I. 4) $\operatorname{Res}\left[\frac{z+1}{(z+2i)^2(z-1)}; z_k\right]$

1) $\operatorname{Res}\left[\frac{z+1}{(z+2i)^2(z-1)}; -2i\right]$

$\lim\limits_{z\to -2i} \frac{z+1}{(z+2i)^2(z-1)} = \infty$ - полюс $z_1 = -2i$

$\lim\limits_{z\to -2i} \frac{(z+1)(z+2i)^m}{(z+2i)^2(z-1)} = |m=2| = \lim\limits_{z\to -2i} \frac{(z+1)(z+2i)^2}{(z+2i)^2(z-1)} =$

$= \frac{1-2i}{-1-2i}$.

$\operatorname{Res}\left[\frac{z+1}{(z+2i)^2(z-1)}; -2i\right] = \frac{1}{(2-1)!} \lim\limits_{z\to -2i} \left(\frac{(z+1)(z+2i)^2}{(z+2i)^2(z-1)}\right)'$

$= \lim\limits_{z\to -2i} \left(-\frac{2}{(z-1)^2}\right) = -\frac{2}{(-2i-1)^2} = \frac{-2}{-4+4i+1} =$

$= -\frac{2}{-3+4i} = -\frac{2\cdot(-3-4i)}{(-3+4i)(-3-4i)} = \frac{6+8i}{9-16i^2} =$

$= \frac{6+8i}{25}$

2) $\operatorname{Res}\left[\frac{z+1}{(z+2i)^2(z-1)};1\right]$

$\lim\limits_{z\to1}\frac{z+1}{(z+2i)^2(z-1)}=\infty$ – полюс $z_2=1$

$\lim \frac{(z+1)(z-1)^m}{(z+2i)^2(z-1)}=|m=1|=\lim \frac{(z+1)\cancel{(z-1)}}{(z+2i)^2\cancel{(z-1)}}=$

$=\frac{2}{(1+2i)^2}=-\frac{6+8i}{25}=\operatorname{Res}\left[\frac{z+1}{(z+2i)^2(z-1)};1\right]$

5) $\operatorname{Res}\left(\frac{\sin z}{z};z_k\right)$

$z_k=0$

$\operatorname{Res}\left(\frac{\sin z}{z};0\right)$

$\lim\limits_{z\to0}\frac{\sin z}{z}\sim\lim\limits_{z\to0}\frac{z}{z}=1$ – у.о.т., $=>$,

главн. часть отсутствует $C_{-1}=0$

6) $\operatorname{Res}\left(\frac{\cos z}{4(z-\frac{\pi}{2})^2};\frac{\pi}{2}\right)$

$\lim\limits_{z\to\frac{\pi}{2}}\frac{\cos z}{(2z-\pi)^2}=\left[\frac{0}{0}\right]=\left|\begin{array}{l} z-\frac{\pi}{2}=t\to0 \\ z=\frac{\pi}{2}+t \end{array}\right|=$

$=\lim\limits_{t\to0}\frac{\cos\left(\frac{\pi}{2}+t\right)}{4t^2}=\lim\limits_{t\to0}\frac{-\sin t}{4t^2}=$

$=\lim\limits_{t\to0}\frac{-t}{4t^2}=-\infty$ – полюс 2-ого пор-ка

$$\lim \frac{\cos\left(z-\frac{\pi}{2}\right)^m}{4\left(z-\frac{\pi}{2}\right)^2} = |m=2| = \lim_{z\to\frac{\pi}{2}} \frac{\cos z\left(z-\frac{\pi}{2}\right)}{4\left(z-\frac{\pi}{2}\right)^2} = 0$$

$$\operatorname{Res}\left(\frac{\cos z}{4\left(z-\frac{\pi}{2}\right)^2}; \frac{\pi}{2}\right) = \frac{1}{(2-1)!} \lim_{z\to\frac{\pi}{2}} \left(\frac{\cos z\left(z-\frac{\pi}{2}\right)^2}{4\left(z-\frac{\pi}{2}\right)^2}\right) =$$

$$= \lim_{z\to\frac{\pi}{2}} \left(-\frac{\sin z}{4}\right) = -\frac{1}{4}$$

7) $\operatorname{Res}\left(z^4 \cdot e^{\frac{1}{z}}; 0\right)$

$\lim z^4 \cdot e^{\frac{1}{z}} - \nexists$

$\operatorname{Res}\left(z^4 \cdot e^{\frac{1}{z}}; 0\right) = C_{-1} = \frac{1}{120}$

$e^u = 1 + \frac{u}{1!} + \frac{u^2}{2!} + \frac{u^3}{3!} + \dots ; \; u = \frac{1}{z}$

$f(z) = z^4\left(1 + \frac{1}{z} + \frac{1}{z^2 2!} + \frac{1}{z^3 \cdot 3!} + \frac{1}{z^4 \cdot 4!} + \dots\right) =$

$= z^4 + z^3 + \frac{z^2}{2} + \frac{z}{6} + \frac{1}{24} + \frac{1}{120z} + \frac{1}{720z} + \dots$

2) $\operatorname{Res}\left[\frac{z+1}{(z+2i)^2(z-1)}; 1\right]$

$\lim\limits_{z\to 1} \frac{z+1}{(z+2i)^2(z-1)} = \infty$ – полюс $z_2 = 1$

$\lim \frac{(z+1)(z-1)^m}{(z+2i)^2(z-1)} = |m=1| = \lim \frac{(z+1)\cancel{(z-1)}}{(z+2i)^2\cancel{(z-1)}} =$

$= \frac{2}{(1+2i)^2} = -\frac{6+8i}{25} = \operatorname{Res}\left[\frac{z+1}{(z+2i)^2(z-1)}; 1\right]$

5) $\operatorname{Res}\left(\frac{\sin z}{z}; z_k\right)$

$z_k = 0$

$\operatorname{Res}\left(\frac{\sin z}{z}; 0\right)$

$\lim\limits_{z\to 0} \frac{\sin z}{z} \sim \lim\limits_{z\to 0} \frac{z}{z} = 1$ – У.О.Т. $\Rightarrow$

главн. часть отсутствует $C_{-1} = 0$

6) $\operatorname{Res}\left(\frac{\cos z}{4\left(z-\frac{\pi}{2}\right)^2}; \frac{\pi}{2}\right)$

$\lim\limits_{z\to\frac{\pi}{2}} \frac{\cos z}{(2z-\pi)^2} = \left[\frac{0}{0}\right] = \left| \begin{matrix} z-\frac{\pi}{2} = t \to 0 \\ z = \frac{\pi}{2} + t \end{matrix} \right| =$

$= \lim\limits_{t\to 0} \frac{\cos\left(\frac{\pi}{2}+t\right)}{4t^2} = \lim\limits_{t\to 0} \frac{-\sin t}{4t^2} =$

$= \lim\limits_{t\to 0} \frac{-t}{4t^2} = -\infty$ – полюс 2-ого пор-ка

$$\lim \frac{\cos\left(z-\frac{\pi}{2}\right)^m}{4\left(z-\frac{\pi}{2}\right)^2} = |m=2| = \lim_{z\to\frac{\pi}{2}} \frac{\cos z\left(z-\frac{\pi}{2}\right)}{4\left(z-\frac{\pi}{2}\right)^2} = 0$$

$$\operatorname{Res}\left(\frac{\cos z}{4\left(z-\frac{\pi}{2}\right)^2}; \frac{\pi}{2}\right) = \frac{1}{(2-1)!} \lim_{z\to\frac{\pi}{2}} \left(\frac{\cos z\left(z-\frac{\pi}{2}\right)^2}{4\left(z-\frac{\pi}{2}\right)^2}\right) =$$

$$= \lim_{z\to\frac{\pi}{2}} \left(-\frac{\sin z}{4}\right) = -\frac{1}{4}$$

7) $\operatorname{Res}\left(z^4 \cdot e^{\frac{1}{z}}; 0\right)$

$$\lim z^4 \cdot e^{\frac{1}{z}} - \nexists$$

$$\operatorname{Res}\left(z^4 \cdot e^{\frac{1}{z}}; 0\right) = c_{-1} = \frac{1}{120}$$

$$e^u = 1 + \frac{u}{1!} + \frac{u^2}{2!} + \frac{u^3}{3!} + \dots; \quad u = \frac{1}{z}$$

$$f(z) = z^4 \left(1 + \frac{1}{z} + \frac{1}{z^2 2!} + \frac{1}{z^3 \cdot 3!} + \frac{1}{z^4 \cdot 4!} + \dots\right)$$

$$= z^4 + z^3 + \frac{z^2}{2} + \frac{z}{6} + \frac{1}{24} + \frac{1}{120z} + \frac{1}{720z} + \dots$$