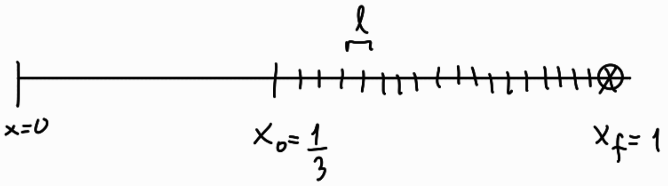
Simulação de caminhante: calcular MFPT

começa com ℓ grande e vai diminuindo



$$\frac{\partial \rho}{\partial t} = D \frac{\partial^2 \rho}{\partial x^2} \qquad D = \frac{\ell^2}{2\tau}$$

$$\rho(x, t=0) = N\,\delta(x - x_0)$$

$$\left.\frac{\partial \rho}{\partial x}\right|_{x=0} = 0 \,, \quad \rho(x = x_f, t) = 0$$

resolver com séries de Fourier → e depois calcula o τ teórico

$$\boxed{\tau = \frac{1}{N}\int_0^{x_f} dx \int_0^{\infty} \rho(x,t)\, dt}$$

uso separação de variáveis

$$\rho = X(x)\,T(t) \longrightarrow \frac{XT'}{XT} = \frac{D X'' T}{XT} \longrightarrow \frac{T'}{DT} = \frac{X''}{X} = -\lambda^2$$

$$\begin{cases} T' = -\lambda^2 D T \longrightarrow T(t) = A e^{-D\lambda^2 t} \\ X'' = -\lambda^2 X \longrightarrow X(x) = B\cos(\lambda x) + C\sin(\lambda x) \end{cases}$$

→ resolvo cada uma

$$X'(0) = -B\lambda\sin(0) + C\lambda\cos(0)$$

$$0 = C\lambda \Rightarrow \boxed{C = 0}$$

$$X(x_f) = \underbrace{B}_{\neq 0}\underbrace{\cos(\lambda x_f)}_{=0} = 0 \qquad \cos(\lambda x_f) = 0$$

$$\lambda x_f = (2m+1)\frac{\pi}{2}$$

$$\boxed{\lambda_m = \frac{(2m+1)\pi}{2x_f}}$$

$$\begin{cases} T(t) = A e^{-D\lambda^2 t} \\ X(x) = B\cos(\lambda_m x) \end{cases} \qquad \lambda_m = \frac{(2m+1)\pi}{2x_f} \quad \forall\, m \in \mathbb{N}^*$$

$$\rho = \sum_{m=0}^{\infty} A_m \cos(\lambda_m x)\, e^{-D\lambda_m^2 t}$$

$$\rho(x, t=0) = N\,\delta(x - x_0) = \sum_{m=0}^{\infty} A_m \cos(\lambda_m x)$$

aplico condição inicial

$$\int_0^1 N\,\delta(x-x_0)\cos(\lambda_m x)\,dx = \sum_{m=0}^{\infty}\int_0^1 A_m \cos(\lambda_m x)\cos(\lambda_m x)\,dx$$

$$N\cos(\lambda_m x_0) = \cancel{\sum_{m=0}^{\infty}} A_m \frac{1}{2}\,\cancel{\delta_{m,m}}$$

$$N\cos(\lambda_m x_0) = A_m\frac{1}{2} \Longrightarrow A_m = 2N\cos(\lambda_m x_0)$$

$$\boxed{\rho(x,t) = \sum_{m=0}^{\infty} 2N\cos(\lambda_m x_0)\cos(\lambda_m x)\, e^{-D\lambda_m^2 t}}$$

$\longrightarrow$ A solução

agora aplico aqui

para achar MFPT

$$\boxed{\tau = \frac{1}{N}\int_0^{x_f} dx \int_0^{\infty} \rho(x,t)\,dt}$$

$$\tau = \frac{1}{N}\int_0^{x_f} dx \int_0^{\infty} \rho(x,t)\,dt$$

$$\tau = \sum_{m=0}^{\infty} 2\cos(\lambda_m x_0)\int_0^{x_f}\cos(\lambda_m x)\,dx\int_0^{\infty} e^{-D\lambda_m^2 t}\,dt$$

$$\tau = \sum_{m=0}^{\infty} 2\cos(\lambda_m x_0)\int_0^{1}\cos(\lambda_m x)\,dx\int_0^{\infty} e^{-D\lambda_m^2 t}\,dt$$

$$\int_0^1 \cos\left[\frac{(2m+1)\pi}{2}x\right]dx$$

$$\tau = \sum_{m=0}^{\infty}\frac{(-1)^m}{\lambda_m}\, 2\cos\left(\frac{(2m+1)\pi}{2}\frac{1}{3}\right)(-1)^m\int_0^{\infty} e^{-D\lambda_m^2 t}\,dt$$

$$\int_0^{\infty} e^{-D\lambda_m^2 t}\,dt = -\frac{e^{-D\lambda_m^2 t}}{D\lambda_m^2}\Bigg|_0^{\infty} = -\left[\frac{e^{-\infty}}{D\lambda_m^2} - \frac{e^{0}}{D\lambda_m^2}\right]$$

MFPT $\longrightarrow$ $\tau = \sum_{m=0}^{\infty}\frac{(-1)^m}{D\lambda_m^3}\, 2\cos(\lambda_m x_0)$

$$\boxed{\lambda_m = \frac{(2m+1)\pi}{2x_f}}$$

$$\boxed{D = \frac{\ell^2}{2d\tau}}$$