Slide 8

January 29, 2018 2:55 PM

Magnetically Coupled Circuits

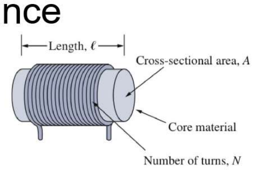
Learning goals:

- Understand the concept of mutual Inductance
- Analyze circuits with mutual inductance
- Analyze circuits with ideal transformers

Self-Inductance

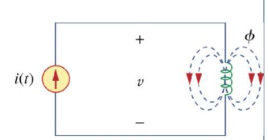
Recall the way an inductor works:

1. A wire current i results in a magnetic field (Ampere's law).
2. For a coil, this results in a magnetic flux ϕ through the coil.
3. A voltage v is "induced" across the coil when ϕ changes (Faraday's Law):



N= number of coils

$$v = N\frac{d\phi}{dt} = N \cdot \frac{d\phi}{di} \cdot \frac{di}{dt}$$



$$V = L\frac{di}{dt}$$

$$L = N\frac{d\phi}{dt}$$ self-inductance

Mutual Inductance

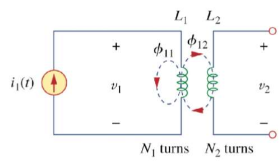
The changing ϕ may also result from a nearby coil. Consider the induced voltages in the 2 coils (side 2 is OC so $i_2(t) \equiv 0$ A):

$\phi_1 = \phi_{11} + \phi_{12}$

$V_1 = N_1 \frac{d\phi_1}{dt} = L_1 \frac{di_1}{dt}$

$V_2 = N_2 \frac{d\phi_{12}}{dt} = N_2 \frac{d\phi_{12}}{di_1} \frac{di_1}{dt}$

$= M_{21} \frac{di_1}{dt}$ $\quad M_{21} = N_2 \frac{d\phi_{12}}{di_1}$ mutual inductance



Mutual inductance M (in henrys) is the ability of one inductor to induce a voltage across a neighboring inductor.

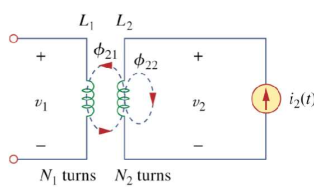
Mutual Inductance

Similarly, if $i_1(t) \equiv 0$ but $i_2(t)$ can change,

$V_2 = L_2 \frac{di_2}{dt}$

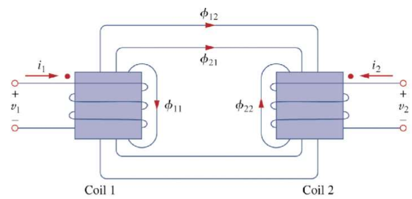
$V_1 = M_{12} \frac{di_2}{dt}$

Mutual inductance $M_{12} = M_{21} = M$



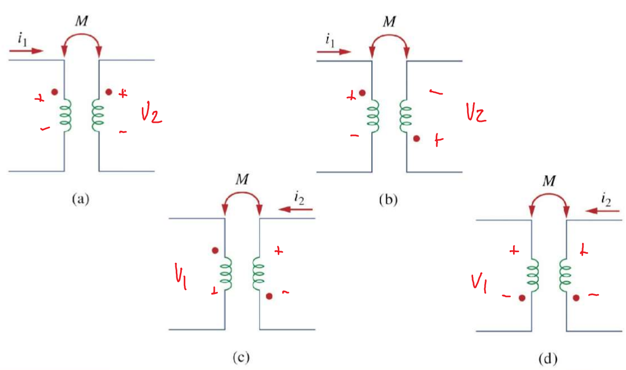
Dot Convention

M>0 but induced voltage polarity depends on physical coil windings. For convenience on schematics, the dot convention is employed to indicate polarity.



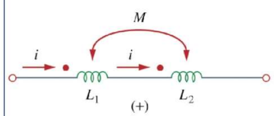
If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.

Dot Convention



Coupled Coils in Series

Calculate the total equivalent inductance including self and mutual inductances.



(a)

In phasor domain

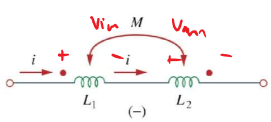
$Z_L = jwL \Rightarrow V = jwL I$

$Z_M = jwM \Rightarrow V = jwM I$

$V = Z \cdot I$

Coupled Coils in Series

Calculate the total equivalent inductance including self and mutual inductances.



(b)

In phasor domain

$Z_L = jwL \qquad V = jwL \cdot I$

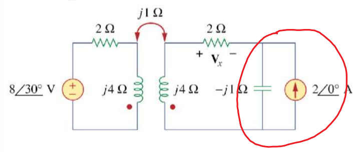
Total voltage $V = V_L + V_2 + V_m + V_{2m}$

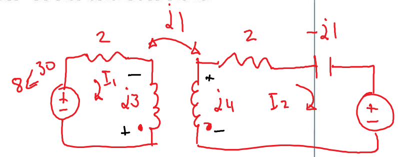
$= jwL_1 I + jwL_2 I + jm I + jwM I$

Total inductance $= L = L_1 + L_2 + 2M$

Example: Mutual Inductance

Find $\mathbf{V}_x$.





$(-j1)\cdot 2$
$= -j2$ V

KVL: $8\angle 30 = 2I_1 + j4I_1 - j1\cdot I_2$

$j4 I_2 + (2 - j1)I_2 - j2 - j_1 I_1 = 0$

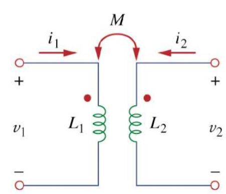
$\Rightarrow I_2 = 1.037\angle 21.12°$ A

$V_x = 2\cdot I_2 = 2.074\angle 21.12°$ V

Energy in Coupled Circuits

For the coupled circuit shown, instantaneous energy stored is:

$W = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M i_1 i_2$

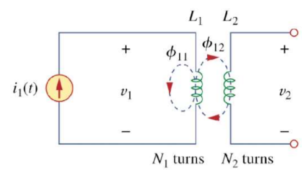


The positive sign is selected for the mutual term if both currents enter or leave the dotted terminals of the coils; the negative sign is selected otherwise.

Coupling Coefficient

Energy Conservation puts a bound on M

$M \leqslant \sqrt{L_1 L_2}$



Define Coupling Coefficient

$$k = \frac{M}{\sqrt{L_1 L_2}} \Rightarrow M = k\sqrt{L_1 L_2}$$

$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{12}}{\phi_{11} + \phi_{12}} \qquad 0 \leqslant k \leqslant 1$$

k is a measure of the magnetic coupling between two coils.

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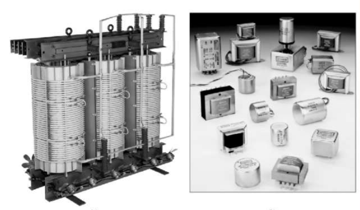
Linear Transformers

A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils.



Transformers can be very efficient at transferring power (efficiencies typically range from 95-99%).

1.Change AC voltage amplitudes to a useful value.
2.Electrically isolate two sides of a circuit (can reduce shock hazards).
3.Transform impedances to maximize power transfer.

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Ideal Transformers

An ideal transformer is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances.

$\to 1$ $\quad$ $\to$ Resistance = 0 $\quad$ $\to L_1, L_2 = \infty$

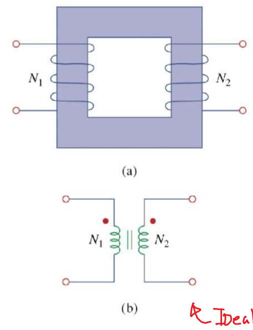
1.) Unity Coupling $k = 1$

2.) Lossless

$R_1 = R_2 = 0\,\Omega$

3.) infinite self inductance

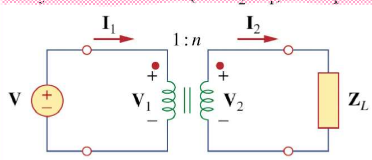
$L_1, L_2, M \to \infty$



← Ideal Transformer

Ideal Transformers

Only the turns ratio ($n = N_2/N_1$) is important.



V_1 and V_2 are the total voltage drops across the coils.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n \quad , \quad \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

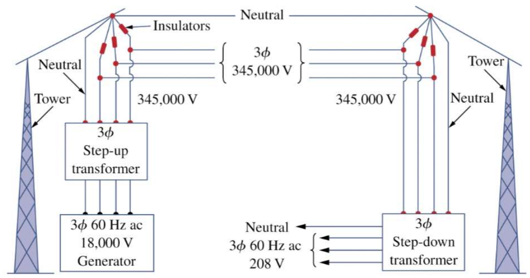
$V_2 > V_1 \to$ step up transformer

$V_2 < V_1 \to$ step down transformer

Application: Power Distribution

Transmitting power at higher voltages has two significant advantages, both due to the reduced current required for a given power:

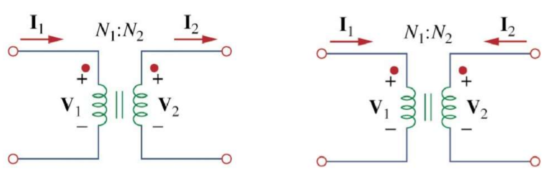
1. Conductor size can be reduced leading to material resource savings.
2. Transmission I^2R losses are reduced.



Ideal Transformer Polarity

Proper voltage polarity and current directions:

Use dot convention!

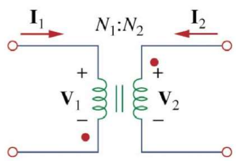


(a): $\frac{V_2}{V_1} = \frac{N_2}{N_1}$, $\frac{I_2}{I_1} = \frac{N_1}{N_2}$

(b): $\frac{V_2}{V_1} = \frac{N_2}{N_1}$, $\frac{I_2}{I_1} = -\frac{N_1}{N_2}$

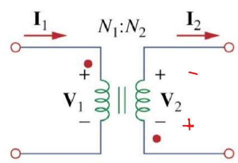
(a) (b)

Ideal Transformer Polarity



$$\frac{V_2}{V_1} = -\frac{N_2}{N_1}, \quad \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

(c)

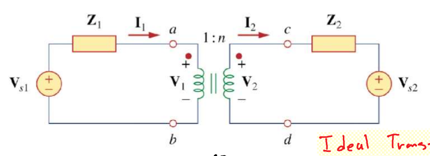


$$\frac{V_2}{V_1} = -\frac{N_2}{N_1} \quad \frac{I_2}{I_1} = -\frac{N_1}{N_2}$$

(d)

Ideal Xformer Equivalent Networks

$n = \frac{N_2}{N_1}$



KVL:

$$V_{s_1} = Z_1 I_1 + V_1 \quad (1)$$

$$V_2 + Z_2 I_2 = V_{s2} \quad (2)$$

Ideal Transformer

$$V_2 = nV_1$$

$$I_2 = \frac{1}{n} I_1$$

$$\Rightarrow nV_1 = \frac{Z_2}{n} I_1 + V_{s_2} \quad (3)$$

substitute (3) into (1)

$$V_{s_1} = \underbrace{\left(Z_1 + \frac{Z_2}{n^2}\right)}_{\text{Impedance}} I_1 + \frac{V_{s2}}{n}$$

reflected from secondary loop

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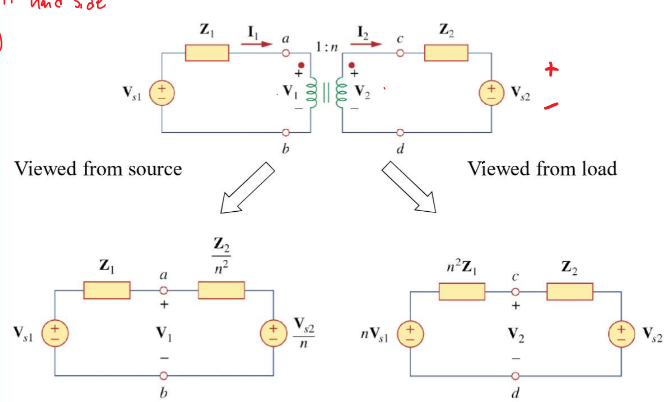
usually left hand side is primary

Ideal Xformer Equivalent Networks



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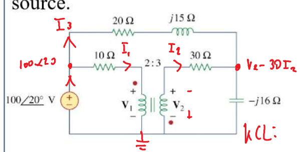
Example: Ideal Xformer

Obtain $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit. Find the complex power supplied by the source.

$n = \frac{3}{2}$



$\frac{V_2}{V_1} = -n \quad , \quad \frac{I_2}{I_1} = -\frac{1}{n}$

KCL: $\frac{100\angle 20 - (V_2 - 30 I_2)}{20 + j15} + I_2 = \frac{V_2 - 30 I_2}{-j16}$

$\frac{100\angle 20 - V_1}{10} = I_1$

Solve for V_1, V_2, I_1, I_2

$V_1 = 63.09\angle 28.65°$ V

$V_2 = 94.64\angle -151.4°$ V

$I_1 = 3.98\angle 5.85°$ A

$I_2 = 2.59\angle -174.1$ A

$I_3 = 4.91\angle -1.24°$ A

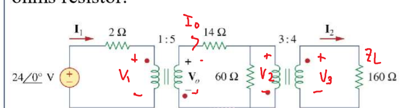
$I_s = I_1 + I_3$

$S_s = V_s \cdot I_s^* = 877\angle 18.11°$ VA

Example: Ideal Xformer

Obtain $\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{V}_o$ in the circuit. Find the average power received by the 160 ohms resistor.

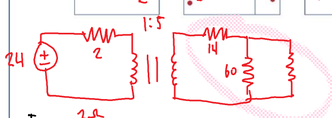
Use equivalent circuit to reflect the impedance from 3rd loop to middle



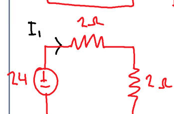
$n_1 = 5, \quad n_2 = \frac{4}{3}$

reflect the 160 Ω to the middle circuit

$Z_R = \frac{Z_L}{n_2^2} = 90\,\Omega$



$Z_L' = 14 + 60 /\!/ 90 = 50\,\Omega$



Reflect Z_L' to the primary side

$Z_R' = \frac{Z_L'}{n_1^2} = 2\,\Omega$

$I_1 = \frac{24}{2+2} = 6$ A

$V_1 = 24 - I_1 \cdot 2 = 12$ V

$$\frac{V_0}{V_1} = -n_1 \Rightarrow V_0 = -n_1 V_1 = -60\,V$$

$$\frac{I_0}{I_1} = -\frac{1}{n_1} \Rightarrow I_0 = -\frac{I_1}{n_1} = -1.2\,A$$

$$V_2 = V_0 - I_0 \cdot 14 = -43.2\,V$$

$$V_3 = -n_2 V_2 = 57.6\,V$$

$$I_2 = \frac{V_3}{160} = 0.36\,A$$

$$P = |I_2|^2 \cdot Z_L = 0.36^2 \cdot 160 = 20.736\,W$$

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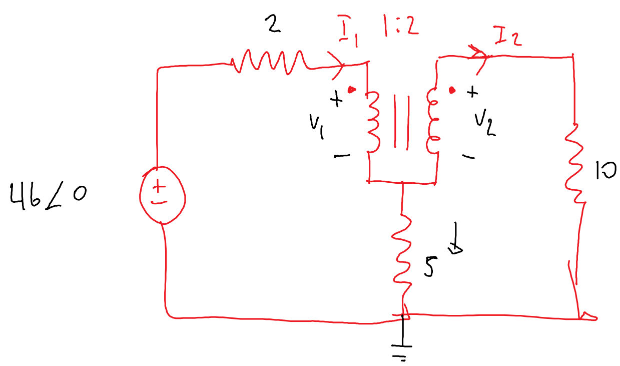
$$I_1 = I_2 + I_3$$

$$n = 2$$



$$\frac{V_2}{V_1} = n$$

$$\frac{I_2}{I_1} = \frac{1}{n}$$

$$46\angle 0 - 2I_1 - V_1 - 5(I_1 - I_2) = 0$$

$$5(I_1 - I_2) + V_2 = 10 I_2$$

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