

```

[ > restart;

[ > f1:=a^3-alpha+p;
                                 $f1 := a^3 - \alpha + p$  (1)
[
[ > f2:=alpha^2+b^3-alpha^3*p+2*alpha*beta;
                                 $f2 := \alpha^2 + b^3 - \alpha^3 p + 2 \alpha \beta$  (2)
[
[ > f3:=alpha+3*a*b^2+3*alpha^2*p-2*beta;
                                 $f3 := \alpha + 3 a b^2 + 3 \alpha^2 p - 2 \beta$  (3)
[
[ > f4:=1+3*a^2*b-3*alpha*p;
                                 $f4 := 1 + 3 a^2 b - 3 \alpha p$  (4)
[
[ > solve({f1},[p]);
                                 $[[p = -a^3 + \alpha]]$  (5)
[
[ > assign(%);
[ > f2;f3;f4;
                                 $\alpha^2 + b^3 - \alpha^3 (-a^3 + \alpha) + 2 \alpha \beta$  (6)
                                 $\alpha + 3 a b^2 + 3 \alpha^2 (-a^3 + \alpha) - 2 \beta$ 
                                 $1 + 3 a^2 b - 3 \alpha (-a^3 + \alpha)$ 
[
[ > simplify(f4);
[ solve({f4},b);
[ assign(%);
                                 $1 + 3 a^2 b + 3 \alpha a^3 - 3 \alpha^2$  (7)
                                 $\left\{ b = -\frac{1 + 3 \alpha a^3 - 3 \alpha^2}{3 a^2} \right\}$ 
[
[ > f2;
                                 $\alpha^2 - \frac{(1 + 3 \alpha a^3 - 3 \alpha^2)^3}{27 a^6} - \alpha^3 (-a^3 + \alpha) + 2 \alpha \beta$  (8)
[
[ > f3;
[ solve({%},beta);
[ assign(%);
                                 $\alpha + \frac{(1 + 3 \alpha a^3 - 3 \alpha^2)^2}{3 a^3} + 3 \alpha^2 (-a^3 + \alpha) - 2 \beta$  (9)
                                 $\left\{ \beta = -\frac{-9 \alpha a^3 - 1 + 6 \alpha^2 + 9 \alpha^3 a^3 - 9 \alpha^4}{6 a^3} \right\}$ 
[
[ > simplify(f2);

```

```
subs(a^6=z,%);
z1:=solve(%,z);
```

$$-\frac{-81\alpha^2 a^6 + 1 - 9\alpha^2 + 27\alpha^4 + 27\alpha^4 a^6 - 27\alpha^6}{27a^6} \quad (10)$$

$$-\frac{-81\alpha^2 z + 1 - 9\alpha^2 + 27\alpha^4 + 27\alpha^4 z - 27\alpha^6}{27a^6}$$

$$z1 := \frac{-1 + 9\alpha^2 - 27\alpha^4 + 27\alpha^6}{27\alpha^2(-3 + \alpha^2)}$$

```
> fsolve(z1=1,alpha);
alpha:=%;
```

$$-0.1057627724 \quad (11)$$

$$\alpha := -0.1057627724$$

```
> f1;
simplify(f2);
subs(a^6=z1,%);
simplify(%);
simplify(f3);
simplify(f4);
```

$$0 \quad (12)$$

$$-\frac{1}{a^6} (1.000000000 10^{-30} (-3.343217076 10^{28} a^6 + 3.343217077 10^{28} - 4.572780109 10^{18} a^3 + 2.261658764 10^{17} a^9))$$

$$-\frac{1.000000000 10^{-30} (1. 10^{19} - 4.572780109 10^{18} a^3 + 2.261658764 10^{17} a^9)}{a^6}$$

$$-\frac{2.000000000 10^{-22} (5.000000000 10^{10} - 2.286390054 10^{10} a^3 + 1.130829382 10^9 a^9)}{a^6}$$

$$-\frac{3.333333333 10^{-21} (-1.803957082 10^{10} a^3 + 4.691527759 10^9 + 3.839218416 10^9 a^6)}{a^3}$$

$$1.000000000 10^{-11}$$

```
> #alpha:=2;
z1;
```

$$1.000000000 \quad (13)$$

```
> a:=evalf(z1)^(1/6.0);
a:=z1;
evalf(b);
evalf(p);
evalf(beta);
evalf(alpha);
```

$$a := 1.0000000000^{\{0.1666666667\}} \quad (14)$$

$$\begin{aligned} a &:= 1.0000000000 \\ &-0.2163847969 \\ &-1.105762772 \\ &-0.001201017865 \\ &-0.1057627724 \end{aligned}$$

$$\begin{aligned} &> \text{beta}^2 - \alpha^3 p; \\ &-0.001306716287 \end{aligned} \quad (15)$$

$$\begin{aligned} &> \text{pow}; \\ &pow \end{aligned} \quad (16)$$

$$\begin{aligned} &> \text{restart}; \\ &> \text{with(LinearAlgebra)}: \\ &> V := 1/2 * x2(t)^2 + 1/2 * c * (x1(t) - \alpha * x2(t))^2; \\ &V := \frac{1}{2} x2(t)^2 + \frac{1}{2} c (x1(t) - \alpha x2(t))^2 \end{aligned} \quad (17)$$

$$\begin{aligned} &> \text{diff}(V, t); \\ &V1 := \text{subs}(\text{diff}(x1(t), t) = (a * x1(t) + b * x2(t))^3, \text{diff}(x2(t), t) = x1(t)^3, \%); \\ &x2(t) \left( \frac{d}{dt} x2(t) \right) + c (x1(t) - \alpha x2(t)) \left( \frac{d}{dt} x1(t) - \alpha \left( \frac{d}{dt} x2(t) \right) \right) \\ &V1 := x2(t) x1(t)^3 + c (x1(t) - \alpha x2(t)) ((a x1(t) + b x2(t))^3 - \alpha x1(t)^3) \end{aligned} \quad (18)$$

$$\begin{aligned} &> V2 := \alpha^3 * x2(t)^4 - p * (x1(t) - \alpha * x2(t))^4 + 2 * \text{beta} * (x1(t) - \alpha * x2(t))^2 * x2(t)^2; \\ &V2 := \alpha^3 x2(t)^4 - p (x1(t) - \alpha x2(t))^4 + 2 \beta (x1(t) - \alpha x2(t))^2 x2(t)^2 \end{aligned} \quad (19)$$

$$\begin{aligned} &> \text{collect}(\text{simplify}(V1), \{x1(t), x2(t)\}, 'distributed'); \\ &\text{collect}(\text{simplify}(V2), \{x1(t), x2(t)\}, 'distributed'); \\ &V12 := \text{collect}(\text{simplify}(V1 - V2), \{x1(t), x2(t)\}, 'distributed'); \\ &(1 + 3 c a^2 b - c \alpha a^3 + c \alpha^2) x2(t) x1(t)^3 + (c a^3 - c \alpha) x1(t)^4 - c \alpha x2(t)^4 b^3 + (3 c a b^2 \\ &- 3 c \alpha a^2 b) x2(t)^2 x1(t)^2 + (c b^3 - 3 c \alpha a b^2) x2(t)^3 x1(t) \\ &4 p x1(t)^3 \alpha x2(t) - p x1(t)^4 + (\alpha^3 + 2 \beta \alpha^2 - p \alpha^4) x2(t)^4 + (-6 p \alpha^2 + 2 \beta) x2(t)^2 x1(t)^2 \\ &+ (-4 \beta \alpha + 4 p \alpha^3) x2(t)^3 x1(t) \\ &V12 := (1 - 4 p \alpha + 3 c a^2 b - c \alpha a^3 + c \alpha^2) x2(t) x1(t)^3 + (p + c a^3 - c \alpha) x1(t)^4 + (p \alpha^4 \\ &- \alpha^3 - 2 \beta \alpha^2 - c \alpha b^3) x2(t)^4 + (-2 \beta + 3 c a b^2 + 6 p \alpha^2 - 3 c \alpha a^2 b) x2(t)^2 x1(t)^2 \\ &+ (-3 c \alpha a b^2 + c b^3 + 4 \beta \alpha - 4 p \alpha^3) x2(t)^3 x1(t) \end{aligned} \quad (20)$$

$$\begin{aligned} &> \text{solve}(1 - 4 * p * \alpha + 3 * c * a^2 * b - c * \alpha * a^3 + c * \alpha^2, p); \\ &p := \%; \end{aligned}$$

```

solve(-3*c*alpha*a*b^2+c*b^3+4*beta*alpha-4*p*alpha^3,beta);
beta:=%;
solve(1+3*c*a^2*b+3*c*alpha*a^3-3*c*alpha^2,b);
b:=%;

```

$$\begin{aligned}
 & -\frac{-1-3ca^2b+c\alpha a^3-c\alpha^2}{4\alpha} \\
 p & := -\frac{-1-3ca^2b+c\alpha a^3-c\alpha^2}{4\alpha} \\
 & -\frac{-3c\alpha ab^2+cb^3-\alpha^2-3\alpha^2ca^2b+\alpha^3ca^3-\alpha^4c}{4\alpha} \\
 \beta & := -\frac{-3c\alpha ab^2+cb^3-\alpha^2-3\alpha^2ca^2b+\alpha^3ca^3-\alpha^4c}{4\alpha} \\
 & -\frac{1+3c\alpha a^3-3c\alpha^2}{3ca^2} \\
 b & := -\frac{1+3c\alpha a^3-3c\alpha^2}{3ca^2}
 \end{aligned}
 \tag{21}$$

```

> -27*c^3*alpha^8+27*c^2*alpha^6-9*alpha^4*c-81*alpha^4*c^2*a^6+
alpha^2+27*alpha^6*c^3*a^6;
subs(a^6=z1,%);
solve(%,z1);
z1:=%;

```

$$\begin{aligned}
 & -27c^3\alpha^8+27c^2\alpha^6-9\alpha^4c-81\alpha^4c^2a^6+\alpha^2+27\alpha^6c^3a^6 \\
 & -27c^3\alpha^8+27c^2\alpha^6-9\alpha^4c-81\alpha^4c^2z1+\alpha^2+27\alpha^6c^3z1 \\
 & \frac{27\alpha^6c^3-27\alpha^4c^2+9c\alpha^2-1}{27\alpha^2c^2(-3+c\alpha^2)} \\
 z1 & := \frac{27\alpha^6c^3-27\alpha^4c^2+9c\alpha^2-1}{27\alpha^2c^2(-3+c\alpha^2)}
 \end{aligned}
 \tag{22}$$

```

> -27*alpha^4*c^3*a^6+81*alpha^2*c^2*a^6-1+9*c*alpha^2-27*c^2*
alpha^4+27*c^3*alpha^6;
subs(a^6=z1,%);
simplify(%);

```

$$\begin{aligned}
 & -27\alpha^4c^3a^6+81\alpha^2c^2a^6-1+9c\alpha^2-27\alpha^4c^2+27\alpha^6c^3 \\
 & -\frac{\alpha^2c(27\alpha^6c^3-27\alpha^4c^2+9c\alpha^2-1)}{-3+c\alpha^2} + \frac{3(27\alpha^6c^3-27\alpha^4c^2+9c\alpha^2-1)}{-3+c\alpha^2} - 1 \\
 & + 9c\alpha^2 - 27\alpha^4c^2 + 27\alpha^6c^3
 \end{aligned}
 \tag{23}$$

```
> z1^{1/6};
```

$$\left( \frac{27 \alpha^6 c^3 - 27 \alpha^4 c^2 + 9 c \alpha^2 - 1}{27 \alpha^2 c^2 (-3 + c \alpha^2)} \right)^{\left\{ \frac{1}{6} \right\}}$$

(24)

```
> #solve(z1);
```

```
> (3*c*alpha^2-1)^3;
```

$$(3 c \alpha^2 - 1)^3$$

(25)

```
> #a:=z1^(1/6);
```

```
b;
```

```
alpha;
```

```
p;
```

```
beta;
```

$$-\frac{1 + 3 c \alpha a^3 - 3 c \alpha^2}{3 c a^2}$$

(26)

$$\alpha$$

$$-\frac{4 c \alpha a^3 - 4 c \alpha^2}{4 \alpha}$$

$$-\frac{1}{4 \alpha} \left( -\frac{\alpha (1 + 3 c \alpha a^3 - 3 c \alpha^2)^2}{3 c a^3} - \frac{(1 + 3 c \alpha a^3 - 3 c \alpha^2)^3}{27 c^2 a^6} - \alpha^2 + \alpha^2 (1 + 3 c \alpha a^3 - 3 c \alpha^2) + \alpha^3 c a^3 - \alpha^4 c \right)$$

```
> subs(a=z1^(1/6),simplify(beta)):
```

```
simplify(%);
```

```
#plot(9*alpha^4-alpha^3*3^(1/2)*((3*alpha^2-1)^3/alpha^2/(alpha^2-3))^(1/2)-12*alpha^2+3*alpha*3^(1/2)*((3*alpha^2-1)^3/alpha^2/(alpha^2-3))^(1/2)+3,alpha=-0.01..0);
```

```
fsolve(9*alpha^4-alpha^3*3^(1/2)*((3*alpha^2-1)^3/alpha^2/(alpha^2-3))^(1/2)-12*alpha^2+3*alpha*3^(1/2)*((3*alpha^2-1)^3/alpha^2/(alpha^2-3))^(1/2)+3,alpha=-10..0);
```

$$-\frac{1}{2 (3 c \alpha^2 - 1)} \left( \left( 9 \alpha^4 c^2 - c^2 \alpha^3 \sqrt{3} \sqrt{\frac{(3 c \alpha^2 - 1)^3}{\alpha^2 c^2 (-3 + c \alpha^2)}} - 12 c \alpha^2 + 3 c \alpha \sqrt{3} \sqrt{\frac{(3 c \alpha^2 - 1)^3}{\alpha^2 c^2 (-3 + c \alpha^2)}} + 3 \right) \alpha \right)$$

(27)

$$\text{fsolve}\left(9\alpha^4 - \alpha^3\sqrt{3}\sqrt{\frac{(3\alpha^2-1)^3}{\alpha^2(\alpha^2-3)}} - 12\alpha^2 + 3\alpha\sqrt{3}\sqrt{\frac{(3\alpha^2-1)^3}{\alpha^2(\alpha^2-3)}} + 3, \alpha, -10..0\right)$$

```
> -alpha*c*3^(1/2)*((3*c*alpha^2-1)^3/alpha^2/(c*alpha^2-3))^(1/2)*(c*alpha^2-3)+9*c^2*alpha^4-12*c*alpha^2+3;
```

$$-\alpha c\sqrt{3}\sqrt{\frac{(3c\alpha^2-1)^3}{\alpha^2(-3+c\alpha^2)}}(-3+c\alpha^2) + 9\alpha^4 c^2 - 12c\alpha^2 + 3 \quad (28)$$

```
> solve(9*c^2*alpha^4-12*c*alpha^2+3,alpha);
```

$$\frac{1}{\sqrt{c}}, -\frac{1}{\sqrt{c}}, \frac{\sqrt{3}}{3\sqrt{c}}, -\frac{\sqrt{3}}{3\sqrt{c}} \quad (29)$$

```
> solve(beta,alpha);
```

$$\text{RootOf}\left(-1 + 27\_Z^6 c^3 - 162 c^3\_Z^5 a^3 + (135 c^3 a^6 - 27 c^2)\_Z^4 + 108 c^2\_Z^3 a^3 + (9 c - 81 c^2 a^6)\_Z^2 - 18 c\_Z a^3\right) \quad (30)$$

```
> (alpha^2-1)*(9*alpha^2-3);
```

```
simplify((9*c^2*alpha^4-12*c*alpha^2+3)/(alpha-1));
```

```
simplify(%/(alpha+1));
```

$$\frac{(\alpha^2-1)(9\alpha^2-3)}{\alpha-1} \quad (31)$$

$$\frac{3(3c^2\alpha^4-4c\alpha^2+1)}{\alpha-1}$$

$$\frac{3(3c^2\alpha^4-4c\alpha^2+1)}{\alpha^2-1}$$

```
> #alpha:=.5773502692;
```

```
> #z1;
```

```
> alpha:=1/(c^(1/2));
```

$$\alpha := \frac{1}{\sqrt{c}} \quad (32)$$

```
> z1;
```

$$-\frac{4}{27c} \quad (33)$$

```
> a:=-z1^(1/6);
```

```
b;
```

```
simplify(beta);
```

```
p;
```

$$a := -\left(-\frac{4}{27c}\right)^{1/6} \quad (34)$$

$$\begin{aligned}
& - \frac{-2 - 3\sqrt{c} \sqrt{-\frac{4}{27c}}}{3c \left(-\frac{4}{27c}\right)^{1/3}} \\
& - \frac{\sqrt{3}}{c \sqrt{-\frac{1}{c}}} \\
& - \frac{\left(-4 - 4\sqrt{c} \sqrt{-\frac{4}{27c}}\right) \sqrt{c}}{4}
\end{aligned}$$

```
> Matrix([[alpha^3,beta],[beta,-p]]);
Determinant(%);
```

$$\begin{bmatrix} -8.000 & 3.255437355 \\ 3.255437355 & -1.510566061 \end{bmatrix}$$

1.48665612

(35)

```
> simplify(V12);
subs(a^6=z1,%);
simplify(%);
collect(simplify(V1-V2),{x1(t),x2(t)},'distributed');
```

$$\begin{aligned}
& \frac{1}{54} \frac{1}{c^2 a^6 \alpha} \left( (27 \alpha^4 c^3 a^6 - 81 \alpha^2 c^2 a^6 + 1 - 9 c \alpha^2 + 27 c^2 \alpha^4 - 27 c^3 \alpha^6) x2(t)^2 (\alpha^2 x2(t)^2 \right. \\
& \quad \left. - x1(t)^2) \right) \\
& \frac{1}{54} \frac{1}{c^2 a^6 \alpha} \left( (27 \alpha^4 c^3 z1 - 81 \alpha^2 c^2 z1 + 1 - 9 c \alpha^2 + 27 c^2 \alpha^4 - 27 c^3 \alpha^6) x2(t)^2 (\alpha^2 x2(t)^2 \right. \\
& \quad \left. - x1(t)^2) \right) \\
& - \frac{1}{54} \frac{1}{c^2 a^6 \alpha} \left( (-27 \alpha^4 c^3 z1 + 81 \alpha^2 c^2 z1 - 1 + 9 c \alpha^2 - 27 c^2 \alpha^4 \right. \\
& \quad \left. + 27 c^3 \alpha^6) x2(t)^2 (\alpha^2 x2(t)^2 - x1(t)^2) \right) \\
& \frac{1}{54} \frac{(-27 c^3 \alpha^8 + 27 c^2 \alpha^6 - 9 \alpha^4 c - 81 \alpha^4 c^2 a^6 + \alpha^2 + 27 \alpha^6 c^3 a^6) x2(t)^4}{c^2 a^6 \alpha} \\
& + \frac{1}{54} \frac{(-27 \alpha^4 c^3 a^6 + 81 \alpha^2 c^2 a^6 - 1 + 9 c \alpha^2 - 27 c^2 \alpha^4 + 27 c^3 \alpha^6) x1(t)^2 x2(t)^2}{c^2 a^6 \alpha}
\end{aligned}$$

(36)

```
> #v1-(v2);
```

```
simplify(%);
-4.520371862 10-9 x2(t) x1(t)3 - 5.287522485 10-10 x1(t)4 - 1.770184594 10-8 x2(t)2 x1(t)2
- 3.324259645 10-8 x2(t)3 x1(t) - 2.338074807 10-8 x2(t)4 (37)
```

```
> u:=(a*x1(t)+b*x2(t))^3;
u := (-1.519820798 x1(t) - 1.452240544 x2(t))3 (38)
```

```
> sys_ode:=diff(x1(t),t)=u,x2(t)=x1(t)^3;
sys_ode :=  $\frac{d}{dt} x1(t) = (-1.519820798 x1(t) - 1.452240544 x2(t))^3$ , x2(t) = x1(t)3 (39)
```

```
> ics:=x1(0)=-1,x2(0)=1;
ics := x1(0) = -1, x2(0) = 1 (40)
```

```
> dsol:=dsolve({sys_ode,ics},numeric);
Warning, Initial value of x2(t) changed from 1.000000 to -1.000000 (41)

dsol := proc(x_rkf45_dae) ... end proc
```

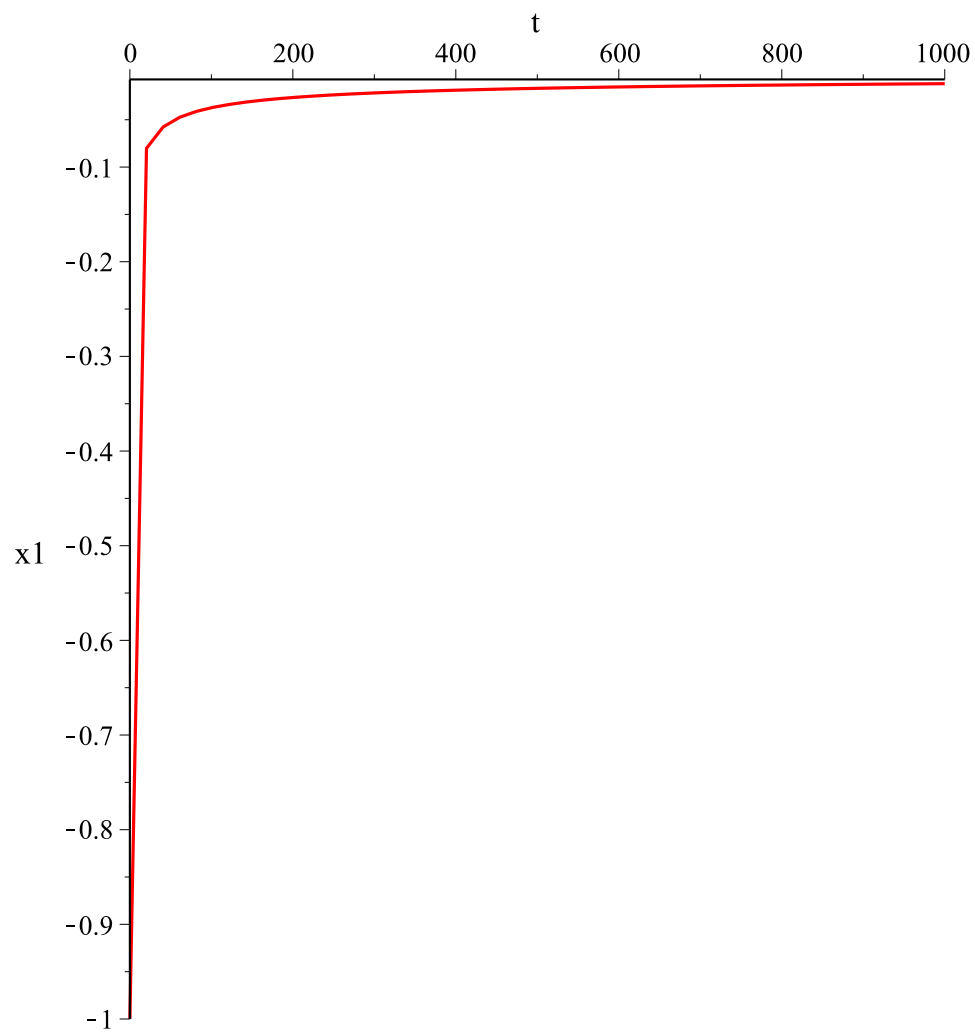
```
> dsol(0);
[t=0., x1(t) = -1., x2(t) = -1.] (42)
```

```
> dsol(1000);
[t=1000., x1(t) = -0.0119146295464185228, x2(t) = -0.00000169138190950595982] (43)
```

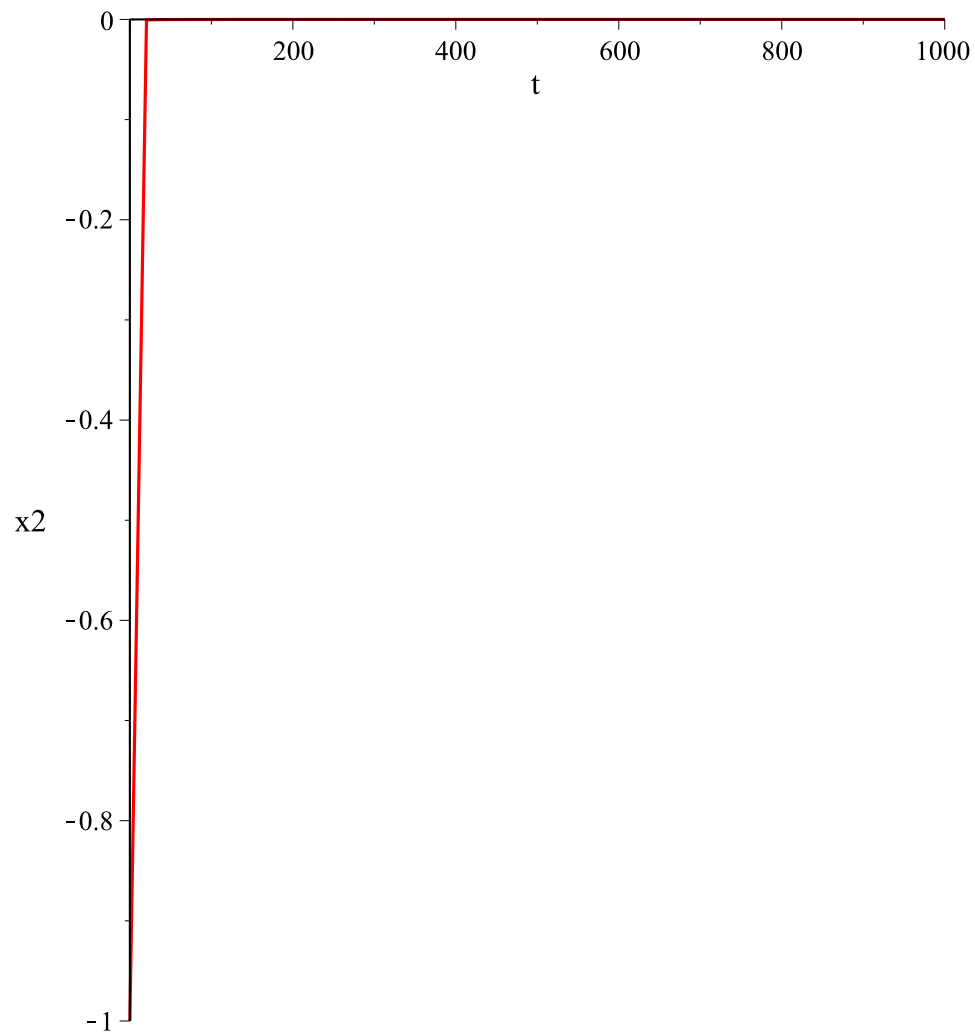
```
> t0:=1000;
t0 := 1000 (44)
```

```
> with(plots):
> odeplot(dsol,[t,x1(t)],t=0..t0);
```





```
> odeplot(dsol,[t,x2(t)],t=0..t0);
```



```
> odeplot(dsol,[x1(t),x2(t)],t=0..t0);
```

