Modelos de Regressão Linear Mistos para dados discretos: Uma abordagem utilizando MCMC através do Stan integrado ao R.

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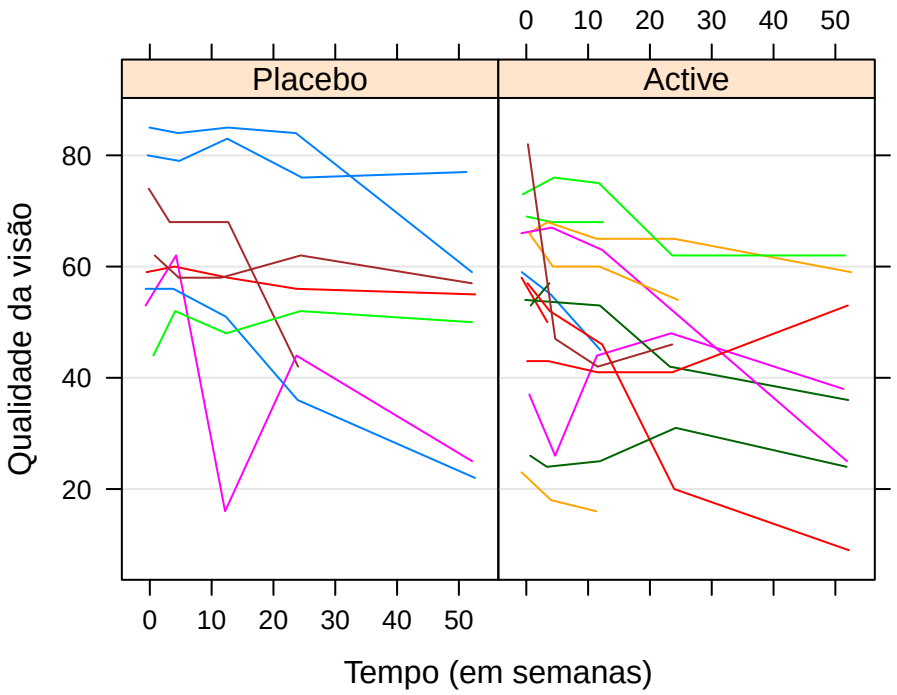
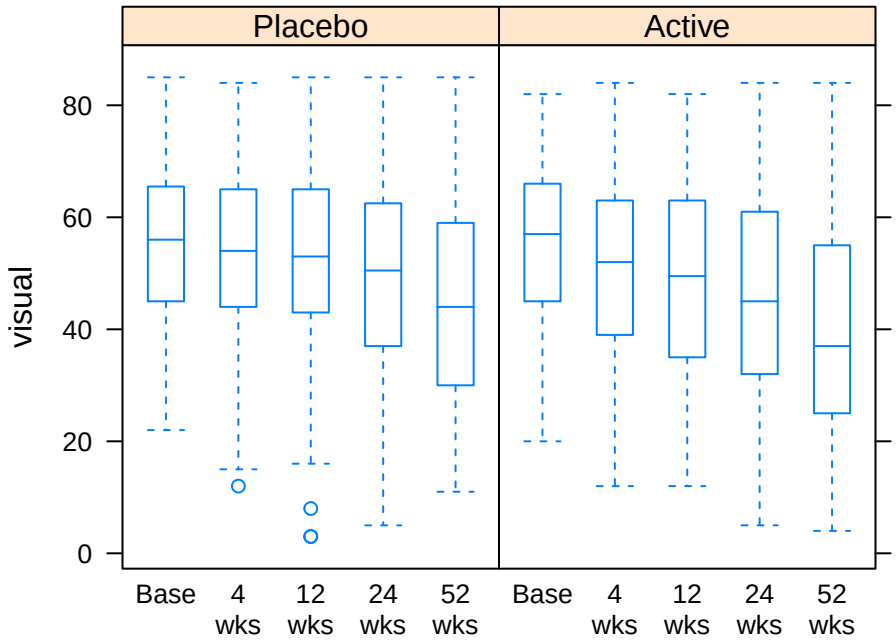



```
##          Placebo Active
## Baseline     119    121
## 4wks         117    114
## 12wks        117    110
## 24wks        112    102
## 52wks        105     90
```

```
## [1] "Placebo" "Active"  "Placebo" "Active"  "Placebo" "Active"
```

```
##          P:n A:n   P:Mean   A:Mean P:Mdn A:Mdn
## Baseline 119 121 55.33613 54.57851  56.0  57.0
## 4wks     117 114 53.96581 50.91228  54.0  52.0
## 12wks    117 110 52.87179 48.67273  53.0  49.5
## 24wks    112 102 49.33036 45.46078  50.5  45.0
## 52wks    105  90 44.43810 39.10000  44.0  37.0
```




```
##           visual0  visual4 visual12 visual24 visual52
## visual0  220.3055 206.7096 196.2439 193.3099 152.7141
## visual4  206.7096 246.2204 224.7933 221.2677 179.2284
## visual12 196.2439 224.7933 286.2072 257.7738 222.6830
## visual24 193.3099 221.2677 257.7738 334.4456 285.2327
## visual52 152.7141 179.2284 222.6830 285.2327 347.4311
```

```
##          visual0 visual4 visual12 visual24 visual52
## visual0     1.00    0.89     0.78     0.71     0.55
## visual4     0.89    1.00     0.85     0.77     0.61
## visual12    0.78    0.85     1.00     0.83     0.71
## visual24    0.71    0.77     0.83     1.00     0.84
## visual52    0.55    0.61     0.71     0.84     1.00
```

1 MODELO NORMAL INDEPENDENTE HOMOCEDASTICO

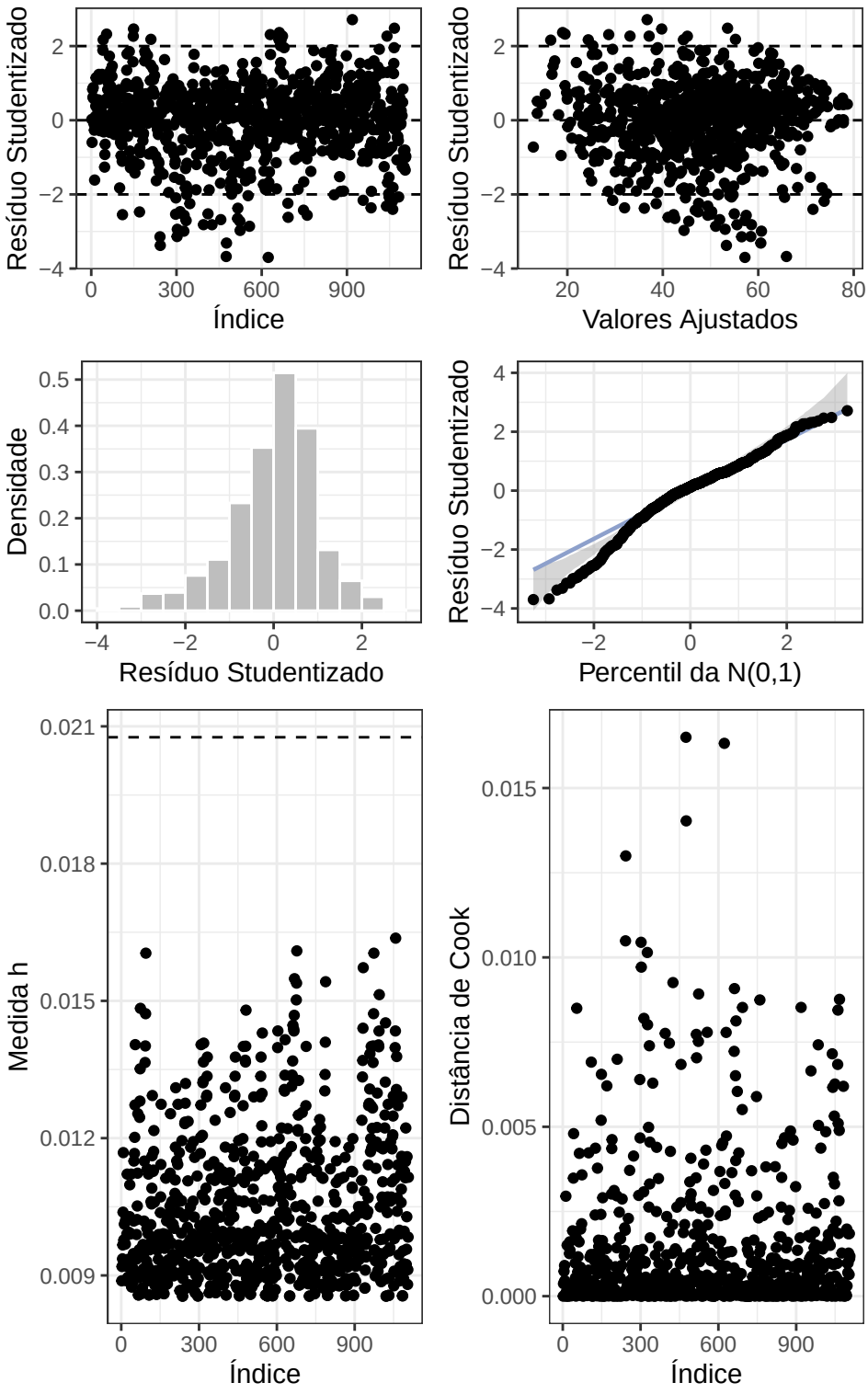
```
##
## Call:
## lm(formula = lm.form, data = armd)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -45.210  -6.459   1.532   7.512  33.283
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## visual0                    0.83037    0.02842  29.213  < 2e-16 ***
## time.f4wks                 8.07531    1.94341   4.155 3.58e-05 ***
```

```
## time.f12wks                 7.08066    1.94066   3.649  0.00028 ***
## time.f24wks                 3.63022    1.95316   1.859  0.06342 .
## time.f52wks                -1.74643    1.98952  -0.878  0.38029
## time.f4wks:treat.fActive   -2.35278    1.62894  -1.444  0.14900
## time.f12wks:treat.fActive -3.70852    1.64378  -2.256  0.02432 *
## time.f24wks:treat.fActive -3.44915    1.69399  -2.036  0.04205 *
## time.f52wks:treat.fActive -4.47345    1.77811  -2.516  0.01206 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.38 on 858 degrees of freedom
## Multiple R-squared:  0.9432, Adjusted R-squared:  0.9426
## F-statistic:  1583 on 9 and 858 DF,  p-value: < 2.2e-16
```

```
## [1] 12.37649
```

```
##                                2.5 %     97.5 %
## visual0                    0.7745832  0.8861617
## time.f4wks                 4.2609239 11.8897036
## time.f12wks                3.2716615 10.8896534
## time.f24wks               -0.2033236  7.4637556
## time.f52wks               -5.6513208  2.1584611
## time.f4wks:treat.fActive  -5.5499518  0.8443996
## time.f12wks:treat.fActive -6.9348245 -0.4822195
## time.f24wks:treat.fActive -6.7740045 -0.1243018
## time.f52wks:treat.fActive -7.9634126 -0.9834943
```

```
## Analysis of Variance Table
##
## Response: visual
##                  Df  Sum Sq Mean Sq    F value    Pr(>F)
## visual0           1 2165776 2165776 14138.9886 < 2.2e-16 ***
## time.f            4   14434    3608    23.5574 < 2.2e-16 ***
## time.f:treat.f    4    2703     676     4.4109  0.001555 **
## Residuals       858  131426     153
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```


Resíduo Studentizado
Índice
Valores Ajustados
Densidade
Resíduo Studentizado
Percentil da N(0,1)
Medida h
Distância de Cook

2 MODELO MISTO FREQUENTISTA

```
## Linear mixed-effects model fit by REML
##   Data: armd
##   Log-restricted-likelihood: -3288.986
##   Fixed: lme.form
##         (Intercept)            visual0               time      treat.fActive
##          9.28807837         0.82643987        -0.21221595         -2.42200012
## time:treat.fActive
##         -0.04959058
##
## Random effects:
##  Formula: ~1 | subject
##          (Intercept) Residual
## StdDev:    8.978212 8.627514
##
## Number of Observations: 867
## Number of Groups: 234
```

```
##                          Value  Std.Error         DF t-value   p-value
## (Intercept)           9.288078   2.681889 631.000000  3.4633 0.0005698 ***
## visual0               0.826440   0.044667 231.000000 18.5022 < 2.2e-16 ***
## time                 -0.212216   0.022929 631.000000 -9.2552 < 2.2e-16 ***
## treat.fActive        -2.422000   1.499967 231.000000 -1.6147 0.1077402
## time:treat.fActive   -0.049591   0.033562 631.000000 -1.4776 0.1400155
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
## subject 2
## Conditional variance covariance matrix
##        1      2      3      4
## 1 74.434  0.000  0.000  0.000
## 2  0.000 74.434  0.000  0.000
## 3  0.000  0.000 74.434  0.000
## 4  0.000  0.000  0.000 74.434
##   Standard Deviations: 8.6275 8.6275 8.6275 8.6275
```

```
##           1         2         3         4
## 1 1.0000000 0.5199116 0.5199116 0.5199116
## 2 0.5199116 1.0000000 0.5199116 0.5199116
## 3 0.5199116 0.5199116 1.0000000 0.5199116
## 4 0.5199116 0.5199116 0.5199116 1.0000000
```

```
##       AIC       BIC      AICc     SABIC     HQCIC -2log.lik
##  6833.788  6881.438  6833.998  6846.091  6848.200  6813.788
```

```
##       AIC       BIC      AICc     SABIC     HQCIC -2log.lik
##  6591.971  6625.286  6592.026  6597.137  6597.444  6577.971
```

3 MODELO MISTO BAYESIANO

Segundo Andrew Gelman (2014) blablabla

```
##
## SAMPLING FOR MODEL 'matrixModel' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.000528 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 5.28 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 1: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 1: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 1: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 1: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 1: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 1: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 1: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 1: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 1: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 1: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 1: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 1:
## Chain 1:  Elapsed Time: 30.16 seconds (Warm-up)
## Chain 1:                15.1695 seconds (Sampling)
## Chain 1:                45.3294 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'matrixModel' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0.000248 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 2.48 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 2: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 2: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 2: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 2: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 2: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 2: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 2: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 2: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 2: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 2: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 2: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 2:
```

```
## Chain 2:  Elapsed Time: 30.8609 seconds (Warm-up)
## Chain 2:                15.4453 seconds (Sampling)
## Chain 2:                46.3062 seconds (Total)
## Chain 2:
##
## SAMPLING FOR MODEL 'matrixModel' NOW (CHAIN 3).
## Chain 3:
## Chain 3: Gradient evaluation took 0.000515 seconds
## Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 5.15 seconds.
## Chain 3: Adjust your expectations accordingly!
## Chain 3:
## Chain 3:
## Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 3: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 3: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 3: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 3: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 3: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
## Chain 3:
## Chain 3:  Elapsed Time: 28.5564 seconds (Warm-up)
## Chain 3:                16.1118 seconds (Sampling)
## Chain 3:                44.6681 seconds (Total)
## Chain 3:
##
## SAMPLING FOR MODEL 'matrixModel' NOW (CHAIN 4).
## Chain 4:
## Chain 4: Gradient evaluation took 0.000231 seconds
## Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 2.31 seconds.
## Chain 4: Adjust your expectations accordingly!
## Chain 4:
## Chain 4:
## Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
## Chain 4: Iteration:  200 / 2000 [ 10%]  (Warmup)
## Chain 4: Iteration:  400 / 2000 [ 20%]  (Warmup)
## Chain 4: Iteration:  600 / 2000 [ 30%]  (Warmup)
## Chain 4: Iteration:  800 / 2000 [ 40%]  (Warmup)
## Chain 4: Iteration: 1000 / 2000 [ 50%]  (Warmup)
## Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
## Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
## Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
## Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
## Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
## Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)
```

```
## Chain 4:
## Chain 4:  Elapsed Time: 27.9232 seconds (Warm-up)
## Chain 4:                16.1807 seconds (Sampling)
## Chain 4:                44.1038 seconds (Total)
## Chain 4:
```

Andrew Gelman, Hal S. Stern, John B. Carlin. 2014. *Bayesian Data Analysis*. Taylor & Francis Group.