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Three-Phase Circuits

- Learning goals:
  - Three-phase circuits
  - Basic wye and delta three-phase connections
  - Voltage and currents in three-phase circuits
  - Complex power in three-phase circuits

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Single-Phase vs Three-Phase Systems

Single-phase system

Three-phase two-wire system

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Generation of Three-Phase Voltages

A three-phase generator consists of a rotating magnet surrounded by three stationary winding coils(a-a', b-b', c-c')

Rotating magnet

A three-phase generator

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What is a Three-Phase Circuit?

It is a system of three sinusoidal voltages of the same frequency having the same amplitude and differing in phase by an angle of 120°

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Advantages of a Three-Phase Circuit

- (1) a) The instant power is generated and b) is constant (non-pulsating)
- (2) The **rotating magnet** used in a three-phase system can be **powered** by three-phase voltages. The three-phase system is more **economical** than the single-phase system.
- (3) Two **three-phase** motors required fewer **wires** than two single-phase motors. The three-phase transmission system also requires less material compared to single-phase systems.

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Balanced Three-Phase Voltages

The **phase sequence** is the order in which the phasors pass a fixed point in the plane when rotated through their respective maximum values.

The sign of a phase sequence

$v_a = V_m \cos(\omega t)$

$v_b = V_m \cos(\omega t - 120°)$

$v_c = V_m \cos(\omega t - 240°)$

$v_a = V_m \cos(\omega t + 120°)$

$v_b = V_m \cos(\omega t)$

$v_c = V_m \cos(\omega t - 120°)$

$v_a = V_m \cos(\omega t - 120°)$

$v_b = V_m \cos(\omega t + 120°)$

$v_c = V_m \cos(\omega t)$

positive, or abc sequence

negative, or acb sequence

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Balanced Three-Phase Voltages

Balanced phase voltages are equal in magnitude and are shifted from one another by 120°

Balanced three-phase refers to the phase magnitudes and **phases** of a balanced system

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Balance Three-Phase Connection

- Four possible connections
  - 1. Y-Y connection (Y-connected source with a Y-connected load)
  - 2. Y-Δ connection (Y-connected source with a Δ-connected load)
  - 3. Δ-Δ connection
  - 4. Δ-Y connection

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Balanced Y-Y Connection

Phase voltages (line to neutral)

$V_{an} = V_m \angle 0°$

$V_{bn} = V_m \angle -120°$

$V_{cn} = V_m \angle -240°$

Line voltages (line to line)

$V_{ab} = V_{an} - V_{bn} = V_m \angle 0° - V_m \angle -120°$

$V_{bc} = V_{bn} - V_{cn} = V_m \angle -120° - V_m \angle -240°$

$V_{ca} = V_{cn} - V_{an} = V_m \angle -240° - V_m \angle 0°$

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Balanced Y-Y Connection

Line voltages (line to line)

$V_{ab} = V_{an} - V_{bn} = V_m \angle 0° - V_m \angle -120°$

$V_{bc} = V_{bn} - V_{cn} = V_m \angle -120° - V_m \angle -240°$

$V_{ca} = V_{cn} - V_{an} = V_m \angle -240° - V_m \angle 0°$

Phase voltages (line to neutral)

$V_{an} = V_{ab} - V_{cn} = V_m \angle 0° - V_m \angle -240°$

$V_{bn} = V_{bc} - V_{an} = V_m \angle -120° - V_m \angle 0°$

$V_{cn} = V_{ca} - V_{bn} = V_m \angle -240° - V_m \angle -120°$

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Balanced Y-Y Connection

Line current = phase current

$I_a = \frac{V_{an}}{Z_Y} = \frac{V_m \angle 0°}{Z_Y} = I_m \angle \theta$

$I_b = \frac{V_{bn}}{Z_Y} = \frac{V_m \angle -120°}{Z_Y} = I_m \angle -120° - \theta$

$I_c = \frac{V_{cn}}{Z_Y} = \frac{V_m \angle -240°}{Z_Y} = I_m \angle -240° - \theta$

Phase current (current in each phase of the source or load) = line current (current in each line connecting the source and the load)

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Balanced Y-Y Connection

Line current

$I_a = \frac{V_{an}}{Z_Y} = \frac{V_m \angle 0°}{Z_Y} = I_m \angle \theta$

$I_b = \frac{V_{bn}}{Z_Y} = \frac{V_m \angle -120°}{Z_Y} = I_m \angle -120° - \theta$

$I_c = \frac{V_{cn}}{Z_Y} = \frac{V_m \angle -240°}{Z_Y} = I_m \angle -240° - \theta$

Phase current

$I_{an} = \frac{V_{an}}{Z_Y} = \frac{V_m \angle 0°}{Z_Y} = I_m \angle \theta$

$I_{bn} = \frac{V_{bn}}{Z_Y} = \frac{V_m \angle -120°}{Z_Y} = I_m \angle -120° - \theta$

$I_{cn} = \frac{V_{cn}}{Z_Y} = \frac{V_m \angle -240°}{Z_Y} = I_m \angle -240° - \theta$

The phase voltage (voltage across each phase) is also. The neutral line can be connected or disconnected without affecting the system

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Balanced Y-Y Connection

Neutral line current = 0 (neutral) and the return of the source or load

$I_n = I_a + I_b + I_c = I_m \angle \theta + I_m \angle -120° - \theta + I_m \angle -240° - \theta$

$= I_m \angle \theta + I_m \angle -120° - \theta + I_m \angle 120° - \theta$

$= 0$

In the Y-Y system, line current is the same as phase current

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Balanced Y-Δ Connection

Phase current

Line current

$I_a = I_{ab} - I_{ca} = I_m \angle \theta - I_m \angle -240° - \theta$

$I_b = I_{bc} - I_{ab} = I_m \angle -120° - \theta - I_m \angle \theta$

$I_c = I_{ca} - I_{bc} = I_m \angle -240° - \theta - I_m \angle -120° - \theta$

$I_a = \sqrt{3} I_{ab}$

Line current lags the phase current by 30°

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Balanced Y-Δ Connection

Phase current

$I_{ab} = \frac{V_{ab}}{Z_\Delta} = \frac{V_m \angle 0°}{Z_\Delta} = I_m \angle \theta$

$I_{bc} = \frac{V_{bc}}{Z_\Delta} = \frac{V_m \angle -120°}{Z_\Delta} = I_m \angle -120° - \theta$

$I_{ca} = \frac{V_{ca}}{Z_\Delta} = \frac{V_m \angle -240°}{Z_\Delta} = I_m \angle -240° - \theta$

Line current

$I_a = I_{ab} - I_{ca} = \sqrt{3} I_{ab} \angle -30°$

$I_b = I_{bc} - I_{ab} = \sqrt{3} I_{bc} \angle -30°$

$I_c = I_{ca} - I_{bc} = \sqrt{3} I_{ca} \angle -30°$

$I_a = \sqrt{3} I_{ab} \angle -30°$

$I_b = \sqrt{3} I_{bc} \angle -30°$

$I_c = \sqrt{3} I_{ca} \angle -30°$

Line current lags the phase current by 30°

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Example

In the balanced two-phase circuit shown below, find the voltage, and the current in each phase.

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$I_{ab} = \frac{V_{ab}}{Z_\Delta} = \frac{V_m \angle 0°}{Z_\Delta} = I_m \angle \theta$

$I_{bc} = I_{ab} \angle -120° = I_m \angle -120° - \theta$

$I_{ca} = I_{ab} \angle -240° = I_m \angle -240° - \theta$

$I_a = \sqrt{3} I_{ab} \angle -30° = \sqrt{3} I_m \angle -30° - \theta$

$I_b = \sqrt{3} I_{bc} \angle -30° = \sqrt{3} I_m \angle -150° - \theta$

$I_c = \sqrt{3} I_{ca} \angle -30° = \sqrt{3} I_m \angle -270° - \theta$

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What about Δ-Δ and Δ-Y Connections?

Connect to Y-Y

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What about Unbalanced Three-Phase System?

An unbalanced system is due to unbalanced voltage sources or unbalanced loads

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Impedance Phasor

Z is a complex number $Z = R + jX$

$Z = |Z| \angle \theta = |Z| \cos\theta + j|Z| \sin\theta$

$|Z| = \sqrt{R^2 + X^2}$ $\theta = \tan^{-1}\left(\frac{X}{R}\right)$

$Z = |Z| \angle \theta = |Z| \cos\theta + j|Z| \sin\theta$

$Z = |Z| \angle \theta = |Z| \cos\theta - j|Z| \sin\theta$

$Z = |Z| \angle \theta = |Z| \cos\theta + j|Z| \sin\theta$

$Z = |Z| \angle \theta = |Z| \cos\theta - j|Z| \sin\theta$

Impedance phasor is a complex number, therefore it can be represented in rectangular form or polar form

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Power Per Phase

Average power $P = \frac{V_m I_m}{2} \cos\theta$

Reactive power $Q = \frac{V_m I_m}{2} \sin\theta$

Apparent power $S = \frac{V_m I_m}{2}$

Complex power $S = \frac{V_m I_m}{2} \angle \theta$

$S = \frac{V_m I_m}{2} \angle \theta$

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Total Power

$P_{total} = \frac{3 V_m I_m}{2} \cos\theta$

Reactive power $Q_{total} = \frac{3 V_m I_m}{2} \sin\theta$

Apparent power $S_{total} = \frac{3 V_m I_m}{2}$

Complex power $S_{total} = \frac{3 V_m I_m}{2} \angle \theta$

$P_{total} = \frac{3 V_m I_m}{2} \cos\theta$

$Q_{total} = \frac{3 V_m I_m}{2} \sin\theta$

$S_{total} = \frac{3 V_m I_m}{2} \angle \theta$

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Power in a Balanced System

Let's determine the power delivered to a single-phase system in a balanced three-phase system

$P_{total} = 3 P_{phase}$

$P_{total} = 3 V_{phase} I_{phase} \cos\theta$

$P_{total} = \sqrt{3} V_{line} I_{line} \cos\theta$

$P_{total} = 3 V_{phase} I_{phase} \cos\theta$

$P_{total} = \sqrt{3} V_{line} I_{line} \cos\theta$

Line voltage

Line current

Phase voltage

Phase current

$P_{total} = 3 V_{phase} I_{phase} \cos\theta$

$P_{total} = \sqrt{3} V_{line} I_{line} \cos\theta$

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Residential Wiring

Single-phase two-wire residential wiring

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Example

The balanced load is connected to a balanced three-phase source. Find the line current, the phase current, the phase voltage, the line voltage, and the total power.

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$V_{an} = V_m \angle 0°$

$V_{bn} = V_m \angle -120°$

$V_{cn} = V_m \angle -240°$

$V_{ab} = V_{an} - V_{bn} = V_m \angle 0° - V_m \angle -120°$

$V_{bc} = V_{bn} - V_{cn} = V_m \angle -120° - V_m \angle -240°$

$V_{ca} = V_{cn} - V_{an} = V_m \angle -240° - V_m \angle 0°$

$V_{an} = V_{ab} - V_{cn} = V_m \angle 0° - V_m \angle -240°$

$V_{bn} = V_{bc} - V_{an} = V_m \angle -120° - V_m \angle 0°$

$V_{cn} = V_{ca} - V_{bn} = V_m \angle -240° - V_m \angle -120°$

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If the light bulb rated 0.5 A, 100W/120V, find:

1. Identify lighting as phase, (PHASE) Each lighting is connected to a phase

2. Phase Voltage (Voltage) is the VOLTAGE Voltage Rated for the Bulb

$V_{phase} = 120 V$ (Rated Voltage)

3. Phase Current (Current) is the Rated Current of the Bulb

$I_{phase} = 0.5 A$ (Rated Current)

4. Line Voltage (Voltage) $V_{line} = \sqrt{3} V_{phase}$

$V_{line} = \sqrt{3} \times 120 = 208 V$

5. Line Current (Current) $I_{line} = I_{phase}$

$I_{line} = 0.5 A$

6. Total Power $P_{total} = 3 V_{phase} I_{phase} \cos\theta$

$P_{total} = 3 \times 120 \times 0.5 \times \cos(0°) = 180 W$

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Wye-wye

Wye-delta

Delta-delta

Delta-wye