From annular cavity to rotor-stator flow: nonlinear dynamics of axisymmetric rolls

Artur Gesla [1,2,*]

Patrick Le Quéré [2]

Yohann Duguet [2]

Laurent Martin Witkowski [3]

[1] Sorbonne Université

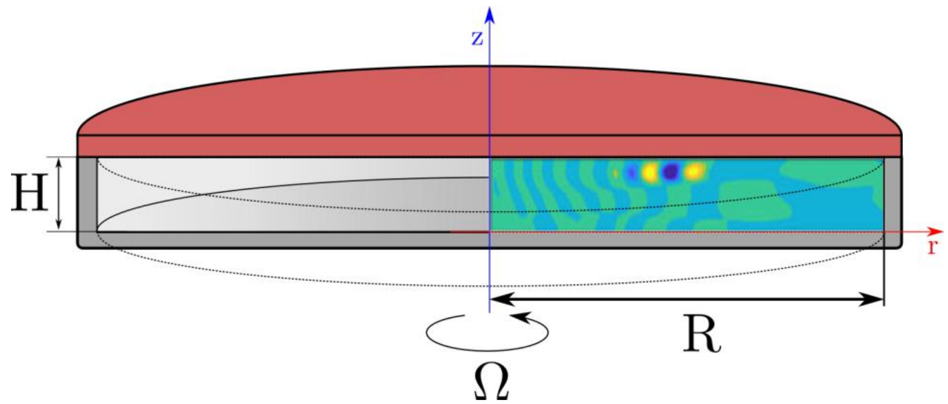
[2] Université Paris-Saclay, LISN-CNRS

[3] Université Claude Bernard Lyon 1, LMFA

* now at EPFL, group HEAD with Eunok Yim

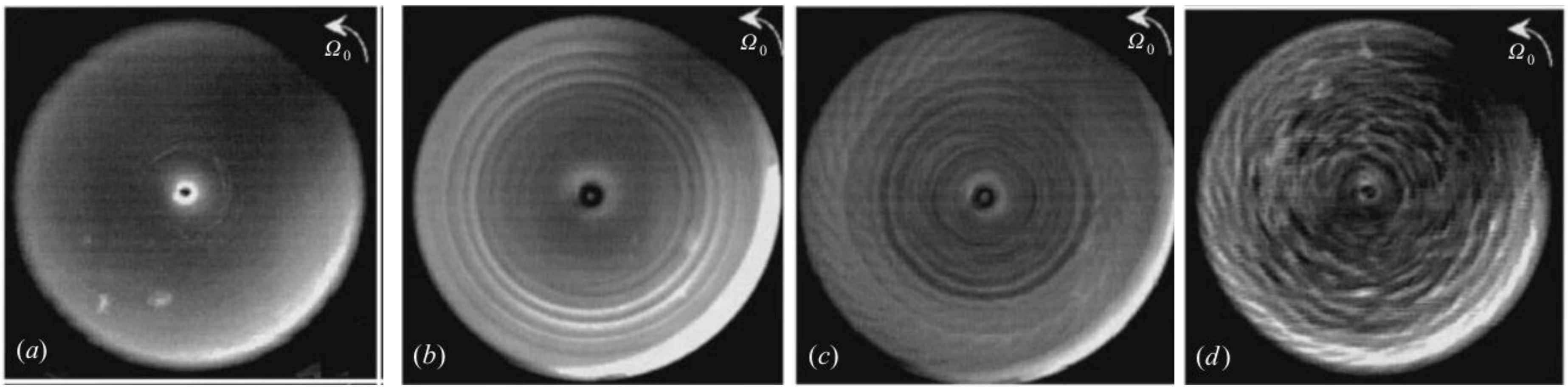
$$Re = H^2\Omega/\nu$$

Aspect ratio : $\Gamma = R\ /H$



Circular rolls

Circular + spiral rolls



Re=150

Re=500

Re=700

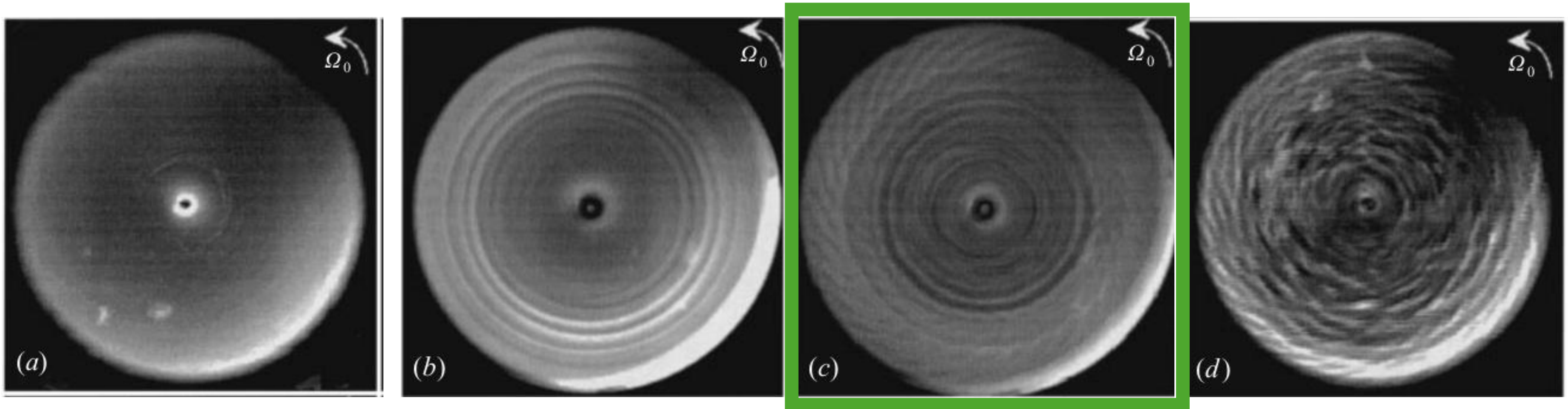
Re=1200

Re

Gauthier *et al.*, 1999

Circular rolls

Circular + spiral rolls

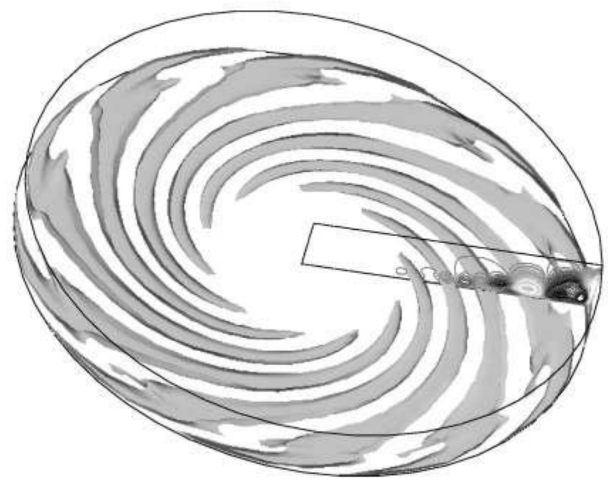


Gauthier *et al.*, 1999

First linearly unstable mode, R/H=10

Gelfgat, Fluid Dyn. Res. 2015

Saturated state, DNS, R/H=5



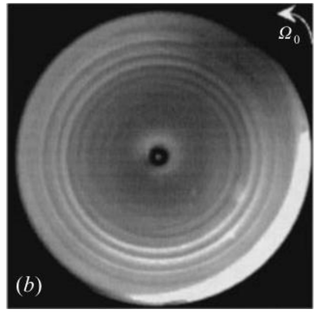
Serre *et al.*, PoF 2004

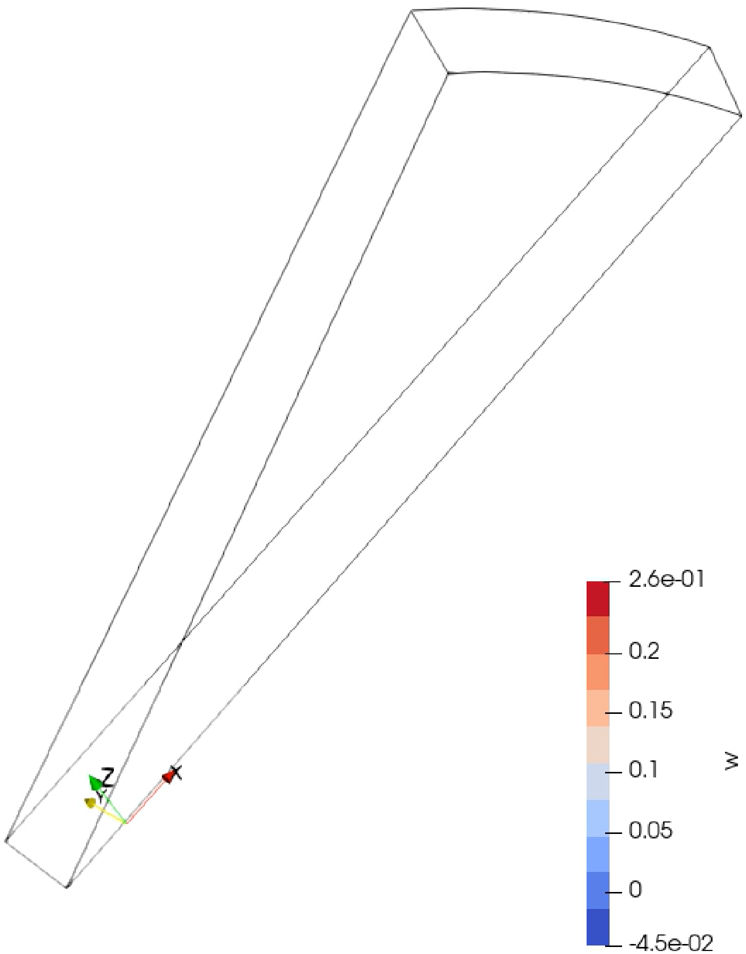
Challenge: No clear explanation for the circular rolls.

DNS

Re=500 Time: 0.000000

Experiment

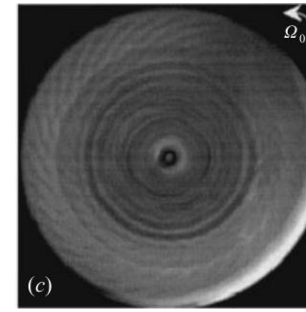


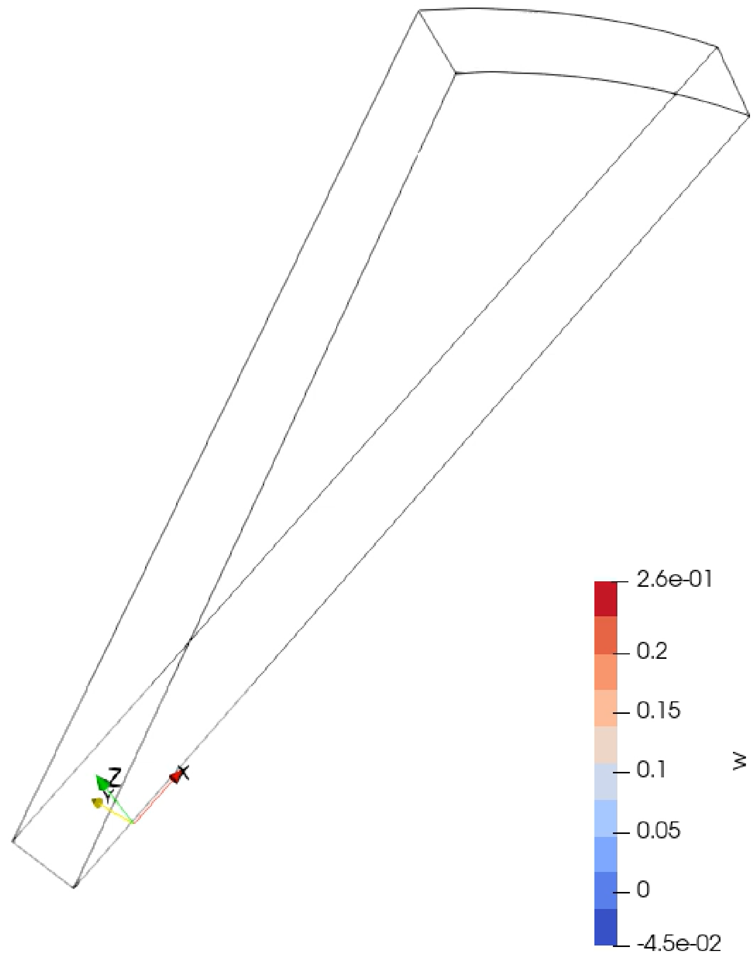


DNS

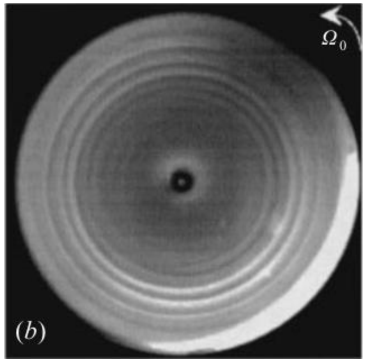
Re=700 Time: 0.000000

Experiment





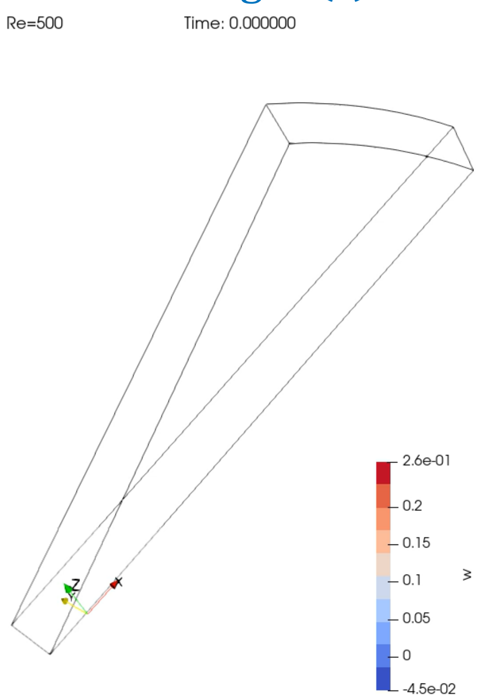
Experiment

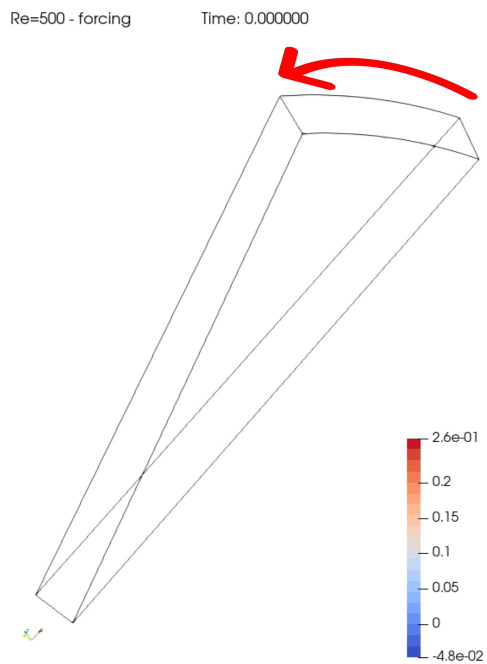


Circular rolls seen experimentally are a linear response to forcing.

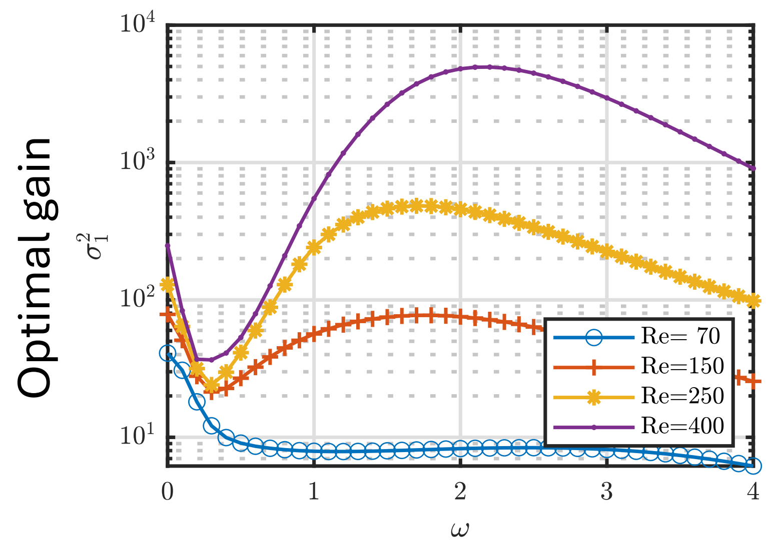
No forcing, $\Omega(t) = 1$

$\Omega(t) = 1 +$ white noise

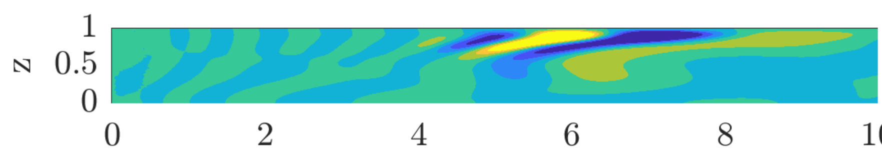




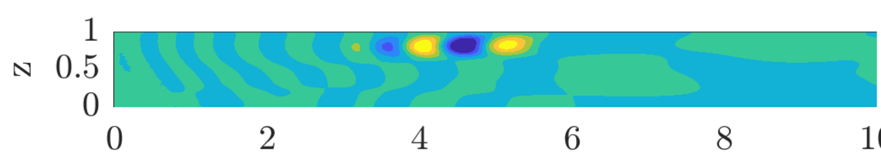
Resolvent analysis (axisymmetric)



f_θ optimal forcing



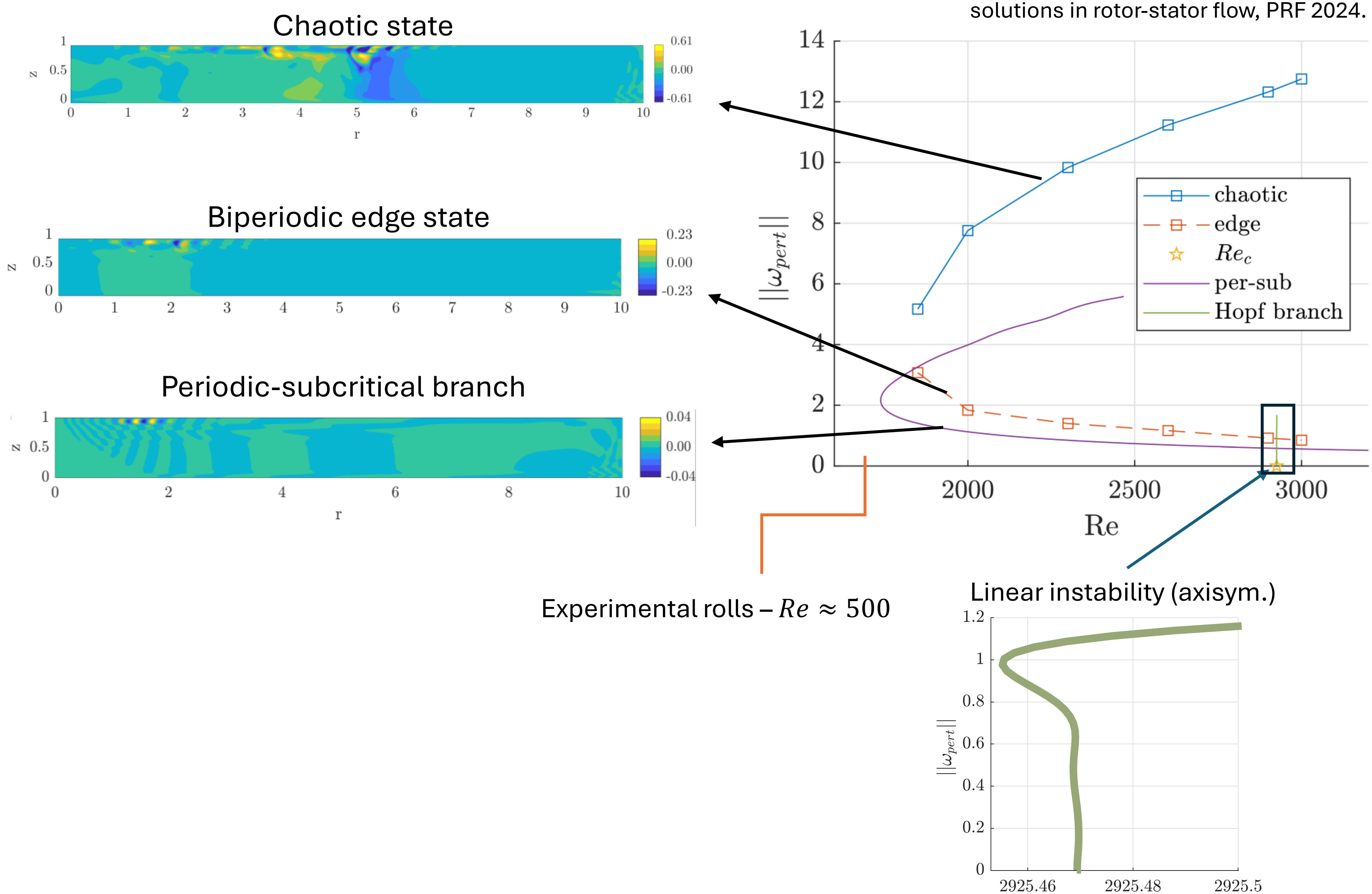
u_θ linear response to forcing



Gesla *et al.* On the origin of circular rolls in rotor-stator flow, JFM 2024.

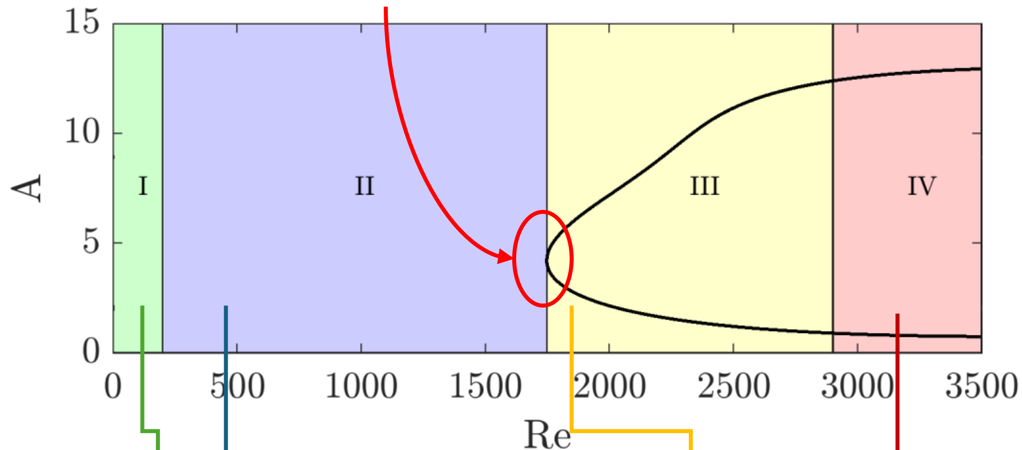
Self-sustained axisymmetric solutions

Gesla *et al*. Subcritical axisymmetric solutions in rotor-stator flow, PRF 2024.



Dynamics of circular rolls

Finite lifetimes



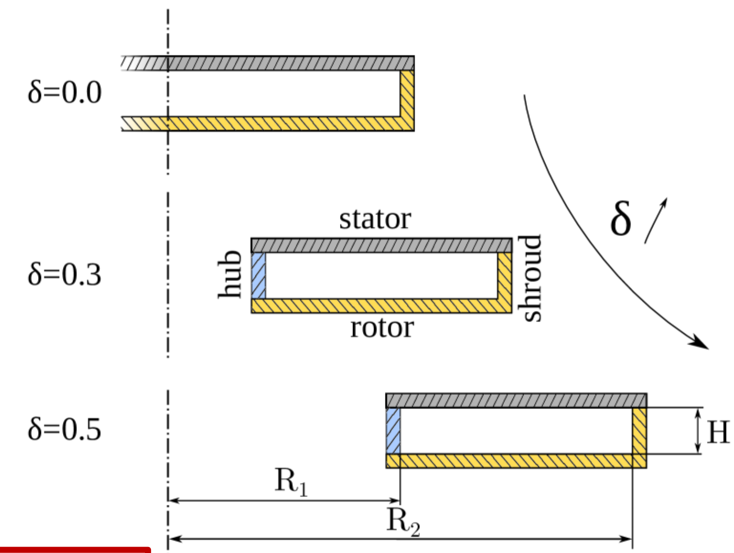
Stable flow, no rolls

Stable flow, circular rolls appear as a response to forcing

Lin. stable flow, self-sustained solutions exist

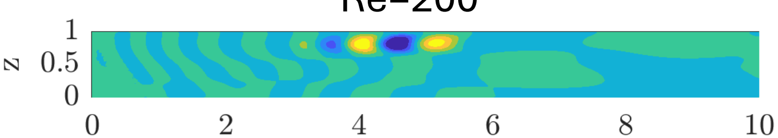
Unstable flow, evolution to chaotic dynamics

Use **homotopy** to explain the origin of the self-sustained states.

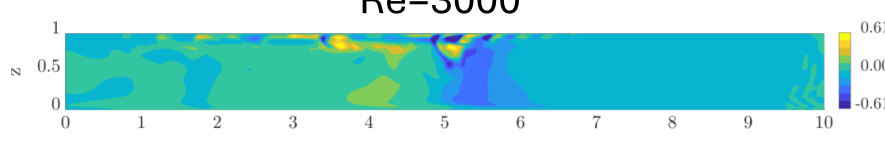


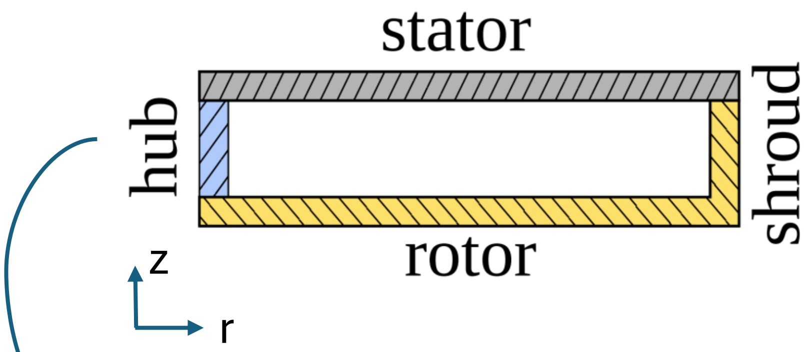
$$\delta = R_1/R_2$$

Re=200



Re=3000





$$\frac{\partial \mathbf{u}}{\partial t} + \nabla(\mathbf{u} \otimes \mathbf{u}) = -\nabla p + \frac{1}{Re}\nabla^2 \mathbf{u}$$

$$\nabla \cdot \mathbf{u} = 0$$

Stress-free on the hub :

$$\left(u_r\,, \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\,, \frac{\partial u_z}{\partial r}\right) = (0,0,0) \text{ at } r_1 = R_1/H.$$

If $R_1 \to 0$ (rotor-stator) :

$$\left(u_r\,, u_\theta\,, \frac{\partial u_z}{\partial r}\right) = (0,0,0).$$

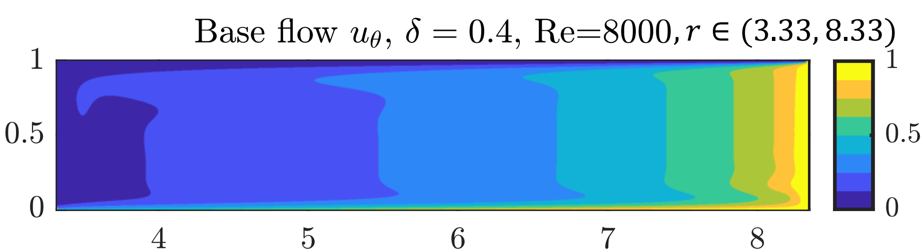
Aspect ratio R/H=5

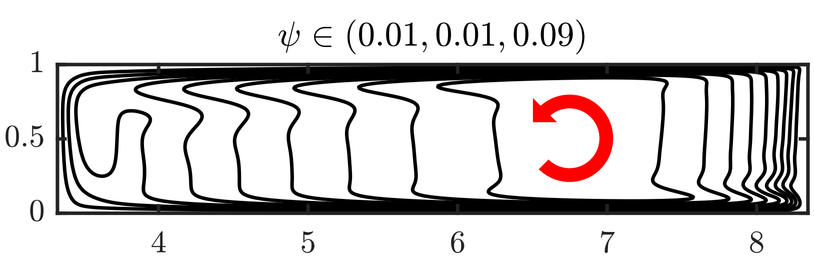
Boundary conditions :

$$\boldsymbol{u} = \mathbf{0} \text{ at } z = 1 \qquad \text{stator,}$$

$$\boldsymbol{u} = H\, r/R_2\, \boldsymbol{e}_\theta \text{ at } z = 0 \qquad \text{rotor,}$$

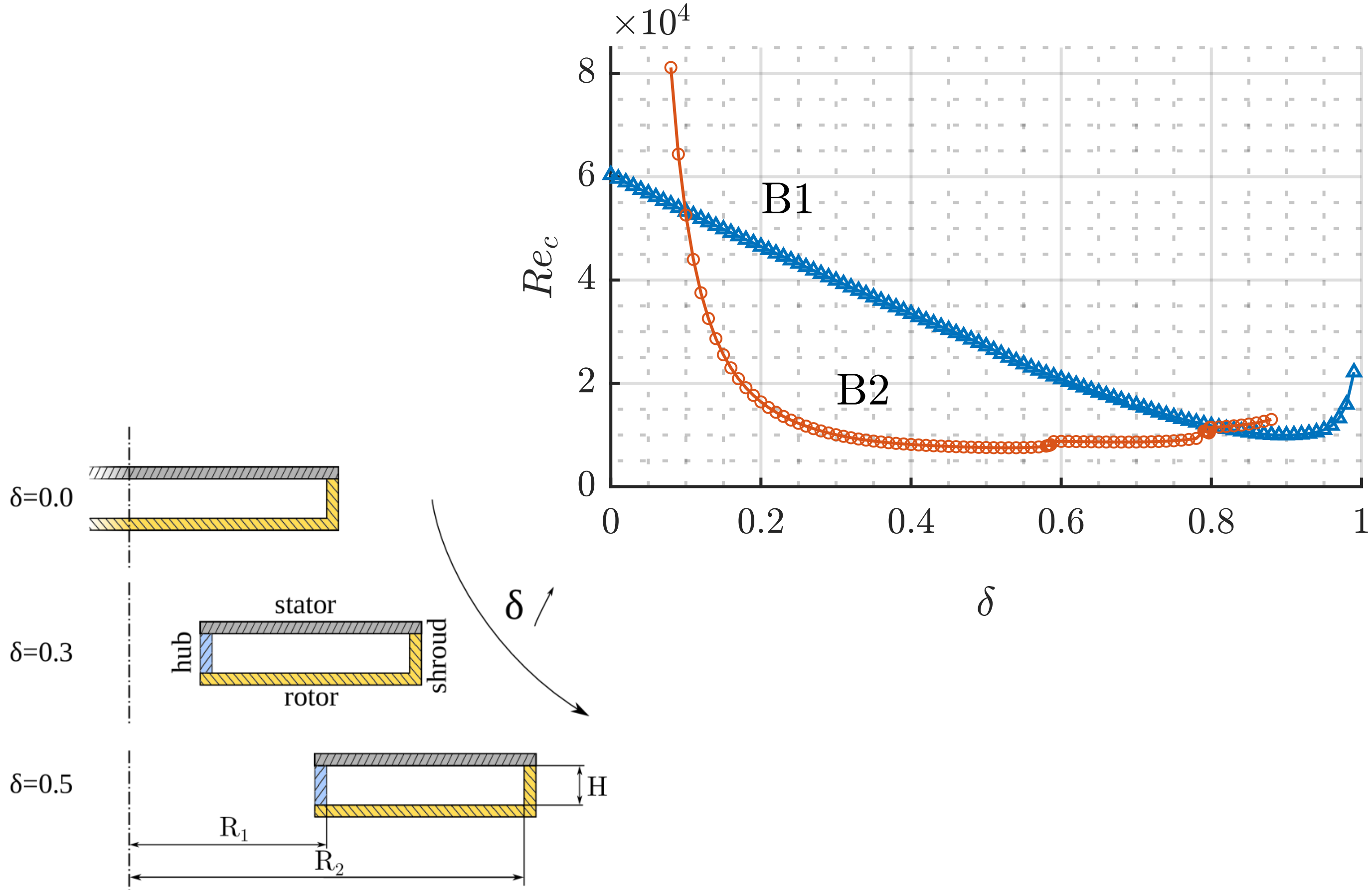
$$\boldsymbol{u} = \boldsymbol{e}_\theta \text{ at } r_2 = R_2/H, \qquad \text{shroud.}$$



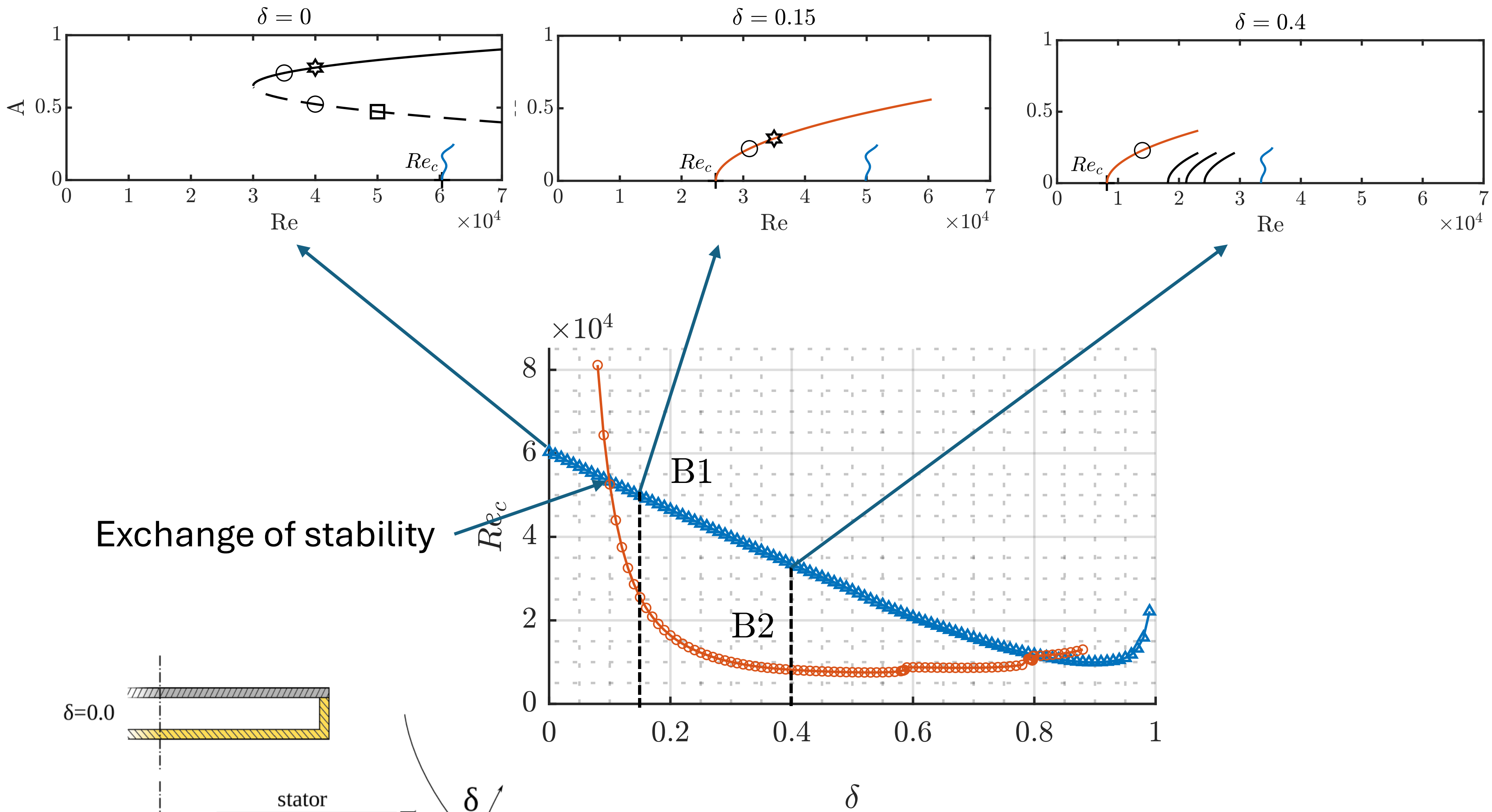


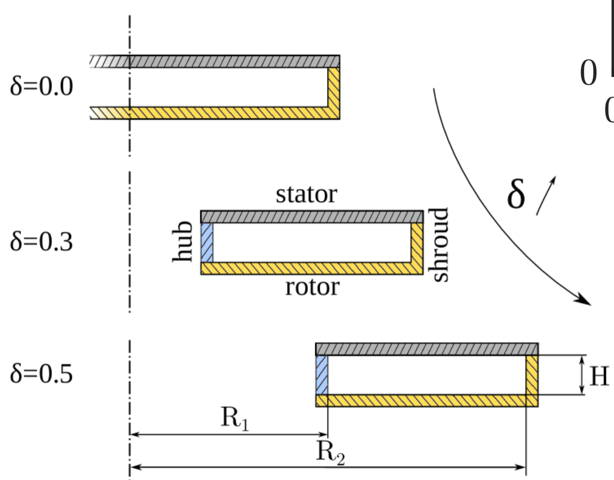
Numerical methods:

- Finite Volume discretization r-z
- Steady state - Newton method + continuation
- Stability – eigenvalue solver (ARPACK)
- Time integration with BDF2 scheme
- Bisection

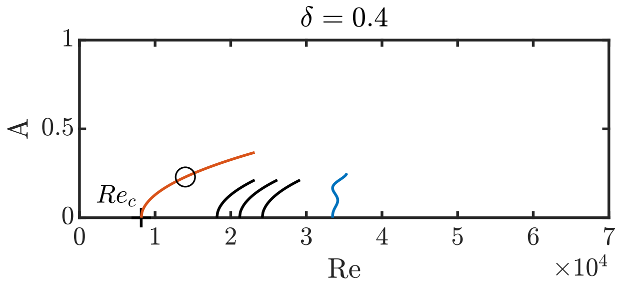


$\times 10^4$
8
6
4
2
0
Re_c
B1
B2
0
0.2
0.4
0.6
0.8
1
δ
δ=0.0
δ=0.3
δ=0.5
δ
stator
hub
shroud
rotor
H
R_1
R_2

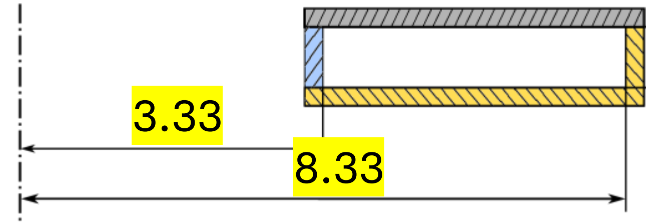


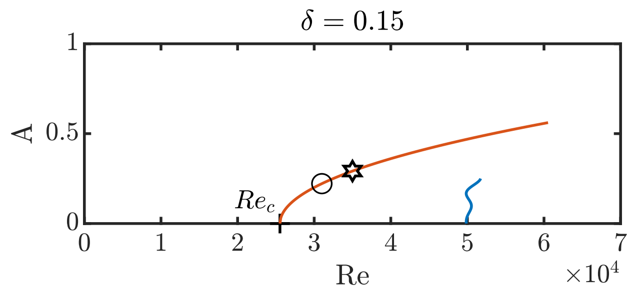


A supercritical branch is replaced by a steep Hopf branch for decreasing δ.
Laminar-turbulent transition becomes subcritical.

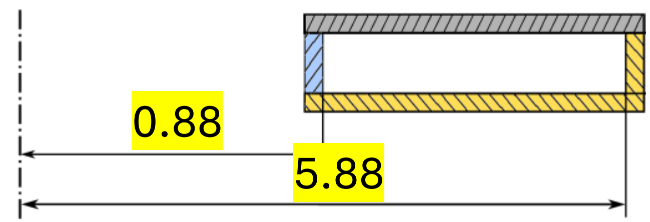


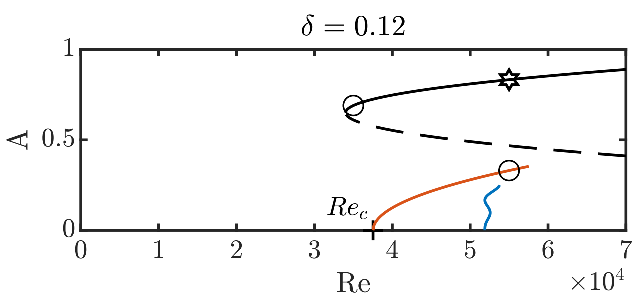
Supercritical transition
∘ – periodic state



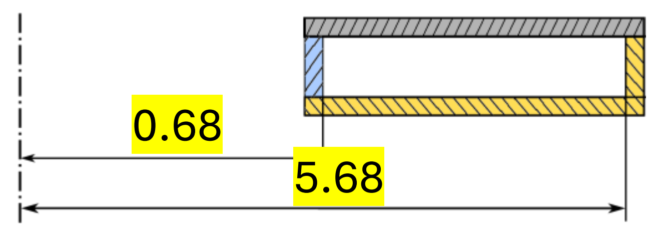


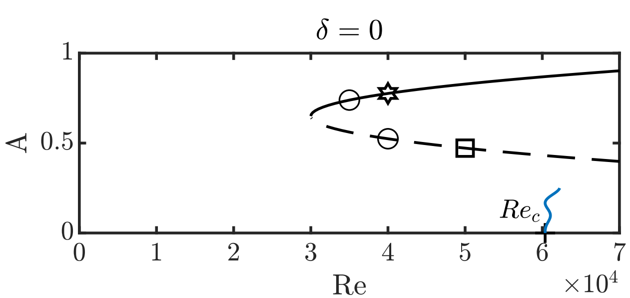
Supercritical transition
∘ – periodic state
✶ - chaotic state



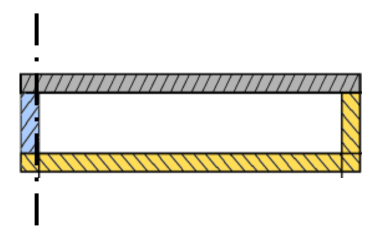


Competition between branches
∘ – periodic state
✶ - chaotic state

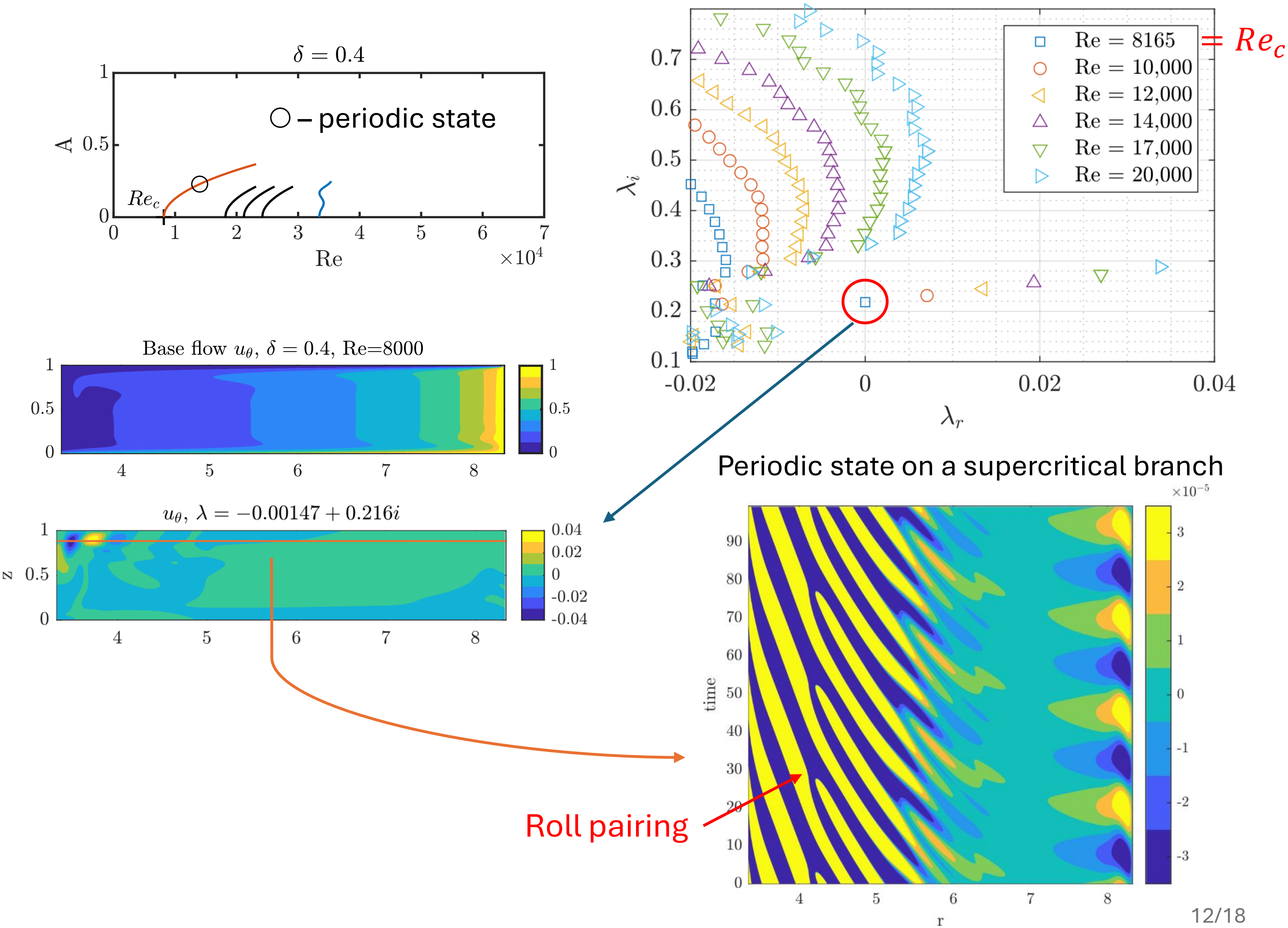




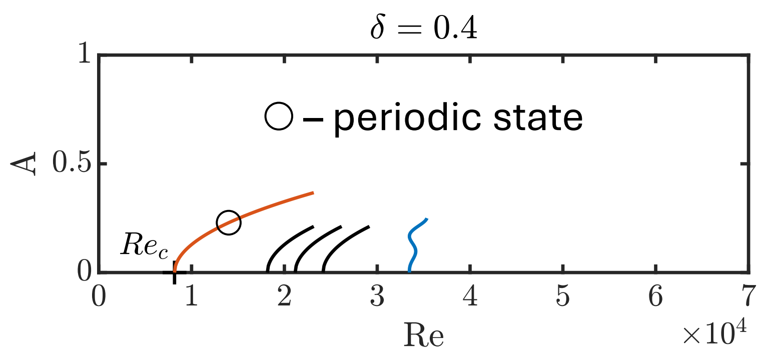
Subcritical transition
∘ – periodic state
□ - biperiodic state
✶ - chaotic state

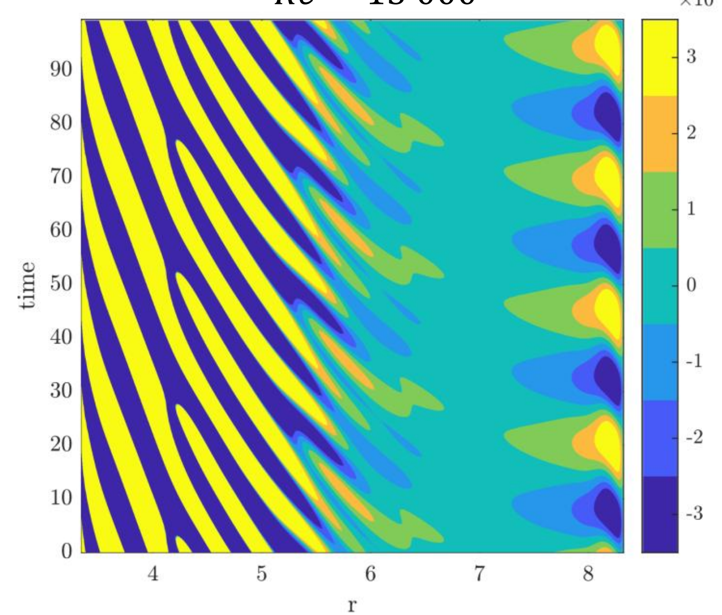


*Here $Re = \Omega R H / \nu$

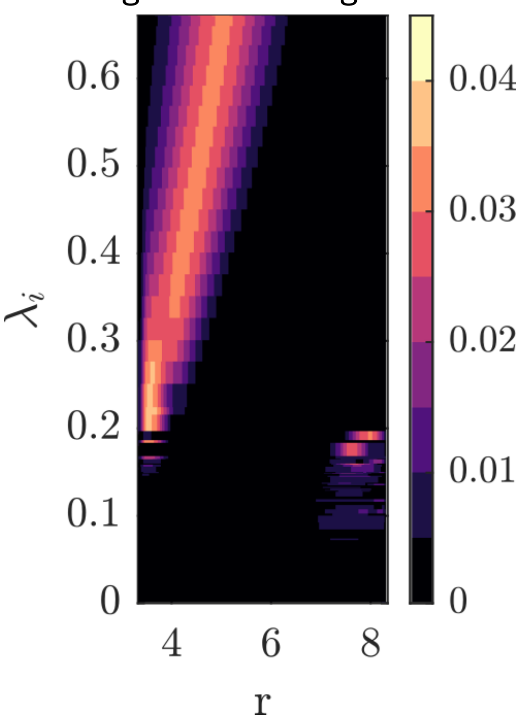

$\delta = 0.4$
○ – periodic state
A
Re
Re_c
$\times 10^4$
Re = 8165
Re = 10,000
Re = 12,000
Re = 14,000
Re = 17,000
Re = 20,000
= Re_c
λ_i
λ_r
Base flow u_θ, $\delta = 0.4$, Re=8000
u_θ, $\lambda = -0.00147 + 0.216i$
z
Periodic state on a supercritical branch
$\times 10^{-5}$
time
r
Roll pairing

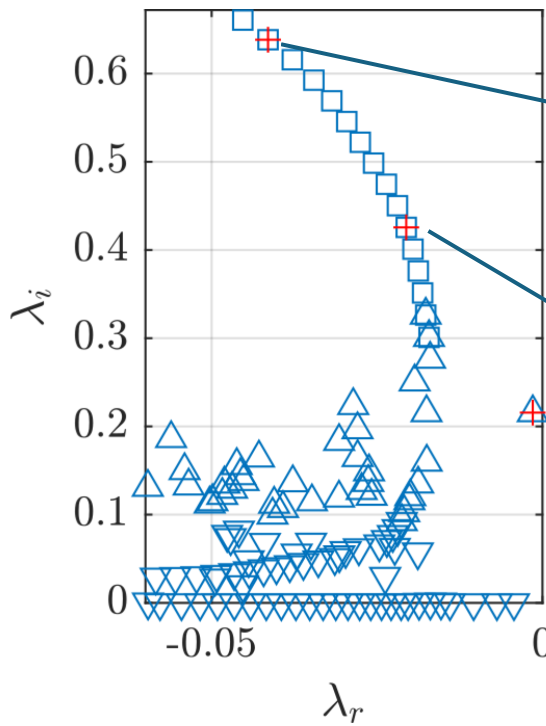
Pairing scenario

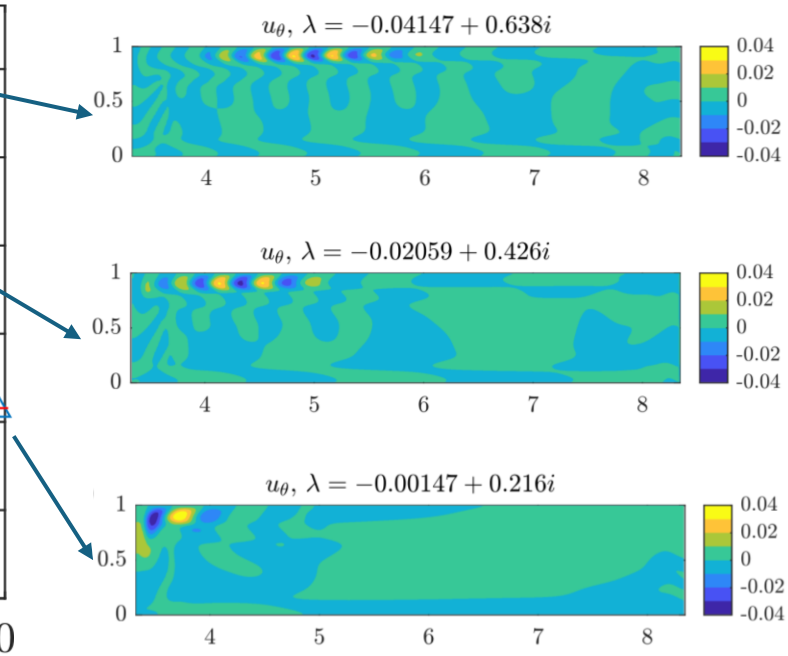




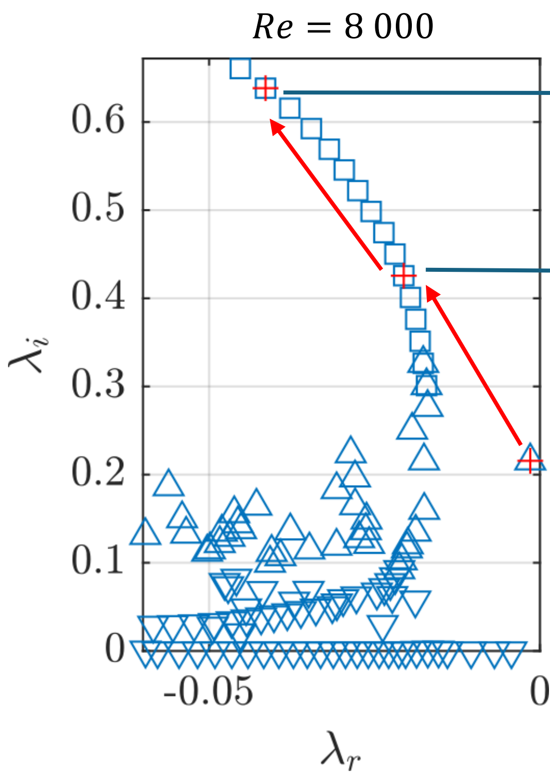
Eigenvector magnitude



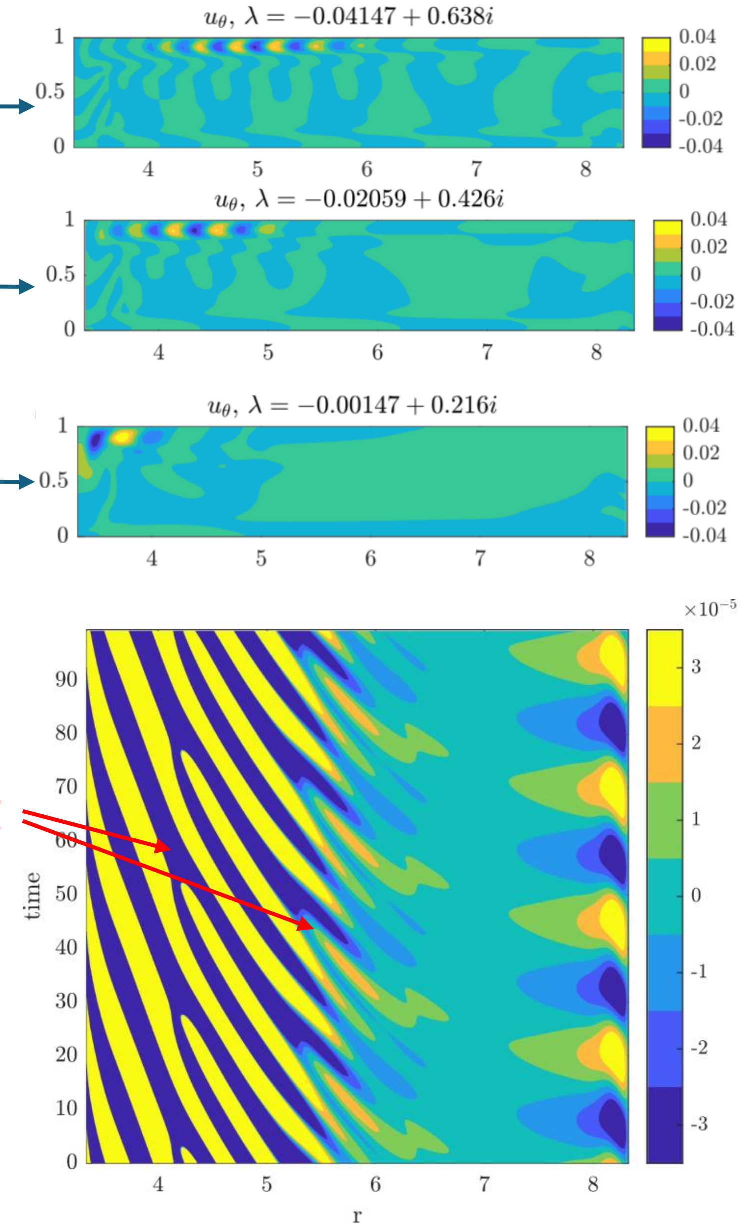




Merging scenario

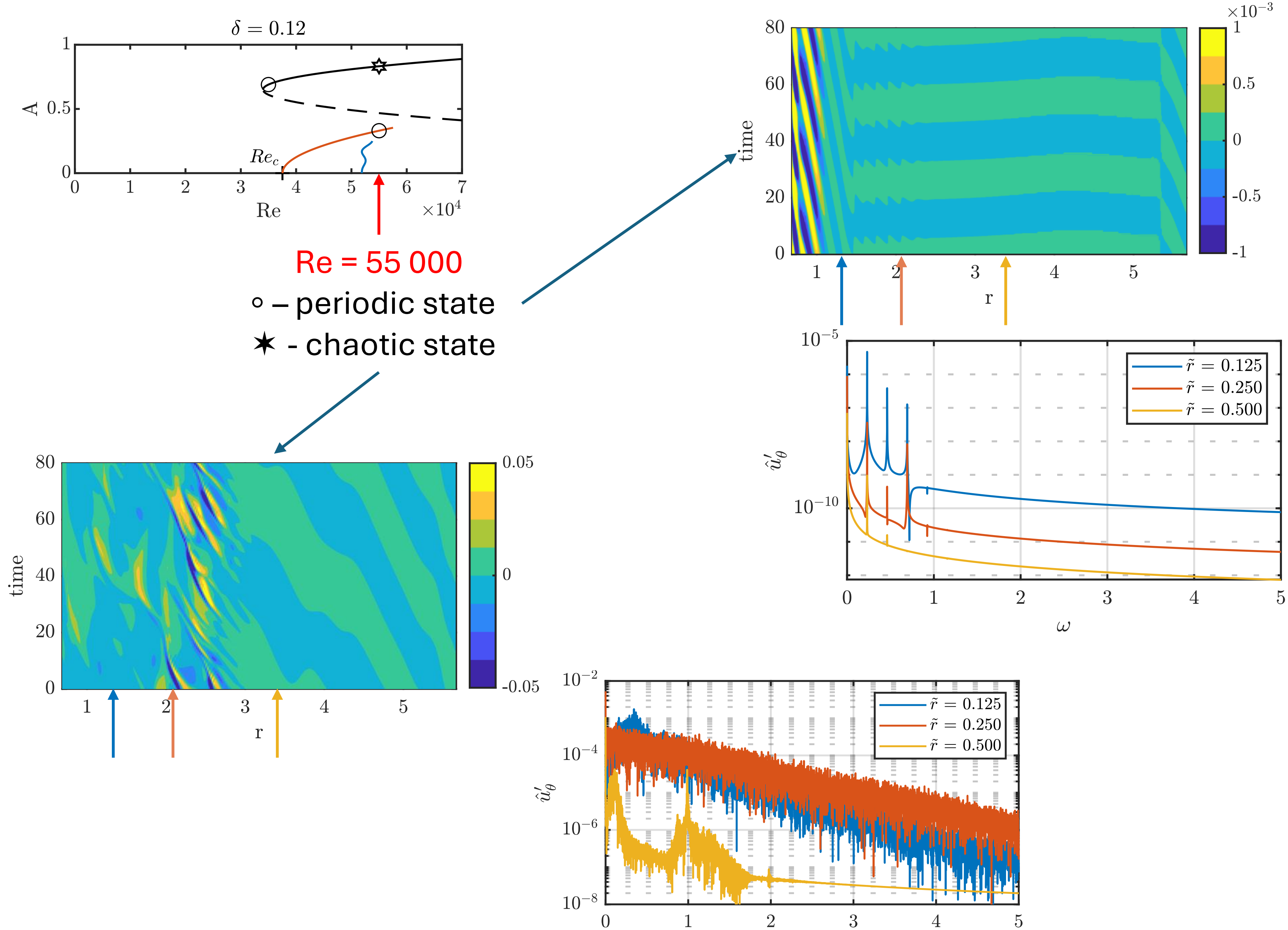


$u \propto e^{\lambda t}$

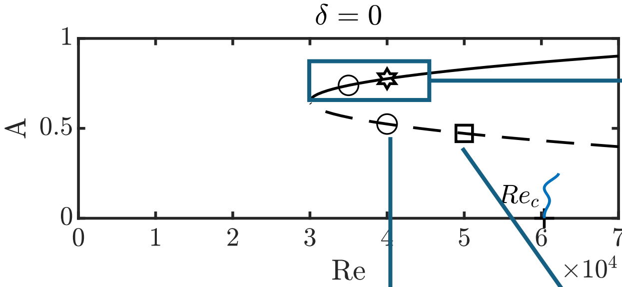


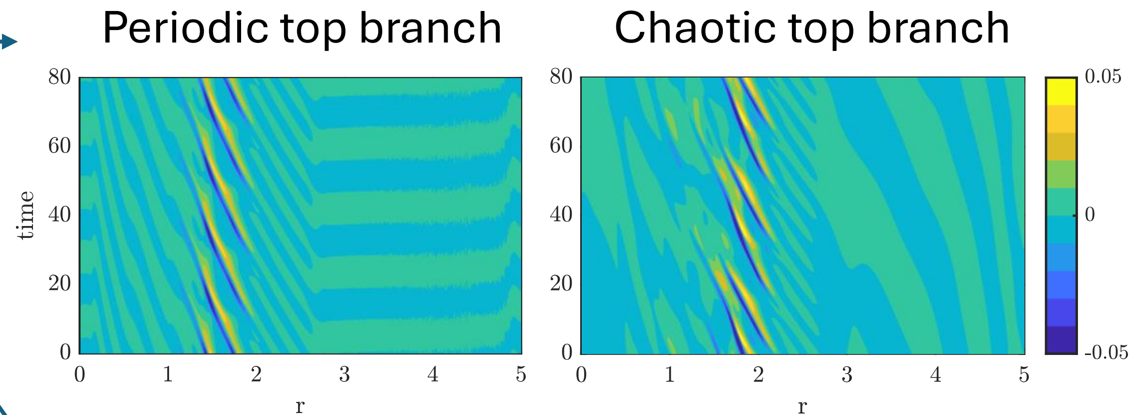
Roll merging

- Interaction of $\lambda_i = 0.216i$ with itself forces the $\lambda_i = 0.426i$
- Interaction of these two forces the $\lambda_i = 0.638i$
- Different spatial support, phase velocity and number of rolls appears as merging



$\delta = 0.12$
A
Re
$\times 10^4$
Re_c
Re = 55 000
∘ – periodic state
✶ - chaotic state
time
r
$\times 10^{-3}$
$\hat{u}'_\theta$
ω
$\tilde{r}$ = 0.125
$\tilde{r}$ = 0.250
$\tilde{r}$ = 0.500



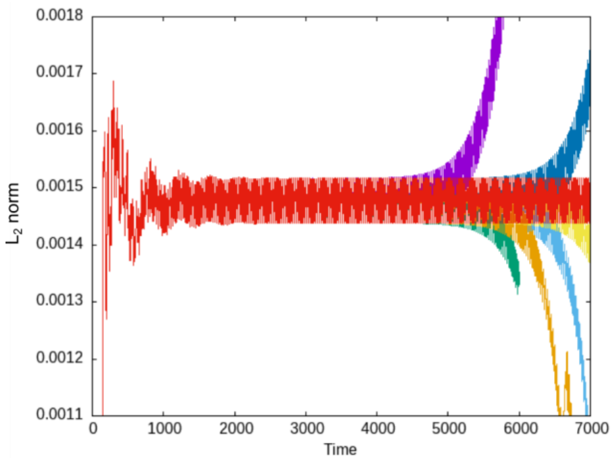


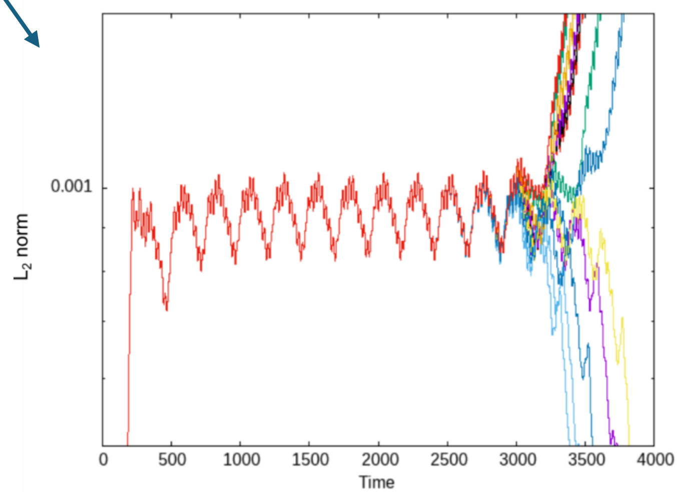
Subcritical transition

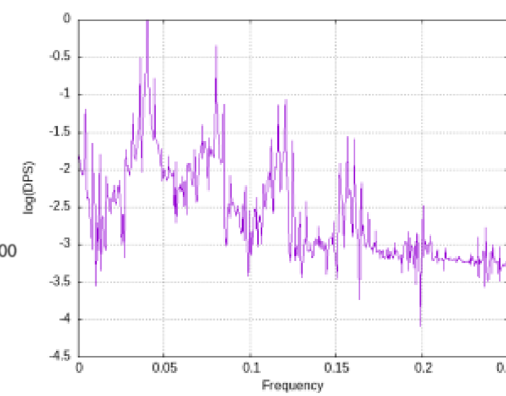
∘ – periodic state

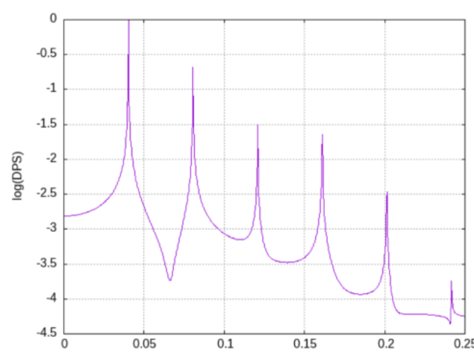
□ - biperiodic state

✶ - chaotic state



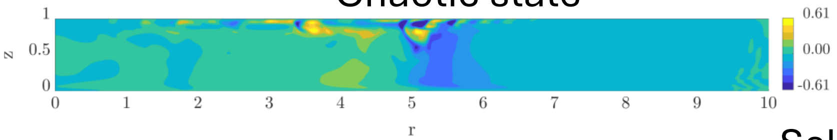




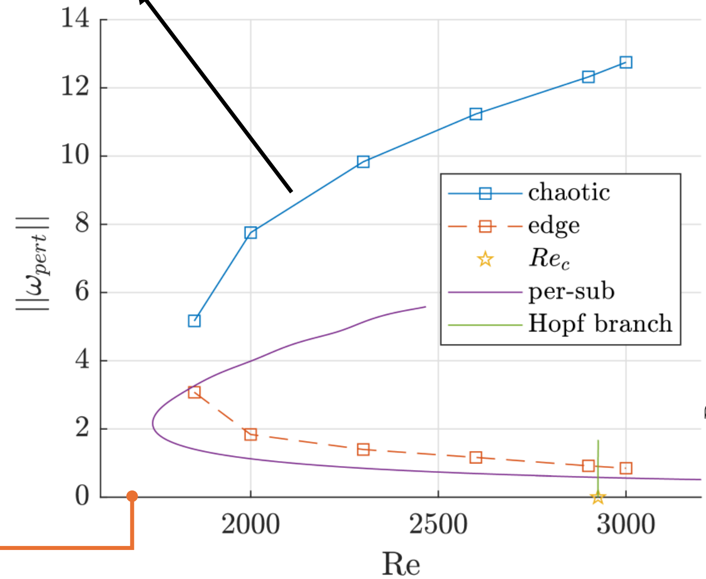


Summary

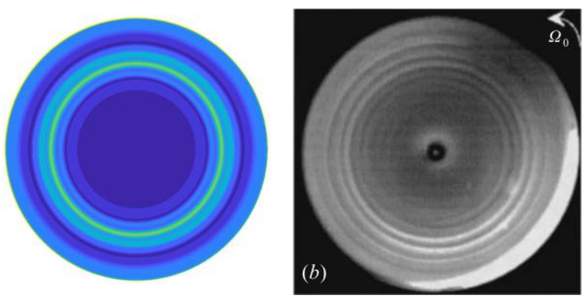
Chaotic state



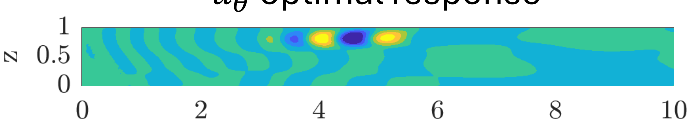
Self-sustained states



Experimental rolls – $Re \approx 500$

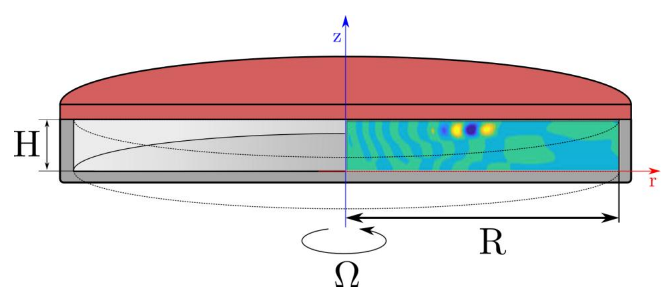


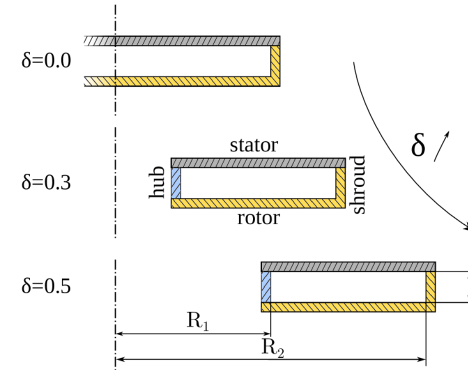
u_θ optimal response

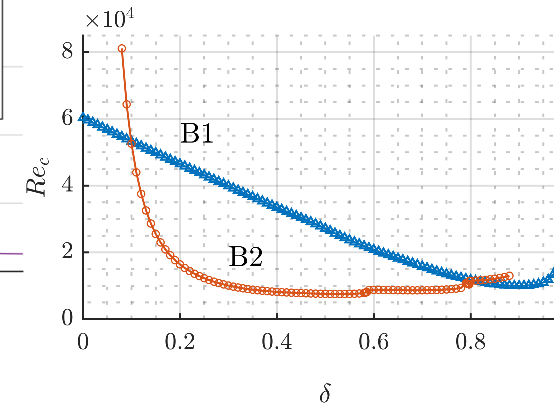


Gesla *et al.* On the origin of circular rolls in rotor-stator flow, JFM 2024.

Gesla *et al.* Subcritical axisymmetric solutions in rotor-stator flow, PRF 2024.

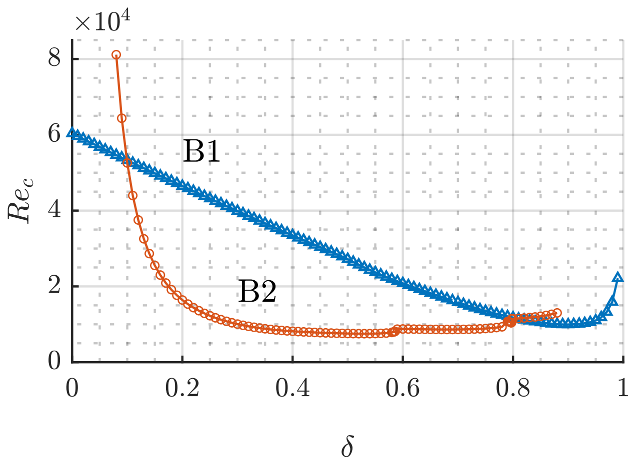




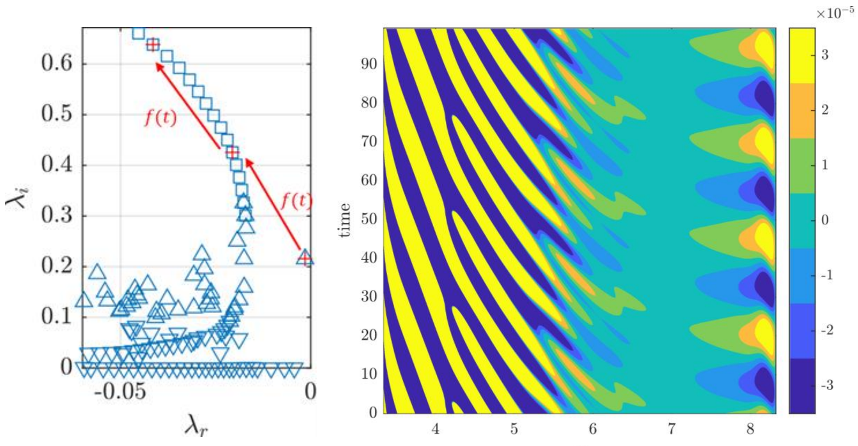


Gesla *et al.* From annular cavity to rotor-stator flow: nonlinear dynamics of axisymmetric rolls, to appear in PRF 2025.

Summary:



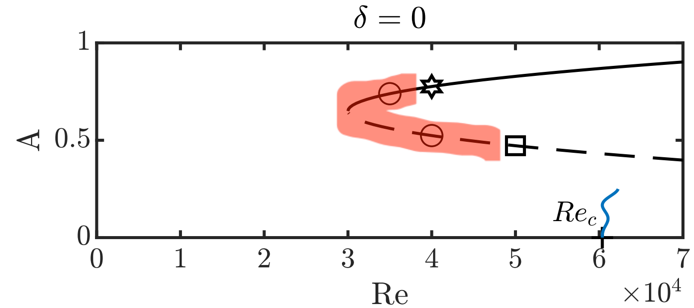
Stability exchange for a radially displaced cavity.



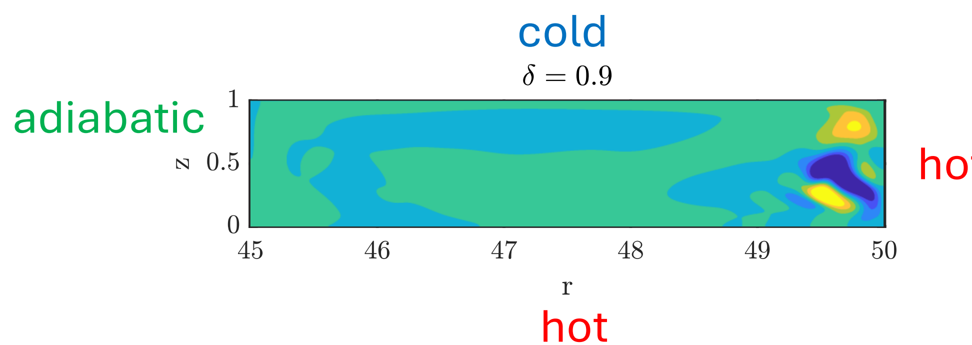
New perspective on the roll merging.

Gesla *et al*. From annular cavity to rotor-stator flow: nonlinear dynamics of axisymmetric rolls, accepted PRF 2025.

Outlooks:

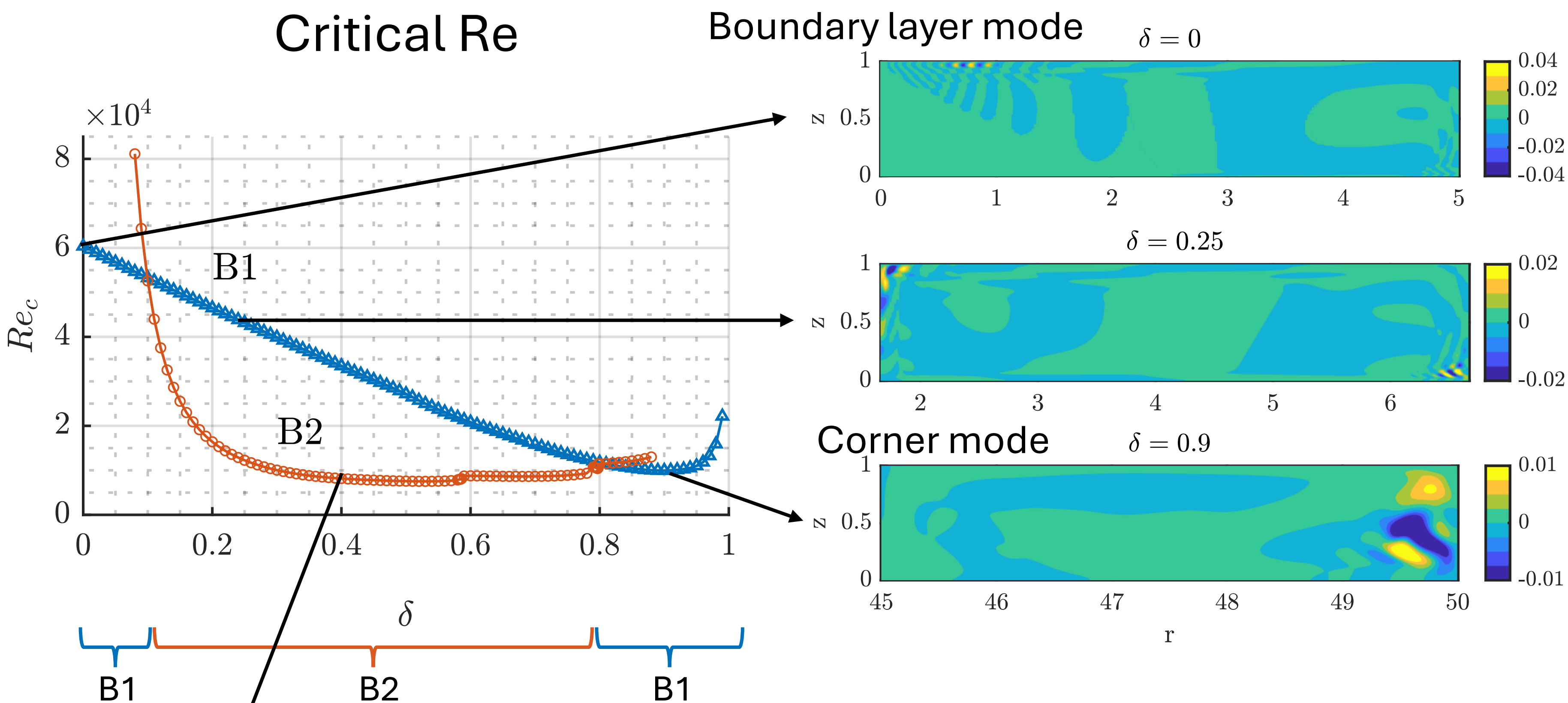


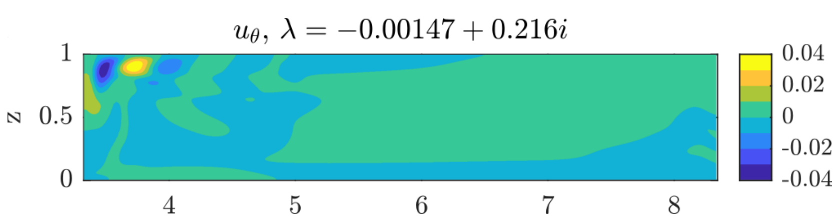
Role of periodic states in edge state/ top branch dynamics (Harmonic Balance Method, Floquet Analysis)



$\delta \to 1$ analysis, similarity to a Differentially Heated Cavity for $\mathrm{Pr} = 1$ and $\Delta\Omega \ll \Omega$, comparison of thresholds in Ra and Re

Critical Re

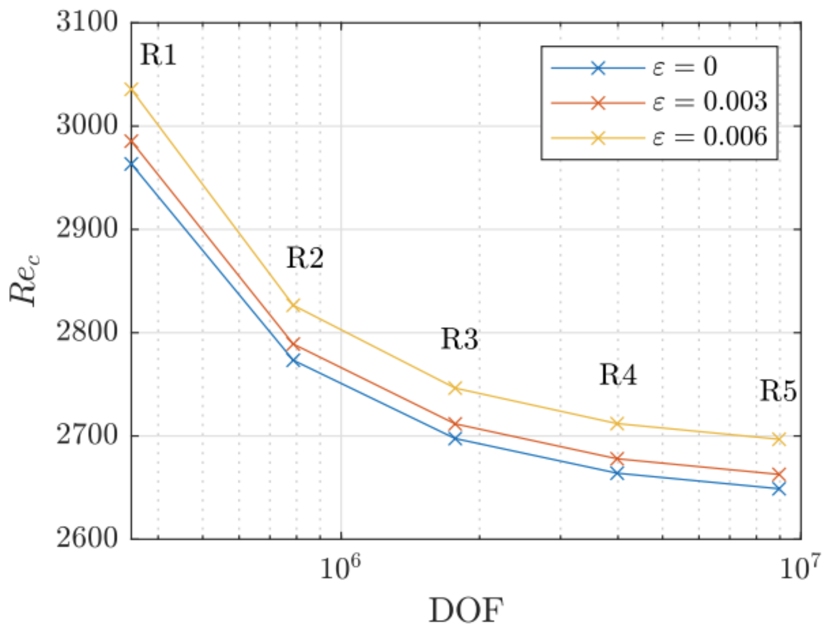


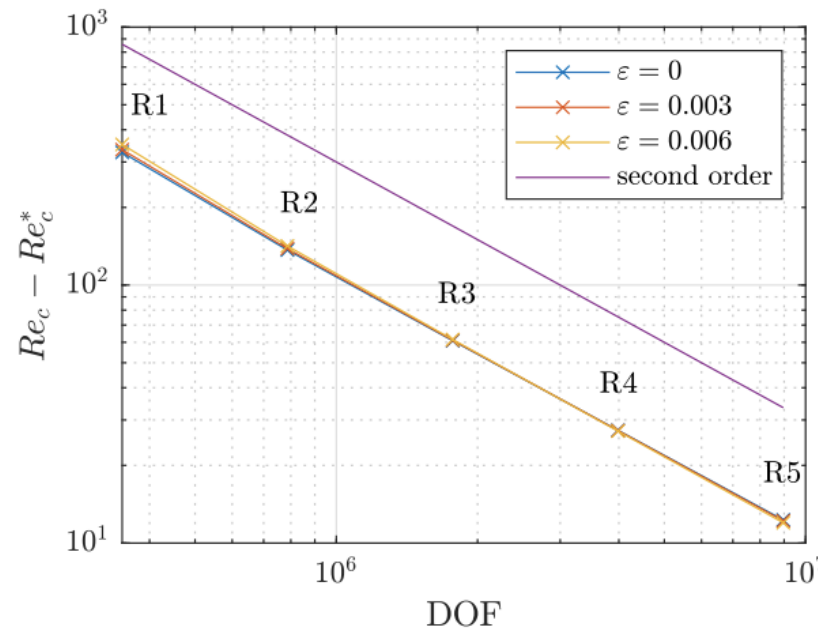


- Branch B1 is responsible for the linear instability at $\delta < 0.1$ and $\delta > 0.8$.
- Eigenmode changes continuously from a boundary layer structure to a corner structure.

Corner singularity

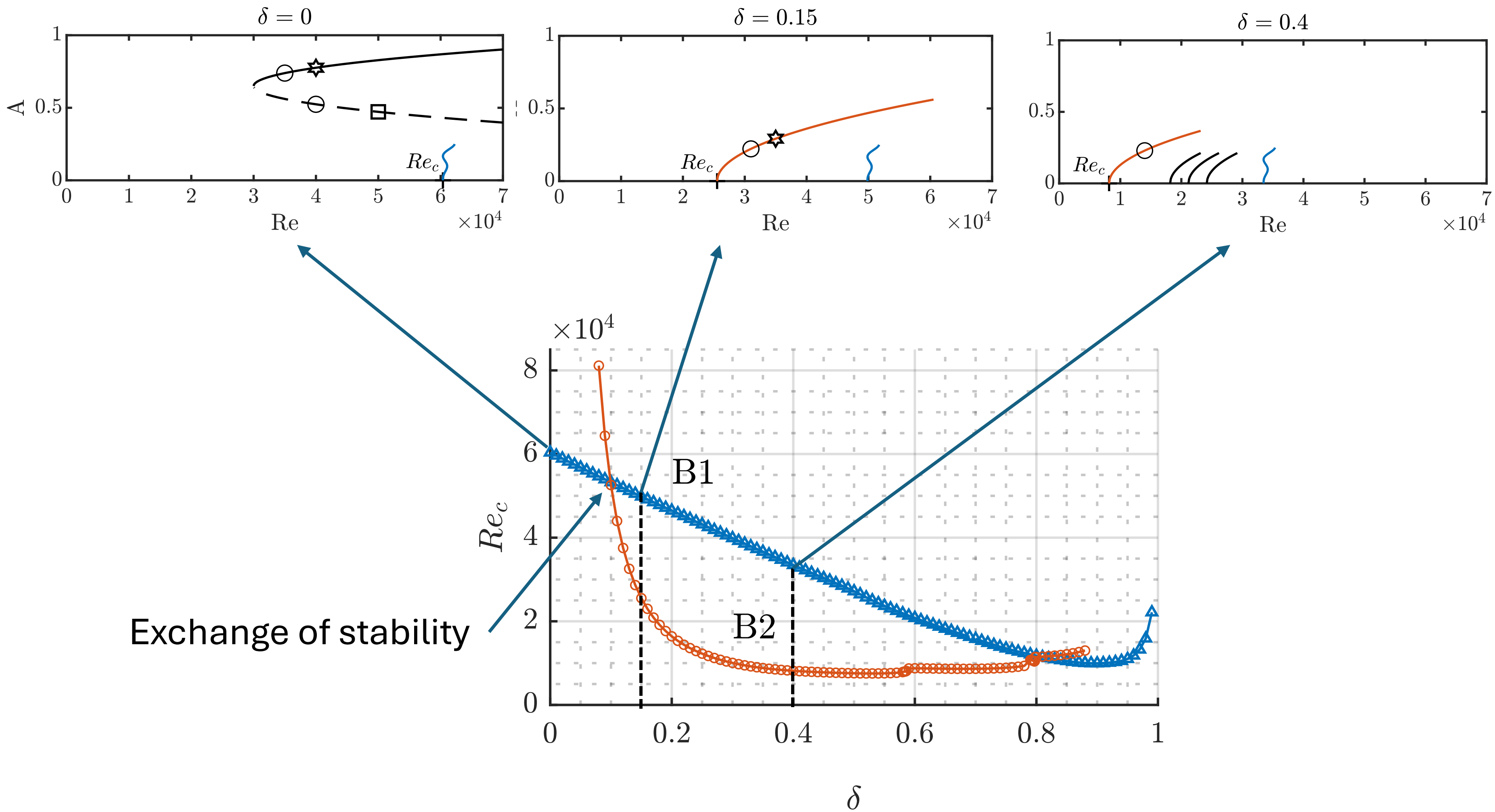
the corner singularity is also considered. This is achieved here by smoothing out the boundary condition at the Bödewadt corner, imposing an exponential velocity profile of the form $u_\theta = r\exp\left(\frac{r-\Gamma}{\varepsilon}\right)$. Two regularisations have been considered : $\varepsilon = 0.003$ and $\varepsilon = 0.006$. The case without any regularization $(u_\theta = 0)$ is referred to as $\varepsilon = 0$ for ease of notation.





resolution	N_r	N_z	type	DOF	$\varepsilon = 0$	$\varepsilon = 0.003$	$\varepsilon = 0.006$
R0	**600**	**160**	uniform	**390 k**	**2925.47**		
R1	683	128	non-uniform	356 k	2963.41	2985.43	3035.61
R2	**1024**	**192**	non-uniform	**796 k**	**2773.3**	2789.01	2826.55
R3	1536	288	non-uniform	1.7 m	2697.48	2711.71	2746.33
R4	2304	432	non-uniform	4.0 m	2663.96	2677.9	2711.97
R5	3456	648	non-uniform	9.0 m	2648.9	2662.8	2696.8
				extrapolation	2636.61	2650.59	2684.81
				order	2.01	2.04	2.09

TABLE V. Critical Reynolds number Re_c depending on the the spatial discretisation. From R1 to R5 the ratio between two consecutive grid resolutions is 1.5 in each direction.



A supercritical branch is replaced by a steep Hopf branch for decreasing δ.
Laminar-turbulent transition becomes subcritical.