I. ① $f(z) = i\bar{z} = i(x-iy) = ix + y$

$\frac{\partial u}{\partial x} = 0 \qquad \frac{\partial v}{\partial y} = 0$ ✓

$\frac{\partial u}{\partial y} = 1 \qquad \frac{\partial v}{\partial x} = 1$ ⊖ $\nexists f'(z)$

② $f(z) = iz^2 - 3z + 1 = i(x+iy)^2 - 3(x+yi) + 1 =$
$= i(x^2 - y^2 + 2ixy) - 3x - 3iy + 1 = (ix^2 - 3iy - iy^2) +$
$+ (-2xy - 3x + 1) = \underbrace{(-2xy - 3x + 1)}_{u} + i\underbrace{(x^2 - 3y - y^2)}_{v}$

$\frac{\partial u}{\partial x} = -2y - 3 \qquad \frac{\partial v}{\partial y} = -3 - 2y$ ✓

$\frac{\partial u}{\partial y} = -2x \qquad \frac{\partial v}{\partial x} = 2x$ ✓

Тогда $z \in \mathbb{C}$

$\exists\ f'(z) = 2iz - 3$

⑥ $f(z) = 2\ln r + 2i\varphi$

$\frac{\partial u}{\partial r} = \frac{2}{r} = \frac{1}{r} \cdot \frac{\partial v}{\partial \varphi} = 2$ ✓

$\frac{\partial u}{\partial \varphi} = 0 = -\frac{1}{r} \cdot \frac{\partial v}{\partial \varphi} = 0$ ✓

Тогда
$\exists\ f'(z) = (2\ln r + 2i\varphi)' = \frac{2}{r}$

(7) $f(z) = \mathrm{Ln}\, z^2 \quad , z \neq 0$

$\mathrm{Ln}\, z = \ln r + i(\arg z + 2\pi k)$

$f(z) = 2\ln r + i2(\arg z + 2\pi k)$

$\frac{\partial u}{\partial r} = \frac{2}{r} \quad = \quad \frac{1}{r} \cdot \frac{\partial v}{\partial \varphi} = 2$ (v)

$\frac{\partial u}{\partial \varphi} = 0 \left(\cancel{\frac{u}{z}}\right) = \frac{\partial v}{\partial \varphi} = 0 \cdot \left(-\frac{1}{r}\right)$ (v)

Тогда

$\exists \quad f'(z) = (2\ln z)' = \frac{r}{z}\left(\frac{2}{r} + i0\right) = \frac{2}{z}$

(8) $f($

8) $f(z) = \cos(iz) = \operatorname{ch} z = \frac{1}{2}(e^{z} + e^{-z}) =$

$= \frac{1}{2} \cdot (e^{x} \cdot (\cos y + i \sin y) + e^{-x} \cdot (\cos y - i \sin y)) =$

$= \frac{1}{2} \cdot (e^{x} \cdot \cos y + i e^{x} \sin y + e^{-x} \cos y - i e^{-x} \cdot \sin y) =$

$= \frac{\cos y\,(e^{x} + e^{-x})}{2} + i\,\frac{\sin y\,(e^{x} - e^{-x})}{2}$

$\frac{\partial u}{\partial x} = \frac{e^{x} \cos y}{2} - \frac{e^{-x} \cos y}{2} = \frac{\partial v}{\partial y} = \frac{\cos y\, e^{x} - \cos y\, e^{-x}}{2}$ ✓

$\frac{\partial u}{\partial y} = -\frac{\sin y\, e^{x} - \sin y \cdot e^{-x}}{2} = -1 \cdot \frac{\partial v}{\partial x} = \frac{\sin y\, e^{x} + \sin y\, e^{-x}}{2}$ ✓

Тогда $\exists\ f'(z) = (\cos iz)' = -i \sin iz$

⑨ $f(z)=\frac{1}{z}=z^{-1}$, $z\neq 0$

$f(z)=r^{-1}(\cos\varphi-i\sin\varphi)=r^{-1}\cos\varphi+ir^{-1}\sin\varphi$

$\frac{du}{dr}=-\frac{\cos\varphi}{r^2}=\frac{1}{r}\cdot\frac{dv}{d\varphi}=-\frac{\cos}{r^2}$ ✓

$\frac{du}{d\varphi}=\frac{-\sin\varphi}{r}\cdot\left(-\frac{1}{r}\right)=\frac{dv}{dr}=\frac{\sin\varphi}{r^2}$ ✓

Тогда $\exists\ f'(z)=-\frac{1}{r^2}$

II ①

$f(z)=\bar{z}^2-z$, $D:|z|<1$

$f(z)=\overline{(x+iy)^2}-x-iy$, $D:-1<z<1$

$f(z)=\overline{x^2-y^2+i2xy}-x-iy=$

$=x^2-y^2-i2xy-x-iy=x^2-x-y^2+i(-2xy-y)$

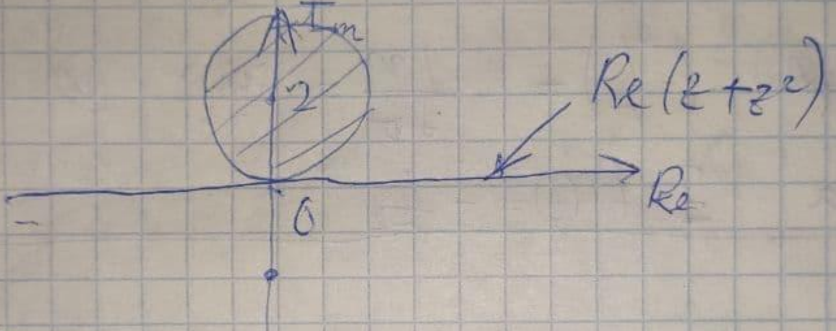
$\frac{du}{dx}=2x-1$ $\frac{dv}{dy}=-2x-1$ ⊖

$f(z)$ - не аналитичная на D

2. $f(z) = Re(z+z^2)$, $D: |z-2i| < 2$

$f(z) = Re(x+iy+x^2+2ixy-y^2) =$

$= x^2+x-y^2$



в т. 0 пересечение с D нач., =>, ф-я не аналитична на D.