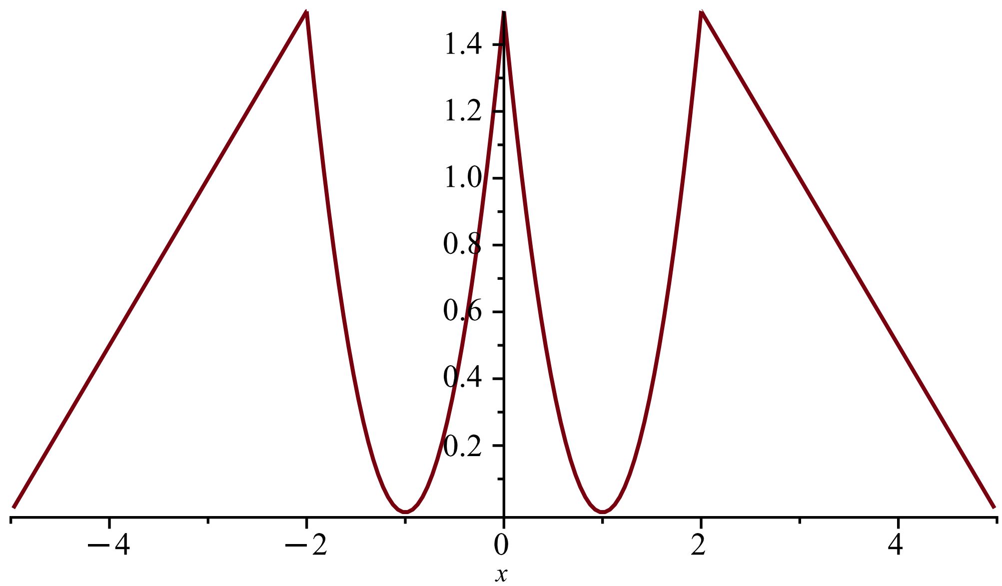
> $plot(S(1000), x = -15..15, discont = true)$



> ***#Определим чётным образом***

> $f\_even := x \to piecewise\left(-5 < x < -2, \frac{1}{2}\cdot x + \frac{5}{2}, -2 \le x \le 0, \frac{3}{2}\cdot(-x-1)^2, 0 \le x \le 2, \frac{3}{2}\cdot(x-1)^2, 2 < x < 5, -\frac{1}{2}\cdot x + \frac{5}{2}\right):$

$plot(f\_even(x), x = -5..5, legend = f\_even(x), discont = true);$



$$\begin{cases} \frac{1}{2}x+\frac{5}{2} & -5<x \textbf{ and } x<-2 \\ \frac{3}{2}(-1-x)^2 & -2\le x \textbf{ and } x\le 0 \\ \frac{3}{2}(x-1)^2 & 0\le x \textbf{ and } x\le 2 \\ -\frac{1}{2}x+\frac{5}{2} & 2<x \textbf{ and } x<5 \end{cases}$$

>

> $a0 := simplify\left(\frac{2}{5}\cdot int(f\_even(x), x=0..5)\right);$

$$a0 := \frac{13}{10} \tag{12}$$

> $an := simplify\left(\frac{2}{5}\cdot int\left(f\_even(x)\cdot\cos\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right), x=0..5\right)\right)$ assuming $n :: posint$

$$an := \frac{-5\,\pi\,(-1)^n\,n+35\,\pi\,n\cos\left(\frac{2\,\pi\,n}{5}\right)+30\,\pi\,n-150\sin\left(\frac{2\,\pi\,n}{5}\right)}{\pi^3\,n^3} \tag{13}$$

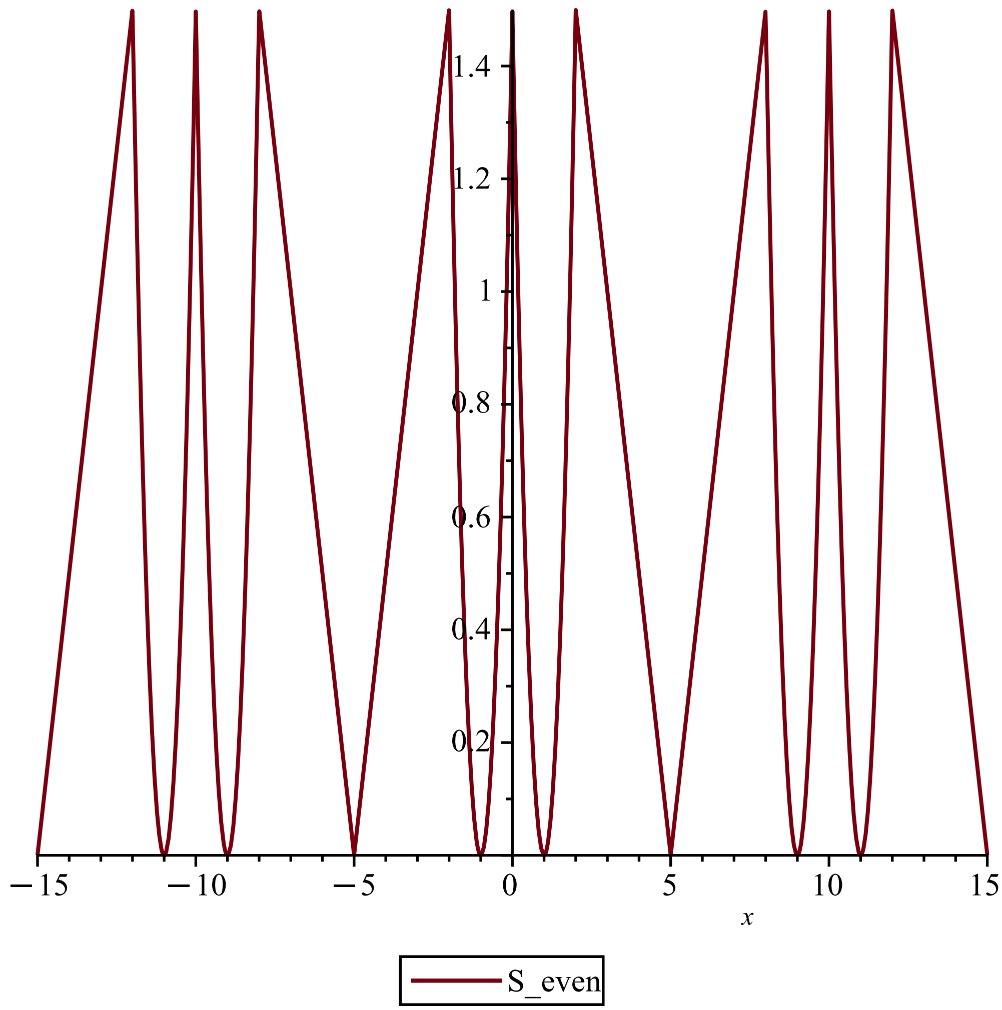
> $bn := simplify\left(\frac{1}{5}\cdot int\left(f\_even(x)\cdot\sin\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right), x=-5..5\right)\right)$ assuming $n :: posint$

$$bn := 0 \tag{14}$$

> $S\_even := k\to\frac{a0}{2}+sum\left(an\cdot\cos\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right)+bn\cdot\sin\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right), n=1..k\right)$

$$S\_even := k\mapsto\frac{a0}{2}+\sum_{n=1}^{k}\left(an\cdot\cos\left(\frac{\pi\cdot n\cdot x}{5}\right)+bn\cdot\sin\left(\frac{\pi\cdot n\cdot x}{5}\right)\right) \tag{15}$$

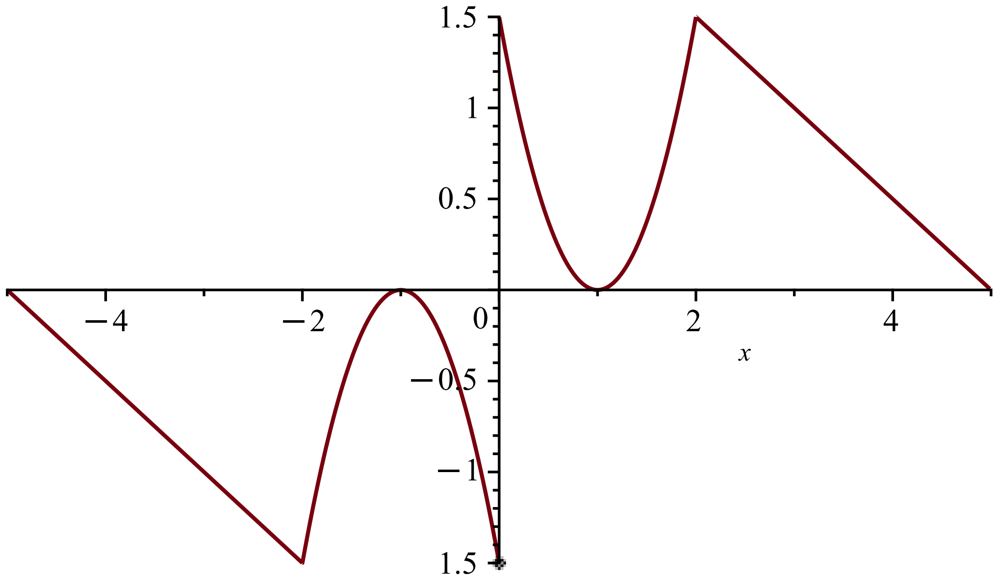
> $plot(S\_even(1000), x = -15..15, discont = true, legend = "S\_even")$



> ***#Определим нечётным образом***

> $f\_odd := x \rightarrow piecewise\left(-5 < x < -2, -\frac{1}{2} \cdot x - \frac{5}{2}, -2 \leq x \leq 0, -\frac{3}{2} \cdot (-x-1)^2, 0 \leq x \leq 2, \frac{3}{2} \cdot (x-1)^2, 2 < x < 5, -\frac{1}{2} \cdot x + \frac{5}{2}\right):$

$plot(f\_odd(x), x = -5..5, legend = f\_odd(x), discont = true);$



$$\begin{cases} \frac{1}{2}x - \frac{5}{2} & -5 < x \textbf{ and } x < -2 \\ -\frac{3}{2}(-1-x)^2 & -2 \le x \textbf{ and } x \le 0 \\ \frac{3}{2}(x-1)^2 & 0 \le x \textbf{ and } x \le 2 \\ -\frac{1}{2}x + \frac{5}{2} & 2 < x \textbf{ and } x < 5 \end{cases}$$

> $a0 := simplify\left(\frac{1}{5} \cdot int(f\_odd(x), x = -5..5)\right);$

$$a0 := 0 \qquad \textbf{(16)}$$

> $an := simplify\left(\frac{1}{5} \cdot int\left(f\_odd(x) \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{5}\right), x = -5..5\right)\right)$ assuming $n :: posint$

$$an := 0 \qquad \textbf{(17)}$$

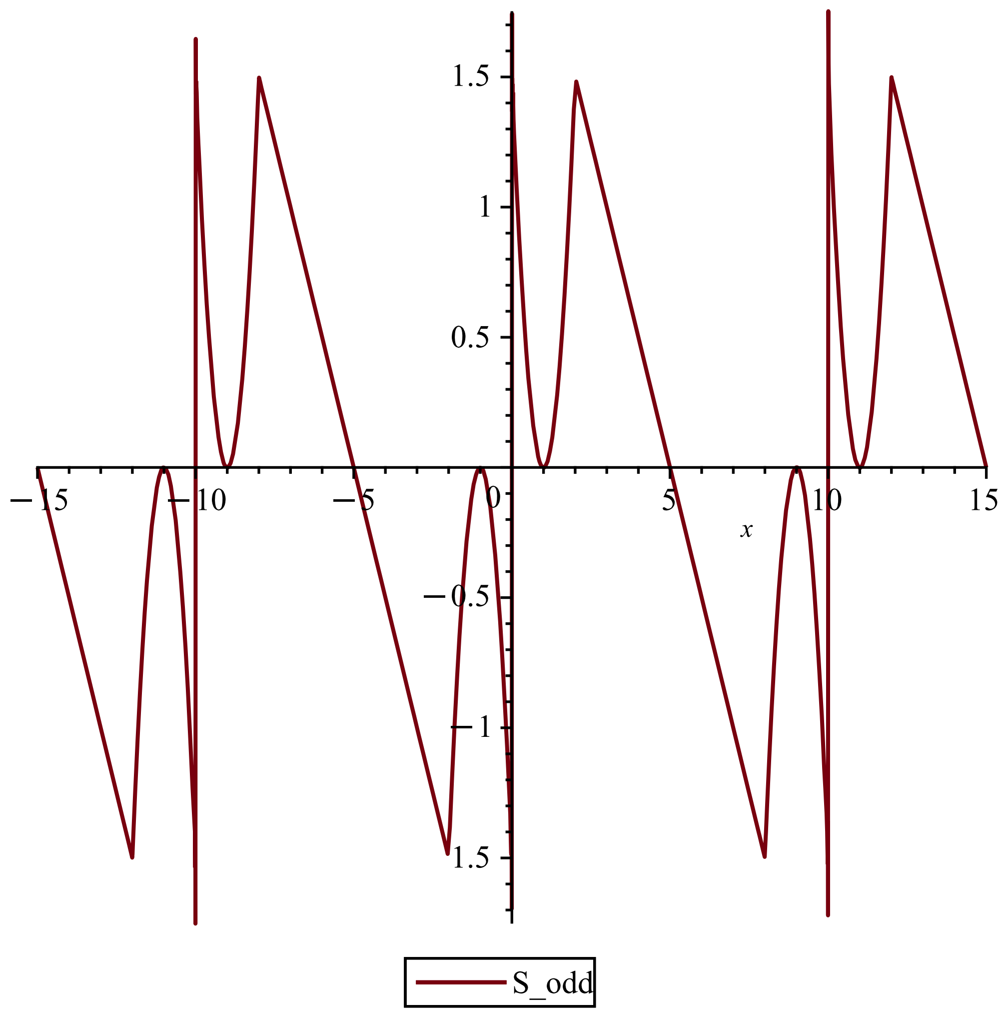
> $bn := simplify\left(\frac{2}{5} \cdot int\left(f\_odd(x) \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{5}\right), x = 0..5\right)\right)$ assuming $n :: posint$

$$bn := \frac{\left(\int_0^5 \sin\left(\frac{\pi n x}{5}\right)\left(\begin{cases} 3(x-1)^2 & x \le 2 \\ 5 - x & 2 < x \end{cases}\right) \mathrm{d}x\right)}{5} \qquad \textbf{(18)}$$

$$= \frac{3\pi^2 n^2 + 35\pi n \sin\left(\frac{2\pi n}{5}\right) + 150\cos\left(\frac{2\pi n}{5}\right) - 150}{\pi^3 n^3}$$

> $S\_odd := k \rightarrow \frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{5}\right) + bn \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{5}\right), n = 1..k\right):$

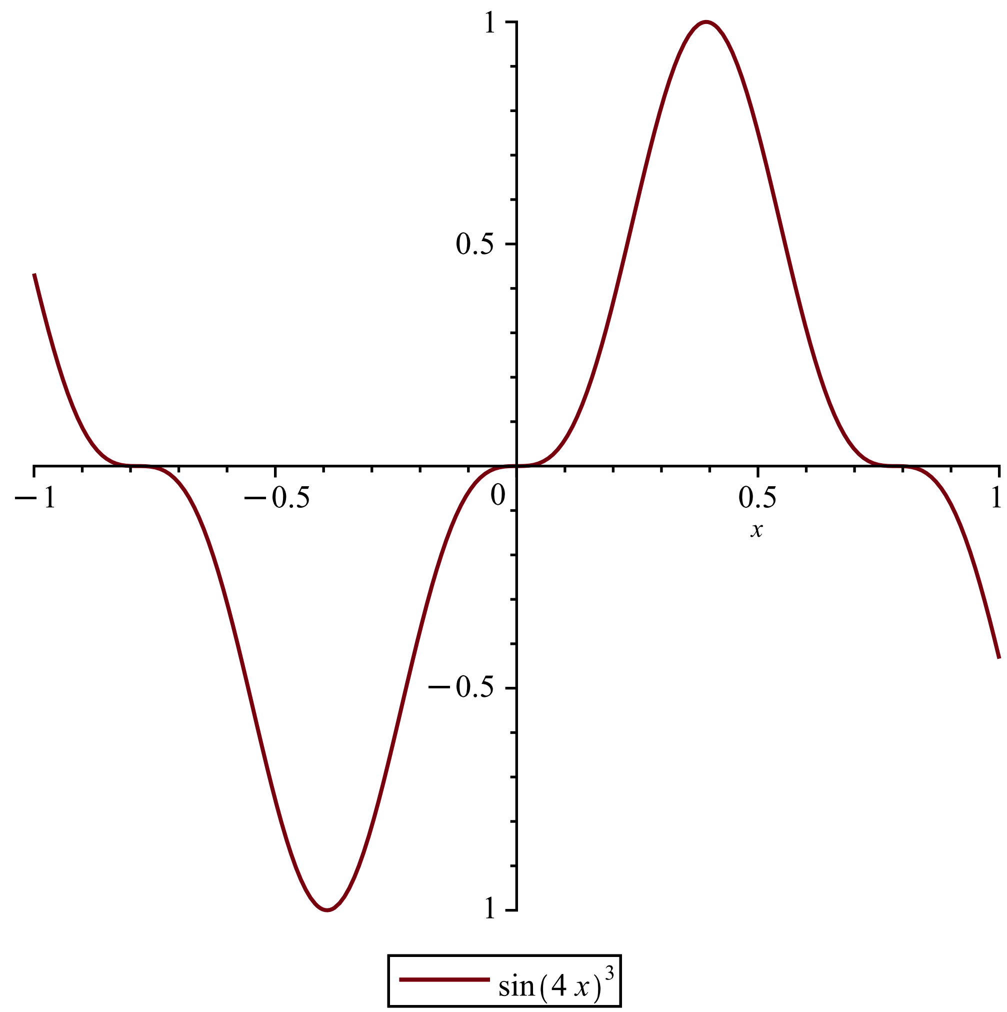
$plot(S\_odd(1000), x = -15..15, discont = true, legend = \text{"S\_odd"})$



>

> **#Задание 4. Разложите функцию в ряд Фурье по многочленам Лежандра и Чебышёва на промежутке [1, 1].**

> $f := \sin^3(4 \cdot x)$:
$our\_function := plot(f(x), x = -1\,..1, discont = true, legend = f)$



> $with(orthopoly)$

$$[G, H, L, P, T, U] \quad (19)$$

> **#По многочлену Лежандра**

> **for** n **from** 0 **to** 11 **do** $c[n] := \dfrac{\int_{-1}^{1} f \cdot P(n, x)\, \mathrm{d}x}{\int_{-1}^{1} P(n, x)^2 \mathrm{d}x}$; **end do**

$$c_0 := 0$$

$$c_1 := -\frac{\sin(4)^2 \cos(4)}{4} - \frac{\cos(4)}{2} + \frac{\sin(4)^3}{48} + \frac{\sin(4)}{8}$$

$$c_2 := 0$$

$$c_3 := -\frac{301 \sin(4)^2 \cos(4)}{576} + \frac{7 \cos(4)}{144} + \frac{833 \sin(4)}{576} + \frac{1981 \sin(4)^3}{6912}$$

$$c_4 := 0$$

$$c_5 := -\frac{9405 \sin(4)}{2048} + \frac{2013 \cos(4)}{512} - \frac{297 \sin(4)^2 \cos(4)}{1024} + \frac{3795 \sin(4)^3}{4096}$$

$$c_6 := 0$$

$$c_7 := \frac{461585 \sin(4)^2 \cos(4)}{442368} + \frac{6338395 \sin(4)}{221184} - \frac{1889815 \cos(4)}{55296} + \frac{5025055 \sin(4)^3}{5308416}$$

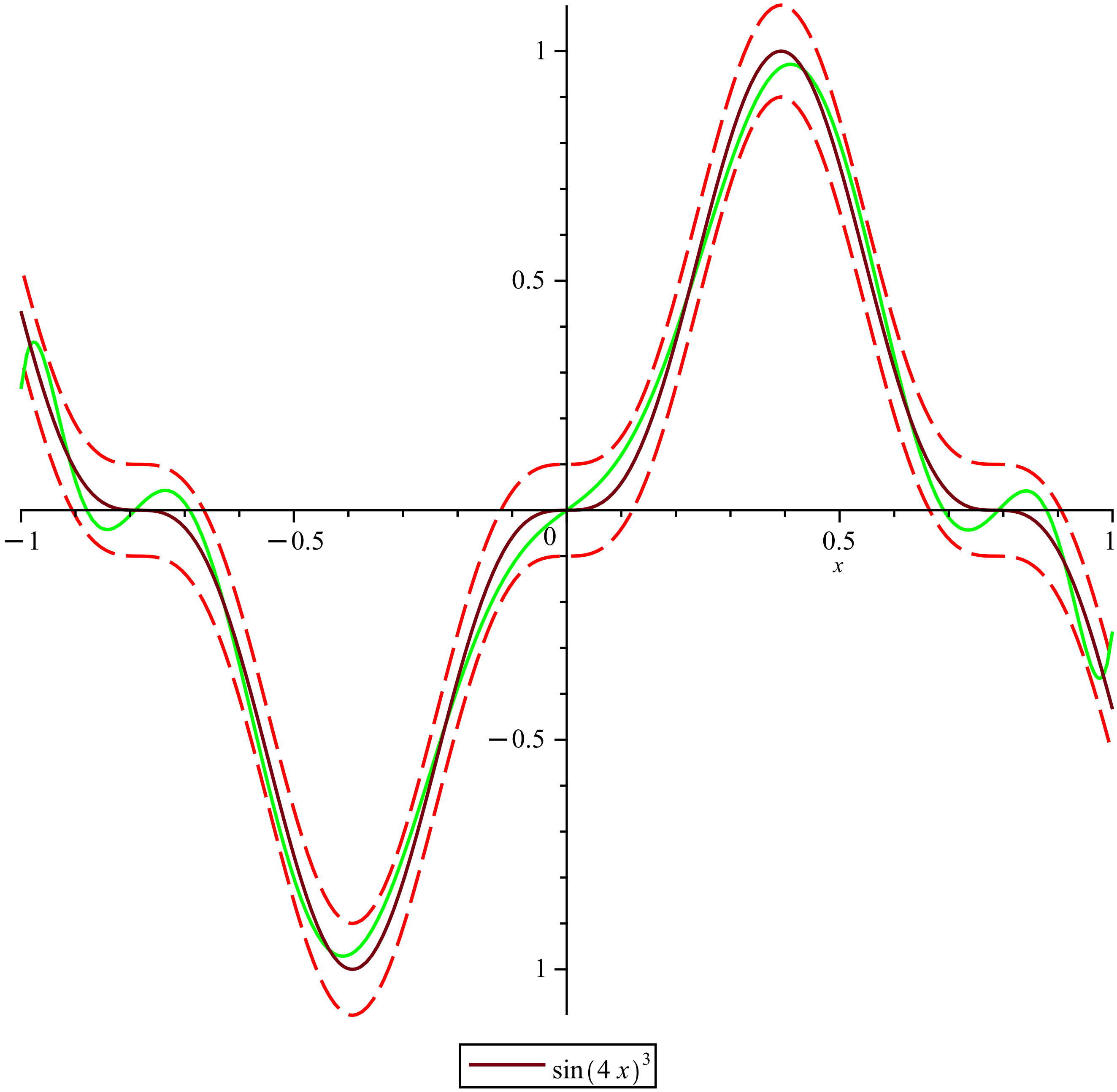
$$c_8 := 0$$

$$c_9 := \frac{86931821 \sin(4)^2 \cos(4)}{63700992} - \frac{63332529665 \sin(4)}{127401984} + \frac{18360959681 \cos(4)}{31850496} - \frac{1107657725 \sin(4)^3}{764411904}$$

$$c_{10} := 0$$

$$c_{11} := \frac{41694728879507 \sin(4)}{3057647616} - \frac{12067647528107 \cos(4)}{764411904} - \frac{31321027331 \sin(4)^3}{36691771392} - \frac{8748249709 \sin(4)^2 \cos(4)}{3057647616} \tag{20}$$

```
> lejandra_graf := plot(add(c[n]·P(n, x), n = 0 ..11), x = −1 ..1, color = green) :
>
> f1 := plot(f + 0.1, x = −1 ..1, linestyle = dash, color = red) :
> f2 := plot(f − 0.1, x = −1 ..1, linestyle = dash, color = red) :
> plots[display]([f1, f2, lejandra_graf,  our_function])
```

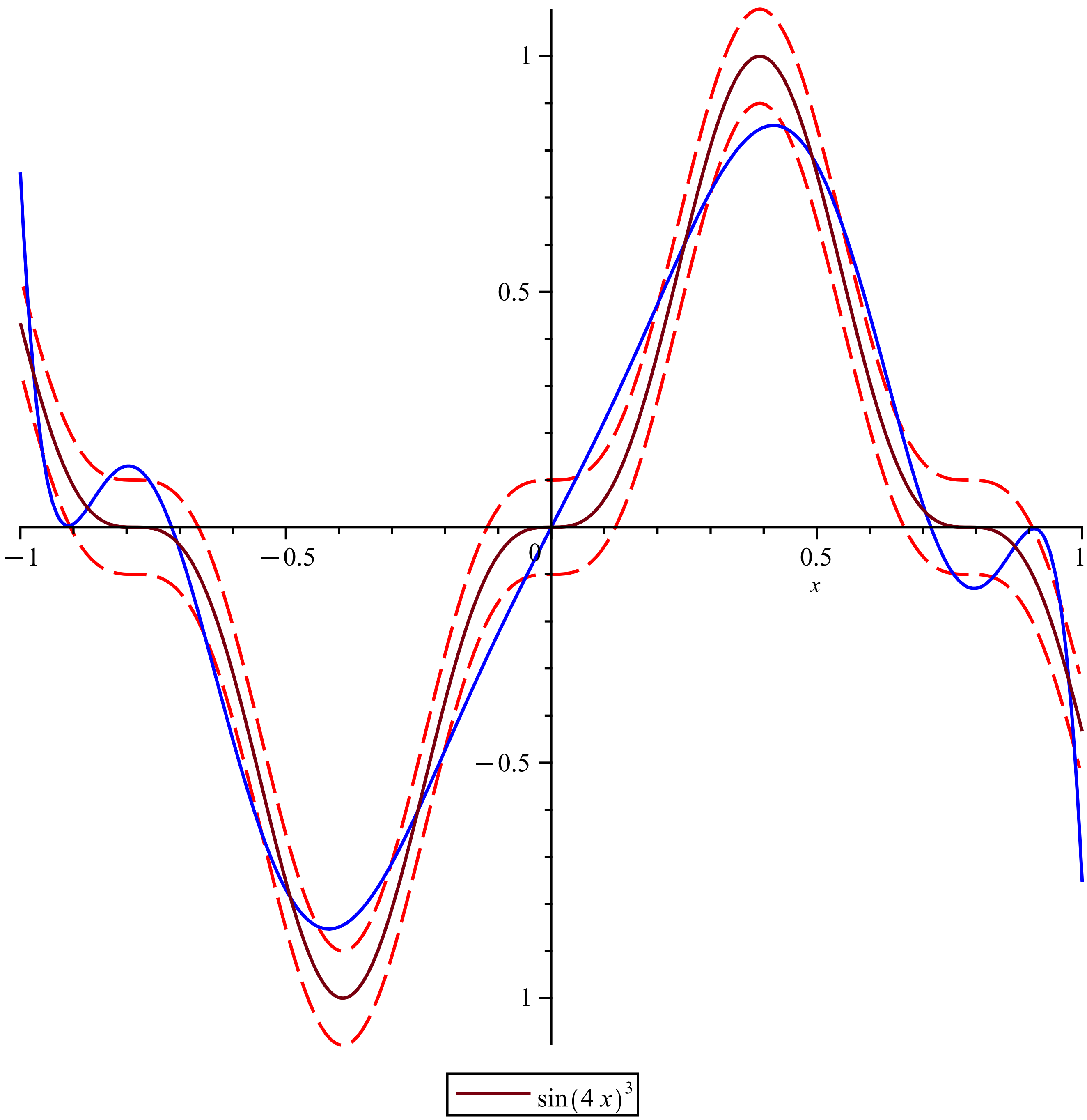


>

> $nmin := plot(add(c[n]\cdot P(n,x),\ n=0..10),\ x=-1..1,\ color=blue)$:

> $plots[display](f1, f2, nmin, our\_function)$

#можем заметить, что когда n = 10 функция отклоняется больше чем на 0,
1 (***Лежандр***)



> ***#По многочлену Чебышёва***

> $$\textbf{for } n \textbf{ from } 0 \textbf{ to } 11 \textbf{ do } c[n] := \frac{\displaystyle\int_{-1}^{1} \frac{f \cdot T(n, x)}{\mathrm{sqrt}(1 - x^2)}\, \mathrm{d}x}{\displaystyle\int_{-1}^{1} \frac{T(n, x)^2}{\mathrm{sqrt}(1 - x^2)}\, \mathrm{d}x}; \textbf{ end do}$$

$$c_0 := \frac{\int_{-1}^{1} \frac{\sin(4x)^3}{\sqrt{-x^2+1}}\, dx}{\pi}$$

$$c_1 := \frac{2\left(\int_{-1}^{1} \frac{\sin(4x)^3 x}{\sqrt{-x^2+1}}\, dx\right)}{\pi}$$

$$c_2 := \frac{2\left(\int_{-1}^{1} \frac{\sin(4x)^3 (2x^2-1)}{\sqrt{-x^2+1}}\, dx\right)}{\pi}$$

$$c_3 := \frac{2\left(\int_{-1}^{1} \frac{\sin(4x)^3 (4x^3-3x)}{\sqrt{-x^2+1}}\, dx\right)}{\pi}$$

$$c_4 := \frac{2\left(\int_{-1}^{1} \frac{\sin(4x)^3 (8x^4-8x^2+1)}{\sqrt{-x^2+1}}\, dx\right)}{\pi}$$

$$c_5 := \frac{2\left(\int_{-1}^{1} \frac{\sin(4x)^3 (16x^5-20x^3+5x)}{\sqrt{-x^2+1}}\, dx\right)}{\pi}$$

$$c_6 := \frac{2\left(\int_{-1}^{1} \frac{\sin(4x)^3 (32x^6-48x^4+18x^2-1)}{\sqrt{-x^2+1}}\, dx\right)}{\pi}$$

$$c_7 := \frac{2\left(\int_{-1}^{1} \frac{\sin(4x)^3 (64x^7-112x^5+56x^3-7x)}{\sqrt{-x^2+1}}\, dx\right)}{\pi}$$

$$c_8 := \frac{2\left(\int_{-1}^{1} \frac{\sin(4x)^3 (128x^8-256x^6+160x^4-32x^2+1)}{\sqrt{-x^2+1}}\, dx\right)}{\pi}$$

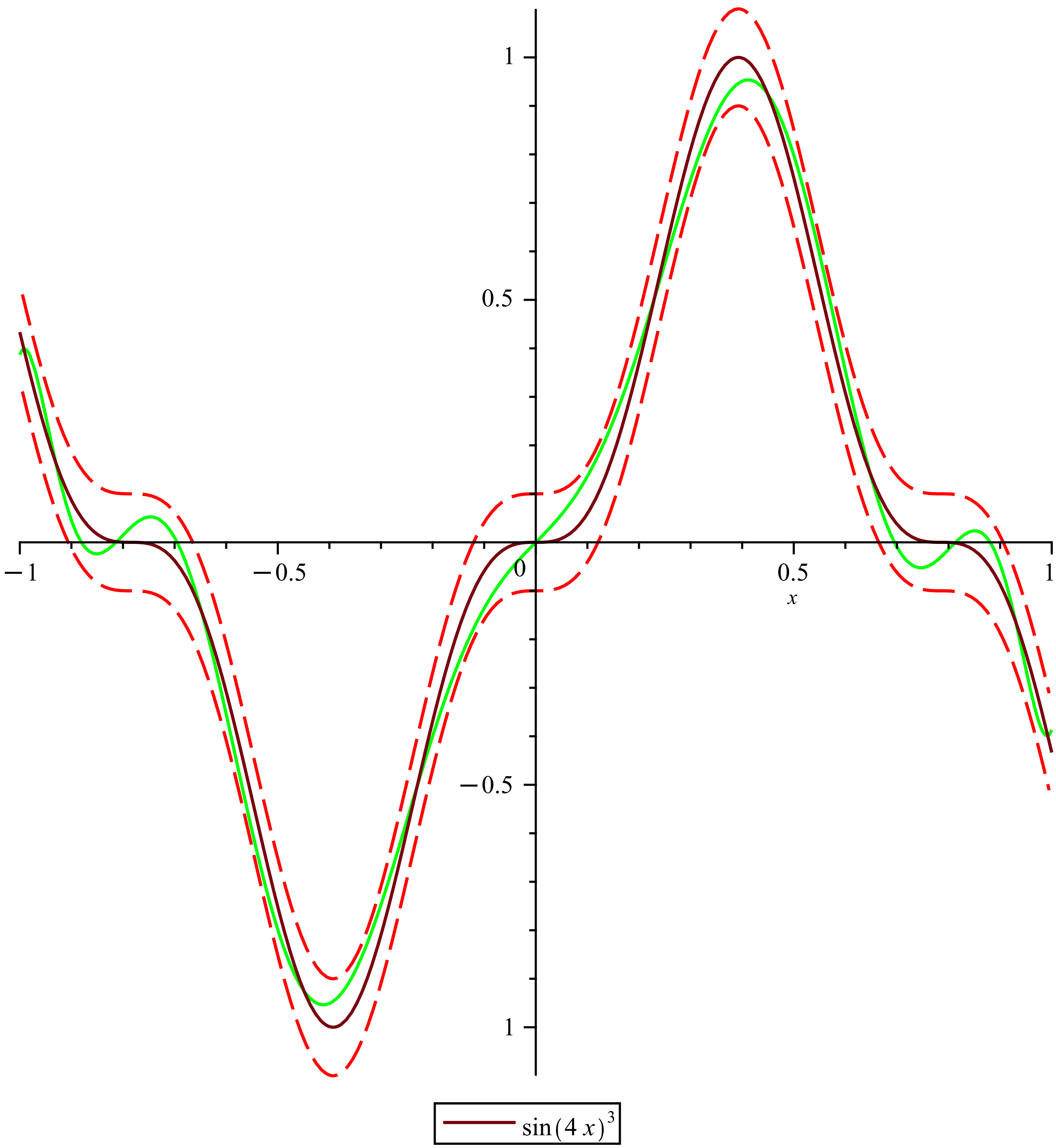
$$c_9 := \frac{2\left(\int_{-1}^{1} \frac{\sin(4\,x)^3\,(256\,x^9 - 576\,x^7 + 432\,x^5 - 120\,x^3 + 9\,x)}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

$$c_{10} := \frac{2\left(\int_{-1}^{1} \frac{\sin(4\,x)^3\,(512\,x^{10} - 1280\,x^8 + 1120\,x^6 - 400\,x^4 + 50\,x^2 - 1)}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

$$c_{11} := \frac{2\left(\int_{-1}^{1} \frac{\sin(4\,x)^3\,(1024\,x^{11} - 2816\,x^9 + 2816\,x^7 - 1232\,x^5 + 220\,x^3 - 11\,x)}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi} \qquad \textbf{(21)}$$

> *cheb_graf* := *plot*(*add*(*c*[*n*]·*T*(*n*, *x*), *n* = 1 ..11), *x* = −1 ..1, *color* = *green*) :

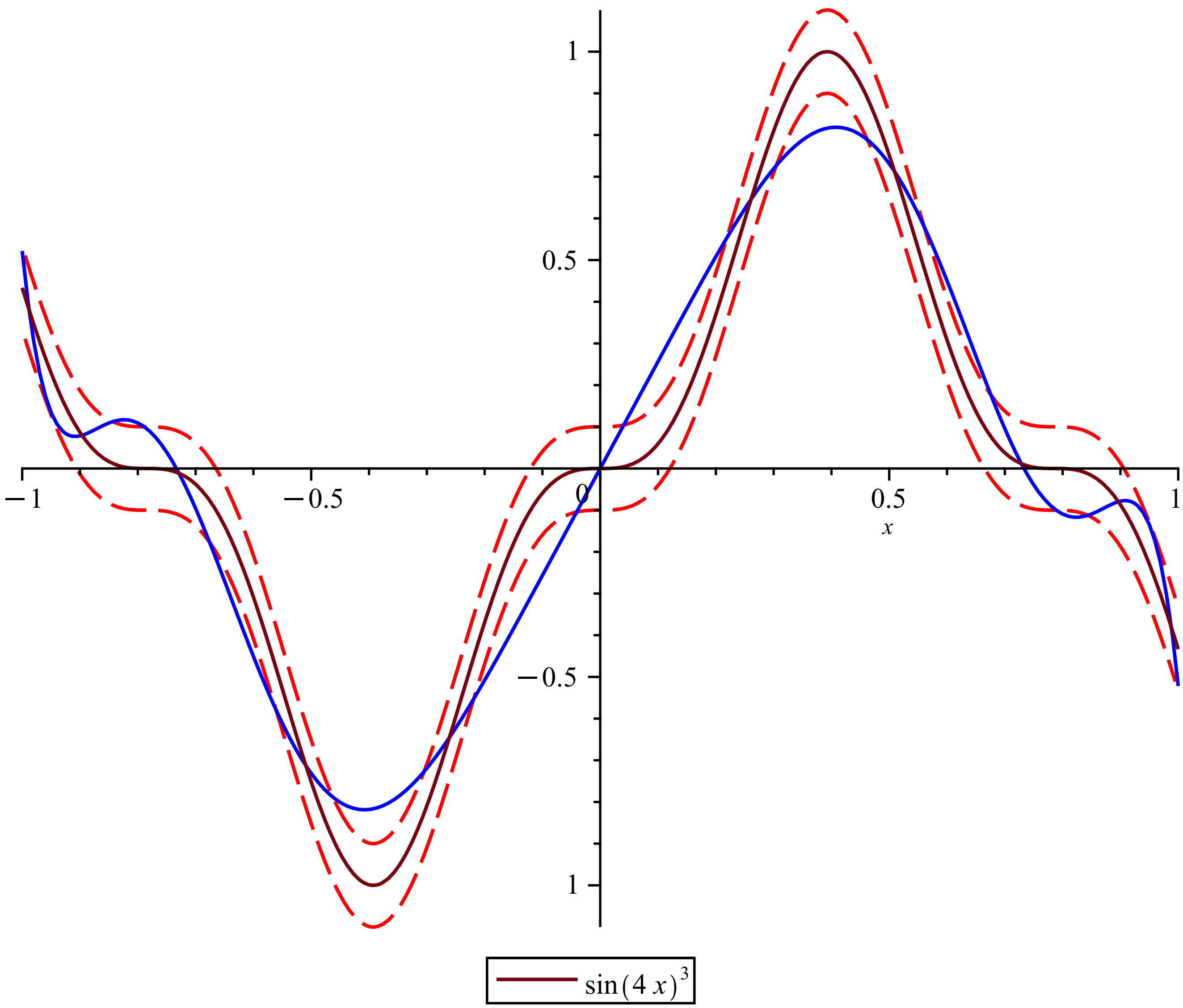
> *plots*[*display*](*f1*, *f2*, *cheb_graf*, *our_function*)



> $nmin := plot(add(c[n]\cdot T(n, x), n = 1..10), x = -1..1, color = blue)$:

> $plots[display](f1, f2, nmin, our\_function)$

#можем заметить, что когда n = 10 функция отклоняется больше чем на 0,
1 (***Чебышев***)



> $bn := simplify(\ int(\mathrm{f}\cdot\sin(\mathrm{Pi}\cdot m\cdot x), x=-1\,..1))$ assuming $m :: posint$

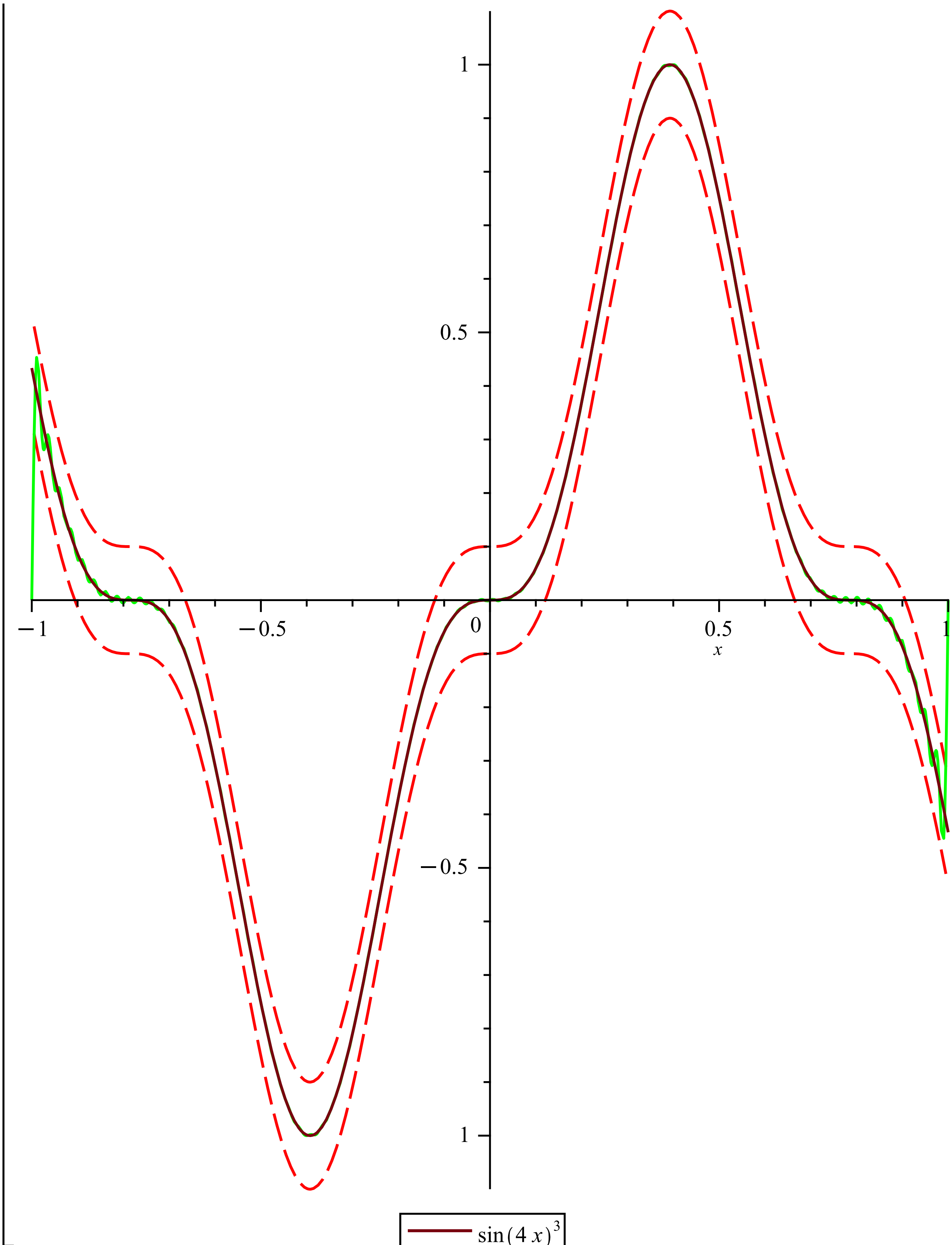
$$bn := -\frac{2\,\pi\,m\,(-1)^m \sin(4)\,\left(\pi^2 m^2 \sin(4)^2 + 8\cos(8) - 104\right)}{\pi^4 m^4 - 160\,\pi^2 m^2 + 2304} \tag{22}$$

> $Sm := \mathrm{k}\rightarrow sum(bn\cdot\sin(\pi\cdot\mathrm{m}\cdot\mathrm{x}), m=1\,..k)$

$$Sm := k \mapsto \sum_{m=1}^{k} bn\cdot\sin(\pi\cdot m\cdot x) \tag{23}$$

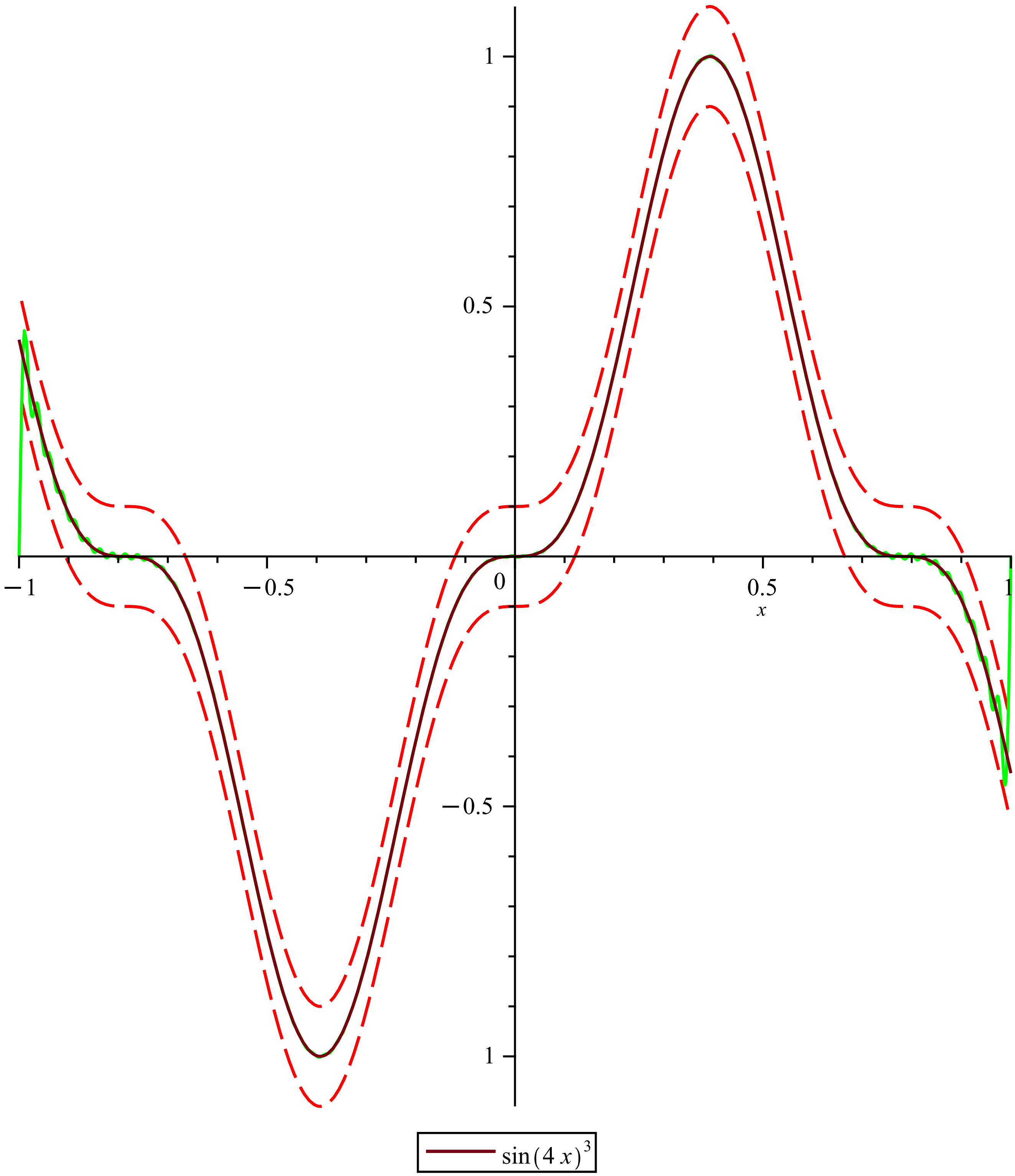
> $fur := plot(Sm(80), x=-1\,..1, discont=true, color=green)$:

> $plots[display](f1, f2, fur, our\_function)$



> minFur := plot(Sm(79), x = −1 ..1, discont = true, color = green) :

> *plots*[*display*](*f1*, *f2*, *minFur*, *our_function*)#можем заметить **с трудом**(„^ ᵕ ^„), что когда n = 79 функция отклоняется больше чем на 0, 1 (***Тригонометрический ряд Фурье***)



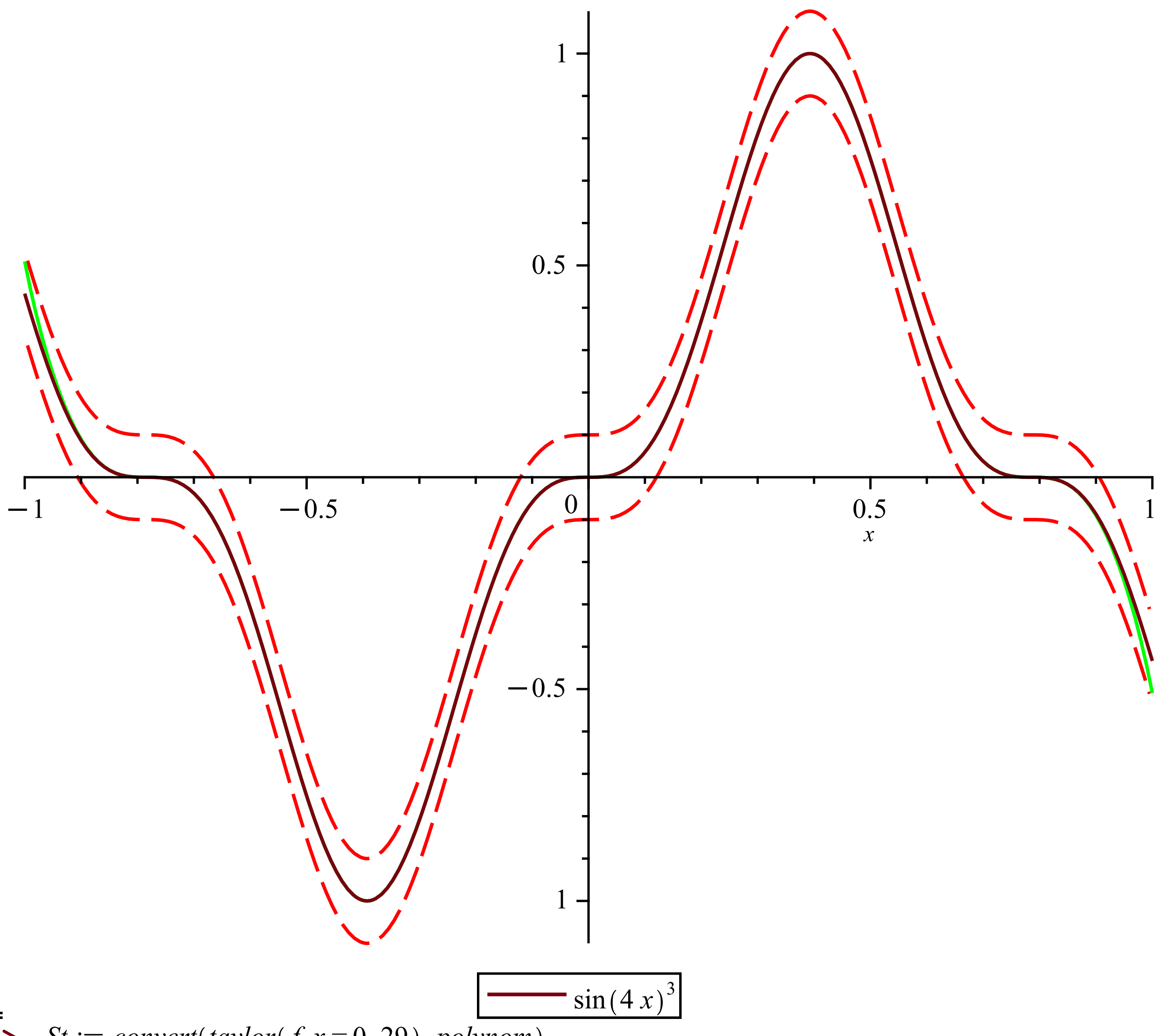
> $St := convert(taylor(f, x = 0, 30), polynom)$

$$St := 64\,x^3 - 512\,x^5 + \frac{26624}{15}\,x^7 - \frac{671744}{189}\,x^9 + \frac{21987328}{4725}\,x^{11} - \frac{19136512}{4455}\,x^{13} \qquad \textbf{(24)}$$

$$+\frac{626913181696}{212837625}x^{15}-\frac{66379055104}{42567525}x^{17}+\frac{234956521472}{357847875}x^{19}-\frac{53180486385664}{236238154425}x^{21}$$
$$+\frac{45781462204547072}{714620417135625}x^{23}-\frac{41656099677405184}{2709275207821875}x^{25}+\frac{1364653825436088795136}{432684797065192546875}x^{27}$$
$$-\frac{338810604935975010304}{605758715891269565625}x^{29}$$

> $StF := plot(St, x=-1..1, color=green)$:

> $plots[display](f1, f2, StF, our\_function)$



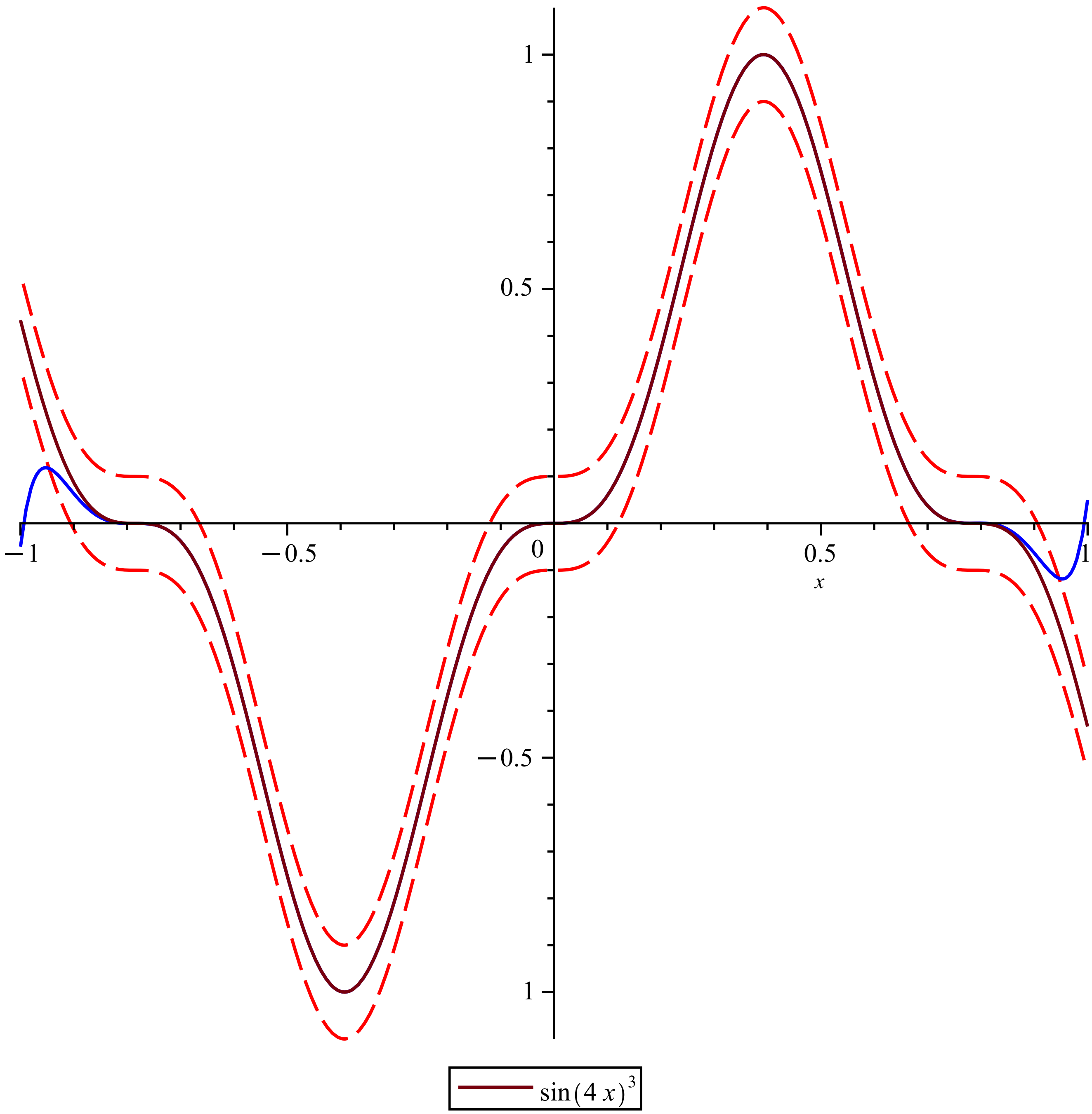
> $St := convert(taylor(f, x=0, 29), polynom)$

$$St := 64x^3-512x^5+\frac{26624}{15}x^7-\frac{671744}{189}x^9+\frac{21987328}{4725}x^{11}-\frac{19136512}{4455}x^{13} \tag{25}$$
$$+\frac{626913181696}{212837625}x^{15}-\frac{66379055104}{42567525}x^{17}+\frac{234956521472}{357847875}x^{19}-\frac{53180486385664}{236238154425}x^{21}$$

$$+\frac{45781462204547072}{714620417135625}x^{23}-\frac{41656099677405184}{2709275207821875}x^{25}+\frac{1364653825436088795136}{43268479706519254687\,5}x^{27}$$

> $StF := plot(St, x = -1\,..1, color = blue)$:

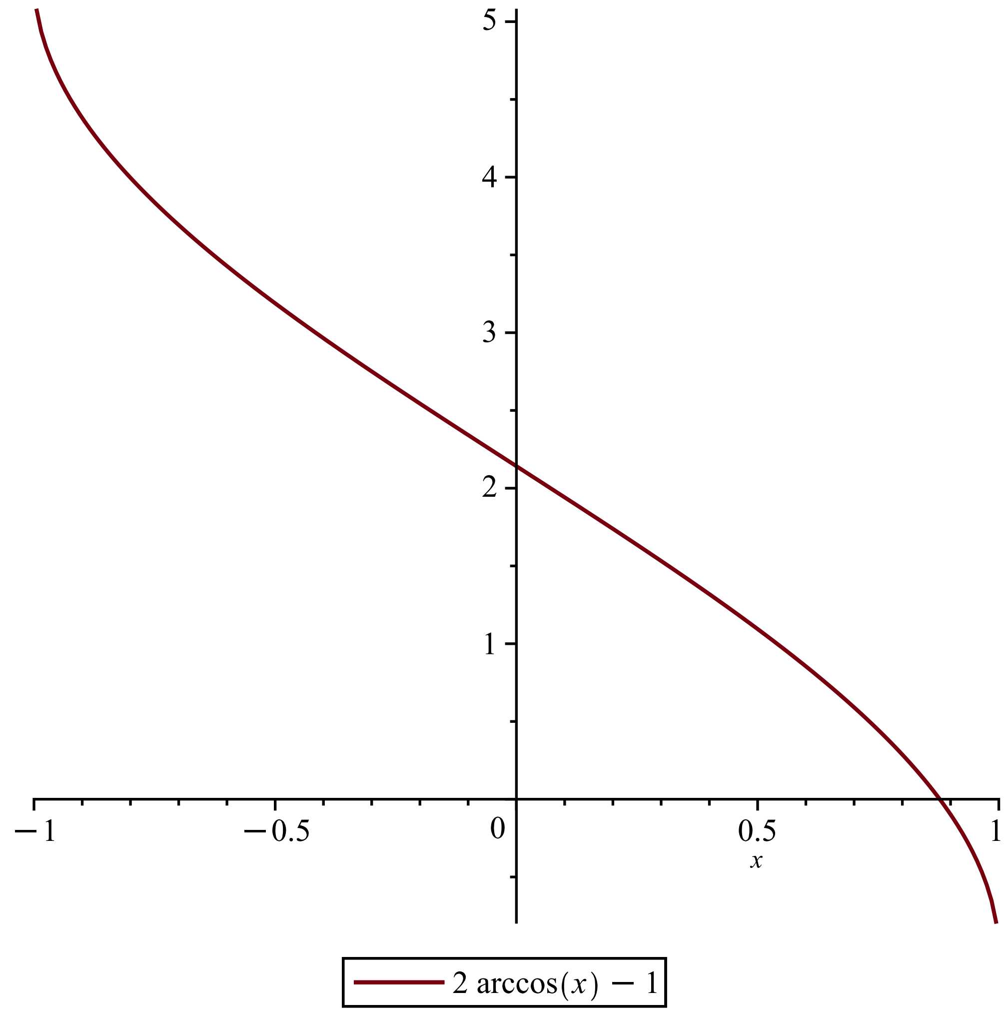
> $plots[display](f1, f2, StF, our\_function)$# при x =0..29 (**Тейлор**`)`



> $restart$

> $f := 2\cdot\arccos(x) - 1$:

> $our\_function1 := plot(f, x = -1\,..1, legend = f)$



> $with(orthopoly)$

$$[G, H, L, P, T, U] \tag{26}$$

> **for** n **from** 0 **to** 7 **do** $c[n] := \dfrac{\int_{-1}^{1} f \cdot P(n, x)\, \mathrm{d}x}{\int_{-1}^{1} P(n, x)^2 \mathrm{d}x}$; **end do**

$$c_0 := -1 + \pi$$

$$c_1 := -\frac{3\,\pi}{4}$$

$$c_2 := 0$$

$$c_3 := -\frac{7\,\pi}{64}$$

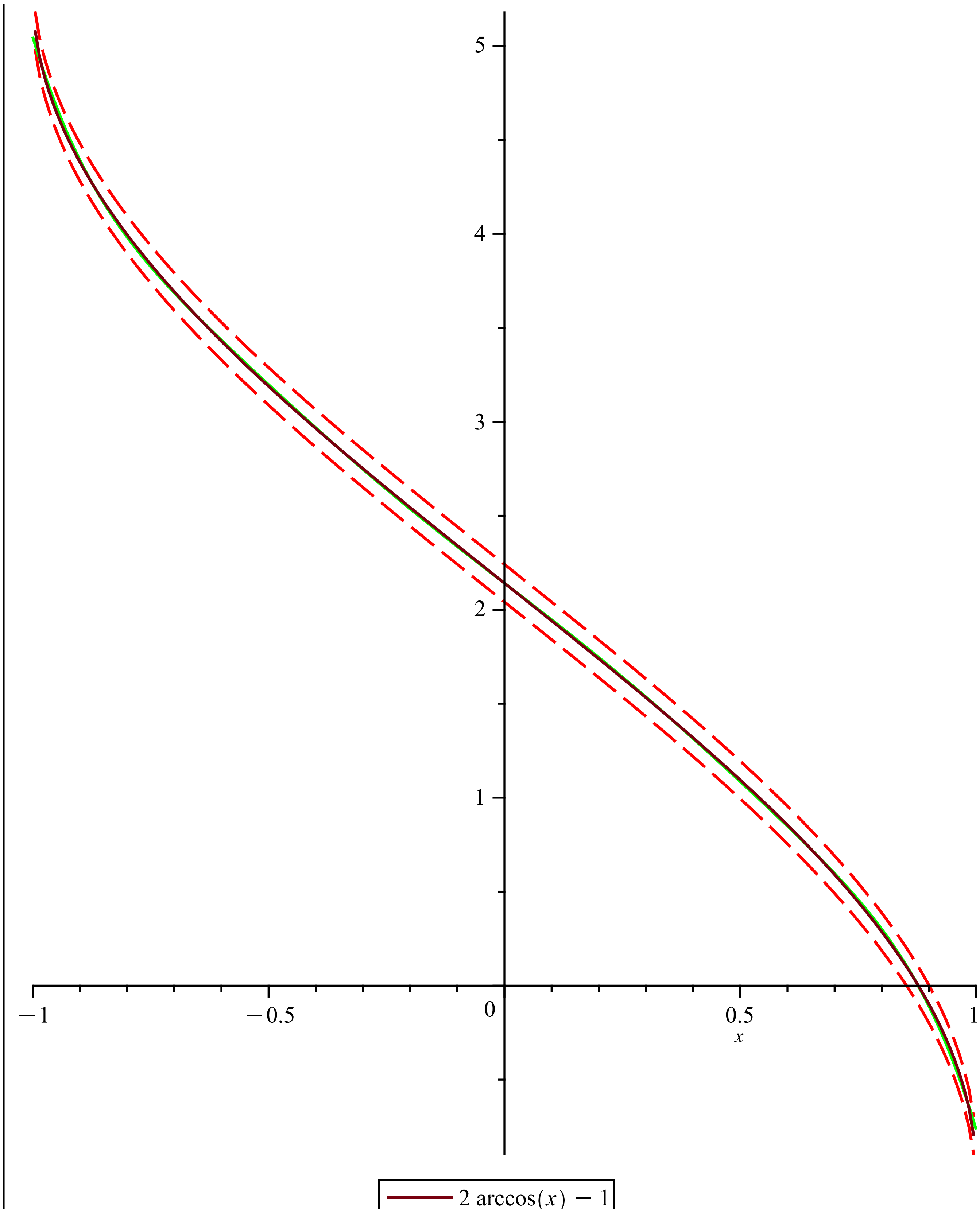
$$c_4 := 0$$

$$c_5 := -\frac{11\,\pi}{256}$$

$$c_6 := 0$$

$$c_7 := -\frac{375\,\pi}{16384} \tag{27}$$

```
> lejandra_graf1 := plot(add(c[n]·P(n, x), n = 0 ..7), x = −1 ..1, color = green) :
>
> f1 := plot(f + 0.1, x = −1 ..1, linestyle = dash, color = red) :
> f2 := plot(f − 0.1, x = −1 ..1, linestyle = dash, color = red) :
> plots[display]([f1, f2, lejandra_graf1, our_function1])
```

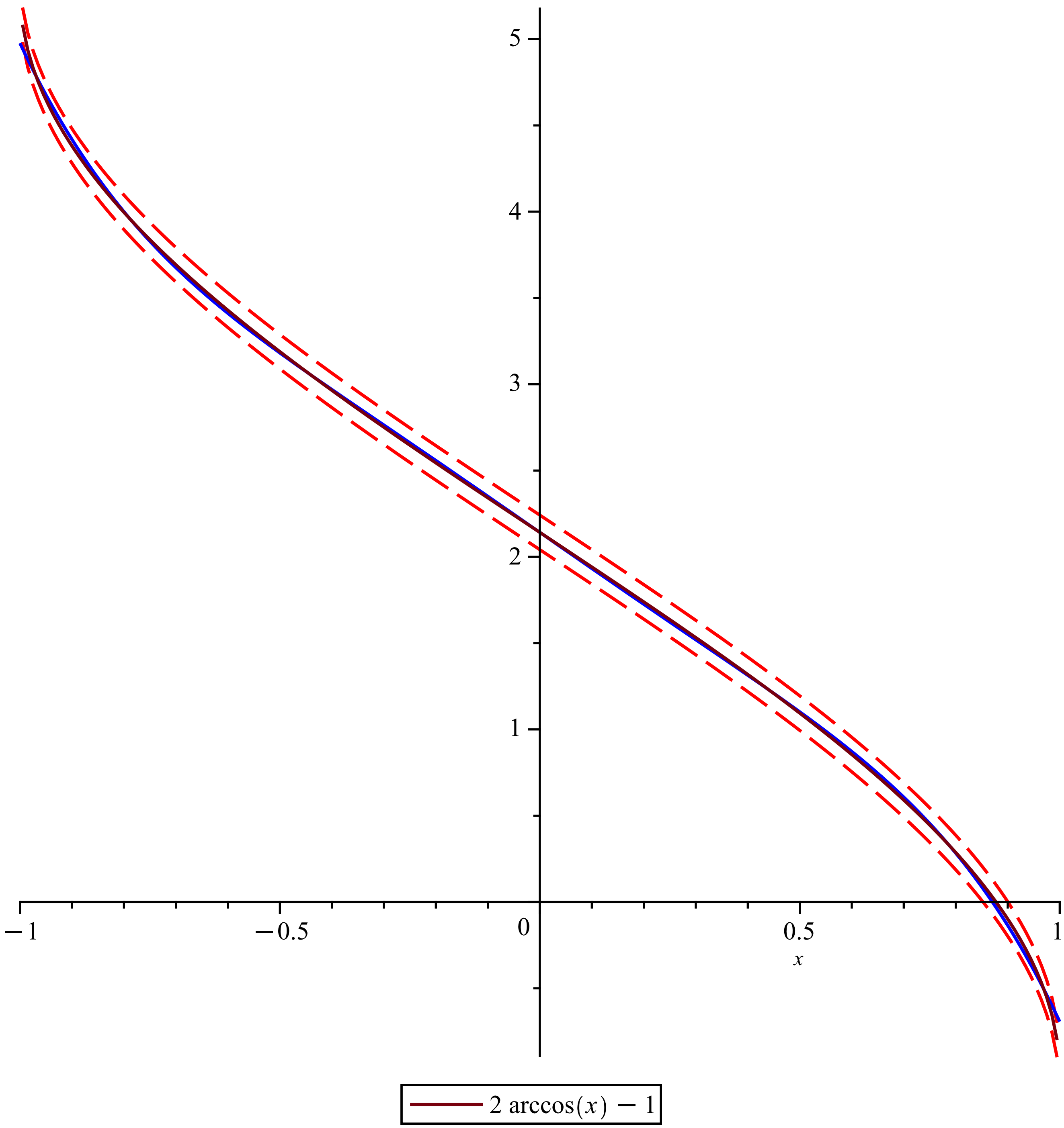


>

> $nmin1 := plot(add(c[n]\cdot P(n, x), n = 0\,..6), x = -1\,..1, color = blue)$:

> $plots[display](f1, f2, nmin1, our\_function1)$

#можем заметить, что когда n = 6 функция отклоняется больше чем на 0, 1 (***Лежандр***)



> **for** n **from** 0 **to** 5 **do** $c[n] := \dfrac{\displaystyle\int_{-1}^{1} \frac{f \cdot T(n, x)}{\mathrm{sqrt}(1 - x^2)}\, \mathrm{d}x}{\displaystyle\int_{-1}^{1} \frac{T(n, x)^2}{\mathrm{sqrt}(1 - x^2)}\, \mathrm{d}x}$; **end do**

$$c_0 := \frac{\pi^2 - \pi}{\pi}$$

$$c_1 := -\frac{8}{\pi}$$

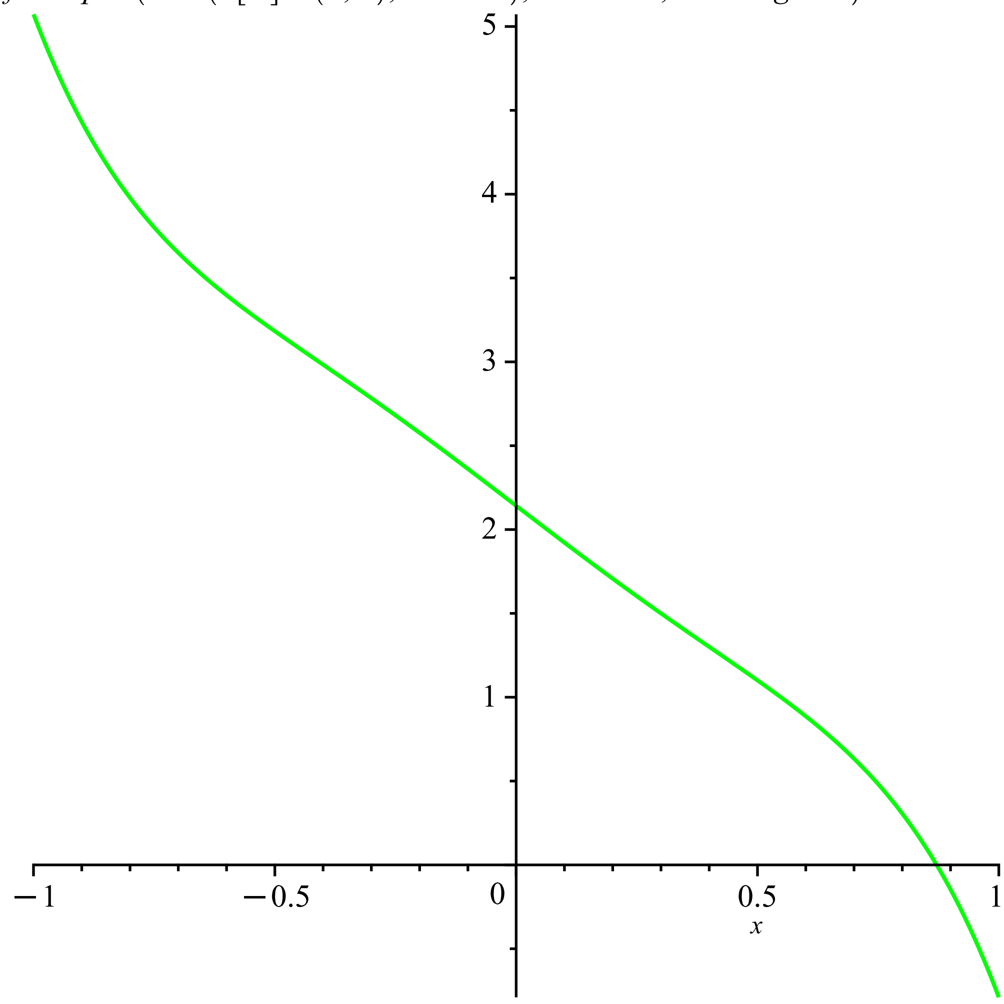
$$c_2 := 0$$

$$c_3 := -\frac{8}{9\,\pi}$$

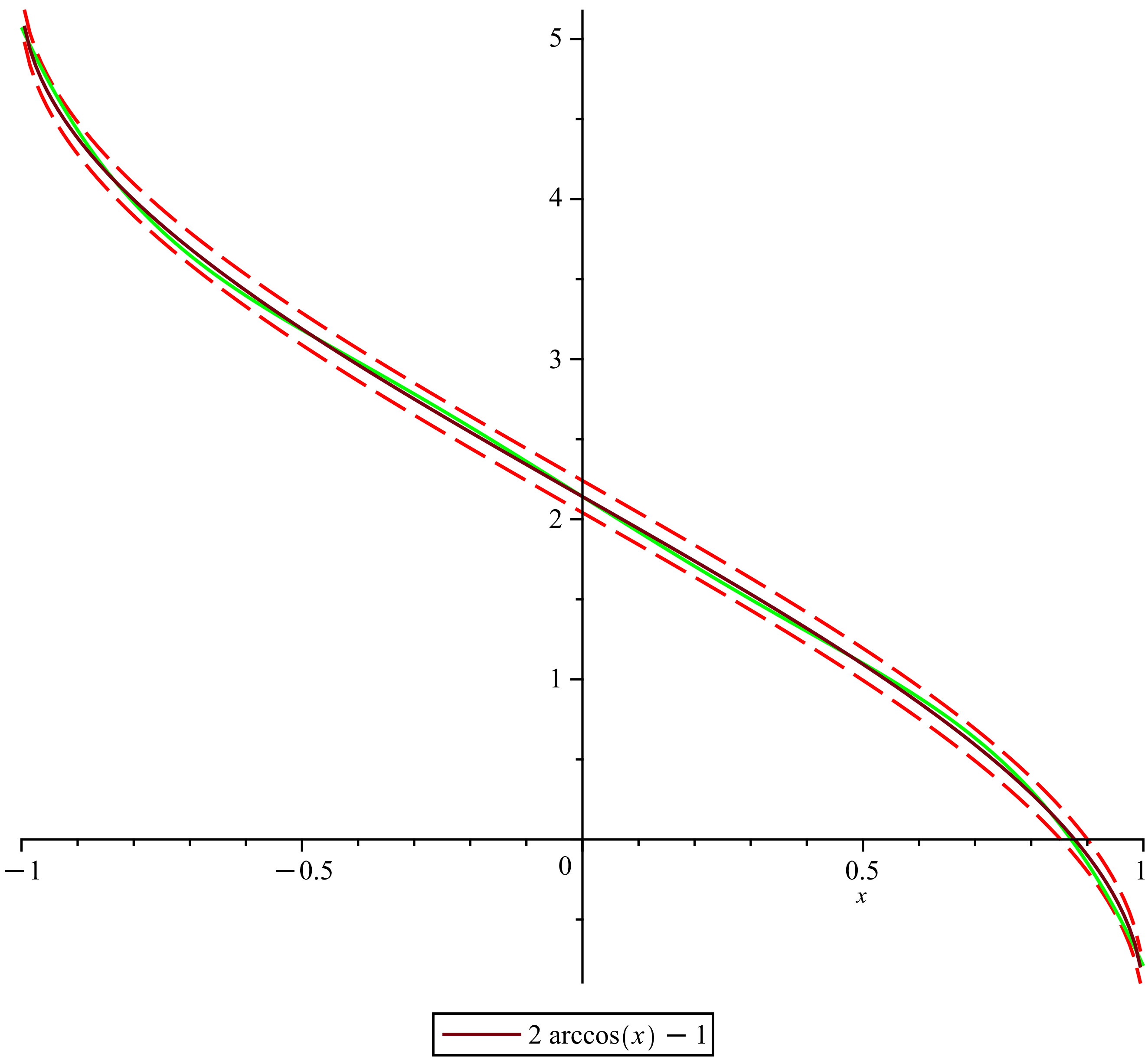
$$c_4 := 0$$

$$c_5 := -\frac{8}{25\,\pi} \quad \textbf{(28)}$$

> $cheb\_graf1 := plot(\; add(c[n] \cdot T(n, x), n = 0..5), x = -1..1, color = green)$



> $plots[display](f1, f2, cheb\_graf1, our\_function1)$

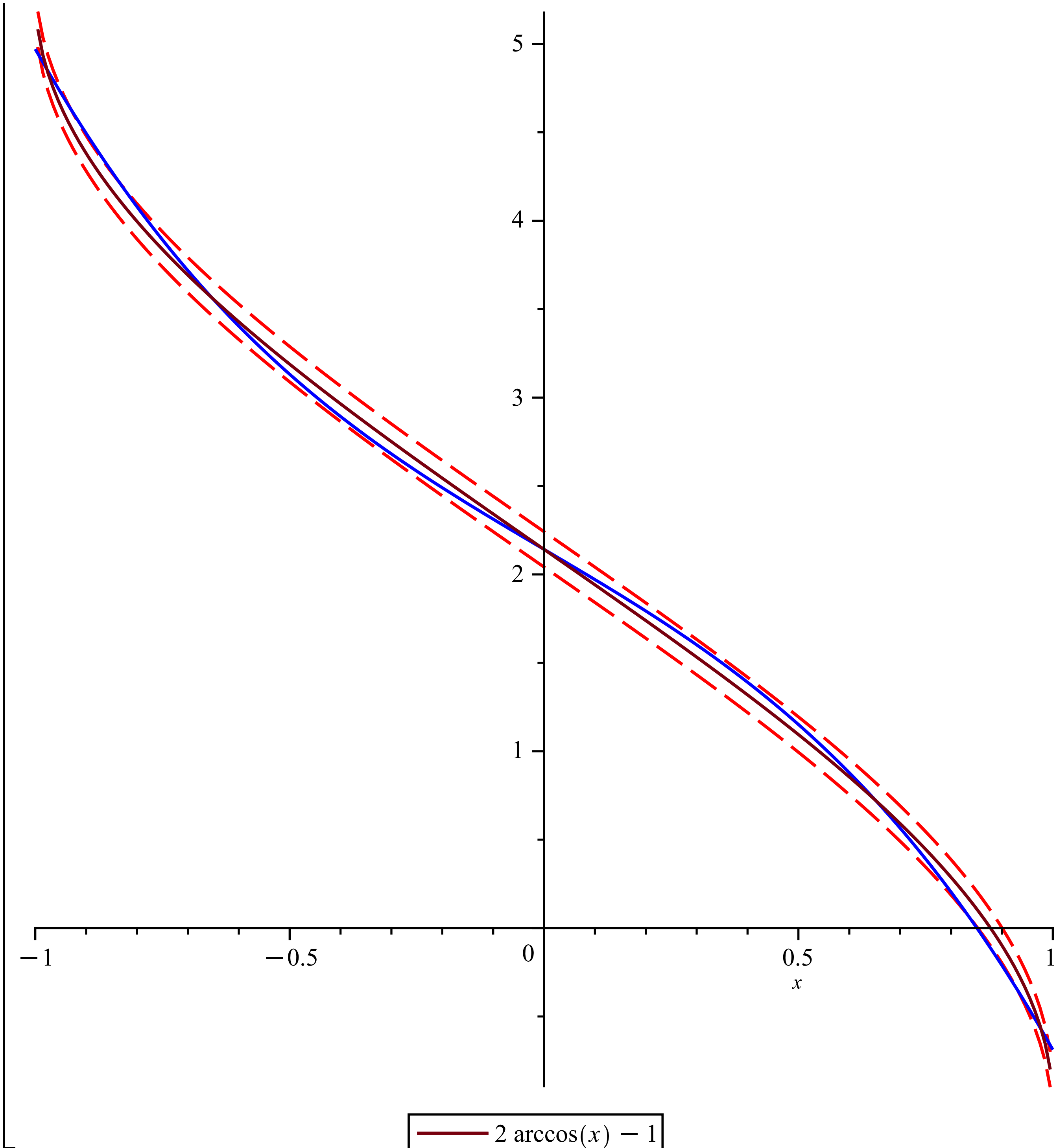


>

> $nmin1 := plot(add(c[n] \cdot T(n, x), n = 0..4), x = -1..1, color = blue)$:

> $plots[display](f1, f2, nmin1, our\_function1)$

#можем заметить, что когда n = 4 функция отклоняется больше чем на 0,
1 (***Чебышев***)



> $a0 := simplify(int(f, x=-1..1))$

$$a0 := -2 + 2\pi \tag{29}$$

> $an := simplify(int(\mathrm{f}\cdot\cos(\mathrm{Pi}\cdot nn\cdot x), x=-1..1))$ assuming $nn :: posint$

$$an := 0 \tag{30}$$

> $bn := simplify(int(\mathrm{f}\cdot\sin(\mathrm{Pi}\cdot nn\cdot x), x=-1..1))$ assuming $nn :: posint$

$$bn := \int_{-1}^{1} (2\arccos(x) - 1)\sin(\pi\, nn\, x)\, \mathrm{d}x \tag{31}$$

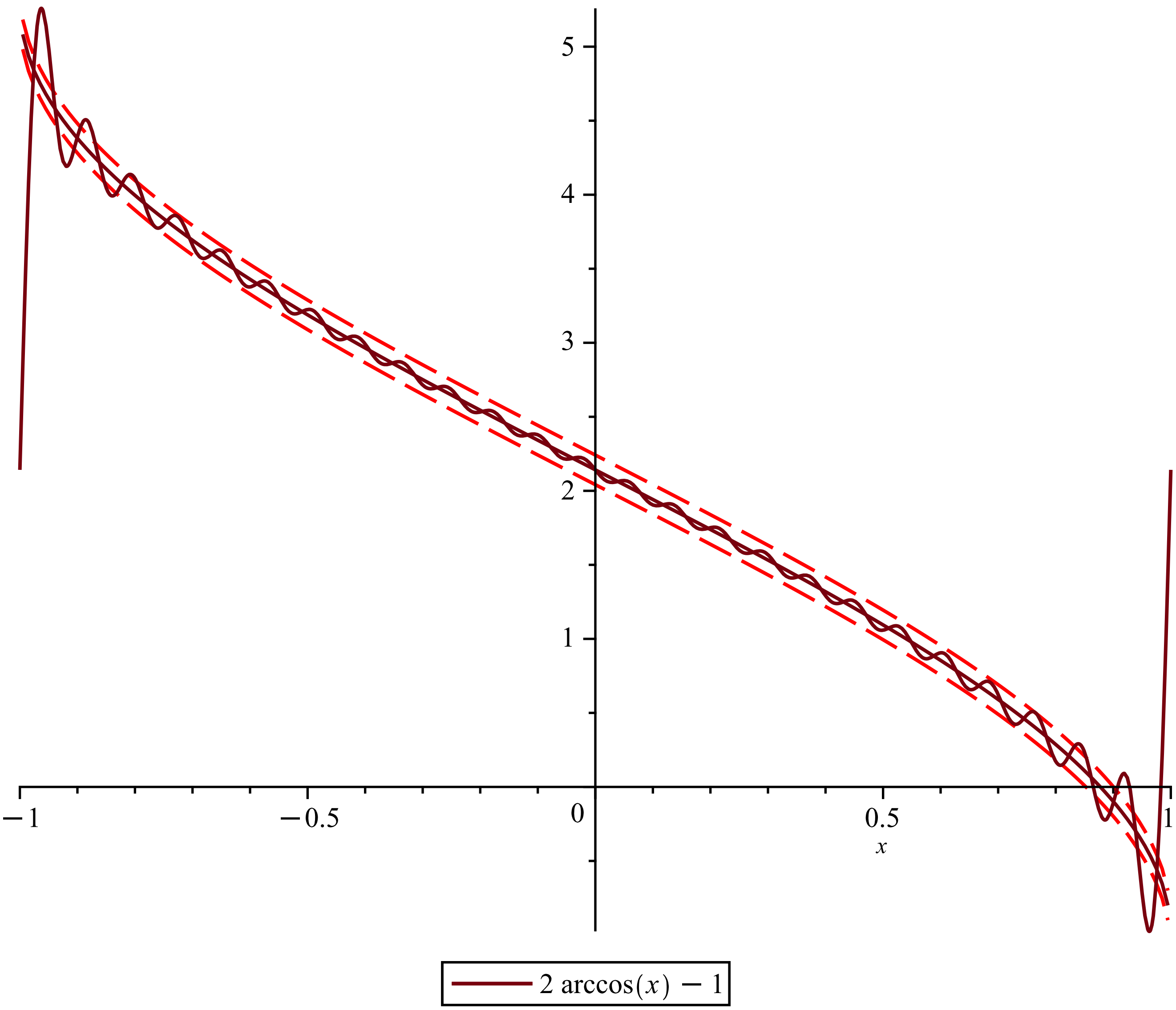
> $Sm := k \rightarrow \frac{a0}{2} + sum(bn \cdot \sin(\pi \cdot nn \cdot x), nn = 1..k)$

$$Sm := k \mapsto \frac{a0}{2} + \left(\sum_{nn=1}^{k} bn \cdot \sin(\pi \cdot nn \cdot x)\right) \qquad \textbf{(32)}$$

> $fur := plot(Sm(25), x = -1..1, discont = true)$:

> $plots[display](f1, f2, fur, our\_function1)$

 # тут можно взять промежуток поменьше, например от -0.75 до 0.75 и показать что он будет внутри f +0.1 и f - 0.1()

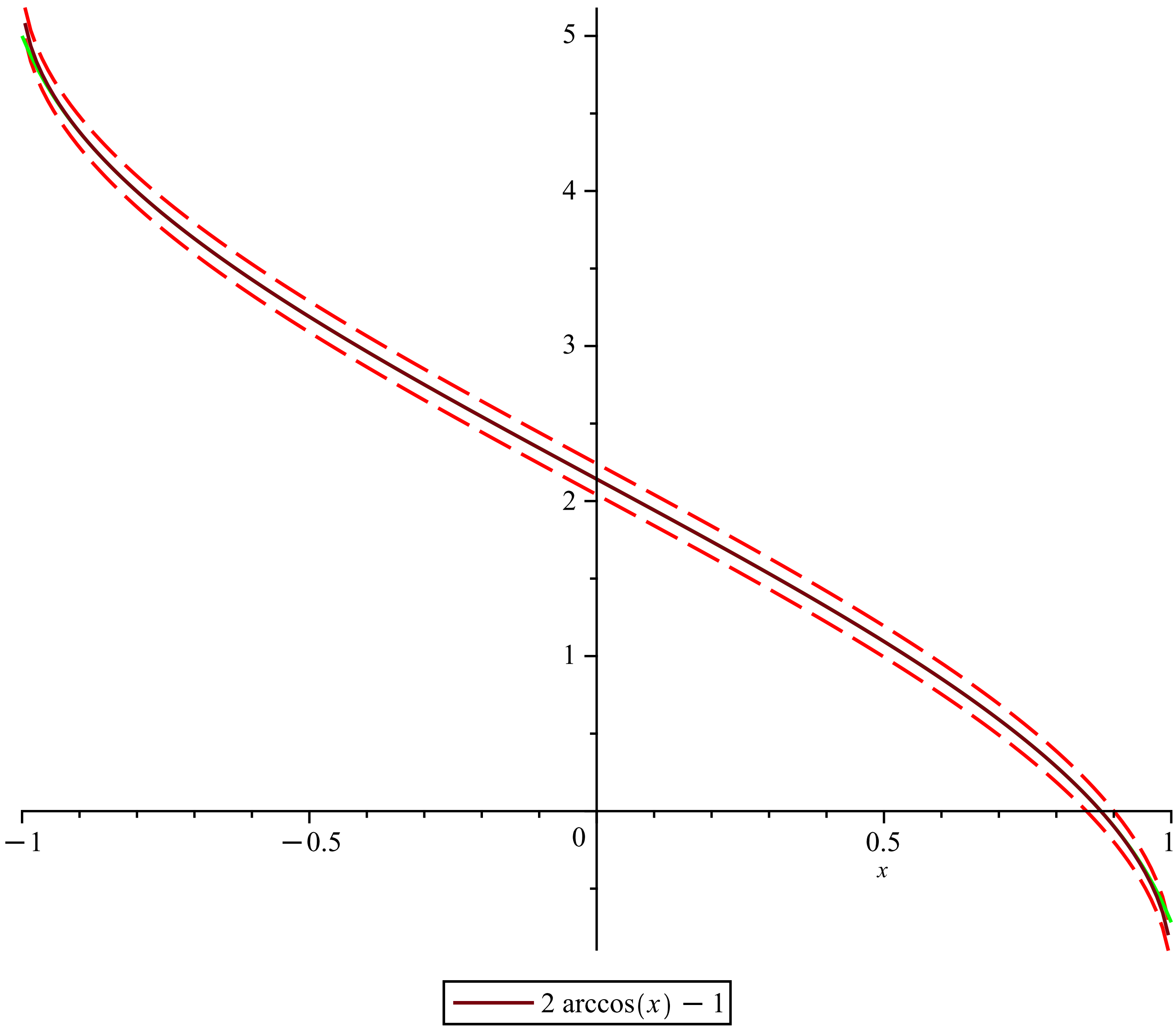


> $St := convert(taylor(f, x = 0, 33), polynom)$

$$St := -1 + \pi - 2x - \frac{1}{3}x^3 - \frac{3}{20}x^5 - \frac{5}{56}x^7 - \frac{35}{576}x^9 - \frac{63}{1408}x^{11} - \frac{231}{6656}x^{13} - \frac{143}{5120}x^{15} - \frac{6435}{278528}x^{17} - \frac{12155}{622592}x^{19} - \frac{46189}{2752512}x^{21} - \frac{88179}{6029312}x^{23} - \frac{676039}{52428800}x^{25} - \frac{1300075}{113246208}x^{27} - \frac{5014575}{486539264}x^{29} - \frac{9694845}{1040187392}x^{31} \qquad \textbf{(33)}$$

> $StF := plot(St, x=-1..1, color=green):$

> $plots[display](f1, f2, StF, our\_function1)$

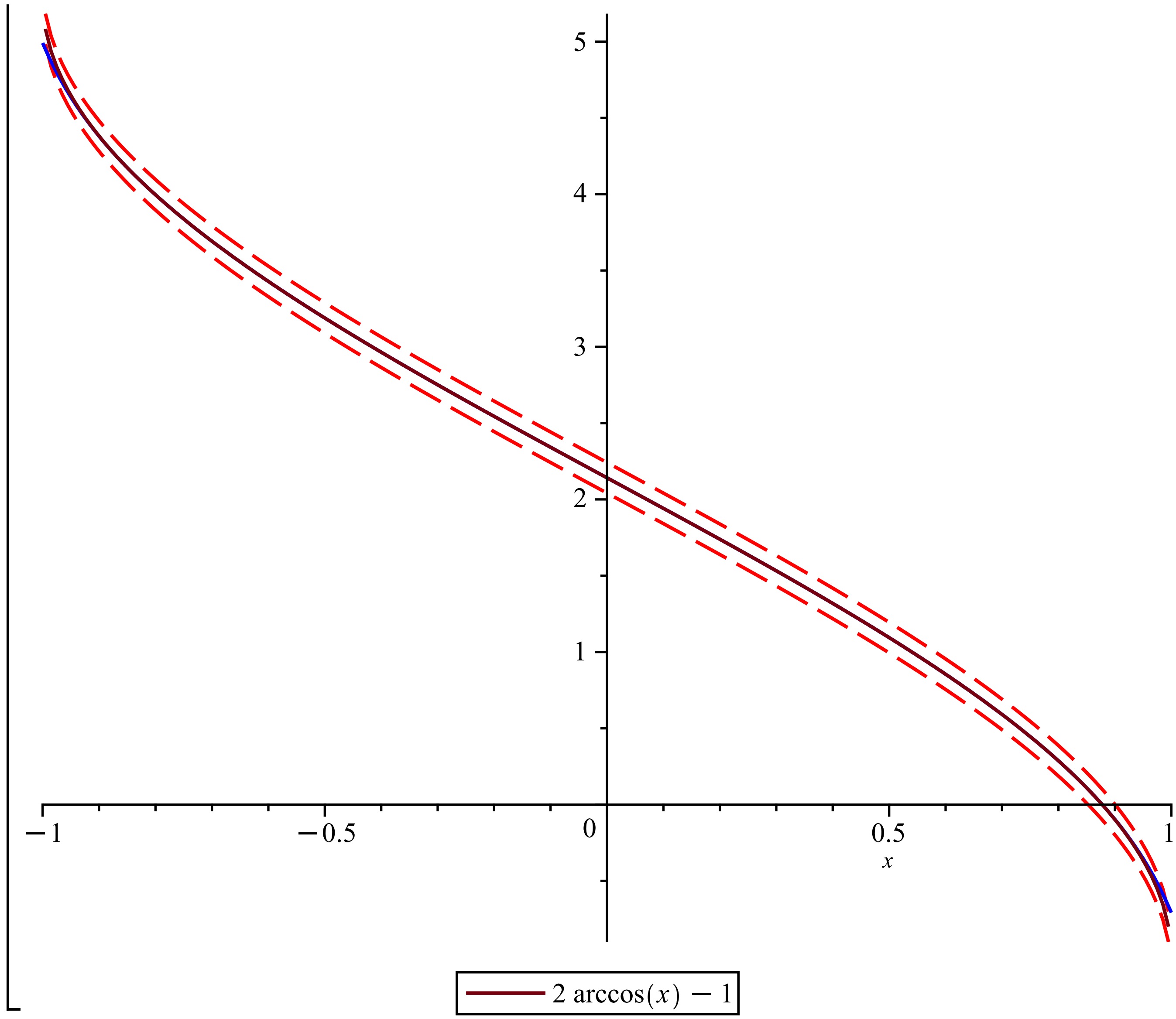


> $minT := convert(taylor(f, x=0, 31), polynom)$

$$minT := -1 + \pi - 2x - \frac{1}{3}x^3 - \frac{3}{20}x^5 - \frac{5}{56}x^7 - \frac{35}{576}x^9 - \frac{63}{1408}x^{11} - \frac{231}{6656}x^{13} - \frac{143}{5120}x^{15} - \frac{6435}{278528}x^{17} - \frac{12155}{622592}x^{19} - \frac{46189}{2752512}x^{21} - \frac{88179}{6029312}x^{23} - \frac{676039}{52428800}x^{25} - \frac{1300075}{113246208}x^{27} - \frac{5014575}{486539264}x^{29} \quad \textbf{(34)}$$

> $minStF := plot(minT, x=-1..1, color=blue):$

> $plots[display](f1, f2, minStF, our\_function1)$


5
4
3
2
1
−1
−0.5
0
0.5
1
x
2 arccos(x) − 1