We are given a piecewise-defined function:

$$V(x) = \max\{V_1(x), V_2(x)\}, \quad \text{where } V_1(x) = x^2 \text{ and } V_2(x) = x.$$

Step 1: Determine where $V_1(x) > V_2(x)$ and vice versa

We analyze:

$$x^2 \geq x \quad \Longleftrightarrow \quad x^2 - x \geq 0 \quad \Longleftrightarrow \quad x(x-1) \geq 0.$$

This inequality holds when:

- $x \leq 0$ or
- $x \geq 1$

So:

- For $x \leq 0$ or $x \geq 1$, we have $V(x) = x^2$
- For $0 < x < 1$, we have $V(x) = x$
- At $x = 0$ and $x = 1$, both expressions are equal: $V_1(x) = V_2(x)$

Step 2: Compute derivative piecewise

- For $x < 0$: $V(x) = x^2 \Rightarrow \frac{dV}{dx} = 2x$
- For $0 < x < 1$: $V(x) = x \Rightarrow \frac{dV}{dx} = 1$
- For $x > 1$: $V(x) = x^2 \Rightarrow \frac{dV}{dx} = 2x$

Step 3: Analyze derivative at non-differentiable points $x = 0$ and $x = 1$

At $x = 0$:

$$\text{Left derivative: } \lim_{x \to 0^-} 2x = 0, \quad \text{Right derivative: } \lim_{x \to 0^+} 1 = 1$$

Since the left and right derivatives differ, $V(x)$ is **not differentiable** at $x = 0$.

At $x = 1$:

$$\text{Left derivative: } \lim_{x \to 1^-} 1 = 1, \quad \text{Right derivative: } \lim_{x \to 1^+} 2x = 2$$

Again, left and right derivatives differ, so $V(x)$ is **not differentiable** at $x = 1$.

Final Answer:

The derivative of $V(x) = \max\{x^2, x\}$ is:

$$\frac{dV}{dx} = \begin{cases} 2x, & x < 0 \\ 1, & 0 < x < 1 \\ 2x, & x > 1 \end{cases} \quad \text{(not differentiable at } x = 0 \text{ and } x = 1)$$