IE400 Project Report

Group 24
Artun Cura - 21703091
Elif Demir - 21601571
Mehmet Alperen Yalçın - 21502273

Introduction

For this project, we modeled four different optimization problems and solved them using solvers. The problem was about helping Santa make decisions so that the children can receive their gifts. We used Google's ORTools library to model linear and integer programming questions and solve them, and pandas library to read data.

For more details of the libraries you can visit:
https://developers.google.com/optimization
https://pandas.pydata.org/

Project Questions and Models

Part A:

Selecting villages as centers by minimizing a condition is actually a classic 'location problem' with small additions. For this part, we are trying to assign 4 villages as centers and minimize the maximum distance a parent must walk.

Parameters:

$d_{i,j}$ = shortest distance between i and j
V = number of villages = 30
m = number of centers = 4

Decision variables:

$y_{i,j} = \{$ 1, if village i is assigned to the village selected as center j
0, o.w $\}$

$c_j = \{$ 1, if village j is selected as center
0, o.w $\}$

Define an additional decision variable T that represents the longest distance between any two villages. This is equal to the maximum distance which a parent should walk.

IP Model:

$$\min \quad T \qquad (1)$$

$$\text{s.t.} \quad \sum_{j=1}^{V} y_{i,j} = 1 \qquad \forall\ i = 1, .. ,V \qquad (2)$$

$$y_{i,j} <= c_j \qquad \forall i,j = 1, .. ,V \qquad (3)$$

$$\sum_{j=1}^{V} c_j = m \qquad (4)$$

$$\sum_{j=1}^{V} d_{i,j} * y_{i,j} <= T \qquad (5)$$

$$x_j \in \{0, 1\} \qquad \forall j = 1, . . . , V \qquad (6)$$

$$y_{i,j} \in \{0, 1\} \qquad \forall i, j = 1, . . . , V \qquad (7)$$

Explanation of the model:

(1) **-** Trying to minimize objective function T.
(2) - All villages must be assigned to a center.
(3) - A village can be assigned to another village only if it is a center.
(4) - Total sum of the centers must be 4.
(5) - The distance between the village and the center should be minimized.
(6) - For all values of x_j, it must be either 1 or 0.
(7) - For all values of $y_{i,j}$ it must be either 1 or 0.

Programming and result:

The code of this part is named as parta.py.
The result of this part is 15 19 24 30.

Part B:

This part is similar to the previous part. For this part, we are trying to assign 4 villages as centers and minimize the maximum distance a parent must walk but also this time we are considering the probability of the road being out of use. We will add another constraint to fulfill this requirement.

Parameters:

$d_{i,j}$ = shortest distance between i and j

$p_{i,j}$ = probability of road between i and j being out of use

V = number of villages = 30

m = number of centers = 4

Decision variables:

$$y_{i,j} = \begin{cases} 1, & \text{if village i is assigned to the village selected as center j} \\ 0, & \text{o.w} \end{cases}$$

$$c_j = \begin{cases} 1, & \text{if village j is selected as center} \\ 0, & \text{o.w} \end{cases}$$

Define an additional decision variable T that represents the longest distance between any two villages. This is equal to the maximum distance which a parent should walk.

IP Model:

$$\min \quad T \qquad (1)$$

$$\text{s.t.} \quad \sum_{j=1}^{V} y_{i,j} = 1 \qquad \forall\ i = 1, .., V \qquad (2)$$

$$y_{i,j} <= c_j \qquad \forall\, i,j = 1, .., V \qquad (3)$$

$$\sum_{j=1}^{V} c_j = m \qquad (4)$$

$$\sum_{j=1}^{V} d_{i,j} * y_{i,j} <= T \qquad (5)$$

$$p_{i,j} * y_{i,j} < 0.60 \qquad \forall\, i,j = 1, . . . , V \qquad (6)$$

$x_j \in \{0, 1\}$ $\forall j = 1, . . . , V$ (7)

$y_{i,j} \in \{0, 1\}$ $\forall i, j = 1, . . . , V$ (8)

Explanation of the model:

(1) - Trying to minimize objective function T.
(2) - All villages must be assigned to a center.
(3) - A village can be assigned to another village only if it is a center.
(4) - Total sum of the centers must be m (4).
(5) - The distance between the village and the center should be minimized.
(6) - Probability of being out of use for a selected road must be smaller than 0.60.
(7)- For all values of x_j, it must be either 1 or 0.
(8)- For all values of $y_{i,j}$ it must be either 1 or 0.4.

Programming and result:

The code of this part is named as partb.py.
The result of this part is 9 13 20 24.

Part C:

This problem is similar to the Traveling Salesman Problem. Santa visits all the places and turns back to his initial position with considering the probability of the road being out of use, in minimum time with a speed of 40km/h.

Parameters:

$d_{i,j}$ = shortest distance between i and j
$p_{i,j}$ = probability of road between i and j being out of use
V = number of villages = 30
t = probability threshold = 0.60

Decision variables:

$x_{i,j} = \{$ 1, if Santa travels from i to j
0, o.w $\}$

Define an additional decision variable $u_{i,j}$, an auxiliary variable to eliminate sub tours.

IP Model:

$$\min \sum_{i=1}^{V} \sum_{j=1}^{V} d_{i,j} * x_{i,j} \quad (1)$$

$$\text{s.t.} \quad \sum_{j=1}^{V} x_{i,j} = 1 \qquad \forall\ i = 1, .. ,V \qquad (2)$$

$$\sum_{i=1}^{V} x_{i,j} = 1 \qquad \forall\ j = 1, .. ,V \qquad (3)$$

$$u_i - u_j + Vx_{i,j} <= V-1 \qquad (4)$$

$$i \mathrel{!=} j \qquad \forall i,j = 2, .. , V \qquad (5)$$

$$x_{i,j} \in \{0, 1\} \qquad \forall i,j = 2, .. , V \qquad (6)$$

$$u_i >= 0 \qquad i = 1, .. , V \qquad (7)$$

$$p_{i,j} * x_{i,j} < t \qquad \forall i,j = 1, . . . , V \qquad (8)$$

Explanation of the model:

(1) - Trying to minimize total distance.
(2) - Must move to only 1 village from any village. (Sum of the villages moved from ith village must be 1.)
(3) - Must come from only 1 village to any village. (Sum of the ways that Santa reaches jth village must be 1).
(4) - Ensuring that elimination of subtours.
(5) - Same village movement elimination. (For all values of i must be different than j)
(6) - For all values of $x_{i,j}$ it must be either 1 or 0.
(7) - For all values of $u_{i,j}$ it must be greater than or equal to 0.
(8) - Probability of being out of use for a selected road must be smaller than 0.60.

Programming and result:

The code of this part is named as partc.py.
The result of this part is 1115 / 40 = 27.875.

Part D:

In this problem instead of Santa, some volunteers visit all the places and turn back to their initial position within 10 hours maximum with a speed of 40km/h. We are trying to minimize the number of helpers.

Parameters:

$d_{i,j}$ = shortest distance between i and j

V = number of villages = 30

D = maximum distance which a volunteer can travel = 10 * 40 = 400

Decision variables:

$$x_{i,j} = \begin{cases} 1, & \text{if a helper travels from i to j} \\ 0, & \text{o.w} \end{cases}$$

$y_{i,j}$ = total distance from the origin to village j traveled by a helper when it goes from i to j

m = number of total helpers

IP Model:

$$\min \quad m \tag{1}$$

$$\text{s.t.} \quad \sum_{j=2}^{V} x_{1,j} = m \tag{2}$$

$$\sum_{i=2}^{V} x_{i,1} = m \tag{3}$$

$$\sum_{i=1}^{V} x_{i,j} = 1 \qquad \forall j = 2, .. ,V \tag{4}$$

$$\sum_{j=1}^{V} x_{i,j} = 1 \qquad \forall i = 2, .. ,V \tag{5}$$

$$\sum_{j=1\ i!=j}^{V} y_{i,j} - \sum_{j=1\ i!=j}^{V} y_{j,i} - \sum_{j=0}^{V} d_{i,j}x_{i,j} = 0 \qquad \forall i = 2, .. ,V \qquad (6)$$

$$y_{i,j} <= D * x_{i,j} \qquad \forall i,j = 2, .. ,V \qquad (7)$$

$$y_{1,j} = d_{1,j} * x_{1,j} \qquad \forall j = 1, .. ,V \qquad (8)$$

$$x_{i,j} \in \{0, 1\} \qquad \forall i,j = 1, . . . ,V \qquad (9)$$

$$y_{i,j} >= 0 \qquad \forall i,j = 1, . . . ,V \qquad (10)$$

Explanation of the model:

(1) - Trying to minimize the number of volunteers.
(2) - m volunteers must leave from the initial village.
(3) - m volunteers must return to the initial village.
(4) - Must move to only 1 village from any village except the first one. (Sum of the villages moved from ith village must be 1).
(5) - Must come from only 1 village to any village except the first one. (Sum of the ways that any volunteer reaches jth village must be 1).
(6) - Connectivity and subtour elimination constraint.
(7) - Total distance for the current path (a path volunteer has taken) can't be more than the maximum (400).
(9) - For all values of $x_{i,j}$ it must be either 1 or 0.
(10) - For all values of $y_{i,j}$ it must be greater than or equal to 0.

Programming and result:

The code of this part is named as partd.py.
The result of this part is 3 volunteers are required.