

Титульный лист по математике
по теме "Кратные интегралы" VII
ст. гр. 950503 Полховский А.П.

Вариант 24

№1 Изменить порядок интегрирования:

$$\int_{-\sqrt{2}}^{-1} dy \int_{-\sqrt{2-y^2}}^0 f dx + \int_{-1}^0 dy \int_y^0 f dx$$

Построим область интегрирования

$$y = -\sqrt{2}; y = -1; x = 0; x = -\sqrt{2-y^2} \Rightarrow$$

$$x^2 + y^2 = 2.$$

$$D_1: \begin{cases} -\sqrt{2} \leq y \leq -1 \\ -\sqrt{2-y^2} \leq x \leq 0 \end{cases}$$

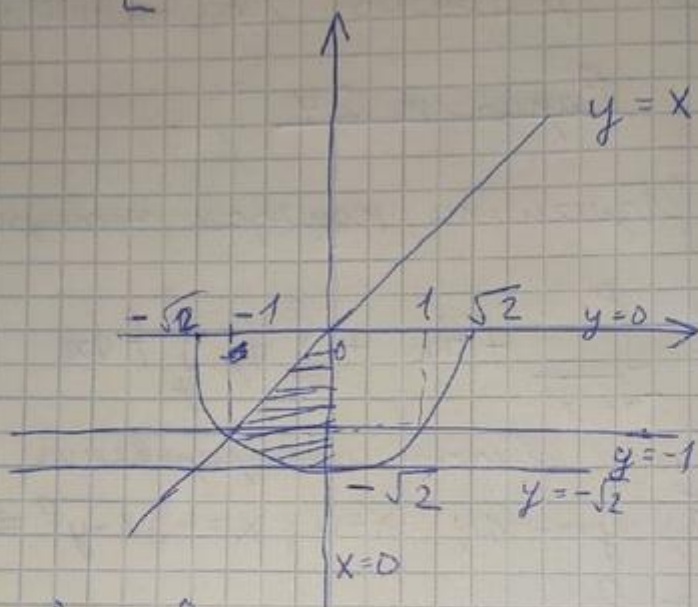
$$y = -1, y = 0; x = 0; x = y$$

$$D_2: \begin{cases} -1 \leq y \leq 0 \\ y \leq x \leq 0 \end{cases}$$

Полховский А.П.
Реш

$$D = D_1 \cup D_2 \text{ область искомая:}$$

$$D: \begin{cases} -1 \leq x \leq 0 \\ -\sqrt{2-x^2} \leq y \leq x \end{cases}$$



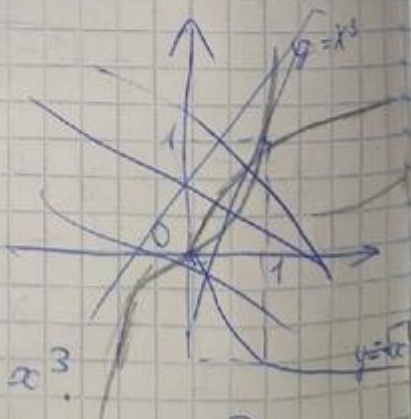
Answer: $\int_{-1}^0 dx \int_{-\sqrt{2-x^2}}^0 f dy$

Nº2. Вывести:

~~1/2~~ $\iint_D (4xy + 176x^3y^3) dx dy$

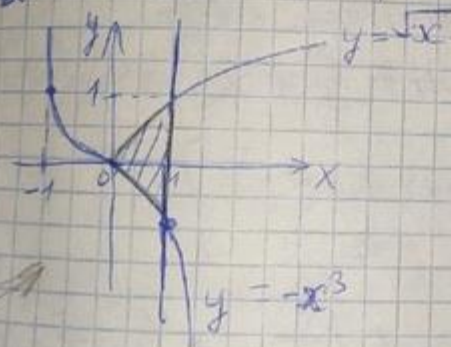
D: $x=1, y=\sqrt{x}, y=-x^3$

Область интегрирования: $D = \begin{cases} 0 \leq x \leq 1 \\ -x^3 \leq y \leq \sqrt{x} \end{cases}$



die Doppelintegral umschreiben d. Ange

$$\begin{aligned} & \int_0^1 dx \int_{-x^3}^{\sqrt{x}} (4xy + 176x^3y^3) dy = \\ &= \int_0^1 dx \left(\frac{4xy^2}{2} + \frac{176x^3y^4}{4} \right) \Big|_{-x^3}^{\sqrt{x}} = \\ &= \int_0^1 dx \left(2x \cdot (x - x^9) + 44x^3(x^2 - x^{12}) \right) dx = \\ &= \int_0^1 (2x^2 - 2x^8 + 44x^5 - 44x^{15}) dx = \\ &= \left(\frac{2x^3}{3} - \frac{2x^9}{9} + \frac{44x^6}{6} - \frac{44x^{16}}{16} \right) \Big|_0^1 = \frac{2}{3} - \frac{2}{9} + \\ &+ \frac{44}{6} - \frac{44}{16} = \frac{2}{3} - \frac{1}{4} + \frac{22}{3} - \frac{11}{4} = 8 - 3 = 5 \end{aligned}$$



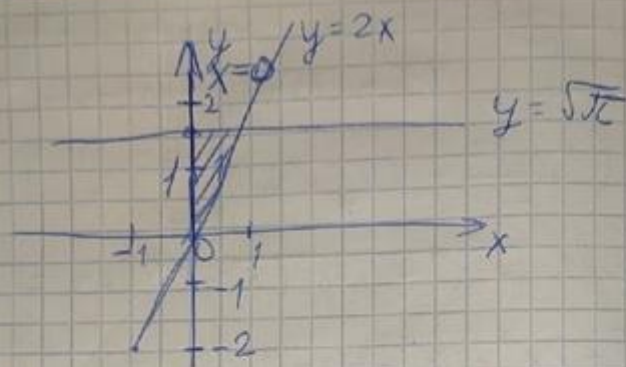
Antwort: 5.

Nº 3 Berechnen:

$$\iint_D y^2 \cos xy \, dx \, dy$$

$$D: x=0, y=\sqrt{x}, y=2x$$

$$\begin{aligned} x &\leq 1 \\ y &\leq \sqrt{x} \end{aligned}$$



Область интегрирования:

$$D: \begin{cases} 0 \leq y \leq \sqrt{x} \\ 0 \leq x \leq \frac{y^2}{2} \end{cases}$$

используем замечание о порядке:

$$\int_0^{\sqrt{x}} dy \int_0^{\frac{y^2}{2}} y^2 \cos xy \, dx = \int_0^{\frac{\sqrt{2}}{2}} y^2 dy \int_0^{\frac{y^2}{2}} \cos(xy) \, dx =$$

$$= \left[t = \frac{y^2}{2}; \text{пределы: } t_1 = \frac{y^2}{2}, t_2 = 0 \right] =$$

$$= \int_0^{\frac{\sqrt{2}}{2}} \sin t \, dt = -\cos t \Big|_0^{\frac{\sqrt{2}}{2}} = -(\cos \frac{\sqrt{2}}{2} - \cos 0) =$$

$$= -(0 - 1) = 1$$

~~Результат~~ Ответ: 1

~~1/04~~ Проверим:

$$\int_0^1 \int_0^{\frac{y^2}{2}} y^2 \cdot 2 \cos \frac{xy^2}{2} \, dx \, dy =$$

$$V: \begin{cases} x=9, y=1, z=2\pi \\ x=0, y=0, z=0 \end{cases}$$



используем замечание о базе

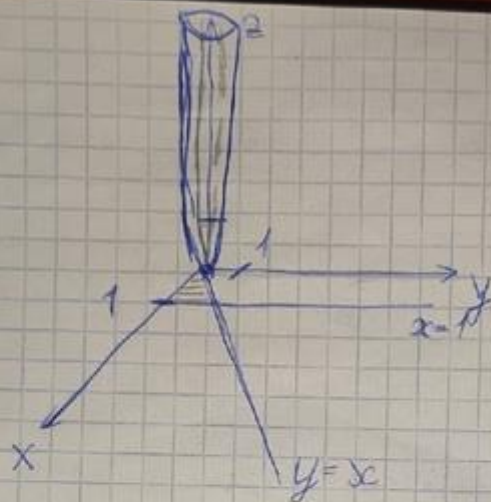
$$\begin{aligned} & \int_0^1 dy \int_0^{2\pi} dz \int_0^9 y^2 z \cos \frac{xyz}{9} dx = \\ & = \int_0^1 dy \int_0^{2\pi} y^2 z dz \int_0^9 \cos \frac{xyz}{9} dx = \int_0^1 dy \int_0^{2\pi} y^2 z \left(\frac{\sin \frac{xyz}{9}}{\frac{y}{9}} \right) \Big|_0^9 dz = \\ & = \int_0^1 dy \int_0^{2\pi} 9y \cdot (\sin(yz) - \sin 0) dy = 9 \int_0^1 y dy \int_0^{2\pi} \sin yz dz = \\ & = 9 \int_0^1 y dy \cdot (-\cos yz) \Big|_0^{2\pi} = -9 \int_0^1 dy \cdot (\cos 2\pi y - \cos 0) = \\ & \cos 0 = -9 \left(\frac{\sin 2\pi y}{2\pi} - y \right) \Big|_0^1 = -9(0 - 1) = 9 \end{aligned}$$

Ответ: 9.

~~№5~~ Борщевик

$$\iiint_V (x+y) dx dy dz$$

$$V: \begin{cases} y=x; y=0; x=1 \\ z=0; z=20x^2+20yz \end{cases}$$



Wollumspas zammennisse 6 luge:

$$\begin{aligned}
 & \int_0^1 dx \int_0^x (x+y) dy \int_0^{30x^2+60y^2} dz = \int_0^1 dx \int_0^x (x+y) dy (30x^2+60y^2) = \\
 & = 30 \int_0^1 dx \int_0^x (x+y) (x^2+y^2) dy = \\
 & = 30 \int_0^1 dx \int_0^x (x^3+x^2y+2xy^2+2y^3) dy = \\
 & = 30 \int_0^1 dx \left(x^3y + \frac{x^2y^2}{2} + \frac{2xy^3}{3} + \frac{2y^4}{4} \right) \Big|_0^x = \\
 & \text{dit} = 30 \int_0^1 dx \left(x^4 + \frac{x^4}{2} + \frac{2x^4}{3} + \frac{2x^4}{2} \right) = \\
 & = 30 \left(\frac{x^5}{5} + \frac{x^5}{10} + \frac{2x^5}{15} + \frac{x^5}{10} \right) \Big|_0^1 = \\
 & = 30 \cdot \frac{8}{3} \cdot \left(\frac{1}{5} - 0 \right) = \frac{80}{5} = 16
 \end{aligned}$$

Antw.: 16.

1/06
gamm...

$x = 5$

$x^2 =$

$x^2 +$

an...

Känge

x^2

$D = 3$

Ton...

1106 Найти площадь фигуры, ограниченной
данными линиями

$$x = \sqrt{72 - y^2}, \quad 6x = y^2, \quad y = 0 (y \geq 0)$$

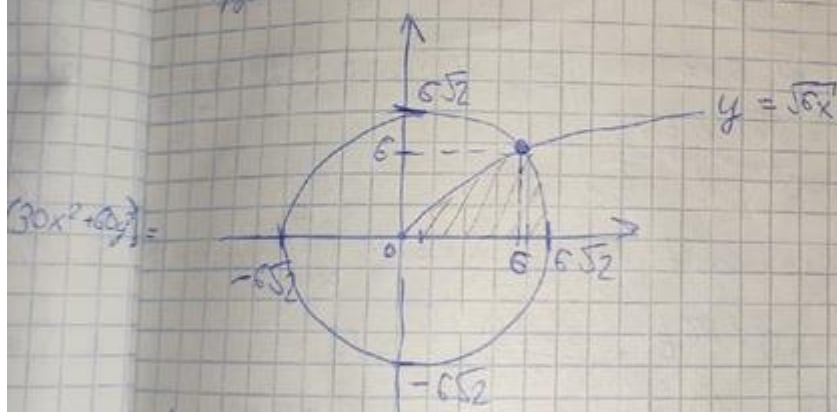
$$x^2 = 72 - y^2$$

$$x^2 + y^2 = 72; \quad y^2 = 6x; \quad y = 0; \quad y \geq 0$$

окружности ($R = \sqrt{72}$) параболы

ось Ox

всё выше
Ох.



Найти и т. пересечение:

$$\begin{cases} y^2 = 6x \\ x^2 + y^2 = 72 \end{cases}$$

$$x^2 + 6x - 72 = 0$$

$$D = 36 + 4 \cdot 72 = (3 \cdot 6)^2$$

$$x_2 = \frac{-6 + 3 \cdot 6}{2} = 6$$

$$x_1 = \frac{-6 - 3 \cdot 6}{2} = -12 - \emptyset, \text{ т.к. } y \geq 0$$

$$\text{Тогда } \begin{cases} x = 6 \\ y = \sqrt{6x} = 6 \end{cases}$$

Область имп. контура:

$$D: \begin{cases} 0 \leq y \leq 6 \\ \frac{y^2}{6} \leq x \leq \sqrt{72-y^2} \end{cases}$$

Площадь:

$$\begin{aligned} S &= \int_0^6 \int_{\frac{y^2}{6}}^{\sqrt{72-y^2}} dx dy = \int_0^6 dy \int_{\frac{y^2}{6}}^{\sqrt{72-y^2}} dx = \\ &= \int_0^6 (\sqrt{72-y^2} - \frac{y^2}{6}) dy = \int_0^6 \sqrt{72-y^2} dy - \frac{y^3}{18} \Big|_0^6 = \\ &= \left[\begin{array}{l} y = \sqrt{72} \sin t \\ dy = \sqrt{72} \cos t dt \\ t_1 = 0, \quad t_2 = \frac{\pi}{4} \end{array} \right] = \int_0^{\frac{\pi}{4}} \sqrt{72} \cdot \cos t \cdot \sqrt{72} \cos t dt - \frac{6^3}{18} = \\ &= \int_0^{\frac{\pi}{4}} 72 \cdot \cos^2 t \cdot dt - \frac{36}{3} = 36 \int_0^{\frac{\pi}{4}} (1 + \cos 2t) dt = \\ &= 12 = 36 \left(t + \frac{\sin 2t}{2} \right) \Big|_0^{\frac{\pi}{4}} - 12 = \\ &= 36 \left(\frac{\pi}{4} + \frac{1}{2} \right) - 12 = 9\pi + 18 - 12 = 9\pi + 6. \end{aligned}$$

Ответ: $9\pi + 6$.

~~12/17~~ №7 Найти область фигуры, ограниченной заданными линиями.

$$x^2 - 4x + y^2 = 0,$$

$$x^2 - 4x + y^2 = 0,$$

$$y=0, \quad y=\sqrt{3}x.$$

$$x^2 - 4x + y^2 = 0$$

$$(x^2 - 2)^2 +$$

$$y=0$$

Введем

$$x = \rho \cos$$

$$x^2 + y^2$$

Заменим

$$(1) \rho^2 = 4$$

$$(2) \rho^2 = 8$$

$$(3) y =$$

$$(4) y$$

$$2\sqrt{3} (3)$$

$$\theta = \frac{\pi}{3}$$

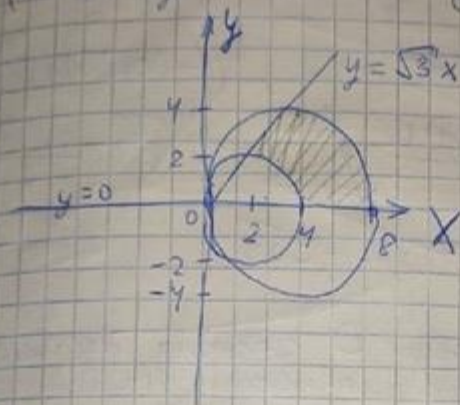
Область

$$x^2 - 4x + 4 + y^2 = 4,$$

$$(x-2)^2 + y^2 = 2^2;$$

$$x^2 - 8x + 16 + y^2 = 16$$

$$(x-4)^2 + y^2 = 4^2$$



$$\cos t \cdot \frac{dt}{dt} =$$

$$-\frac{e^3}{18} =$$

В полярных координатах

2) 1/2 :

$$x = \rho \cos \theta, \quad y = \rho \sin \theta, \Rightarrow$$

$$x^2 + y^2 = \rho^2, \quad dx dy = \rho d\rho d\theta$$

Запишем уравнение окружностей в полярных

$$(1) \rho^2 = 4\rho \cos \theta, \quad \rho = 4 \cos \theta,$$

$$(2) \rho^2 = 16\rho \cos \theta, \quad \rho = 16 \cos \theta;$$

$$(3) y = \sqrt{3}x, \Leftrightarrow \rho \sin \theta = \sqrt{3} \rho \cos \theta$$

$$(4) y = \rho \sin \theta = 0 \quad (\text{линия } \text{при } \theta = 0).$$

$$\text{tg } \theta = \sqrt{3} \Rightarrow \theta = \arctg \sqrt{3} = \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3}$$

Описано интегрирование.

$$D: \begin{cases} 0 \leq \theta \leq \frac{\pi}{3} \\ 4 \cos \theta \leq \rho \leq 16 \cos \theta \end{cases}$$

Тригоном:

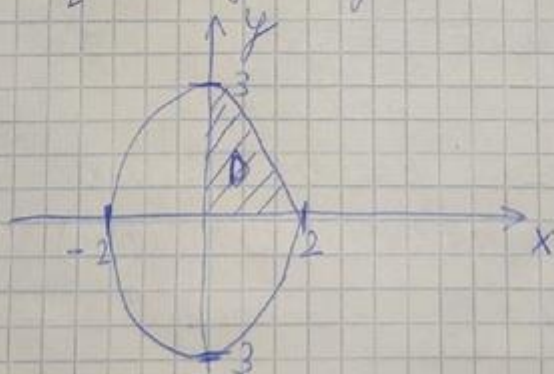
$$\begin{aligned}
 S &= \int_0^{\frac{\pi}{3}} \int_{y \cos \theta}^{8 \cos \theta} dx dy = \int_0^{\frac{\pi}{3}} \int_{y \cos \theta}^{8 \cos \theta} \rho d\rho = \int_0^{\frac{\pi}{3}} \frac{\rho^2}{2} \Big|_{y \cos \theta}^{8 \cos \theta} d\theta \\
 &= 24 \int_0^{\frac{\pi}{3}} \cos^2 \theta d\theta = 12 \int_0^{\frac{\pi}{3}} (1 + \cos 2\theta) d\theta = \\
 &= 12 \left(\theta + \frac{\sin 2\theta}{2} \right) \Big|_0^{\frac{\pi}{3}} = 12 \left(\frac{\pi}{3} + \frac{\sin \frac{2\pi}{3}}{2} \right) = \\
 &= 12 \left(\frac{\pi}{3} + \frac{\sqrt{3}}{4} \right) = 4\pi + 3\sqrt{3}
 \end{aligned}$$

№9 Тригонометрия D задана неравенствами x - полярными координатами. Найти массу, m - масса.

$$D: \frac{x^2}{4} + \frac{y^2}{9} \leq 1; \quad - \text{эллипс}$$

$$x \geq 0, y \geq 0;$$

$$m = \iint_D x^5 y^5 dx dy$$



Перейдем к полярным координатам

$$x = 2R \cdot \cos \varphi, \quad y = 3R \cos \varphi$$

$$\frac{x^2}{9} + \frac{y^2}{9} = 1 \Rightarrow R^2 = 1; R = 1$$

Плотность:

$$\rho = 32 \cdot R^5 = \cos^5 \varphi \cdot 3R \cdot \sin \varphi = 96 R^6 \cos^5 \varphi \sin \varphi$$

Масса:

$$0 \leq \varphi \leq \frac{\pi}{2}$$

$$0 \leq R \leq 1$$

$$M = \int_0^{\frac{\pi}{2}} d\varphi \int_0^1 96 R^6 \cdot \cos^5 \varphi \cdot \sin \varphi \cdot 6R \cdot dR =$$

$$= \int_0^{\frac{\pi}{2}} d\varphi \int_0^1 \frac{96 \cdot 6 \cdot \cos^5 \varphi \cdot \sin \varphi \cdot 1^8}{8} =$$

$$= (-72) \int_0^{\frac{\pi}{2}} \cos^5 \varphi d(\cos \varphi) = -\frac{72}{6} \cdot \cos^6 \varphi \Big|_0^{\frac{\pi}{2}} =$$

$$= -12 \cdot (0 - 1) = 12.$$

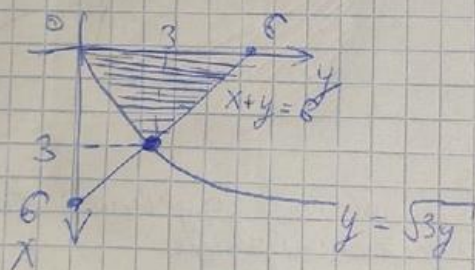
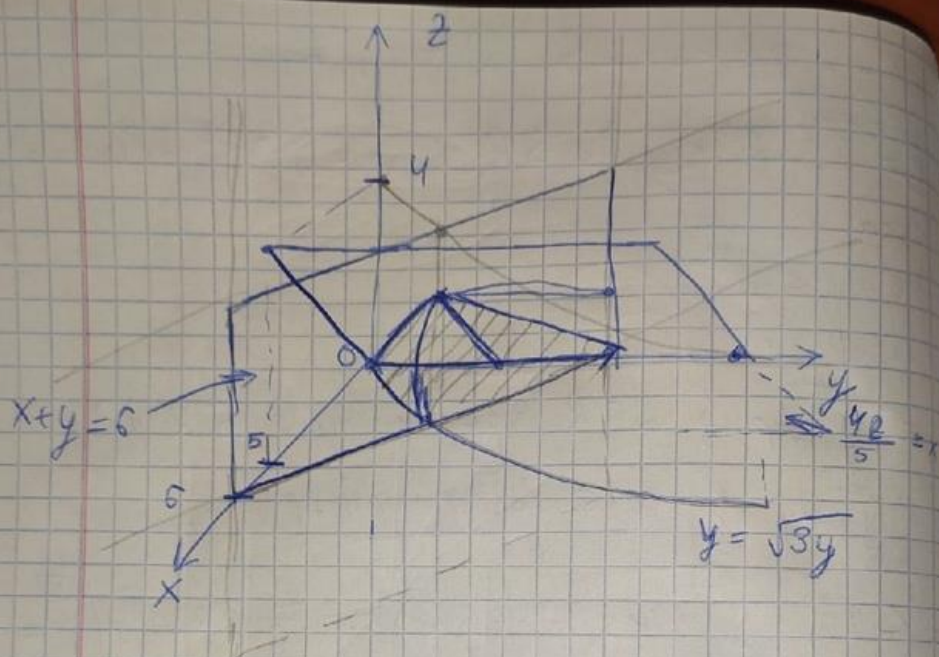
Ответ: 12.

110. Найти объем тела, заданного уравнением его поверхности

$$x + y = 6, \quad x = \sqrt{3y}$$

$$z = 4x/5, \quad z = 0$$

Упростите выражение:



~~18~~ ТЛ-ка пересечения:

$$\begin{cases} y = \sqrt{3x} & x = \sqrt{3y} \\ x + y = 6 \end{cases} \quad x + \frac{x^2}{3} = 6 \quad x^2 + 3x - 18 = 0$$

$$D = 9 + 4 \cdot 18 = 81 = 9^2$$

$$x_2 = \emptyset \quad x_1 = -\frac{3+9}{2} = 3 \quad (x \geq 0)$$

$$y = 3$$

Объем

$V =$

Объем

$V =$

$$= \int_0^3$$

$$= \frac{4}{5}$$

$$= \frac{4}{5}$$

$$= \frac{4}{5}$$

$$= \frac{4}{5}$$

Объем

№1

улан

Объемы тел:

$$V: \begin{cases} 0 \leq x \leq 3 \\ \frac{x^2}{3} \leq y \leq 6-x \\ 0 \leq z \leq \frac{4x}{5} \end{cases}$$

Объем тела:

$$\begin{aligned} V &= \iiint_V dx dy dz = \int_0^3 dx \int_{\frac{x^2}{3}}^{6-x} dy \int_0^{\frac{4x}{5}} dz = \\ &= \int_0^3 dx \int_{\frac{x^2}{3}}^{6-x} \frac{4x}{5} dy = \frac{4}{5} \int_0^3 x dx \left((6-x) - \frac{x^2}{3} \right) = \\ &= \frac{4}{5} \int_0^3 x \left(6-x-\frac{x^2}{3} \right) dx = \frac{4}{5} \int_0^3 \left(6x - x^2 - \frac{x^3}{3} \right) dx = \\ &= \frac{4}{5} \cdot \left(\frac{6x^2}{2} - \frac{x^3}{3} - \frac{x^4}{12} \right) \Big|_0^3 = \\ &= \frac{4}{5} \cdot \left(3 \cdot 9 - \frac{27}{3} - \frac{27 \cdot 3}{4 \cdot 3} \right) = \frac{4}{5} \cdot 9 \left(3 - 1 - \frac{3}{4} \right) = \\ &= \frac{4 \cdot 9}{5} \left(2 - \frac{3}{4} \right) = 9. \end{aligned}$$

Ответ: 9.

№11. Найти объем тела, заданного границей, указанной его проекциями.

$$x^2 + y^2 = 4\sqrt{2}y$$

$$x^2 + y^2 = 9x, \quad x^2 + y^2 = 12x,$$

$$z = \sqrt{x^2 + y^2}, \quad z = 0, \quad y = 0 \quad (y \geq 0)$$

Вычисляем интеграл

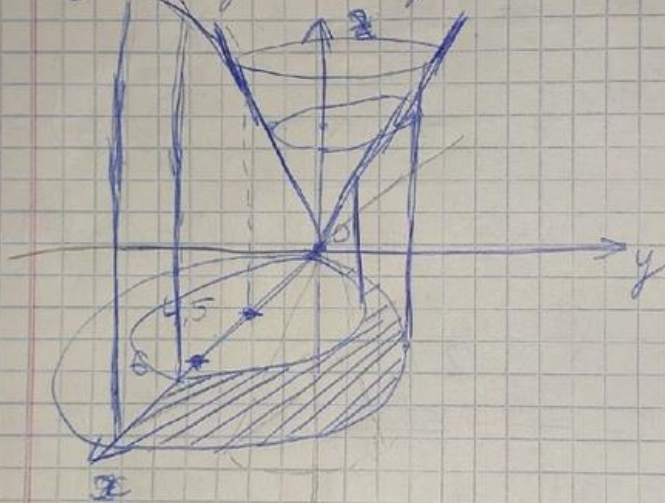
$$x^2 - 9x + \left(\frac{9}{2}\right)^2 + y^2 = \left(\frac{9}{2}\right)^2; \quad x^2 - 12x + 36 + y^2 = 0$$

$$\left(x - \frac{9}{2}\right)^2 + y^2 = \left(\frac{9}{2}\right)^2; \quad (x - 6)^2 + y^2 = 0$$

центр в OXY

центр в OXY

$$z = \sqrt{x^2 + y^2} - \text{конус}$$



В полярных координатах:

$$\rho^2 = 9\rho \cos \theta, \quad \rho = 9 \cos \theta;$$

$$\rho^2 = 12\rho \cos \theta; \quad \rho = 12 \cos \theta; \quad z = \rho - \text{конус}$$

Объем вычитаем:

$$V: \int_0^{\frac{\pi}{6}} \int_{9 \cos \theta}^{12 \cos \theta} \rho \, d\rho \, d\theta$$

Объем

$$V = \int$$

$$= \int_0^{\frac{\pi}{6}} \int_{9 \cos \theta}^{12 \cos \theta} \rho \, d\rho \, d\theta$$

$$= \int_0^{\frac{\pi}{6}} \left[\frac{\rho^2}{2} \right]_{9 \cos \theta}^{12 \cos \theta} d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{2} (144 \cos^2 \theta - 81 \cos^2 \theta) d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{1}{2} (63 \cos^2 \theta) d\theta$$

$$= \frac{63}{4} \int_0^{\frac{\pi}{6}} \cos^2 \theta d\theta$$

$$= \frac{63}{4} \cdot \frac{1}{2} \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{6}}$$

$$= \frac{63}{8} \left[\frac{\pi}{6} + \frac{\sin \frac{\pi}{3}}{2} \right]$$

$$= \frac{63}{8} \left[\frac{\pi}{6} + \frac{\sqrt{3}}{4} \right]$$

Ответ

№13

курса

x

Объем

V:

Объем:

$$\begin{aligned}
 V &= \iiint_V dx dy dz = \int_0^{\frac{\sqrt{12}}{2}} d\theta \int_{9\cos\theta}^{12\cos\theta} \rho d\rho \int_0^{\frac{\sqrt{12}}{2}} \frac{1}{\rho} d\theta = \\
 &= \int_0^{\frac{\sqrt{12}}{2}} d\theta \int_{9\cos\theta}^{12\cos\theta} \rho^2 d\rho = \int_0^{\frac{\sqrt{12}}{2}} \frac{\rho^3}{3} \Big|_{9\cos\theta}^{12\cos\theta} d\theta = \\
 &= \frac{1}{3} \int_0^{\frac{\sqrt{12}}{2}} (12^3 \cos^3 \theta - 9^3 \cos^3 \theta) d\theta = \\
 &= \frac{12^3 - 9^3}{3} \int_0^{\frac{\sqrt{12}}{2}} \cos^3 \theta d\theta = (12^2 \cdot 4 - 9^2 \cdot 3) \times \\
 &\times \int_0^{\frac{\sqrt{12}}{2}} \cos^2 \theta \cos \theta d\theta = 9(64 - 27) \int_0^{\frac{\sqrt{12}}{2}} (1 - \sin^2 \theta) d\sin \theta = \\
 &= 9 \cdot 37 \left(\sin \theta - \frac{\sin^3 \theta}{3} \right) \Big|_0^{\frac{\sqrt{12}}{2}} = 9 \cdot 37 \left(\sin \frac{\sqrt{12}}{2} - \frac{1}{3} \sin^3 \frac{\sqrt{12}}{2} \right) = \\
 &= 9 \cdot 37 \left(1 - \frac{1}{3} \right) = 9 \cdot 37 \cdot \frac{2}{3} = 222.
 \end{aligned}$$

Ответ: 222.

№13 Найти объем тела, заданного уравнениями его поверхностей:

$$z = \sqrt{36 - x^2 - y^2}, \quad z = 2, \quad x^2 + y^2 = 27.$$

$$R = 3\sqrt{3}$$

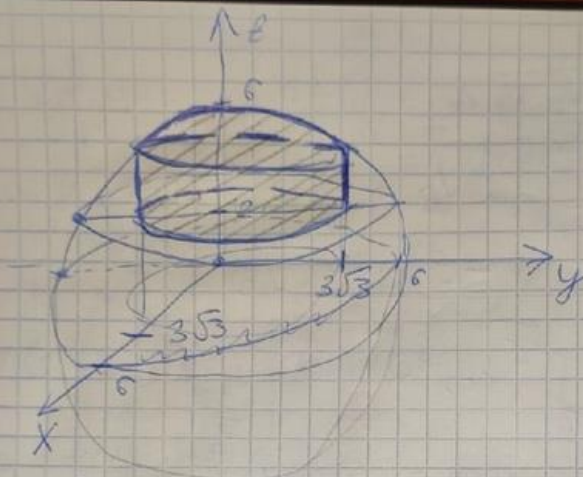
сфера

$$x^2 + y^2 + z^2 = 36$$

цилиндр

Область определения в цилиндрических координатах:

$$V: \begin{cases} 0 \leq \theta \leq 2\pi \\ 0 \leq \rho \leq 3\sqrt{3} \\ 2 \leq z \leq \sqrt{36 - \rho^2} \end{cases}$$



Объем пирамиды

$$V = \int_{3\sqrt{3}}^{2\pi} \int_0^{3\sqrt{3}} \rho d\rho \int_{-3}^{3\sqrt{3}-\rho^2} dz = \left. \frac{1}{2} \rho^2 \right|_0^{3\sqrt{3}} \cdot z \Big|_{-3}^{3\sqrt{3}-\rho^2}$$

$$\times \int_0^{3\sqrt{3}} \rho (36 - \rho^2 - 2) d\rho = 2\pi \left(\int_0^{3\sqrt{3}} (36 - \rho^2) \rho d\rho - \rho^2 \Big|_0^{3\sqrt{3}} \right)$$

$$= \left[\begin{array}{l} t = 36 - \rho^2 \\ dt = -2\rho d\rho \\ t_1 = 36, t_2 = 9 \end{array} \right] \rho d\rho = -\frac{dt}{2}$$

$$= 2\pi \left(-\frac{1}{3} \int_0^9 \sqrt{t} dt - (3\sqrt{3})^2 \right) = 2\pi \left(-\frac{1}{3} \cdot \frac{t^{3/2}}{3/2} \Big|_0^9 - 27 \right)$$

$$= 2\pi \left(-\frac{1}{3} (27 - 0) - 27 \right) = 2\pi \cdot 36 = 72\pi$$

Ответ: 72π .

№15. Найти объем тела, заданного неравенствами

$$49 \leq x$$

$$-\sqrt{\frac{x^2+1}{24}}$$

$$y \geq 0$$

Перемени

$$x = \rho \sin \varphi$$

Тогда

$$7 \leq \rho$$

$$-\sqrt{\frac{\rho^2 \sin^2 \varphi}{24}}$$

$$y = 0, z = 7$$

$$y = \frac{x}{\sqrt{3}}$$

$$0 = \frac{7}{6}$$

$$0 = \frac{7}{6}$$

$$\Rightarrow \frac{7}{6} \leq$$

$$-27$$

Ради

полю

На криво

$$= \sqrt{\frac{\rho^2 \sin^2 \varphi}{\rho^2 \cos^2 \varphi}}$$

$$\beta = \alpha$$

$$49 \leq x^2 + y^2 + z^2 \leq 81;$$

$$-\sqrt{\frac{x^2+y^2}{24}} \leq z \leq 0;$$

$$y \geq 0, y \geq \frac{x}{\sqrt{3}}.$$

Переходим к сферическим координатам:

$$x = \rho \sin \varphi \cos \theta, y = \rho \sin \varphi \sin \theta, z = \rho \cos \varphi$$

Тогда

$$7 \leq \rho \leq 9$$

$$-\sqrt{\frac{\rho^2 \sin^2 \varphi}{24}} \leq \rho \cos \varphi \leq 0$$

$$y=0 \Rightarrow \rho \sin \theta = 0; \theta = 0, \pi$$

$$\rho \sin \theta = \frac{y}{\sqrt{3}}; \rho \sin \theta = \frac{\rho \cos \theta}{\sqrt{3}}; \tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6} \text{ или } \theta = \frac{5\pi}{6}$$

$$\Rightarrow \frac{\pi}{6} \leq \theta \leq \frac{5\pi}{6}$$

2/3

35

10

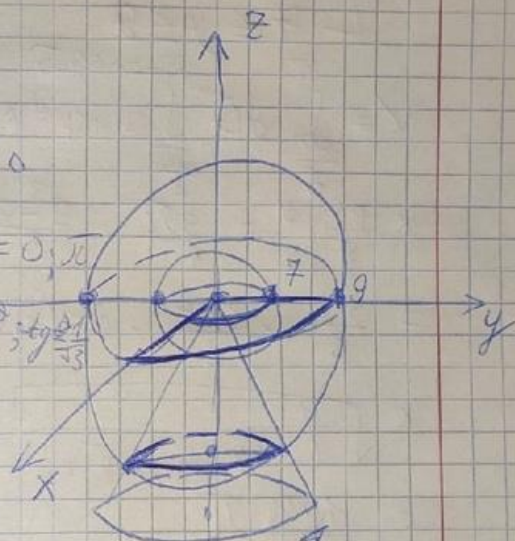
27) Находим $z=0$:

$$\rho \cos \varphi = 0 \Rightarrow \cos \varphi = 0 \Rightarrow \varphi = \frac{\pi}{2}$$

$$\text{Наконец, } \tan \beta = \frac{\sqrt{x^2+y^2}}{z^2} =$$

$$= \sqrt{\frac{\rho^2 \sin^2 \varphi}{\rho^2 \cos^2 \varphi}} = \sqrt{24}$$

$$\beta = \arctg \sqrt{24}; \varphi = \pi - \beta$$



$$\begin{aligned} \text{tg } \varphi &= \text{tg}(\pi - \beta) = \frac{\sin(\pi - \beta)}{\cos(\pi - \beta)} = \frac{\sin \beta}{-\cos \beta} = \\ &= -\text{tg } \beta = -\sqrt{24} \quad ; \quad \varphi = \pi - \arctg \sqrt{24}; \end{aligned}$$

Тогда (один из параметров)

$$V: \begin{cases} \frac{\pi}{6} \leq \theta \leq \pi \\ \frac{\pi}{2} \leq \varphi \leq \pi - \arctg \sqrt{24} \\ 7 \leq \rho \leq 9 \end{cases}$$

$$\begin{aligned} V &= \int_{\frac{\pi}{6}}^{\pi} \int_{\frac{\pi}{2}}^{\pi - \arctg \sqrt{24}} \int_7^9 \sin \varphi \, d\varphi \, d\theta \, d\rho = \theta \left|_{\frac{\pi}{6}}^{\pi} \right| \left(-\cos \varphi \right) \Big|_{\frac{\pi}{2}}^{\pi - \arctg \sqrt{24}} \cdot \rho \Big|_7^9 \\ &= \frac{\pi}{6} \cdot \frac{9^3 - 7^3}{3} \cdot \left(0 - \cos(\pi - \arctg \sqrt{24}) \right) = \\ &= \frac{5\pi}{6} \cdot \frac{386}{3} \cdot \cos(\arctg \sqrt{24}) = \frac{985\pi}{9} \cdot \\ &\times \frac{1}{\sqrt{1 + \text{tg}^2(\arctg \sqrt{24})}} = \frac{985\pi}{9} \cdot \frac{1}{\sqrt{1 + (\sqrt{24})^2}} = \\ &= \frac{193\pi}{9} \end{aligned}$$

Ответ: $\frac{193\pi}{9}$

~~Решение~~