From annular cavity to rotor-stator flow: nonlinear dynamics of axisymmetric rolls

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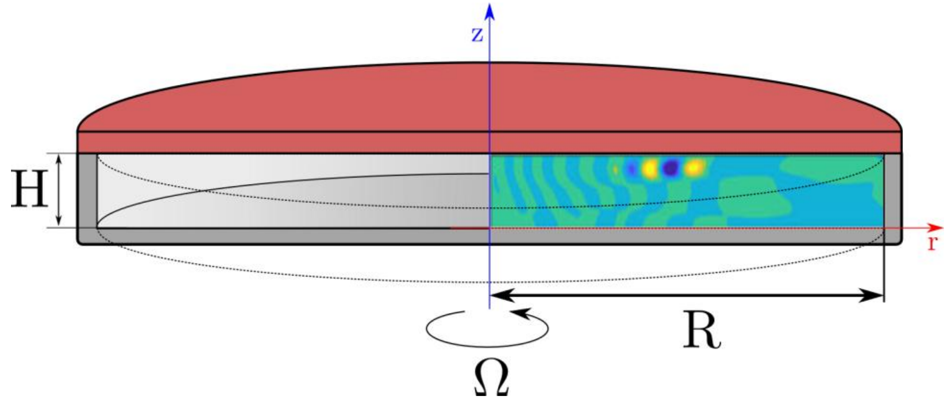
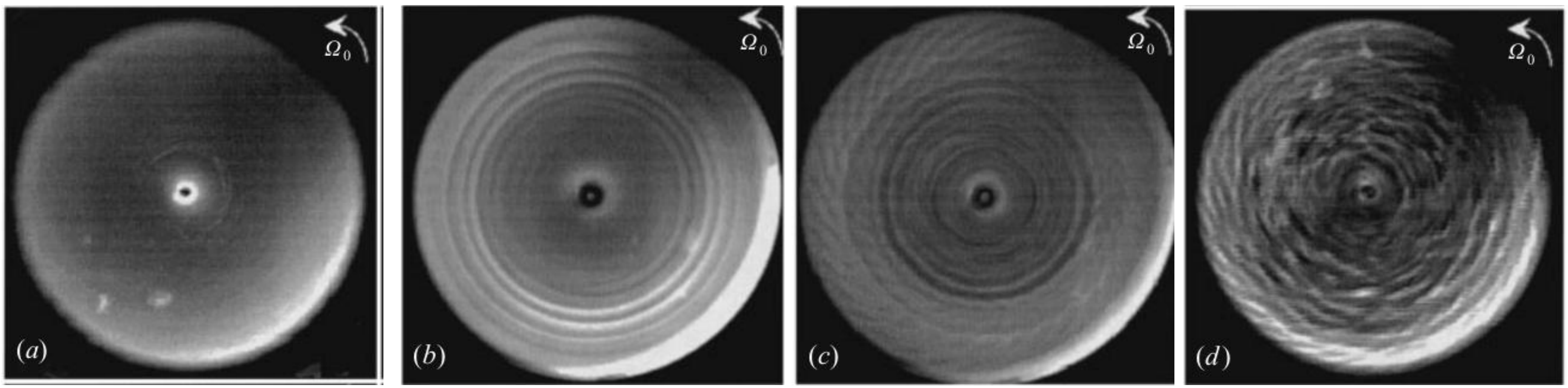
$Re = H^2\Omega/\nu$

Aspect ratio : $\Gamma = R\,/H$



Circular rolls   Circular + spiral rolls



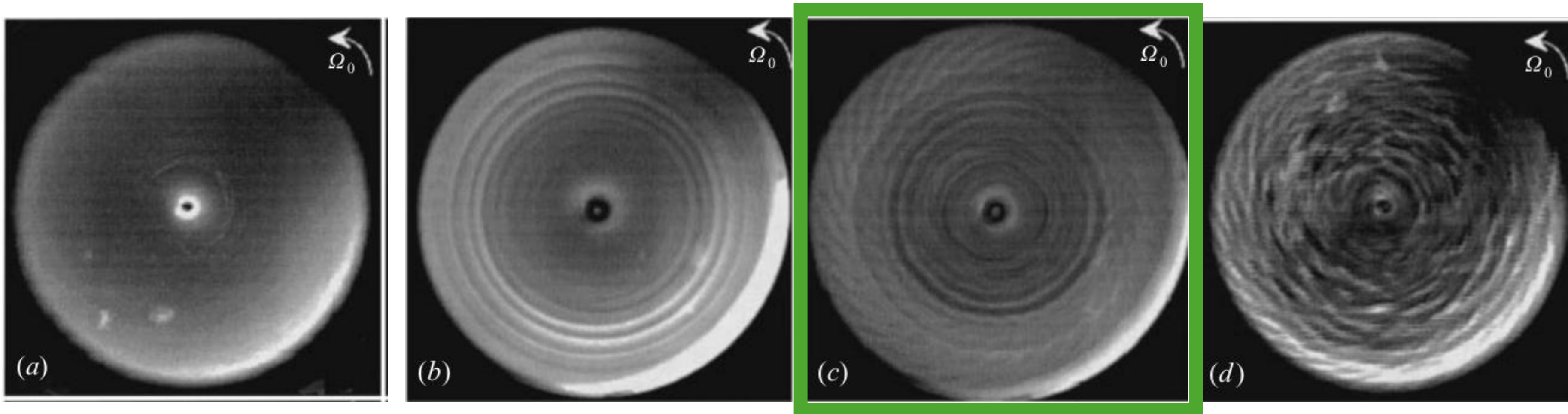
Re=150   Re=500   Re=700   Re=1200

Re

Gauthier *et al.*, 1999

Circular rolls

Circular + spiral rolls



Gauthier *et al.*, 1999

First linearly unstable mode, R/H=10

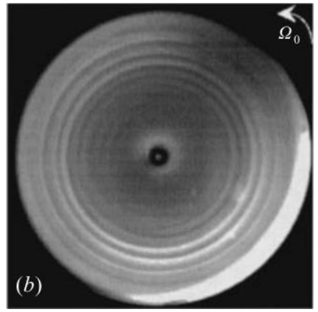
Gelfgat, Fluid Dyn. Res. 2015

Saturated state, DNS, R/H=5

Serre *et al.*, PoF 2004

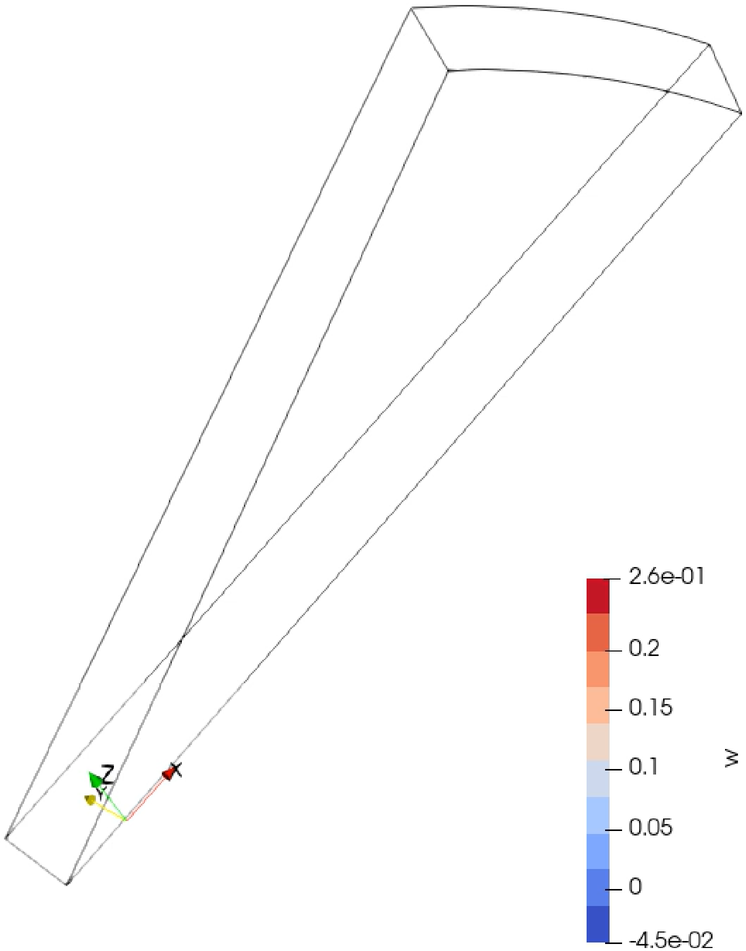
Challenge: No clear explanation for the circular rolls.

Experiment

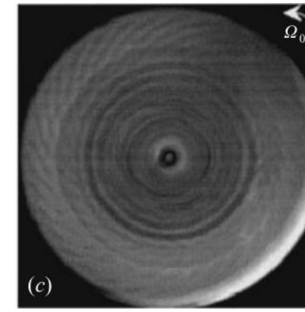


DNS

Re=500 Time: 0.000000

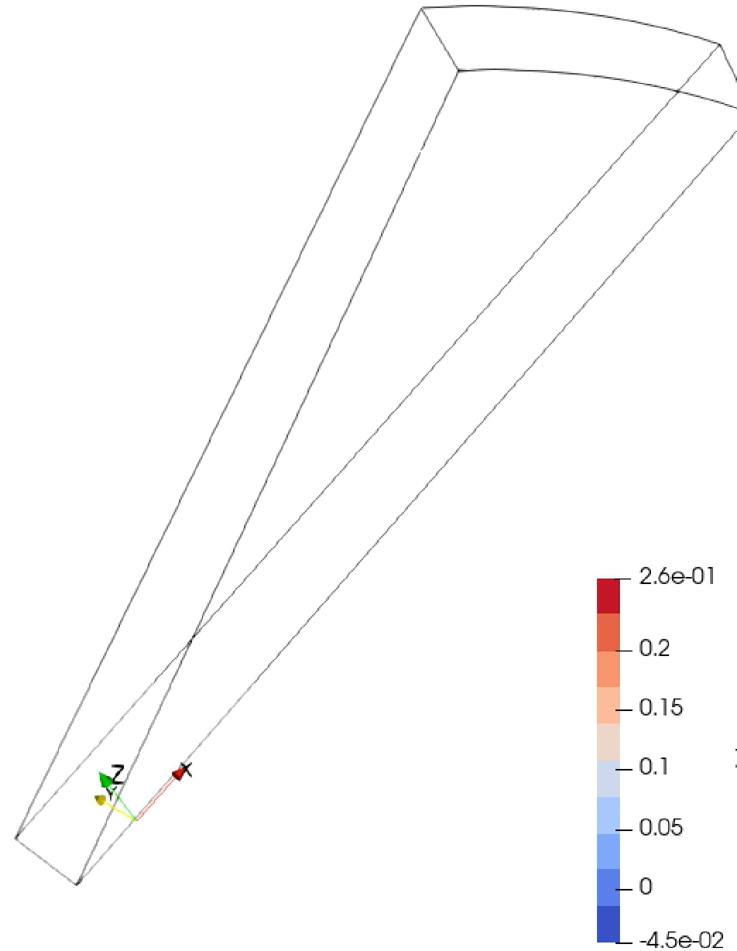


Experiment

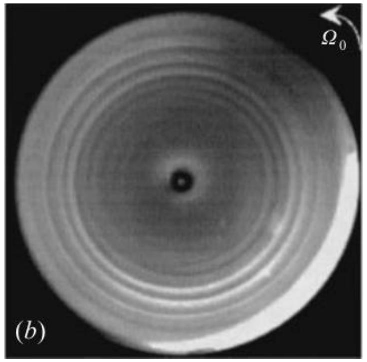


DNS

Re=700 Time: 0.000000

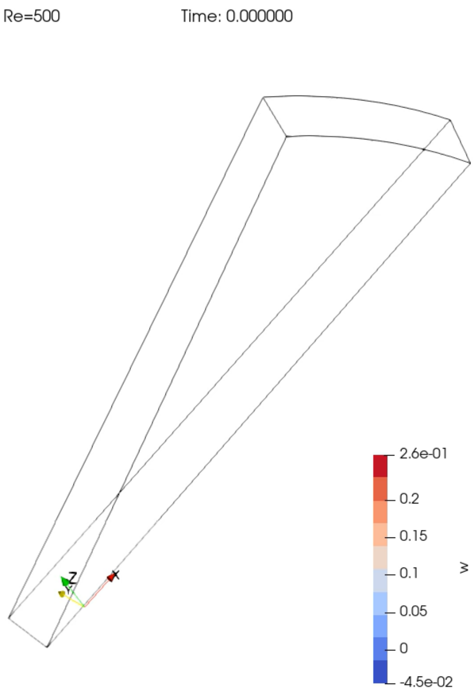


Experiment

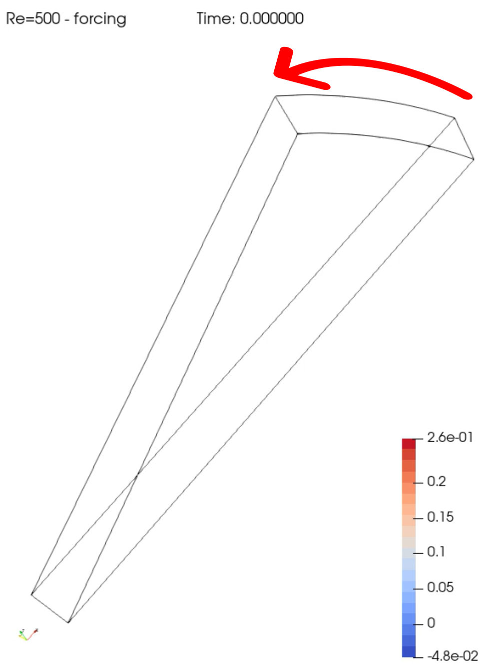


Circular rolls seen experimentally are a linear response to forcing.

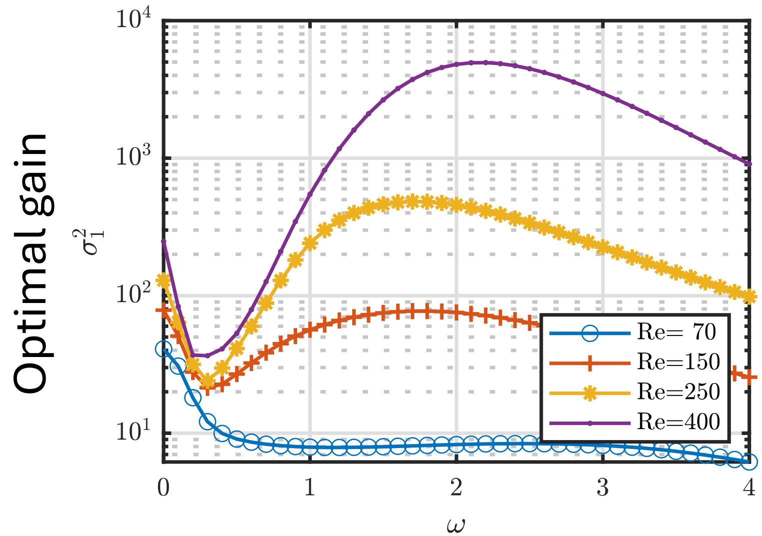
No forcing, $\Omega(t) = 1$



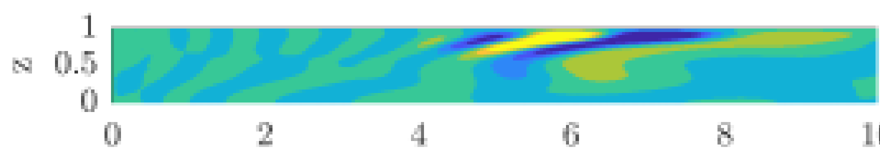
$\Omega(t) = 1 +$ white noise



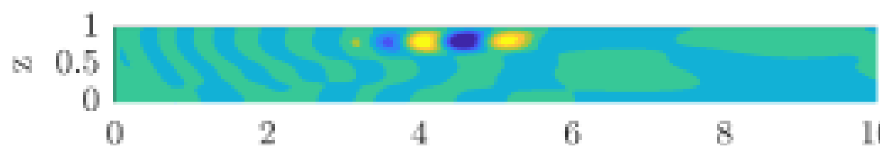
Resolvent analysis (axisymmetric)



f_θ optimal forcing



u_θ linear response to forcing

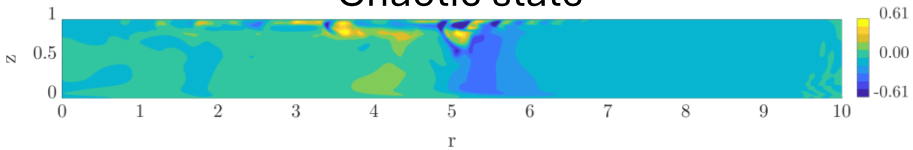


Gesla *et al.* On the origin of circular rolls in rotor-stator flow, JFM 2024.

Self-sustained axisymmetric solutions

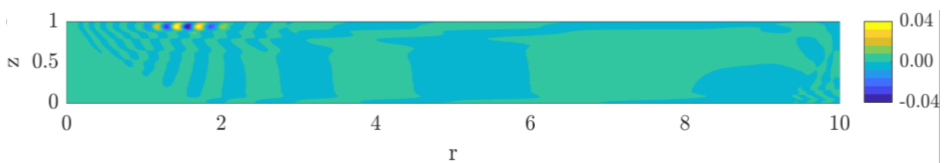
Gesla *et al*. Subcritical axisymmetric solutions in rotor-stator flow, PRF 2024.

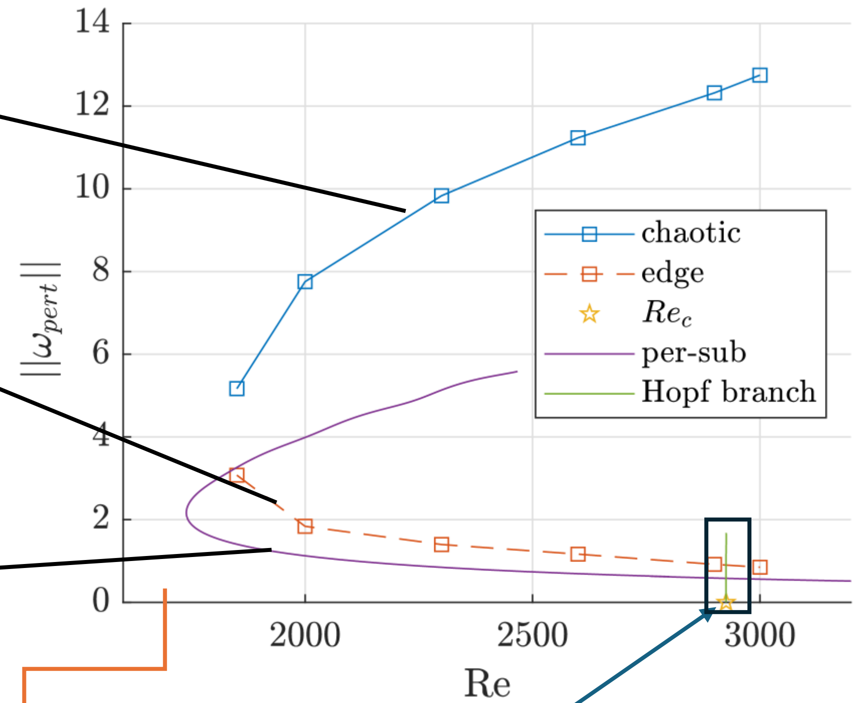
Chaotic state



Biperiodic edge state

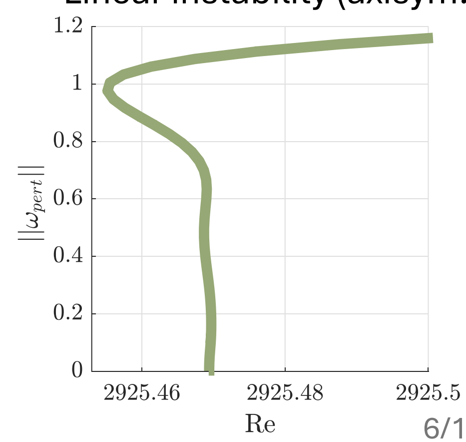
Periodic-subcritical branch





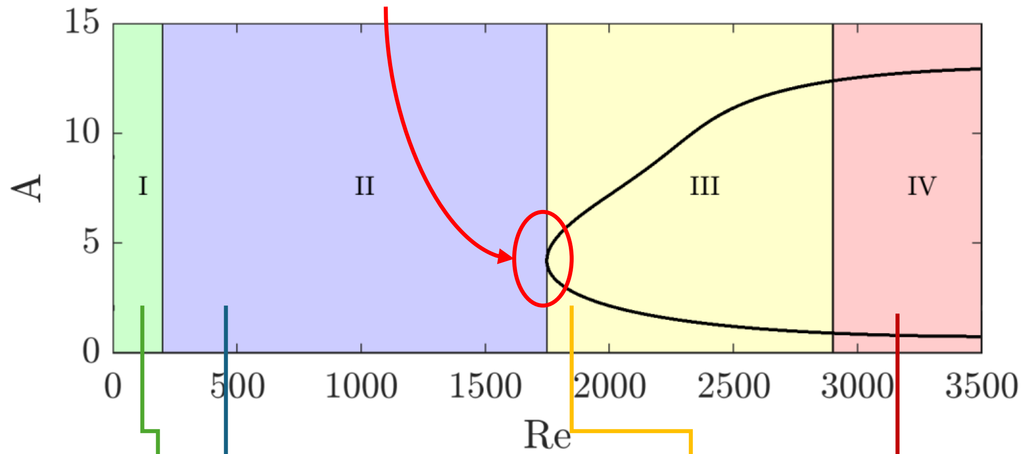
Experimental rolls – $Re \approx 500$

Linear instability (axisym.)

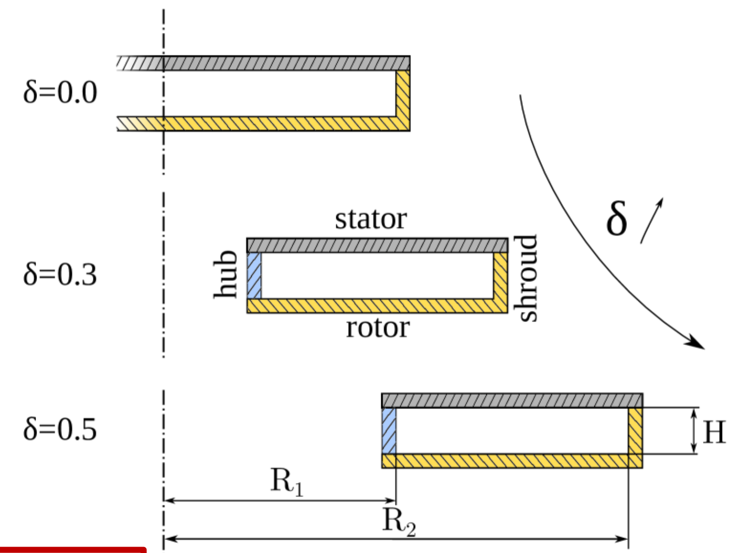


Dynamics of circular rolls

Finite lifetimes



Use **homotopy** to explain the origin of the self-sustained states.



$$\delta = R_1/R_2$$

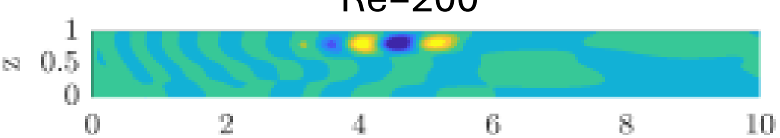
Stable flow, no rolls

Stable flow, circular rolls appear as a response to forcing

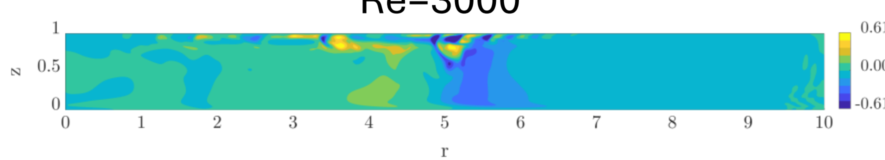
Lin. stable flow, self-sustained solutions exist

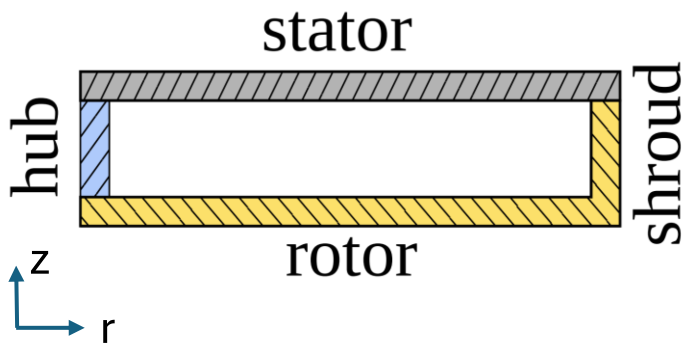
Unstable flow, evolution to chaotic dynamics

Re=200



Re=3000





$$\frac{\partial \mathbf{u}}{\partial t} + \nabla(\mathbf{u} \otimes \mathbf{u}) = -\nabla p + \frac{1}{Re}\nabla^2 \mathbf{u}$$

$$\nabla \cdot \mathbf{u} = 0$$

Stress-free on the hub :

$$(u_r\,, \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\,, \frac{\partial u_z}{\partial r}) = (0,0,0) \text{ at } r_1 = R_1/H.$$

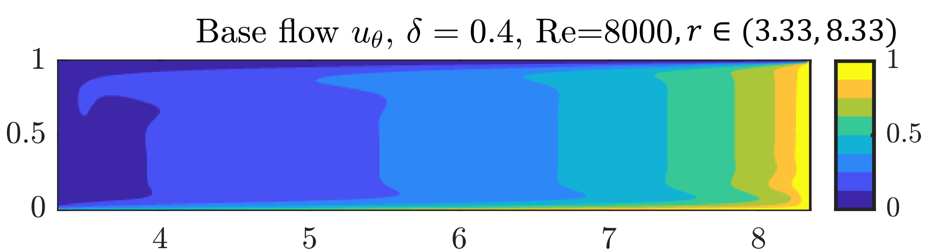
If $R_1 \rightarrow 0$ (rotor-stator) :

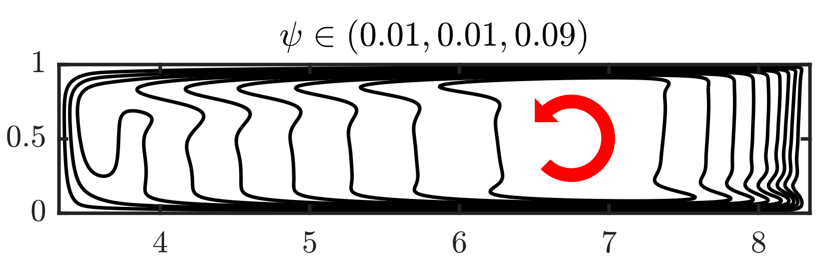
$$(u_r\,, u_\theta\,, \frac{\partial u_z}{\partial r}) = (0,0,0).$$

Aspect ratio R/H=5

Boundary conditions :

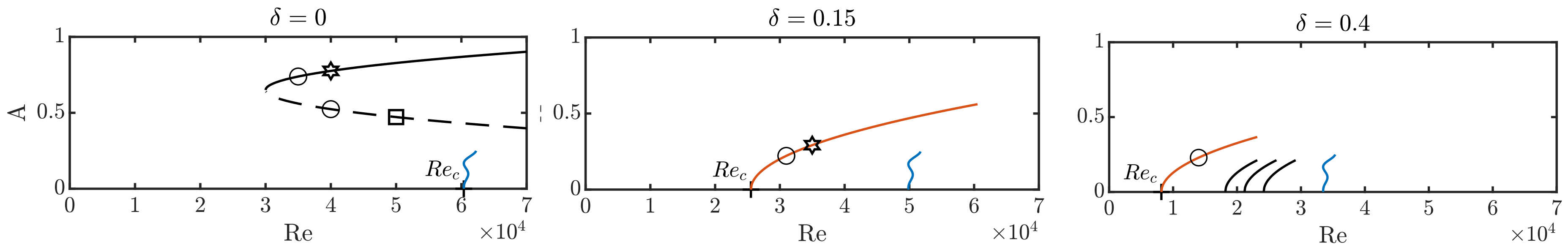
$\boldsymbol{u} = \mathbf{0}$ at $z = 1$	stator,
$\boldsymbol{u} = H\,r/R_2\,\boldsymbol{e}_\theta$ at $z = 0$	rotor,
$\boldsymbol{u} = \boldsymbol{e}_\theta$ at $r_2 = R_2/H,$	shroud.

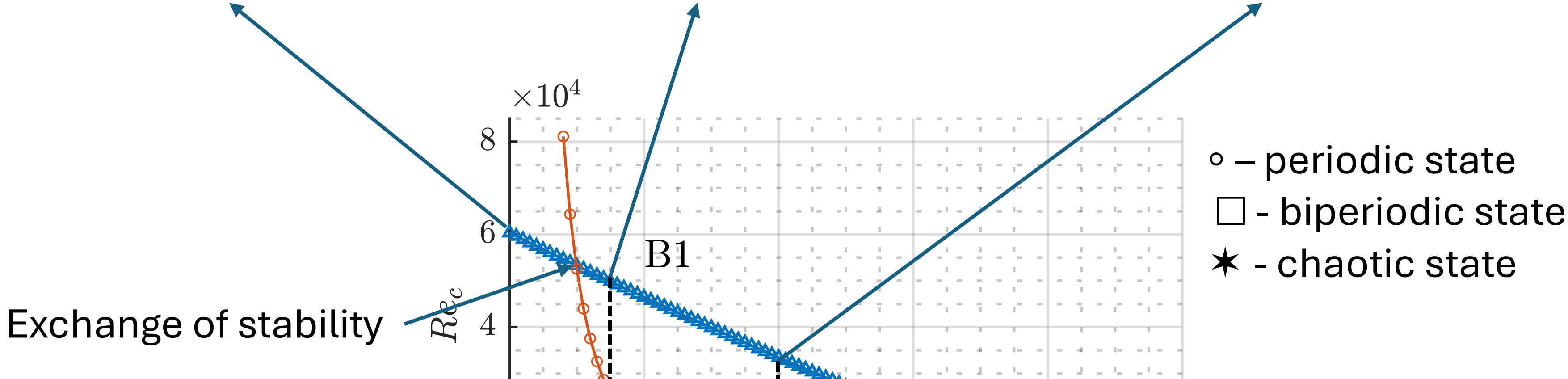


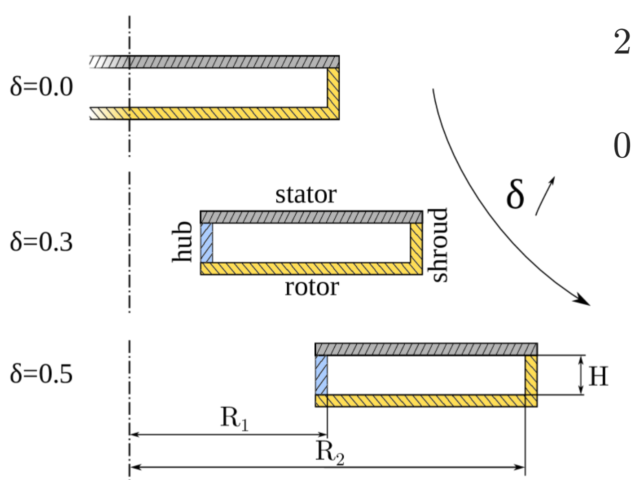


Numerical methods:

- Finite Volume discretization r-z
- Steady state - Newton method + continuation
- Stability – eigenvalue solver (ARPACK)
- Time integration with BDF2 scheme
- Bisection

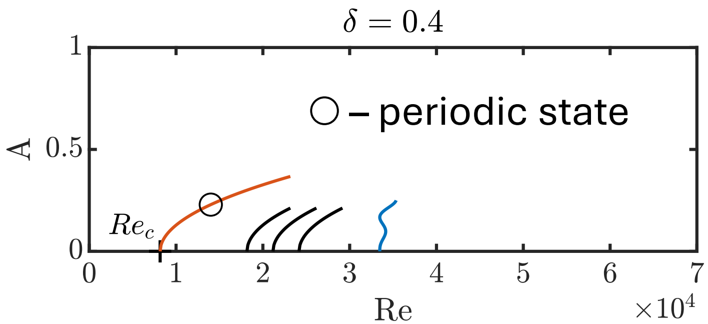


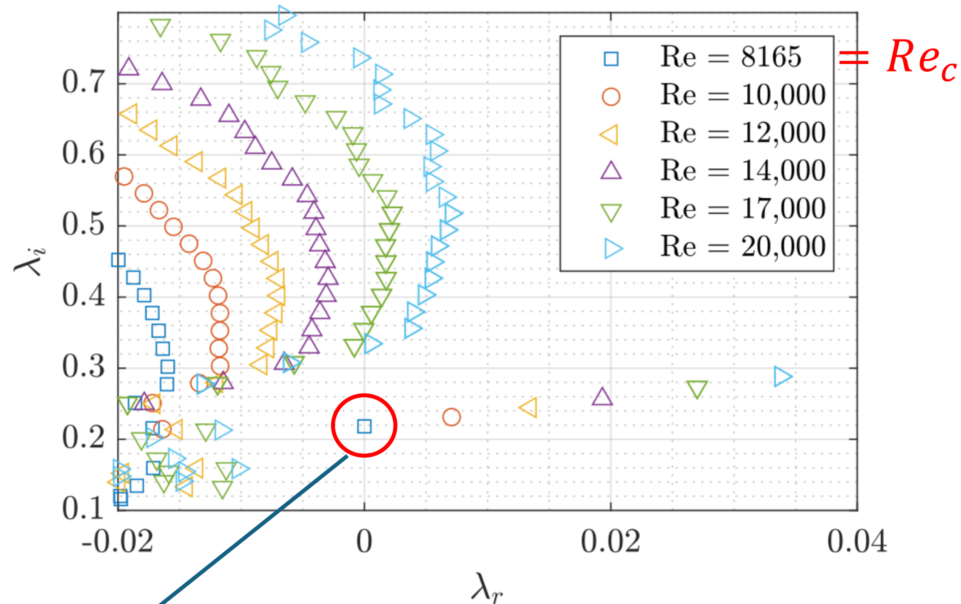


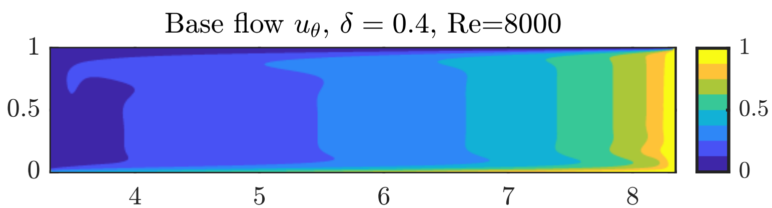


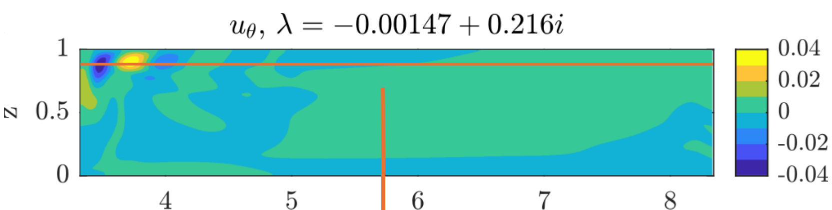
$$\delta = R_1/R_2$$

A supercritical branch is replaced by a steep Hopf branch for decreasing δ. Laminar-turbulent transition becomes subcritical.

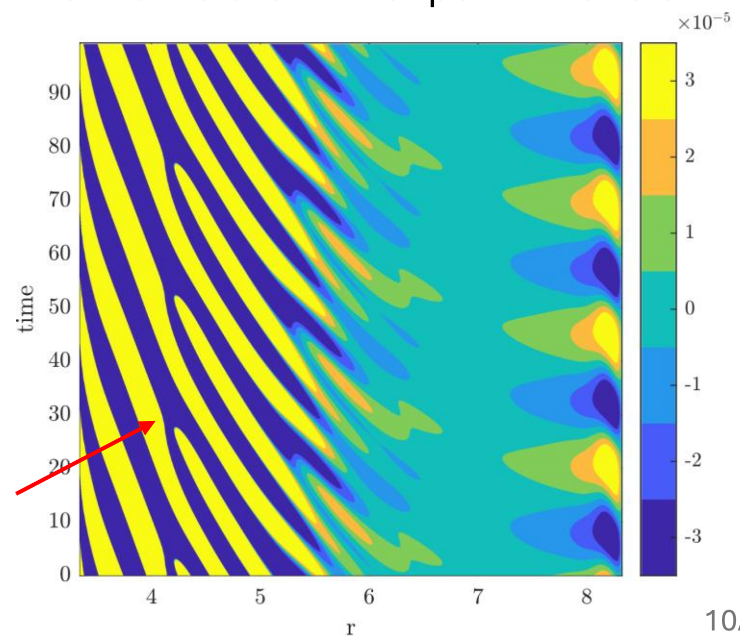






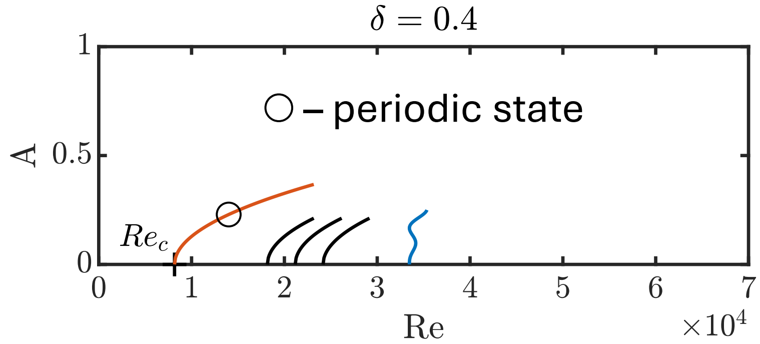


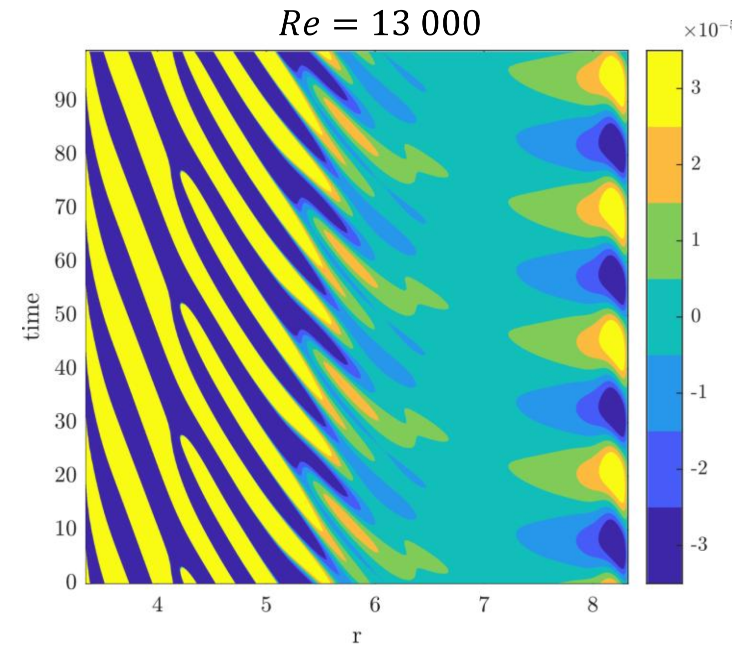
Periodic state on a supercritical branch



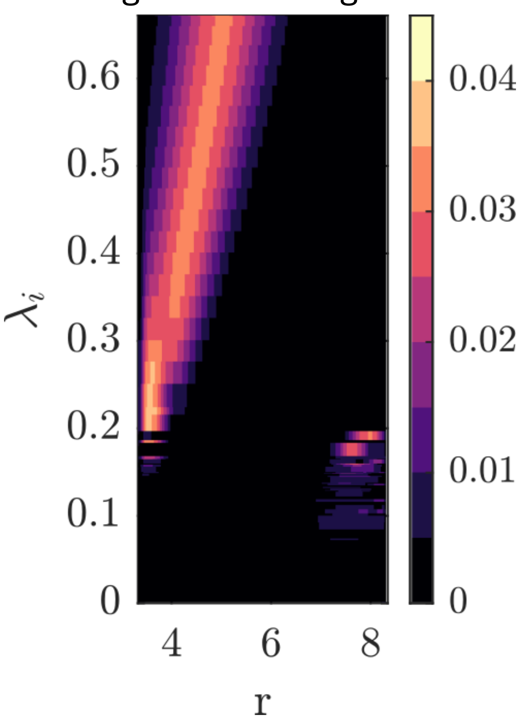
Roll pairing

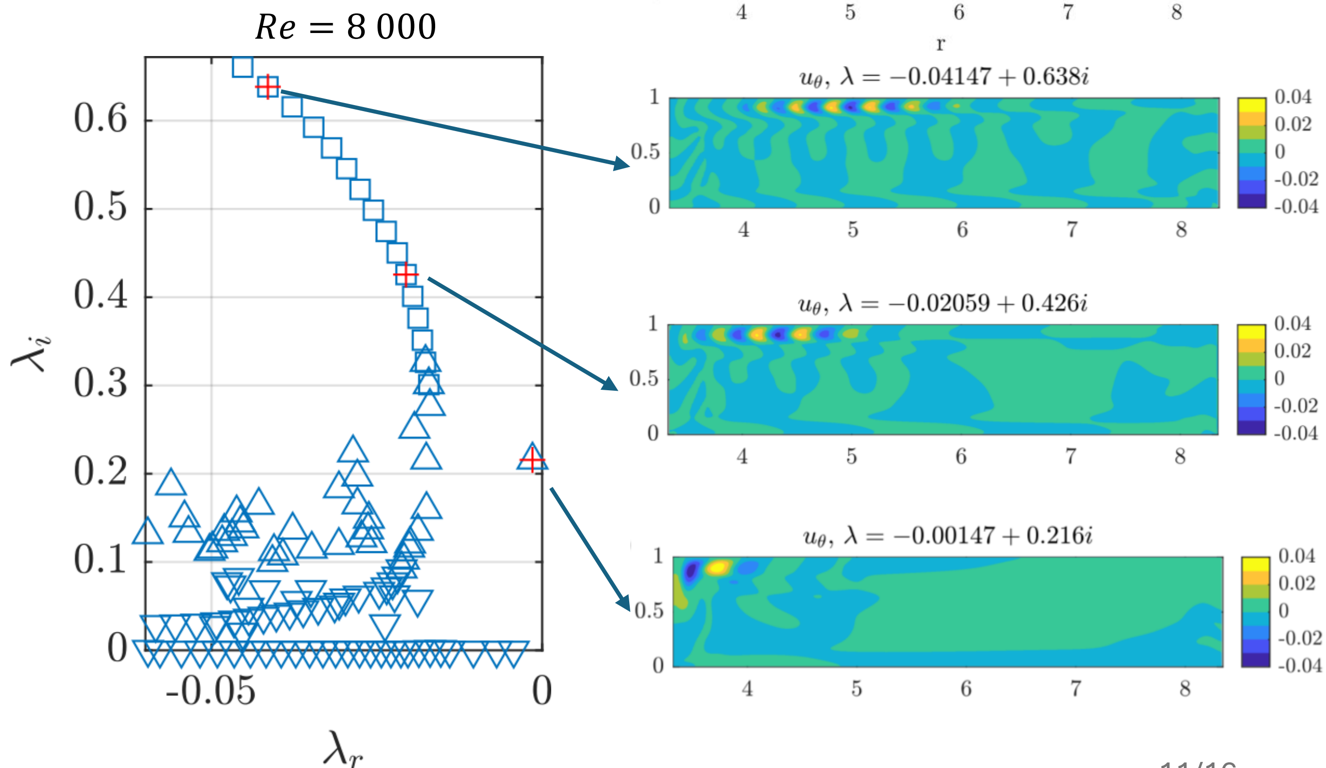
Pairing scenario



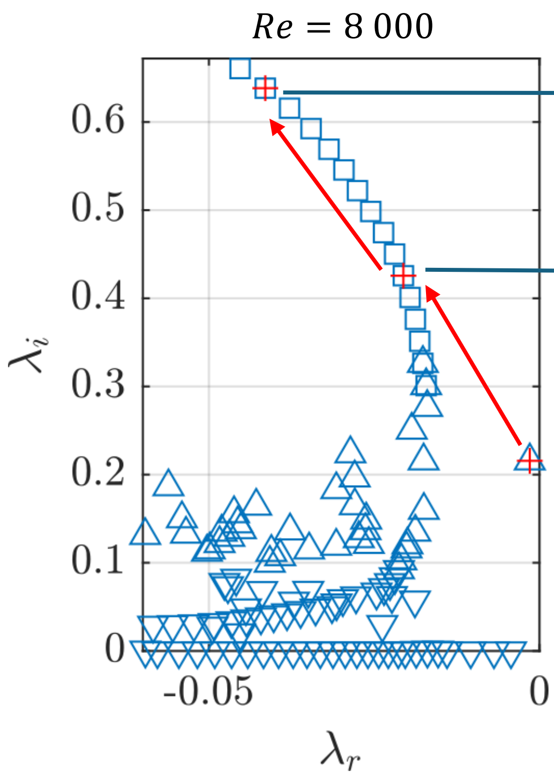


Eigenvector magnitude

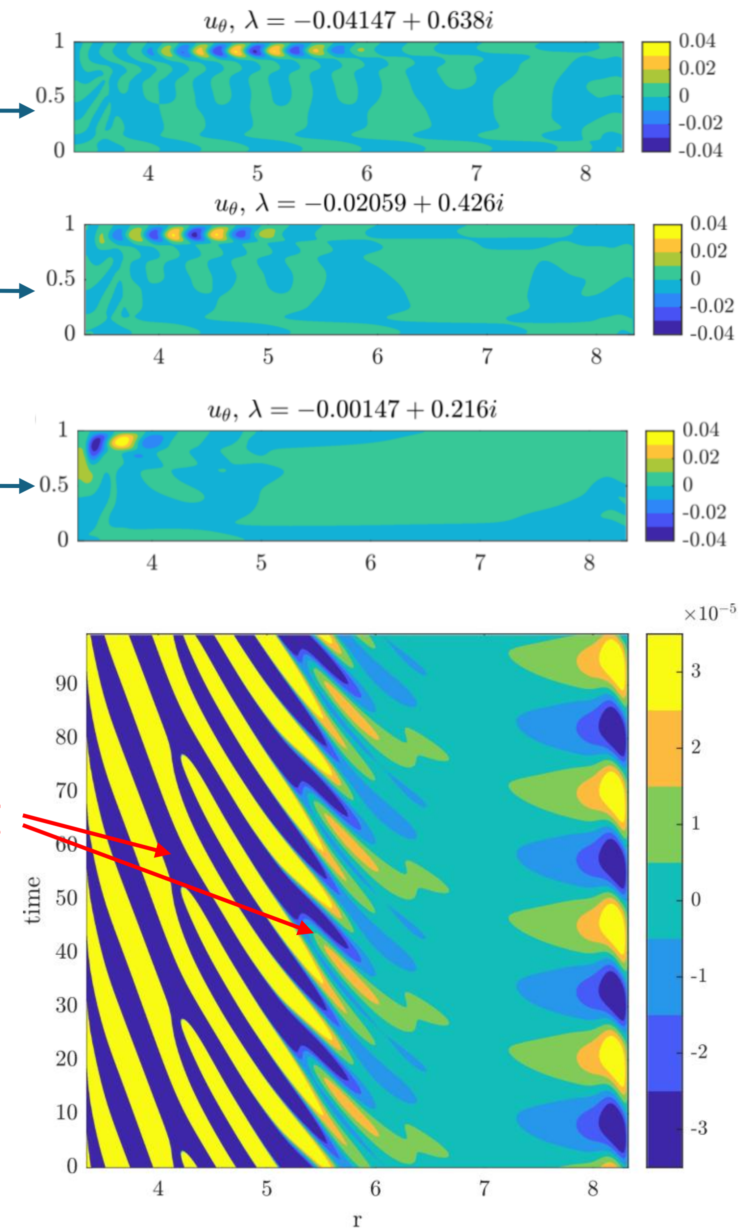




Merging scenario

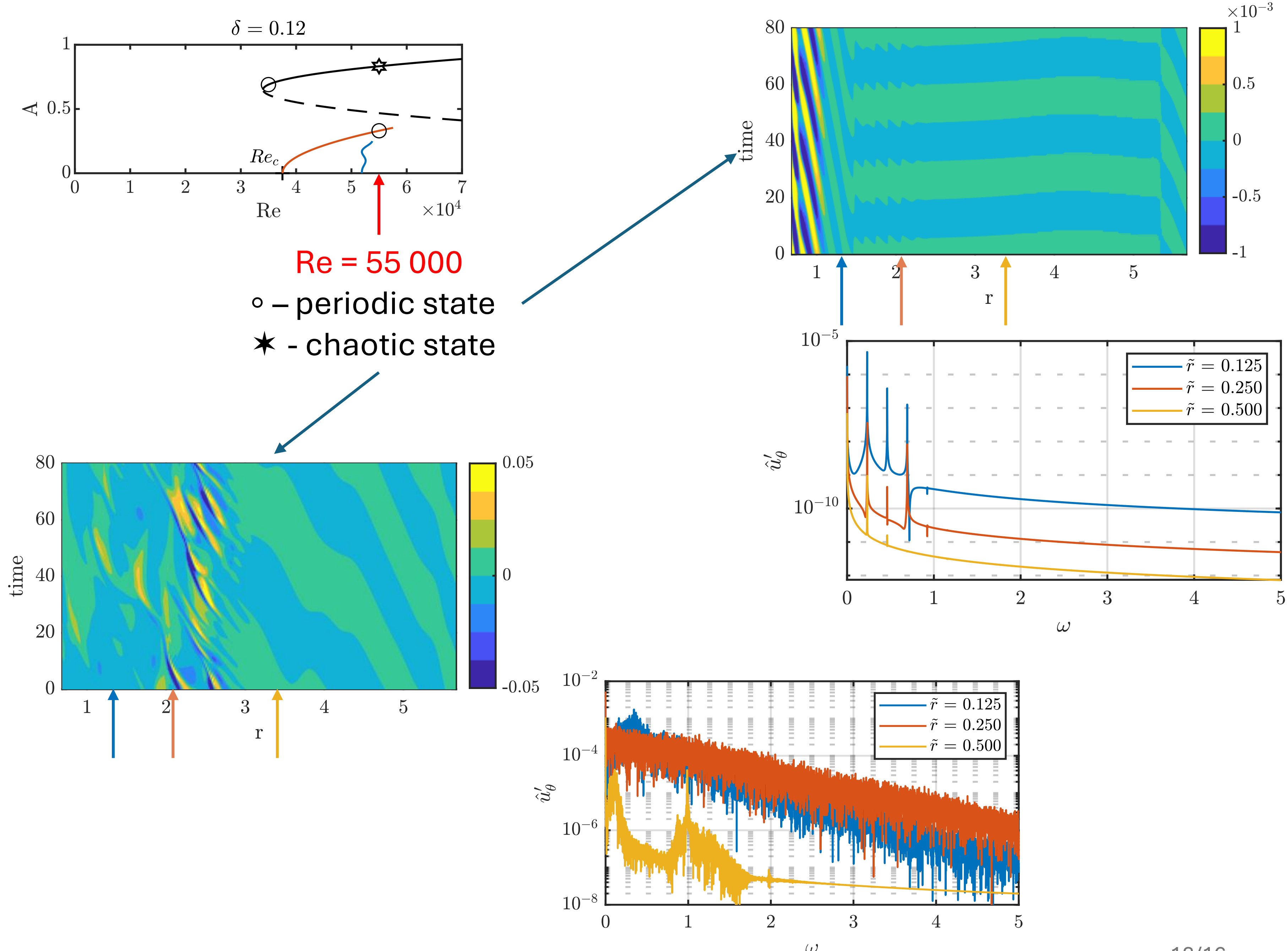


$u \propto e^{\lambda t}$

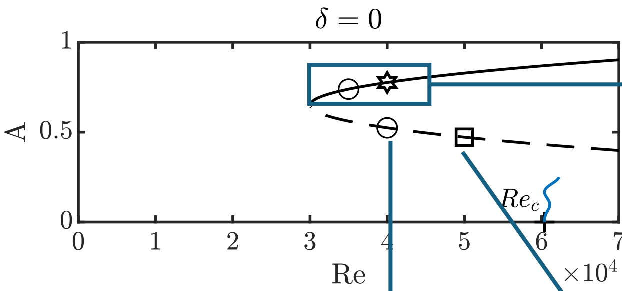


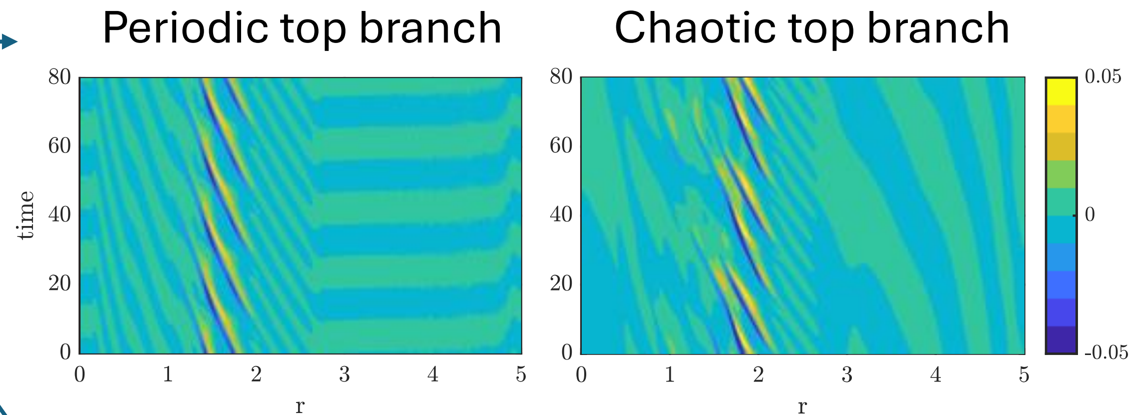
Roll merging

- Interaction of $\lambda_i = 0.216i$ with itself forces the $\lambda_i = 0.426i$
- Interaction of these two forces the $\lambda_i = 0.638i$
- Different spatial support, phase velocity and number of rolls appears as merging



$\delta = 0.12$
A
Re_c
Re
$\times 10^4$
Re = 55 000
∘ – periodic state
✶ - chaotic state
time
r
$\times 10^{-3}$
$\hat{u}'_\theta$
ω
$\tilde{r} = 0.125$
$\tilde{r} = 0.250$
$\tilde{r} = 0.500$



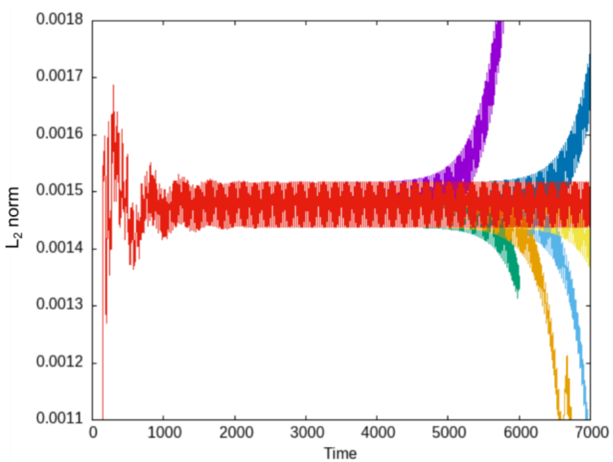


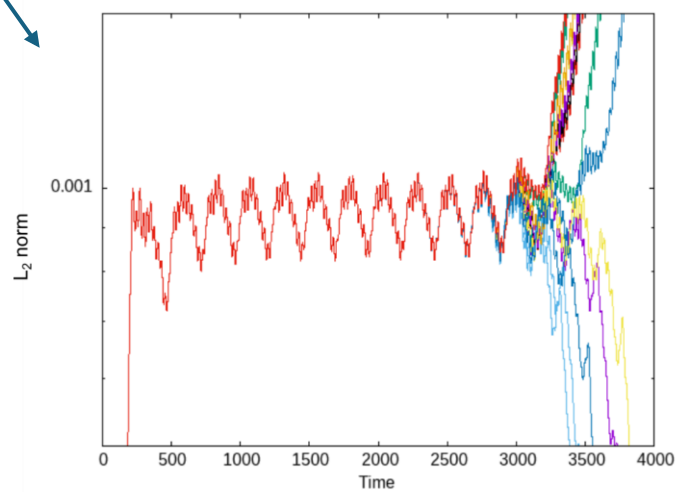
Subcritical transition

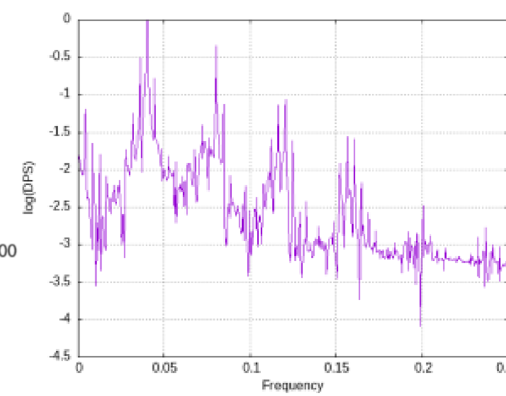
∘ – periodic state

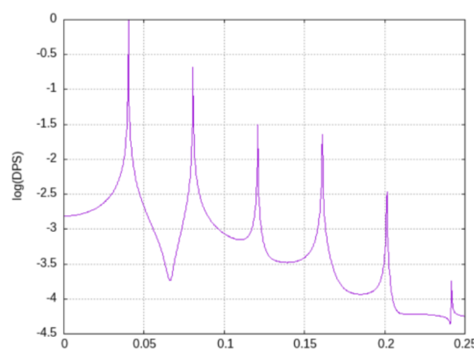
□ - biperiodic state

✶ - chaotic state



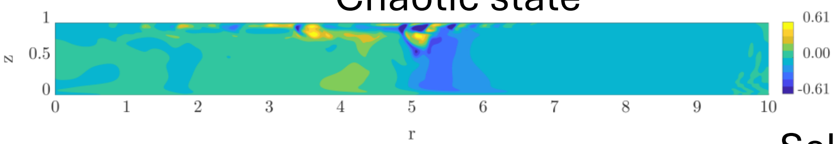




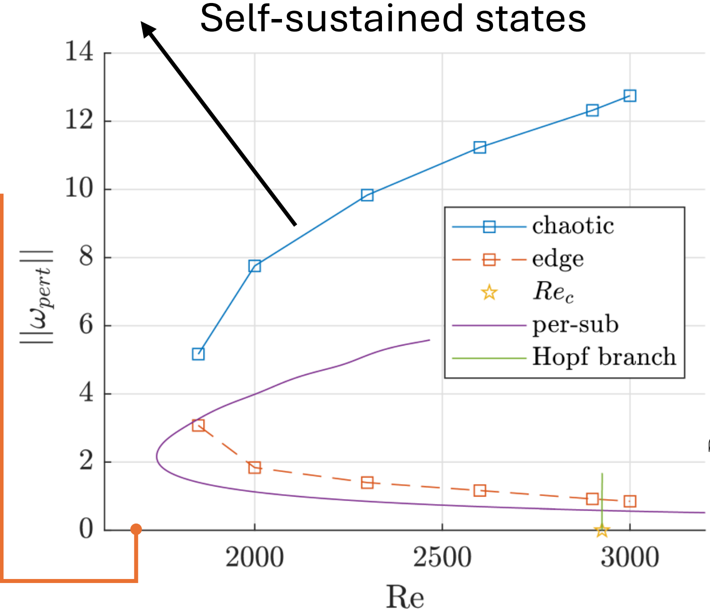


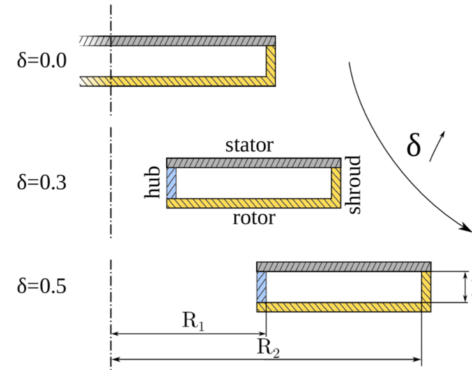
Summary

Chaotic state

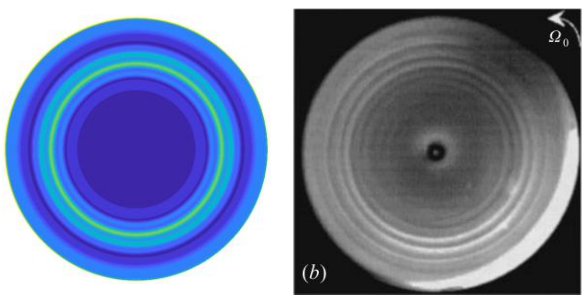


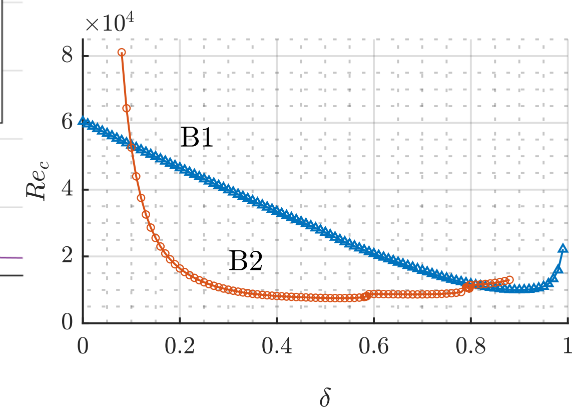
Self-sustained states



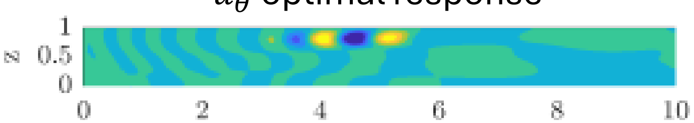


Experimental rolls – $Re \approx 500$





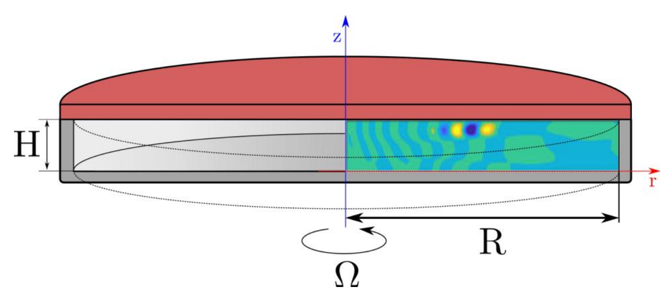
u_θ optimal response



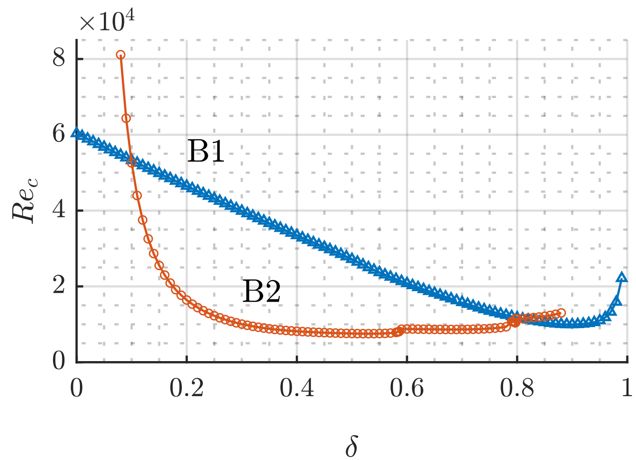
Gesla *et al*. Subcritical axisymmetric solutions in rotor-stator flow, PRF 2024.

Gesla *et al*. From annular cavity to rotor-stator flow: nonlinear dynamics of axisymmetric rolls, to appear in PRF 2025.

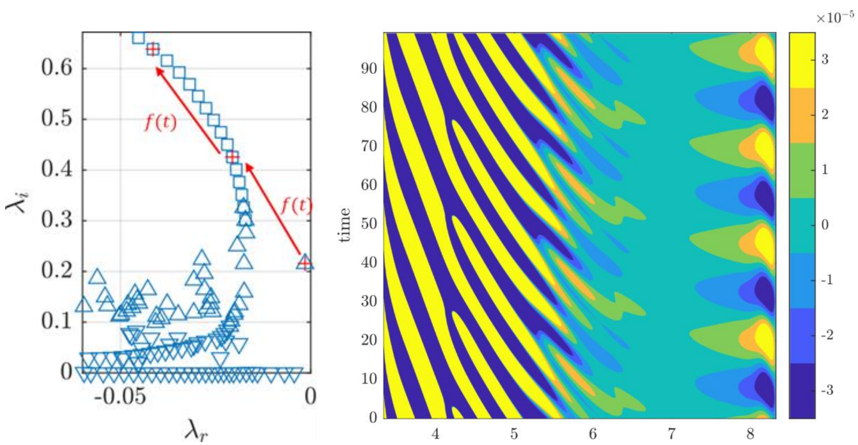
Gesla *et al*. On the origin of circular rolls in rotor-stator flow, JFM 2024.



Summary:



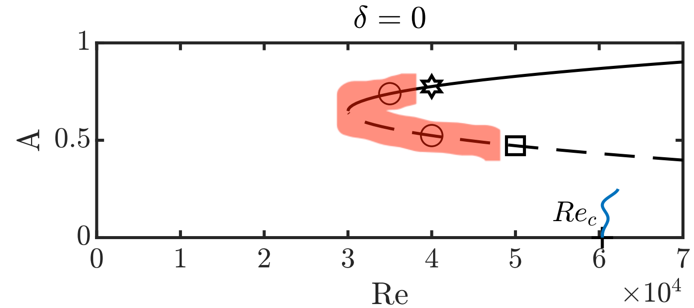
Stability exchange for a radially displaced cavity.



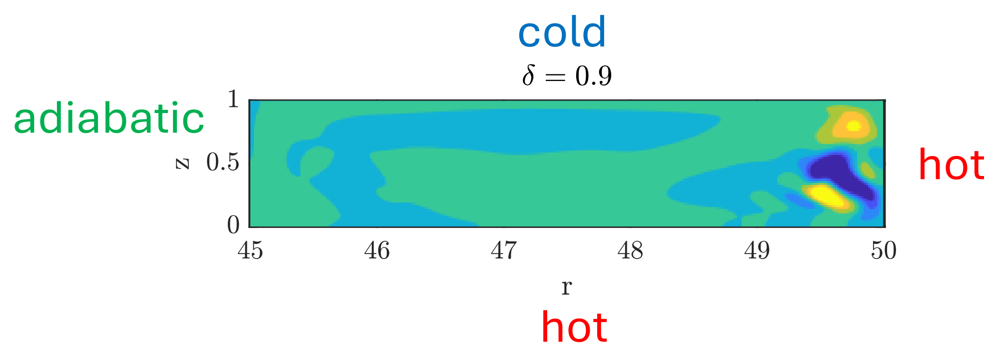
New perspective on the roll merging.

Gesla *et al*. From annular cavity to rotor-stator flow: nonlinear dynamics of axisymmetric rolls, accepted PRF 2025.

Outlooks:

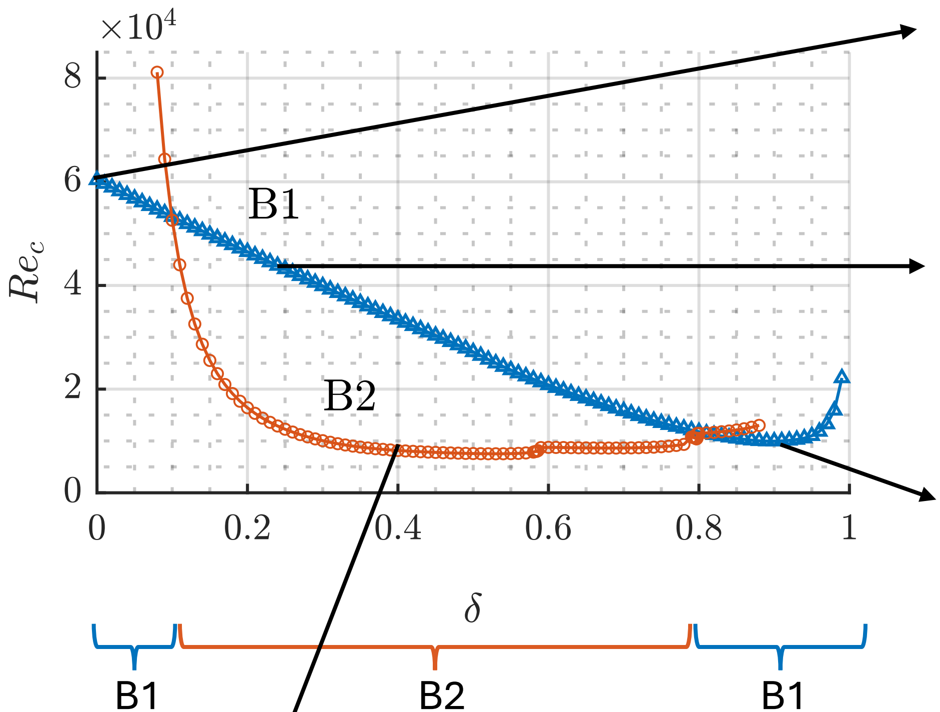


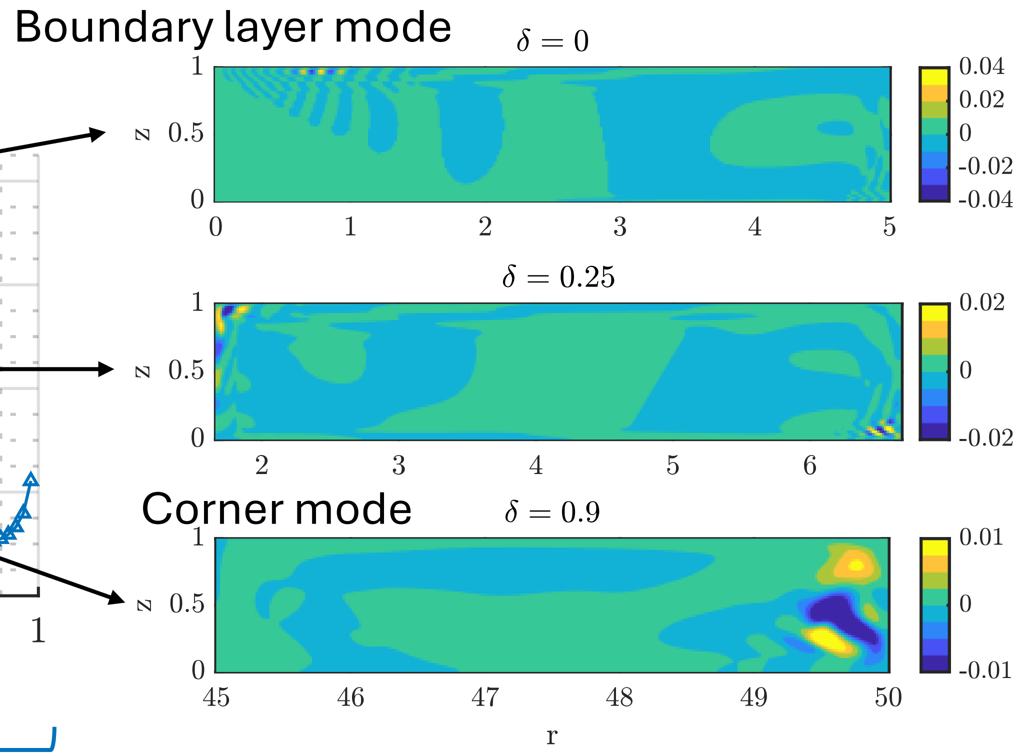
Role of periodic states in edge state/ top branch dynamics (Harmonic Balance Method, Floquet Analysis)

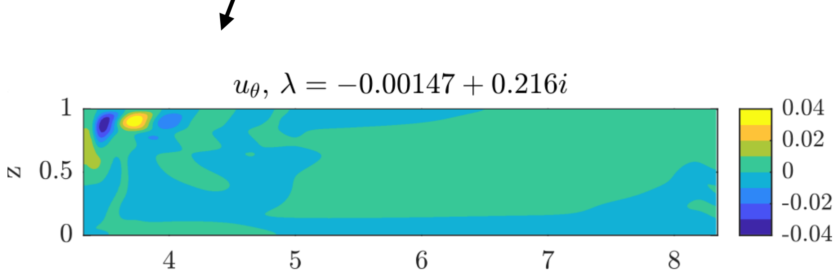


$\delta \to 1$ analysis, similarity to a Differentially Heated Cavity for $\mathrm{Pr} = 1$ and $\Delta\Omega \ll \Omega$, comparison of thresholds in Ra and Re

Critical Re



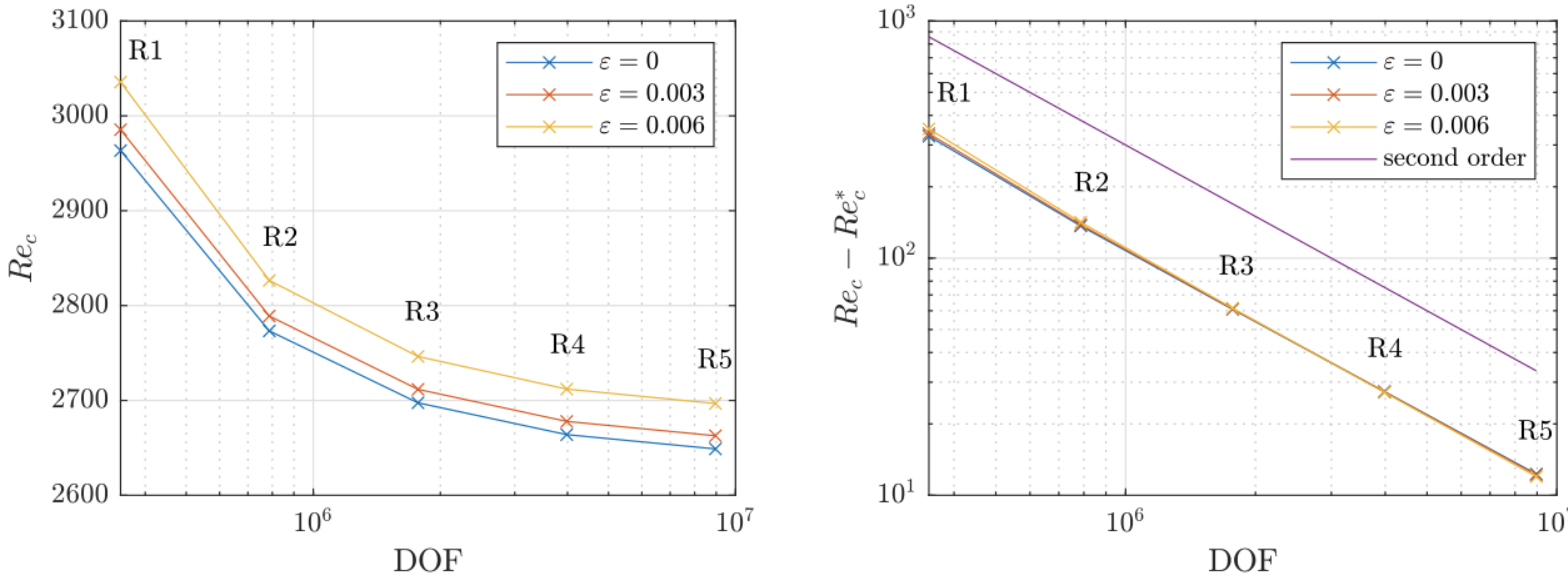




- Branch B1 is responsible for the linear instability at $\delta < 0.1$ and $\delta > 0.8$.
- Eigenmode changes continuously from a boundary layer structure to a corner structure.

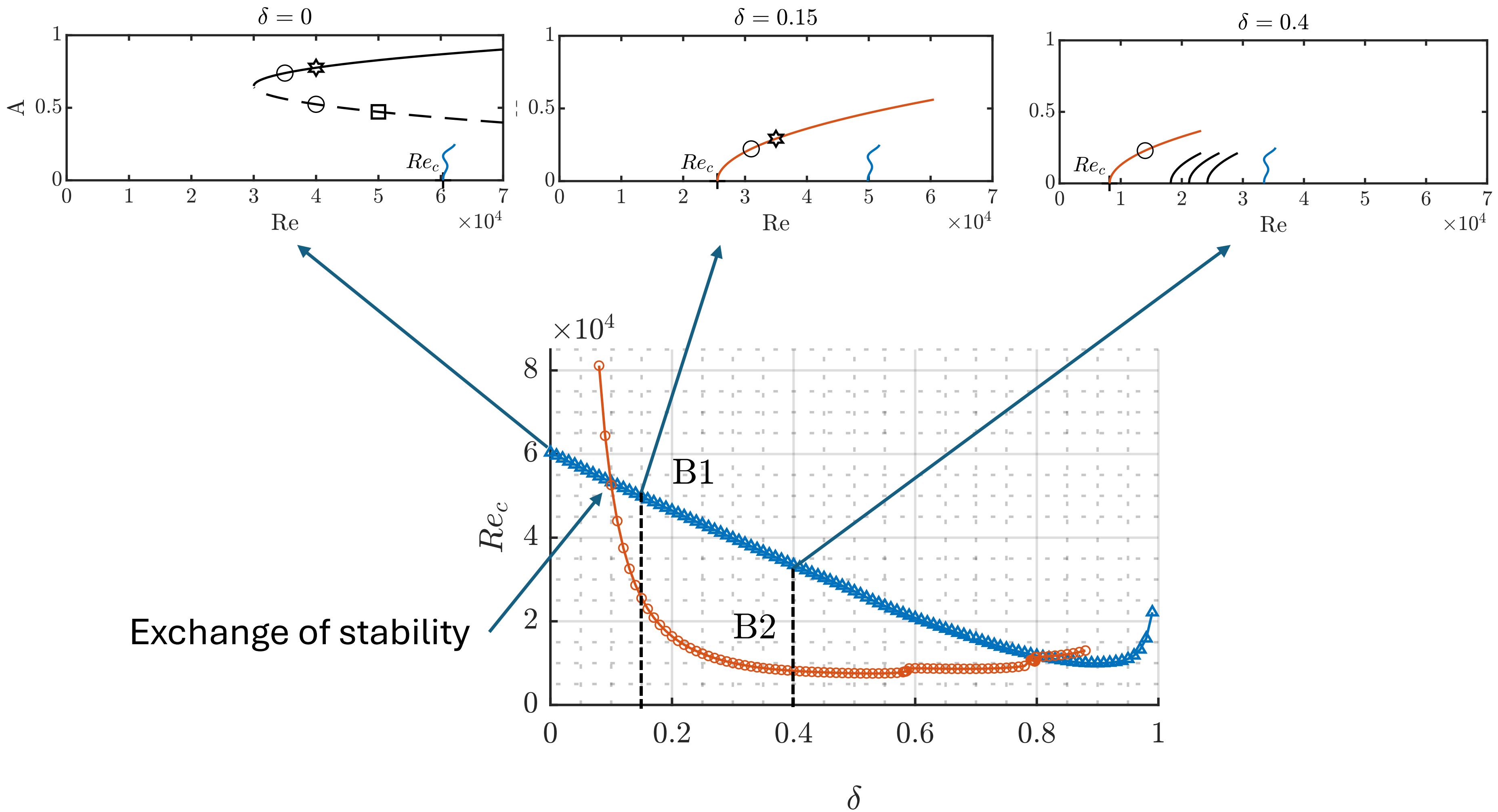
Corner singularity

the corner singularity is also considered. This is achieved here by smoothing out the boundary condition at the Bödewadt corner, imposing an exponential velocity profile of the form $u_\theta = r\exp\left(\frac{r-\Gamma}{\varepsilon}\right)$. Two regularisations have been considered : $\varepsilon = 0.003$ and $\varepsilon = 0.006$. The case without any regularization $(u_\theta = 0)$ is referred to as $\varepsilon = 0$ for ease of notation.

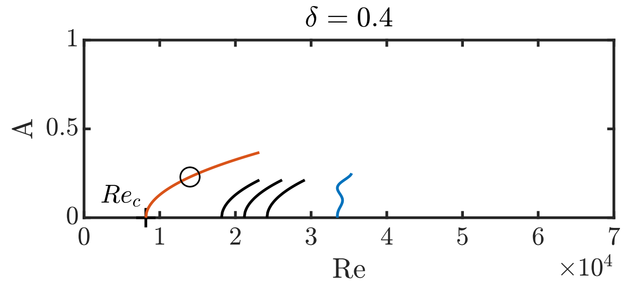


resolution	N_r	N_z	type	DOF	$\varepsilon = 0$	$\varepsilon = 0.003$	$\varepsilon = 0.006$
R0	**600**	**160**	uniform	**390 k**	**2925.47**		
R1	683	128	non-uniform	356 k	2963.41	2985.43	3035.61
R2	**1024**	**192**	non-uniform	**796 k**	**2773.3**	2789.01	2826.55
R3	1536	288	non-uniform	1.7 m	2697.48	2711.71	2746.33
R4	2304	432	non-uniform	4.0 m	2663.96	2677.9	2711.97
R5	3456	648	non-uniform	9.0 m	2648.9	2662.8	2696.8
				extrapolation	2636.61	2650.59	2684.81
				order	2.01	2.04	2.09

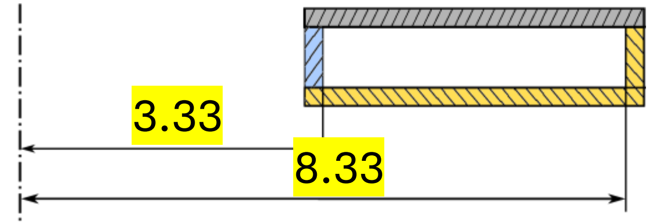
TABLE V. Critical Reynolds number Re_c depending on the the spatial discretisation. From R1 to R5 the ratio between two consecutive grid resolutions is 1.5 in each direction.

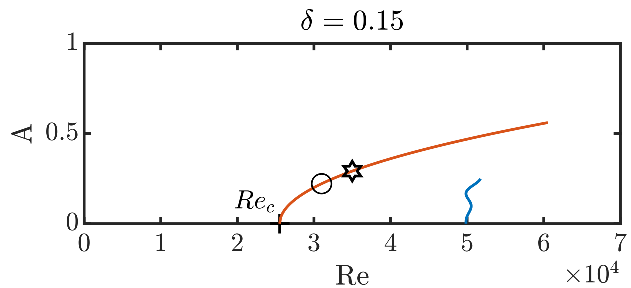


A supercritical branch is replaced by a steep Hopf branch for decreasing δ.
Laminar-turbulent transition becomes subcritical.

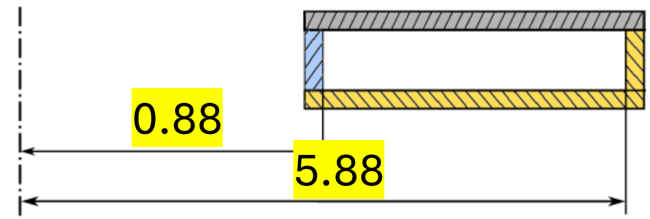


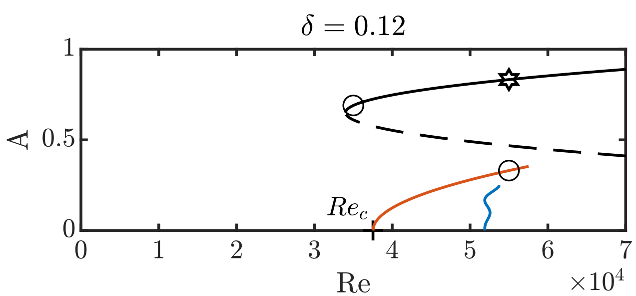
Supercritical transition
∘ – periodic state



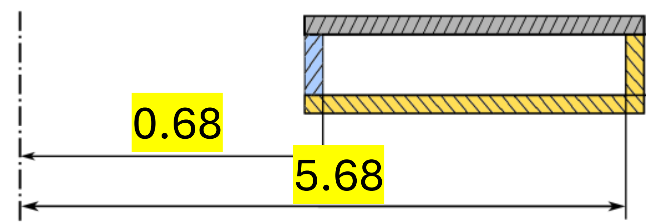


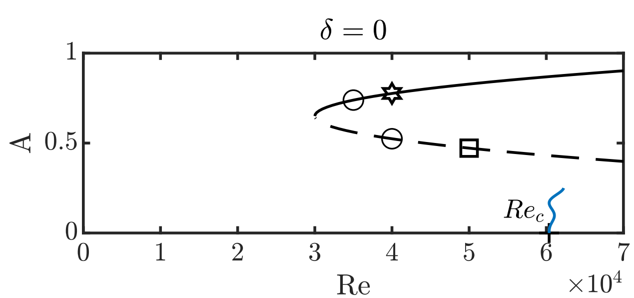
Supercritical transition
∘ – periodic state
✶ - chaotic state



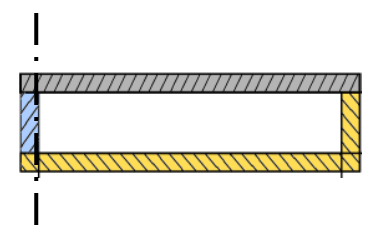


Competition between branches
∘ – periodic state
✶ - chaotic state





Subcritical transition
∘ – periodic state
□ - biperiodic state
✶ - chaotic state



*Here $Re = \Omega RH/\nu$