SSV COMPUTATION [FOR A FIXED FREQ]

$$N(s)=\begin{bmatrix}\dfrac{1}{s+1} & \dfrac{1}{s+2}\\ \dfrac{1}{s+3} & \dfrac{1}{s+5}\end{bmatrix}\rightarrow N(j\omega)=\begin{bmatrix}\dfrac{1}{j\omega+1} & \dfrac{1}{j\omega+2}\\ \dfrac{1}{j\omega+3} & \dfrac{1}{j\omega+5}\end{bmatrix}\rightarrow N(j\omega_x)=\begin{bmatrix}\dfrac{1}{j\omega_x+1} & \dfrac{1}{j\omega_x+2}\\ \dfrac{1}{j\omega_x+3} & \dfrac{1}{j\omega_x+5}\end{bmatrix}$$

$$N(j1)=\begin{bmatrix}\dfrac{1}{j1+1} & \dfrac{1}{j1+2}\\ \dfrac{1}{j1+3} & \dfrac{1}{j1+5}\end{bmatrix}$$

<table>
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```
clear all;close all;clc;
%%
N_s=[tf([1],[1,1]),tf([1],[1,2]);
     tf([1],[1,3]),tf([1],[1,5])];
%% compute the matrix for a given freq
Njw1=evalfr(N_s,1*i)
```

</td><td>

```
>> Njw1=evalfr(N_s,1*i)

Njw1 =

   0.5000 - 0.5000i   0.4000 - 0.2000i
   0.3000 - 0.1000i   0.1923 - 0.0385i
```

</td></tr>
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CASE 1: FULL DELTA [UNSTRUCTURED UNCERTAINTY]

$$\Delta = \{\Delta \in \mathbb{C}^{n\times n}\}$$

$$\mathcal{D} = \{D: D\Delta = \Delta D, \forall \Delta \in \boldsymbol{\Delta}\}$$

$$\mathcal{D} = \{dI_n: d \in \mathbb{R}, d > 0\}$$

$$\mu_{\boldsymbol{\Delta}}(M) = \mu_{\boldsymbol{\Delta}}(DMD^{-1})$$

$$\mu_{\boldsymbol{\Delta}}(M) \leq \inf_{D\in\mathcal{D}} \bar{\sigma}(DMD^{-1})$$

Since, there is a suitable parameterization for $\boxed{D \in \mathcal{D}}$, the problem can be expressed more specifically as,

$$\inf_{D\in\mathcal{D}} \bar{\sigma}(DMD^{-1}) = \inf_{D\in\mathcal{D}} \bar{\sigma}(dI_2Md^{-1}I_2)$$

Matlab code

```
%% ADD YALMIP AND SDPT3 LIBRARIES
%% CASE 1: Full Delta Set ---> single d --> d*eye(n), d in REAL
N_s=[tf([1],[1,1]),tf([1],[1,2]);tf([1],[1,3]),tf([1],[1,5])];
%% compute the matrix for a given freq
Njw1=evalfr(N_s,1*i)
%% CASE 1: Full Delta Set ---> single d --> d*eye(n), d in REAL
% BISECTION PROBLEM
g2=sdpvar(1,1,'symmetric');
eps1=1e-5;

D1=sdpvar(1,1,'symmetric');
% D1=sdpvar(2,2,'hermitian','complex');


% X=blkdiag(D1);
X=[D1*eye(2)];

F=[];
F=[F;Njw1'*X*Njw1-g2*X<=-eps1*eye(2)];
F=[F;eps1<=D1];   % D1 is hermitian, complex,pos-def matrix

% ops = sdpsettings('solver','sdpt3');
ops = sdpsettings('solver','bisection','bisection.solver','sdpt3');
sol = bisection(F,[g2],ops);
sol.info

X=value(X)
g2=value(g2)

gamma=sqrt(g2)
YALMIP_SSV=gamma;
%%
rho_N=max(abs(eig(Njw1)));                  % spectral radius of Njw1
sigma_max_N=max(svd(Njw1));                 % simga-max of Njw1

% COMPUTE THE SSV USING MATLAB-MUSSV FCN
% bounds = mussv(N,BlockStructure)
[bounds,muinfo] = mussv(Njw1,[2,2]);
MATLAB_MUSSV=bounds(2);
clc;
fprintf("YALMIP_SSV \t rho_N \t sigma_max_N \t MATLAB_MUSSV\n");
fprintf("%6.4f \t\t %6.4f \t %6.4f \t %6.4f\n",YALMIP_SSV,rho_N,sigma_max_N,MATLAB_MUSSV);
```

Code OUTPUT

```
YALMIP_SSV     rho_N     sigma_max_N     MATLAB_MUSSV
0.9152         0.8680       0.9152       0.9152
```

CASE 2: repeating-scalar-delta [STRUCTURED UNCERTAINTY]

$$\Delta = \{\delta I_n : \delta \in \mathbb{C}\}$$

$$\mathcal{D} = \{D: D\Delta = \Delta D, \forall \Delta \in \mathbf{\Delta}\}$$

$$\mathcal{D} = \{D: D \in \mathbb{C}^{n\times n}, D = D^H > 0\}$$

$$\mu_{\mathbf{\Delta}}(M) = \mu_{\mathbf{\Delta}}(DMD^{-1})$$

$$\mu_{\mathbf{\Delta}}(M) \le \inf_{D\in\mathcal{D}} \bar{\sigma}(DMD^{-1})$$

Since, there is a suitable parameterization for $\boxed{D \in \mathcal{D}}$, the problem can be expressed more specifically as,

$$\mathcal{D} = \left\{D = \begin{bmatrix} d_1 & d_3 + jd_4 \\ d_3 - jd_4 & d_2 \end{bmatrix} : \boxed{d_1, \dots, d_4 \in \mathbb{R}}, \boxed{d_1, \dots, d_4 > 0}\right\}$$

$$\inf_{D\in\mathcal{D}} \bar{\sigma}(DMD^{-1}) = \inf_{d_1,\dots,d_4>0} \bar{\sigma}\left(\begin{bmatrix} d_1 & d_3 + jd_4 \\ d_3 - jd_4 & d_2 \end{bmatrix} M \begin{bmatrix} d_1 & d_3 + jd_4 \\ d_3 - jd_4 & d_2 \end{bmatrix}^{-1}\right)$$

$$\boxed{\begin{array}{ll} \inf\limits_{\substack{D\in\mathcal{D} \\ \gamma\in\mathbb{R}^+}} & \boxed{\gamma} \\ s.t. & \boxed{\bar{\sigma}(DMD^{-1}) < \gamma} \end{array}}$$

FOR SQUARE CASE	FOR RECTANGULAR CASE
$\bar{\sigma}(DMD^{-1}) < \gamma$ $\|DMD^{-1}\|_2 < \gamma$ $\|DMD^{-1}\|_2^2 < \gamma^2$ $\lambda_{max}([DMD^{-1}]^T[DMD^{-1}]) < \gamma^2$ $[DMD^{-1}]^T[DMD^{-1}] < \gamma^2 I$ $[D^{-T}M^T D^T][DMD^{-1}] < \gamma^2 I$ $D^{-T}M^T[D^T D]MD^{-1} < \gamma^2 I$ $D^{-T}M^T[D^T D]M < \gamma^2 ID$ $M^T[D^T D]M < D^T\gamma^2 ID$ $M^T[D^T D]M < \gamma^2[D^T D]$ $X := D$ $M^T XM < \gamma^2 X$ $\boxed{\begin{array}{ll} \inf\limits_{\substack{X\in\mathcal{X} \\ \gamma\in\mathbb{R}^+}} & \boxed{\gamma} \\ s.t. & \boxed{M^T XM < \gamma^2 X} \end{array}}$	$\bar{\sigma}(D_L MD_R^{-1}) < \gamma$ $\left\|D_L MD_R^{-1}\right\|_2 < \gamma$ $\left\|D_L MD_R^{-1}\right\|_2^2 < \gamma^2$ $\lambda_{max}([D_L MD_R^{-1}]^T[D_L MD_R^{-1}]) < \gamma^2$ $[D_L MD_R^{-1}]^T[D_L MD_R^{-1}] < \gamma^2 I$ $[D_R^{-T}M^T D_L^T][D_L MD_R^{-1}] < \gamma^2 I$ $D_R^{-T}M^T[D_L^T D_L]MD_R^{-1} < \gamma^2 I$ $D_R^{-T}M^T[D_L^T D_L]M < \gamma^2 ID_R$ $M^T[D_L^T D_L]M < D_R^T\gamma^2 ID_R$ $M^T[D_L^T D_L]M < \gamma^2[D_R^T D_R]$ $X_L := [D_L^T D_L], X_R := [D_R^T D_R]$ $M^T X_L M < \gamma^2 X_R$ $\boxed{\begin{array}{ll} \inf\limits_{\substack{(X_L,X_R)\in\mathcal{X} \\ \gamma\in\mathbb{R}^+}} & \boxed{\gamma} \\ s.t. & \boxed{M^T X_L M < \gamma^2 X_R} \end{array}}$

Let us determine the structure of X

$$X = D^H D = [D]^T[D] = [D^H D] \rightarrow [\boldsymbol{D}], \boxed{\boldsymbol{D} \in \mathbb{R}, \boldsymbol{D} = \boldsymbol{D^H} > 0}$$

Matlab code

```
%% ADD YALMIP AND SDPT3 LIBRARIES
%% CASE 2: Structured Delta Set [delta*eye(n)] ---> FULL D,hermitian(like symmetric),complex
N_s=[tf([1],[1,1]),tf([1],[1,2]);tf([1],[1,3]),tf([1],[1,5])];
%% compute the matrix for a given freq
Njw1=evalfr(N_s,1*i)
%% CASE 2: Structured Delta Set [delta*eye(n)] ---> FULL D,hermitian(like symmetric),complex
% BISECTION PROBLEM
g2=sdpvar(1,1,'symmetric');
eps1=1e-5;

% D1=sdpvar(1,1,'symmetric');
D1=sdpvar(2,2,'hermitian','complex');


% X=blkdiag(D1,D2);
X=[D1];

F=[];
F=[F;Njw1'*X*Njw1-g2*X<=-eps1*eye(2)];
F=[F;eps1*eye(2)<=D1];   % D1 is hermitian, complex,pos-def matrix

% ops = sdpsettings('solver','sdpt3');
ops = sdpsettings('solver','bisection','bisection.solver','sdpt3');
sol = bisection(F,[g2],ops);
sol.info

X=value(X)
g2=value(g2)

gamma=sqrt(g2)
YALMIP_SSV=gamma;
%%
rho_N=max(abs(eig(Njw1)));                   % spectral radius of Njw1
sigma_max_N=max(svd(Njw1));                  % simga-max of Njw1

% COMPUTE THE SSV USING MATLAB-MUSSV FCN
% bounds = mussv(N,BlockStructure)
[bounds,muinfo] = mussv(Njw1,[2,0]);
MATLAB_MUSSV=bounds(2);
clc;
fprintf("YALMIP_SSV \t rho_N \t sigma_max_N \t MATLAB_MUSSV\n");
fprintf("%6.4f \t\t %6.4f \t %6.4f \t %6.4f\n",YALMIP_SSV,rho_N,sigma_max_N,MATLAB_MUSSV);
```

Code OUTPUT

```
YALMIP_SSV    rho_N    sigma_max_N    MATLAB_MUSSV
0.8680        0.8680      0.9152      0.8680
```

CASE 3: diagonal-delta [STRUCTURED UNCERTAINTY]

$$\mathbf{\Delta} = \{\text{diag}[\delta_1, \delta_2] : \delta_1, \delta_2 \in \mathbb{C}\}$$

$$\mathcal{D} = \{D : D\Delta = \Delta D, \forall \Delta \in \mathbf{\Delta}\}$$

$$\mathcal{D} = \left\{\text{diag}[\text{d}_1, \text{d}_2] : \boxed{\text{d}_1, \text{d}_2 \in \mathbb{R}}, \boxed{\text{d}_1, \text{d}_2 > 0}\right\}$$

$$\mu_{\mathbf{\Delta}}(M) = \mu_{\mathbf{\Delta}}(DMD^{-1})$$

$$\mu_{\mathbf{\Delta}}(M) \le \inf_{D \in \mathcal{D}} \bar{\sigma}(DMD^{-1})$$

Since, there is a suitable parameterization for $\boxed{D \in \mathcal{D}}$, the problem can be expressed more specifically as,

$$\mathcal{D} = \left\{D = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} : \boxed{d_1, d_2 \in \mathbb{R}}, \boxed{d_1, d_2 > 0}\right\}$$

$$\inf_{D \in \mathcal{D}} \bar{\sigma}(DMD^{-1}) = \inf_{d_1, d_2 > 0} \bar{\sigma}\left(\begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} M \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}^{-1}\right)$$

$$\boxed{\begin{array}{ll} \inf\limits_{\substack{D \in \mathcal{D} \\ \gamma \in \mathbb{R}^+}} & \boxed{\gamma} \\ s.t. & \boxed{\bar{\sigma}(DMD^{-1}) < \gamma} \end{array}}$$

FOR SQUARE CASE	FOR RECTANGULAR CASE
$\bar{\sigma}(DMD^{-1}) < \gamma$ $\lVert DMD^{-1} \rVert_2 < \gamma$ $\lVert DMD^{-1} \rVert_2^2 < \gamma^2$ $\lambda_{max}([DMD^{-1}]^T [DMD^{-1}]) < \gamma^2$ $[DMD^{-1}]^T [DMD^{-1}] < \gamma^2 I$ $[D^{-T} M^T D^T][DMD^{-1}] < \gamma^2 I$ $D^{-T} M^T [D^T D] M D^{-1} < \gamma^2 I$ $D^{-T} M^T [D^T D] M < \gamma^2 I D$ $M^T [D^T D] M < D^T \gamma^2 I D$ $M^T [D^T D] M < \gamma^2 [D^T D]$ $X := D$ $M^T X M < \gamma^2 X$ $\boxed{\begin{array}{ll} \inf\limits_{\substack{X \in \mathcal{X} \\ \gamma \in \mathbb{R}^+}} & \boxed{\gamma} \\ s.t. & \boxed{M^T X M < \gamma^2 X} \end{array}}$	$\bar{\sigma}(D_L M D_R^{-1}) < \gamma$ $\lVert D_L M D_R^{-1} \rVert_2 < \gamma$ $\lVert D_L M D_R^{-1} \rVert_2^2 < \gamma^2$ $\lambda_{max}([D_L M D_R^{-1}]^T [D_L M D_R^{-1}]) < \gamma^2$ $[D_L M D_R^{-1}]^T [D_L M D_R^{-1}] < \gamma^2 I$ $[D_R^{-T} M^T D_L^T][D_L M D_R^{-1}] < \gamma^2 I$ $D_R^{-T} M^T [D_L^T D_L] M D_R^{-1} < \gamma^2 I$ $D_R^{-T} M^T [D_L^T D_L] M < \gamma^2 I D_R$ $M^T [D_L^T D_L] M < D_R^T \gamma^2 I D_R$ $M^T [D_L^T D_L] M < \gamma^2 [D_R^T D_R]$ $X_L := [D_L^T D_L], X_R := [D_R^T D_R]$ $M^T X_L M < \gamma^2 X_R$ $\boxed{\begin{array}{ll} \inf\limits_{\substack{(X_L, X_R) \in \mathcal{X} \\ \gamma \in \mathbb{R}^+}} & \boxed{\gamma} \\ s.t. & \boxed{M^T X_L M < \gamma^2 X_R} \end{array}}$

Let us determine the structure of X

$$X = D^H D = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}^T \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} = \begin{bmatrix} (d_1)^2 & \\ & (d_2)^2 \end{bmatrix} \to \begin{bmatrix} \boldsymbol{D_1} & \\ & \boldsymbol{D_2} \end{bmatrix}, \begin{array}{c} \boxed{\boldsymbol{D_1} \in \mathbb{R}, \boldsymbol{D_1} > 0} \\ \boxed{\boldsymbol{D_2} \in \mathbb{R}, \boldsymbol{D_2} > 0} \end{array}$$

Matlab code

```
%% CASE 3: Structured Delta Set diag[delta1,delta2] ---> diag[d1,d2], d1,d2 REAL
N_s=[tf([1],[1,1]),tf([1],[1,2]);tf([1],[1,3]),tf([1],[1,5])];
%% compute the matrix for a given freq
Njw1=evalfr(N_s,1*i)
%% CASE 3: Structured Delta Set diag[delta1,delta2] ---> diag[d1,d2], d1,d2 REAL
% BISECTION PROBLEM
g2=sdpvar(1,1,'symmetric');
eps1=1e-5;

D1=sdpvar(1,1,'symmetric');
% D1=sdpvar(1,1,'hermitian','complex');
D2=sdpvar(1,1,'symmetric');
% D2=sdpvar(1,1,'hermitian','complex');

% X=blkdiag(D1,D2);
X=diag([D1,D2]);

F=[];
F=[F;Njw1'*X*Njw1-g2*X<=-eps1*eye(2)];
F=[F;eps1<=D1<=1e3];
F=[F;eps1<=D2<=1e3];

% ops = sdpsettings('solver','sdpt3');
ops = sdpsettings('solver','bisection','bisection.solver','sdpt3');
sol = bisection(F,[g2],ops);
sol.info

X=value(X)
g2=value(g2)

gamma=sqrt(g2)
YALMIP_SSV=gamma;
%%
rho_N=max(abs(eig(Njw1)));                    % spectral radius of Njw1
sigma_max_N=max(svd(Njw1));                   % simga-max of Njw1

% COMPUTE THE SSV USING MATLAB-MUSSV FCN
% bounds = mussv(N,BlockStructure)
[bounds,muinfo] = mussv(Njw1,[1,1;1,1]);
MATLAB_MUSSV=bounds(2);
clc;
fprintf("YALMIP_SSV \t rho_N \t sigma_max_N \t MATLAB_MUSSV\n");
fprintf("%6.4f \t\t %6.4f \t %6.4f \t %6.4f\n",YALMIP_SSV,rho_N,sigma_max_N,MATLAB_MUSSV);
```

Code OUTPUT		
	YALMIP_SSV rho_N sigma_max_N MATLAB_MUSSV 0.9057 0.8680 0.9152 0.9057	>> X X = 664.8191 0 0 940.4891

CASE 4: MIXED CASE [SQUARE CASE]

SSV COMPUTATION [FOR A FIXED FREQ]

$$N(s)=\begin{bmatrix}\frac{1}{s+1} & \frac{1}{s+2} & \frac{1}{s+3} & \frac{1}{s+3} & \frac{1}{s+4}\\ \frac{1}{s+5} & \frac{1}{s+1} & \frac{1}{s+2} & \frac{1}{s+3} & \frac{1}{s+4}\\ \frac{1}{s+2} & \frac{1}{s+4} & \frac{1}{s+7} & \frac{1}{s+3} & \frac{1}{s+4}\\ \frac{1}{s+2} & \frac{1}{s+4} & \frac{1}{s+7} & \frac{1}{s+3} & \frac{1}{s+4}\\ \frac{1}{s+2} & \frac{1}{s+4} & \frac{1}{s+7} & \frac{1}{s+3} & \frac{1}{s+4}\end{bmatrix} \rightarrow N(j\omega)=\begin{bmatrix}\frac{1}{j\omega+1} & \frac{1}{j\omega+2} & \frac{1}{j\omega+3} & \frac{1}{j\omega+3} & \frac{1}{j\omega+4}\\ \frac{1}{j\omega+5} & \frac{1}{j\omega+1} & \frac{1}{j\omega+2} & \frac{1}{j\omega+3} & \frac{1}{j\omega+4}\\ \frac{1}{j\omega+2} & \frac{1}{j\omega+4} & \frac{1}{j\omega+7} & \frac{1}{j\omega+3} & \frac{1}{j\omega+4}\\ \frac{1}{j\omega+2} & \frac{1}{j\omega+4} & \frac{1}{j\omega+7} & \frac{1}{j\omega+3} & \frac{1}{j\omega+4}\\ \frac{1}{j\omega+2} & \frac{1}{j\omega+4} & \frac{1}{j\omega+7} & \frac{1}{j\omega+3} & \frac{1}{j\omega+4}\end{bmatrix}$$

$$N(j1)=\begin{bmatrix}\frac{1}{j1+1} & \frac{1}{j1+2} & \frac{1}{j1+3} & \frac{1}{j1+3} & \frac{1}{j1+4}\\ \frac{1}{j1+5} & \frac{1}{j1+1} & \frac{1}{j1+2} & \frac{1}{j1+3} & \frac{1}{j1+4}\\ \frac{1}{j1+2} & \frac{1}{j1+4} & \frac{1}{j1+7} & \frac{1}{j1+3} & \frac{1}{j1+4}\\ \frac{1}{j1+2} & \frac{1}{j1+4} & \frac{1}{j1+7} & \frac{1}{j1+3} & \frac{1}{j1+4}\\ \frac{1}{j1+2} & \frac{1}{j1+4} & \frac{1}{j1+7} & \frac{1}{j1+3} & \frac{1}{j1+4}\end{bmatrix}$$

CASE 4: MIXED CASE [SQUARE CASE]

$$\mathbf{\Delta} = \left\{ \begin{bmatrix} \delta_1 I_2 & & \\ & \delta_2 & \\ & & \Delta_3 \end{bmatrix} : \delta_1, \delta_2 \in \mathbb{C}, \Delta_3 \in \mathbb{C}^{2\times 2} \right\}, \mathcal{D} = \{D : D\Delta = \Delta D, \forall \Delta \in \mathbf{\Delta}\}$$

$$\mathcal{D} = \left\{ \begin{bmatrix} D_1 & & \\ & D_2 & \\ & & D_3 I_2 \end{bmatrix} : \boxed{D_1 \in \mathbb{C}^{2\times 2}, D_1 = D_1^H > 0}, \boxed{D_2 \in \mathbb{R}, D_2 > 0}, \boxed{D_3 \in \mathbb{R}, D_3 > 0} \right\}$$

$$\mu_{\mathbf{\Delta}}(M) = \mu_{\mathbf{\Delta}}(DMD^{-1}) \to \mu_{\mathbf{\Delta}}(M) \le \inf_{D \in \mathcal{D}} \bar{\sigma}(DMD^{-1})$$

Since, there is a suitable parameterization for $\boxed{D \in \mathcal{D}}$, the problem can be expressed more specifically as,

$$\mathcal{D} = \left\{ \begin{bmatrix} \begin{bmatrix} d_1 & d_3 + jd_4 \\ d_3 - jd_4 & d_2 \end{bmatrix} & & \\ & d_5 & \\ & & d_6 I_2 \end{bmatrix} : \boxed{d_1, \dots, d_6 \in \mathbb{R}}, \boxed{d_1, \dots, d_6 > 0} \right\}$$

$$\inf_{D \in \mathcal{D}} \bar{\sigma}(DMD^{-1}) = \inf_{d_1, \dots, d_6 > 0} \bar{\sigma}(DMD^{-1})$$

$$\boxed{\begin{array}{ll} \inf\limits_{\substack{D \in \mathcal{D} \\ \gamma \in \mathbb{R}^+}} & \boxed{\gamma} \\ s.t. & \boxed{\bar{\sigma}(DMD^{-1}) < \gamma} \end{array}}$$

FOR SQUARE CASE	FOR RECTANGULAR CASE
$\bar{\sigma}(DMD^{-1}) < \gamma$ $\Vert DMD^{-1}\Vert_2 < \gamma$ $\Vert DMD^{-1}\Vert_2^2 < \gamma^2$ $\lambda_{max}([DMD^{-1}]^T[DMD^{-1}]) < \gamma^2$ $[DMD^{-1}]^T[DMD^{-1}] < \gamma^2 I$ $[D^{-T}M^T D^T][DMD^{-1}] < \gamma^2 I$ $D^{-T}M^T[D^T D]MD^{-1} < \gamma^2 I$ $D^{-T}M^T[D^T D]M < \gamma^2 ID$ $M^T[D^T D]M < D^T\gamma^2 ID$ $M^T[D^T D]M < \gamma^2[D^T D]$ $X \coloneqq D$ $M^T XM < \gamma^2 X$ $\boxed{\begin{array}{ll} \inf\limits_{\substack{X \in \mathcal{X} \\ \gamma \in \mathbb{R}^+}} & \boxed{\gamma} \\ s.t. & \boxed{M^T XM < \gamma^2 X} \end{array}}$	$\bar{\sigma}(D_L MD_R^{-1}) < \gamma$ $\Vert D_L MD_R^{-1}\Vert_2 < \gamma$ $\Vert D_L MD_R^{-1}\Vert_2^2 < \gamma^2$ $\lambda_{max}([D_L MD_R^{-1}]^T[D_L MD_R^{-1}]) < \gamma^2$ $[D_L MD_R^{-1}]^T[D_L MD_R^{-1}] < \gamma^2 I$ $[D_R^{-T}M^T D_L^T][D_L MD_R^{-1}] < \gamma^2 I$ $D_R^{-T}M^T[D_L^T D_L]MD_R^{-1} < \gamma^2 I$ $D_R^{-T}M^T[D_L^T D_L]M < \gamma^2 ID_R$ $M^T[D_L^T D_L]M < D_R^T\gamma^2 ID_R$ $M^T[D_L^T D_L]M < \gamma^2[D_R^T D_R]$ $X_L \coloneqq [D_L^T D_L], X_R \coloneqq [D_R^T D_R]$ $M^T X_L M < \gamma^2 X_R$ $\boxed{\begin{array}{ll} \inf\limits_{\substack{(X_L, X_R) \in \mathcal{X} \\ \gamma \in \mathbb{R}^+}} & \boxed{\gamma} \\ s.t. & \boxed{M^T X_L M < \gamma^2 X_R} \end{array}}$

Let us determine the structure of X

$$X = D^H D = \begin{bmatrix} D_1 & & \\ & D_2 & \\ & & D_3 I_2 \end{bmatrix}^T \begin{bmatrix} D_1 & & \\ & D_2 & \\ & & D_3 I_2 \end{bmatrix} = \begin{bmatrix} D_1^H D_1 & & \\ & (D_2)^2 & \\ & & (D_3)^2 I_2 \end{bmatrix} \to \begin{bmatrix} \boldsymbol{D_1} & & \\ & \boldsymbol{D_2} & \\ & & \boldsymbol{D_3} I_2 \end{bmatrix}, \begin{matrix} \boxed{\boldsymbol{D_1} \in \mathbb{C}^{2\times 2}, \boldsymbol{D_1} = \boldsymbol{D_1^H} > 0} \\ \boxed{\boldsymbol{D_2} \in \mathbb{R}, \boldsymbol{D_2} > 0} \\ \boxed{\boldsymbol{D_3} \in \mathbb{R}, \boldsymbol{D_3} > 0} \end{matrix}$$

MATLAB code

```
%% ADD YALMIP AND SDPT3 LIBRARIES
%% CASE 4:  DELTA STRUCTURE = blkdiag([delta1*eye(2),delta2,DELTA_3]):delta1,delta2 in COMPLEX, DELTA_3 in 2x2 COMPLEX MATRIX
N_s=[tf([1],[1,1]),tf([1],[1,2]),tf([1],[1,3]),tf([1],[1,3]),tf([1],[1,4]);
     tf([1],[1,5]),tf([1],[1,1]),tf([1],[1,2]),tf([1],[1,3]),tf([1],[1,4]);
     tf([1],[1,2]),tf([1],[1,4]),tf([1],[1,7]),tf([1],[1,3]),tf([1],[1,4]);
     tf([1],[1,2]),tf([1],[1,4]),tf([1],[1,7]),tf([1],[1,3]),tf([1],[1,4]);
     tf([1],[1,2]),tf([1],[1,4]),tf([1],[1,7]),tf([1],[1,3]),tf([1],[1,4])];
%% compute the matrix for a given freq
Njw1=evalfr(N_s,1*i)
%% CASE 4:  DELTA STRUCTURE = blkdiag([delta1*eye(2),delta2,DELTA_3]):delta1,delta2 in COMPLEX, DELTA_3 in 2x2 COMPLEX MATRIX
% BISECTION PROBLEM
g2=sdpvar(1,1,'symmetric');
eps1=1e-3;

D1=sdpvar(2,2,'hermitian','complex');
D2=sdpvar(1,1,'symmetric');
D3=sdpvar(1,1,'symmetric');

X=blkdiag(D1,D2,D3*eye(2));

F=[];
F=[F;Njw1'*X*Njw1-g2*X<=-eps1*eye(5)];

% ops = sdpsettings('solver','sdpt3');
ops = sdpsettings('solver','bisection','bisection.solver','sdpt3');
sol = bisection(F,[g2],ops);
sol.info

X=value(X)
g2=value(g2)

gamma=sqrt(g2)
YALMIP_SSV=gamma;
%%
rho_N=max(abs(eig(Njw1)));                   % spectral radius of Njw1
sigma_max_N=max(svd(Njw1));                  % simga-max of Njw1

% COMPUTE THE SSV USING MATLAB-MUSSV FCN
% bounds = mussv(N,BlockStructure)
[bounds,muinfo] = mussv(Njw1,[2,0;1,1;2,2]);
MATLAB_MUSSV=bounds(2);
clc;
fprintf("YALMIP_SSV \t rho_N \t sigma_max_N \t MATLAB_MUSSV\n");
fprintf("%6.4f \t\t %6.4f \t %6.4f \t %6.4f\n",YALMIP_SSV,rho_N,sigma_max_N,MATLAB_MUSSV);
```

Code OUTPUT	YALMIP_SSV	rho_N	sigma_max_N	MATLAB_MUSSV
	1.6757	1.6313	1.6797	1.6757

CASE 5: MIXED CASE [RECTANGULAR CASE]

SSV COMPUTATION [FOR A FIXED FREQ]

$$N(s)=\begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+2} & \frac{1}{s+3} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+4} \\ \frac{1}{s+5} & \frac{1}{s+1} & \frac{1}{s+2} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+4} \\ \frac{1}{s+2} & \frac{1}{s+4} & \frac{1}{s+7} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+2} \\ \frac{1}{s+2} & \frac{1}{s+4} & \frac{1}{s+7} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+4} \\ \frac{1}{s+2} & \frac{1}{s+4} & \frac{1}{s+7} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+3} \\ \frac{1}{s+5} & \frac{1}{s+1} & \frac{1}{s+2} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+4} \\ \frac{1}{s+2} & \frac{1}{s+4} & \frac{1}{s+7} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+4} \end{bmatrix}$$

$$N(j1)=\begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+2} & \frac{1}{s+3} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+4} \\ \frac{1}{s+5} & \frac{1}{s+1} & \frac{1}{s+2} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+4} \\ \frac{1}{s+2} & \frac{1}{s+4} & \frac{1}{s+7} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+2} \\ \frac{1}{s+2} & \frac{1}{s+4} & \frac{1}{s+7} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+4} \\ \frac{1}{s+2} & \frac{1}{s+4} & \frac{1}{s+7} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+3} \\ \frac{1}{s+5} & \frac{1}{s+1} & \frac{1}{s+2} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+4} \\ \frac{1}{s+2} & \frac{1}{s+4} & \frac{1}{s+7} & \frac{1}{s+3} & \frac{1}{s+4} & \frac{1}{s+3} & \frac{1}{s+4} \end{bmatrix}_{s\leftarrow j1}$$

CASE 5: MIXED CASE [RECTANGULAR CASE]

$$\mathbf{\Delta} = \left\{ \begin{bmatrix} \delta_1 I_2 & & & \\ & \delta_2 & & \\ & & \Delta_3 & \\ & & & \Delta_4 \end{bmatrix} : \begin{matrix} \boxed{\delta_1 \in \mathbb{C}} \\ \boxed{\delta_2 \in \mathbb{C}} \\ \boxed{\Delta_3 \in \mathbb{C}^{2\times 3}} \\ \boxed{\Delta_4 \in \mathbb{C}^{2\times 1}} \end{matrix} \right\}, \mathcal{D} = \{D : D\Delta = \Delta D, \forall \Delta \in \mathbf{\Delta}\}$$

$$\mathcal{D} = \{\mathcal{D}_L, \mathcal{D}_R\} = \left\{ D_L = \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & D_3 I_3 & \\ & & & D_4 \end{bmatrix}, D_R = \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & D_3 I_2 & \\ & & & D_4 I_2 \end{bmatrix} : \begin{matrix} \boxed{D_1 \in \mathbb{C}^{2\times 2}, D_1 = D_1^H > 0} \\ \boxed{D_2 \in \mathbb{R}, D_2 > 0} \\ \boxed{D_3 \in \mathbb{R}, D_3 > 0} \\ \boxed{D_4 \in \mathbb{R}, D_4 > 0} \end{matrix} \right\}$$

$$\mathcal{D} = \{\mathcal{D}_L, \mathcal{D}_R\} = \left\{ D_L = \begin{bmatrix} \boxed{\begin{matrix} d_1 & d_3 + jd_4 \\ d_3 - jd_4 & d_2 \end{matrix}} & & & \\ & \boxed{d_5} & & \\ & & \boxed{d_6 I_3} & \\ & & & \boxed{d_7} \end{bmatrix}, D_R = \begin{bmatrix} \boxed{\begin{matrix} d_1 & d_3 + jd_4 \\ d_3 - jd_4 & d_2 \end{matrix}} & & & \\ & \boxed{d_5} & & \\ & & \boxed{d_6 I_2} & \\ & & & \boxed{d_7 I_2} \end{bmatrix} : \begin{matrix} \boxed{d_1, \ldots, d_7 \in \mathbb{R}} \\ \boxed{d_1, \ldots, d_7 \in \mathbb{R}} \end{matrix} \right\}$$

$$\mu_{\mathbf{\Delta}}(M) = \mu_{\mathbf{\Delta}}(DMD^{-1}) \rightarrow \mu_{\mathbf{\Delta}}(M) \leq \inf_{D \in \mathcal{D}} \bar{\sigma}(DMD^{-1}) = \inf_{d_1,\ldots,d_7 > 0} \bar{\sigma}(DMD^{-1})$$

$$\boxed{\begin{matrix} \inf\limits_{\substack{D \in \mathcal{D} \\ \gamma \in \mathbb{R}^+}} & \boxed{\gamma} \\ s.t. & \boxed{\bar{\sigma}(DMD^{-1}) < \gamma} \end{matrix}}$$

FOR SQUARE CASE	FOR RECTANGULAR CASE
$\bar{\sigma}(DMD^{-1}) < \gamma$	$\bar{\sigma}(D_L M D_R^{-1}) < \gamma$
$\Vert DMD^{-1} \Vert_2 < \gamma$	$\Vert D_L M D_R^{-1} \Vert_2 < \gamma$
$\Vert DMD^{-1} \Vert_2^2 < \gamma^2$	$\Vert D_L M D_R^{-1} \Vert_2^2 < \gamma^2$
$\lambda_{max}([DMD^{-1}]^T [DMD^{-1}]) < \gamma^2$	$\lambda_{max}([D_L M D_R^{-1}]^T [D_L M D_R^{-1}]) < \gamma^2$
$[DMD^{-1}]^T [DMD^{-1}] < \gamma^2 I$	$[D_L M D_R^{-1}]^T [D_L M D_R^{-1}] < \gamma^2 I$
$[D^{-T} M^T D^T][DMD^{-1}] < \gamma^2 I$	$[D_R^{-T} M^T D_L^T][D_L M D_R^{-1}] < \gamma^2 I$
$D^{-T} M^T [D^T D] M D^{-1} < \gamma^2 I$	$D_R^{-T} M^T [D_L^T D_L] M D_R^{-1} < \gamma^2 I$
$D^{-T} M^T [D^T D] M < \gamma^2 I D$	$D_R^{-T} M^T [D_L^T D_L] M < \gamma^2 I D_R$
$M^T [D^T D] M < D^T \gamma^2 I D$	$M^T [D_L^T D_L] M < D_R^T \gamma^2 I D_R$
$M^T [D^T D] M < \gamma^2 [D^T D]$	$M^T [D_L^T D_L] M < \gamma^2 [D_R^T D_R]$
$X \coloneqq D$	$X_L \coloneqq [D_L^T D_L], X_R \coloneqq [D_R^T D_R]$
$M^T X M < \gamma^2 X$	$M^T X_L M < \gamma^2 X_R$
$\boxed{\begin{matrix} \inf\limits_{\substack{X \in \mathcal{X} \\ \gamma \in \mathbb{R}^+}} & \boxed{\gamma} \\ s.t. & \boxed{M^T X M < \gamma^2 X} \end{matrix}}$	$\boxed{\begin{matrix} \inf\limits_{\substack{(X_L, X_R) \in \mathcal{X} \\ \gamma \in \mathbb{R}^+}} & \boxed{\gamma} \\ s.t. & \boxed{M^T X_L M < \gamma^2 X_R} \end{matrix}}$

Let us determine the structure of $X_L \& X_R$

$$X_L = D_L^H D_L = \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & D_3 I_3 & \\ & & & D_4 \end{bmatrix}^T \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & D_3 I_3 & \\ & & & D_4 \end{bmatrix} = \begin{bmatrix} D_1^H D_1 & & & \\ & (D_2)^2 (D_2) & & \\ & & (D_3)^2 (D_3) I_3 & \\ & & & (D_4)^2 (D_4) \end{bmatrix} \to \begin{bmatrix} \boldsymbol{D_1} & & & \\ & \boldsymbol{D_2} & & \\ & & \boldsymbol{D_3} I_3 & \\ & & & \boldsymbol{D_4} \end{bmatrix}$$

$$X_R = D_L^H D_L = \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & D_3 I_2 & \\ & & & D_4 I_2 \end{bmatrix}^T \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & D_3 I_2 & \\ & & & D_4 I_2 \end{bmatrix} = \begin{bmatrix} D_1^H D_1 & & & \\ & (D_2)^2 (D_2) & & \\ & & (D_3)^2 (D_3) I_2 & \\ & & & (D_4)^2 (D_4) I_2 \end{bmatrix} \to \begin{bmatrix} \boldsymbol{D_1} & & & \\ & \boldsymbol{D_2} & & \\ & & \boldsymbol{D_3} I_2 & \\ & & & \boldsymbol{D_4} I_2 \end{bmatrix}$$

$$\mathcal{D} = \{\mathcal{D}_L, \mathcal{D}_R\} = \left\{ D_L = \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & D_3 I_3 & \\ & & & D_4 \end{bmatrix}, D_R = \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & D_3 I_2 & \\ & & & D_4 I_2 \end{bmatrix} : \begin{matrix} \boxed{D_1 \in \mathbb{C}^{2\times 2}, D_1 = D_1^H > 0} \\ \boxed{D_2 \in \mathbb{R}, D_2 > 0} \\ \boxed{D_3 \in \mathbb{R}, D_3 > 0} \\ \boxed{D_4 \in \mathbb{R}, D_4 > 0} \end{matrix} \right\}$$

MATLAB code-FEASIBILITY (for a given γ, does $\exists X_L, X_R$?)

```
%% ADD YALMIP AND SDPT3 LIBRARIES
%% CASE 5: Mixed DELTA,rectangular case
clear all;close all;clc;
N_s=[tf([1],[1,1]),tf([1],[1,2]),tf([1],[1,3]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,4]);
     tf([1],[1,5]),tf([1],[1,1]),tf([1],[1,2]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,4]);
     tf([1],[1,2]),tf([1],[1,4]),tf([1],[1,7]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,2]);
     tf([1],[1,2]),tf([1],[1,4]),tf([1],[1,7]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,4]);
     tf([1],[1,2]),tf([1],[1,4]),tf([1],[1,7]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,3]);
     tf([1],[1,5]),tf([1],[1,1]),tf([1],[1,2]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,4]);
     tf([1],[1,2]),tf([1],[1,4]),tf([1],[1,7]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,4])];
%% compute the matrix for a given freq
Njw1=evalfr(N_s,1*i)
%% CASE 5: mixed DELTA - RECTANGULAR CASE
% FEASIBILITY PROBLEM

% COST FUNCTION
% For full delta   --> d*eye(n)
% For delta*eye(n) --> FULL D
% For diag[delta1*eye(s1),delta2*eye(s2),delta3*eye(s3)] --> diag[D1,D2,D3]

gamma=100;
eps1=1e-3;

D1=sdpvar(2,2,'hermitian','complex');
D2=sdpvar(1,1,'symmetric');
D3=sdpvar(1,1,'symmetric');
D4=sdpvar(1,1,'symmetric');

XL=blkdiag(D1,D2,D3*eye(3),D4);
XR=blkdiag(D1,D2,D3*eye(2),D4*eye(2));

F=[];
% F=[F;M'*XL*M-gamma^2*XR<=-eps1*eye(7)]
F=[F;Njw1'*XL*Njw1-gamma^2*XR<=-eps1*eye(7)]

% F=[F;0<=vec(XL)<=1e3];

ops = sdpsettings('solver','sdpt3');
sol = optimize(F,[],ops);
sol.info

XL=value(XL)
XR=value(XR)
```

Code OUTPUT

```
 Total CPU time (secs)  = 0.20
 CPU time per iteration = 0.02
 termination code       =  0
 DIMACS errors: 1.4e-10  0.0e+00  2.6e-09
-------------------------------------------


ans =

    'Successfully solved (SDPT3-4)'
```

the bisection part

MATLAB code-BISECTION (Find min γ, s.t. $\exists X_L, X_R$)	

```
%% ADD YALMIP AND SDPT3 LIBRARIES
%% CASE 5: Mixed DELTA,rectangular case
clear all;close all;clc;
N_s=[tf([1],[1,1]),tf([1],[1,2]),tf([1],[1,3]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,4]);
     tf([1],[1,5]),tf([1],[1,1]),tf([1],[1,2]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,4]);
     tf([1],[1,2]),tf([1],[1,4]),tf([1],[1,7]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,2]);
     tf([1],[1,2]),tf([1],[1,4]),tf([1],[1,7]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,4]);
     tf([1],[1,2]),tf([1],[1,4]),tf([1],[1,7]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,3]);
     tf([1],[1,5]),tf([1],[1,1]),tf([1],[1,2]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,4]);
     tf([1],[1,2]),tf([1],[1,4]),tf([1],[1,7]),tf([1],[1,3]),tf([1],[1,4]),tf([1],[1,3]),tf([1],[1,4])];
%% compute the matrix for a given freq
Njw1=evalfr(N_s,1*i)
% CASE 5: mixed DELTA - RECTANGULAR CASE
% BISECTION PROBLEM
g2=sdpvar(1,1,'symmetric');
eps1=1e-3;

D1=sdpvar(2,2,'hermitian','complex');
D2=sdpvar(1,1,'symmetric');
D3=sdpvar(1,1,'symmetric');
D4=sdpvar(1,1,'symmetric');

XL=blkdiag(D1,D2,D3*eye(3),D4);
XR=blkdiag(D1,D2,D3*eye(2),D4*eye(2));

F=[];
% F=[F;M'*XL*M-gamma^2*XR<=-eps1*eye(7)]
F=[F;Njw1'*XL*Njw1-g2*XR<=-eps1*eye(7)]

% F=[F;0<=vec(XL)<=1e3];

% ops = sdpsettings('solver','sdpt3');
ops = sdpsettings('solver','bisection','bisection.solver','sdpt3');
sol = bisection(F,[g2],ops);
sol.info

XL=value(XL)
XR=value(XR)
g2=value(g2)

gamma=sqrt(g2)
YALMIP_SSV=gamma;
%%
rho_N=max(abs(eig(Njw1)));                  % spectral radius of Njw1
sigma_max_N=max(svd(Njw1));                 % simga-max of Njw1

% COMPUTE THE SSV USING MATLAB-MUSSV FCN
% bounds = mussv(N,BlockStructure)
[bounds,muinfo] = mussv(Njw1,[2,0;1,1;2,3;2,1]);
MATLAB_MUSSV=bounds(2);
clc;
fprintf("YALMIP_SSV \t rho_N \t sigma_max_N \t MATLAB_MUSSV\n");
fprintf("%6.4f       \t %6.4f \t %6.4f       \t %6.4f\n",YALMIP_SSV,rho_N,sigma_max_N,MATLAB_MUSSV);
```

Code OUTPUT	
	YALMIP_SSV rho_N sigma_max_N MATLAB_MUSSV 2.1974 2.2120 2.2646 2.1974

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INFO ABOUT MATLAB MUSSV

$\gg$ **[bounds,rowd] = mu(M,blk)**

where the structure of the Δ is specified by a two-column matrix **blk**. for example, a

$$\Delta = \begin{bmatrix} \delta_1 I_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_5 I_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Delta_6 \end{bmatrix}$$

$$\delta_1, \delta_2, \delta_5, \in \mathbb{C}, \ \Delta_3 \in \mathbb{C}^{2\times 3}, \Delta_4 \in \mathbb{C}^{3\times 3}, \Delta_6 \in \mathbb{C}^{2\times 1}$$

can be specified by

$$\mathbf{blk} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ 2 & 3 \\ 3 & 3 \\ 3 & 0 \\ 2 & 1 \end{bmatrix}.$$

Note that Δ_j is not required to be square. The outputs of the program include a 2×1 vector **bounds** containing the upper and lower bounds of $\mu_{\boldsymbol{\Delta}}(M)$ and the row vector **rowd** containing the scaling D. The D matrix can be recovered by

$\gg [\mathbf{D}_\ell, \mathbf{D_r}] = \mathbf{unwrapd}(\mathbf{rowd}, \mathbf{blk})$

where D_ℓ and D_r denote the left and right scaling matrices used in computing the upper-bound $\inf \overline{\sigma}\left(D_\ell M D_r^{-1}\right)$ when some full blocks are not necessarily square and they are equal if all full blocks are square.

Kemin ZHOU,John C Doyle "Essentials of Robust Control" EXAMPLE 10.2

INFO ABOUT MATLAB MUSSV

INFO ABOUT D_L, D_R

$$\mu_{\mathbf{\Delta}}(M) \leq \inf_{D \in \mathcal{D}} \bar{\sigma}(DMD^{-1})$$

$$\mu_{\mathbf{\Delta}}(M) \leq \inf_{D \in \mathcal{D}} \bar{\sigma}(D_L M D_R^{-1})$$

and

$$\mathbf{\Delta} = \left\{ \Delta = \begin{bmatrix} \delta_1 I_2 & & & \\ & \delta_2 & & \\ & & \Delta_3 & \\ & & & \Delta_4 \end{bmatrix} : \ \delta_1, \delta_2 \in \mathbb{C}, \Delta_3 \in \mathbb{C}^{2\times 3}, \Delta_4 \in \mathbb{C}^{2\times 1} \right\}.$$

Then blk $= \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ 2 & 3 \\ 2 & 1 \end{bmatrix}$ and the MATLAB program gives bounds $= \begin{bmatrix} 10.5955 & 10.5518 \end{bmatrix}$

and

$$D_\ell = \begin{bmatrix} D_1 & & & \\ & 0.7638 & & \\ & & 0.8809 I_3 & \\ & & & 1.0293 \end{bmatrix}$$

$$D_r = \begin{bmatrix} D_1 & & & \\ & 0.7638 & & \\ & & 0.8809 I_2 & \\ & & & 1.0293 I_2 \end{bmatrix}$$