From annular cavity to rotor-stator flow: nonlinear dynamics of axisymmetric rolls

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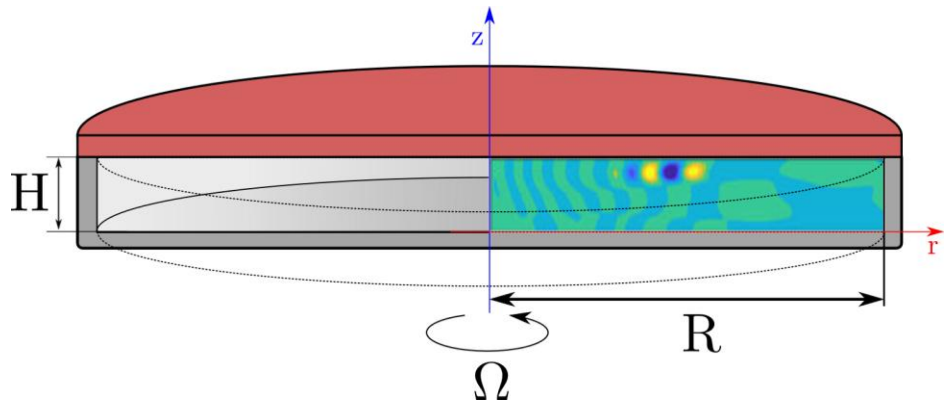
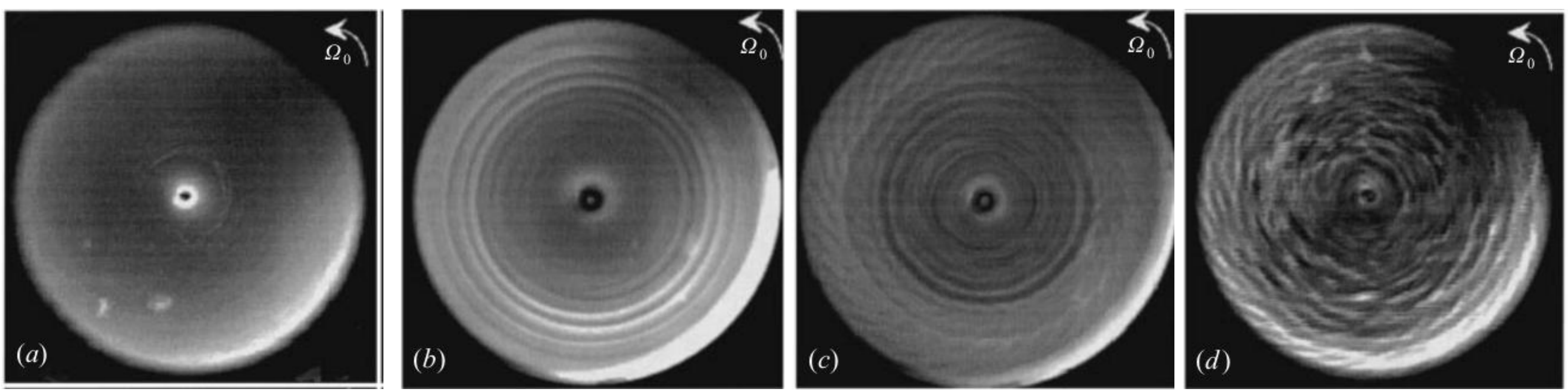
$$Re = H^2\Omega/\nu$$

Aspect ratio : $\Gamma = R\ /H$



Circular rolls

Circular + spiral rolls



Re=150

Re=500

Re=700

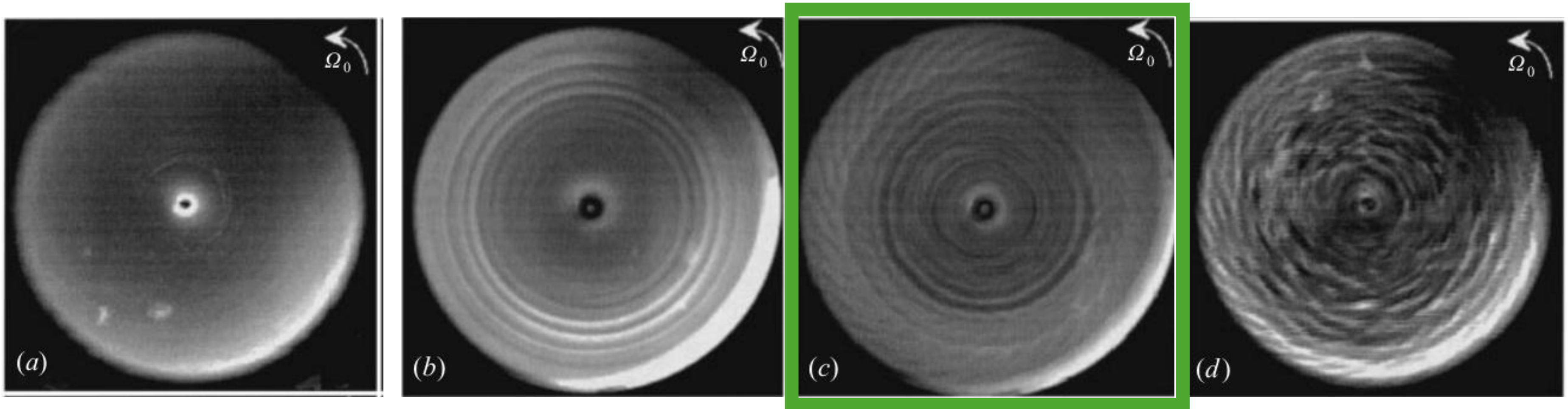
Re=1200

Re

Gauthier *et al.*, 1999

Circular rolls

Circular + spiral rolls

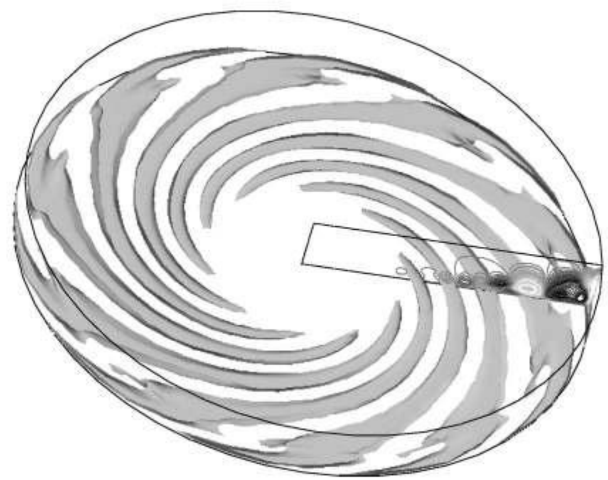


Gauthier *et al.*, 1999

First linearly unstable mode, R/H=10

Gelfgat, Fluid Dyn. Res. 2015

Saturated state, DNS, R/H=5

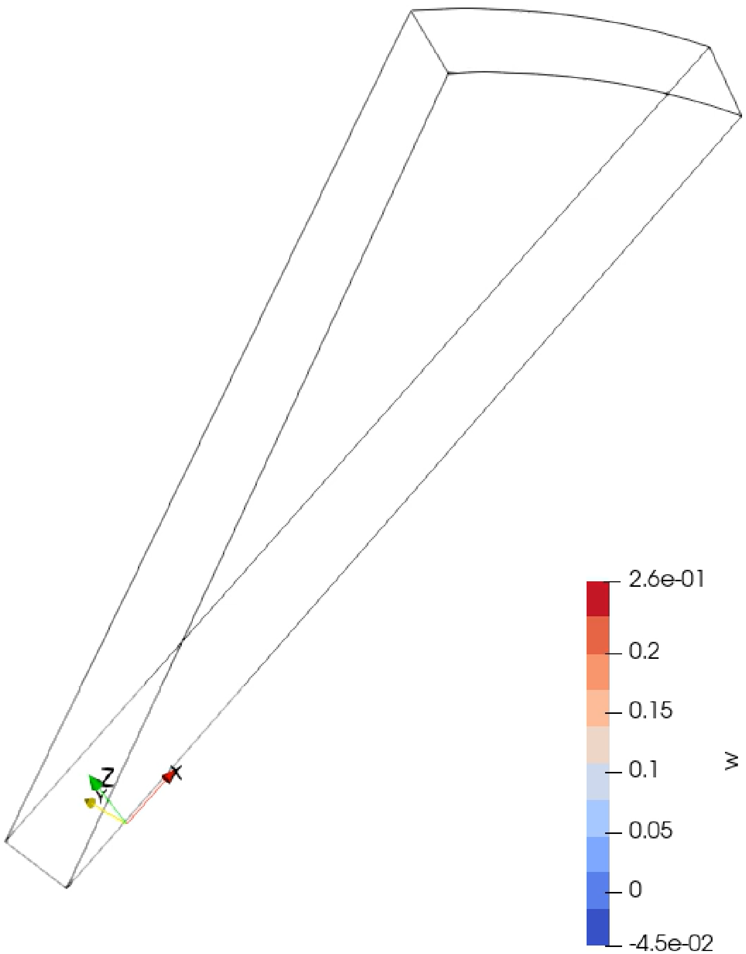


Serre *et al.*, PoF 2004

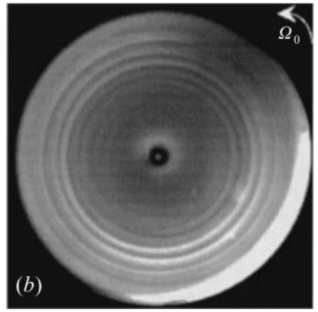
Challenge: No clear explanation of the circular rolls.

DNS

Re=500 Time: 0.000000

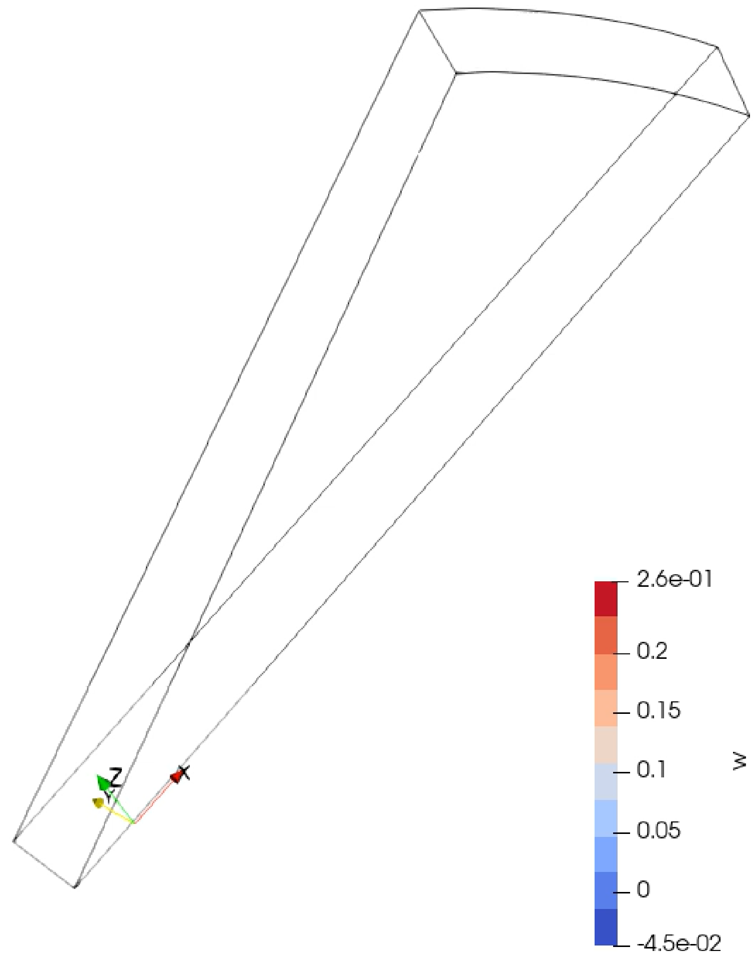


Experiment

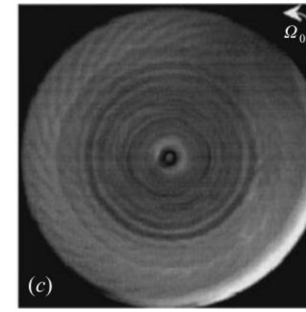


DNS

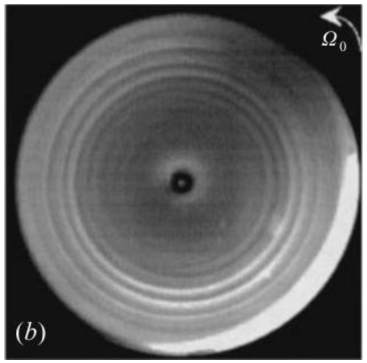
Re=700 Time: 0.000000



Experiment

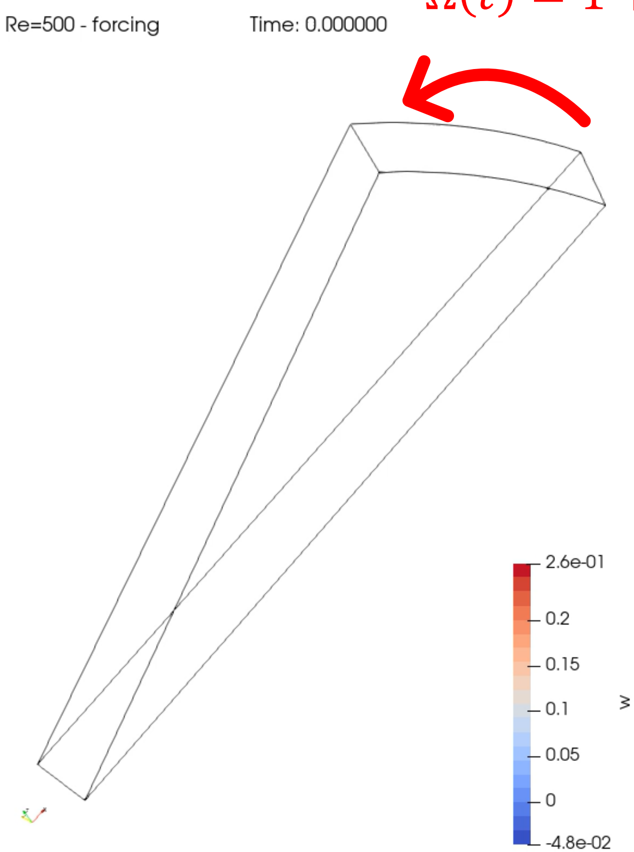


Experiment

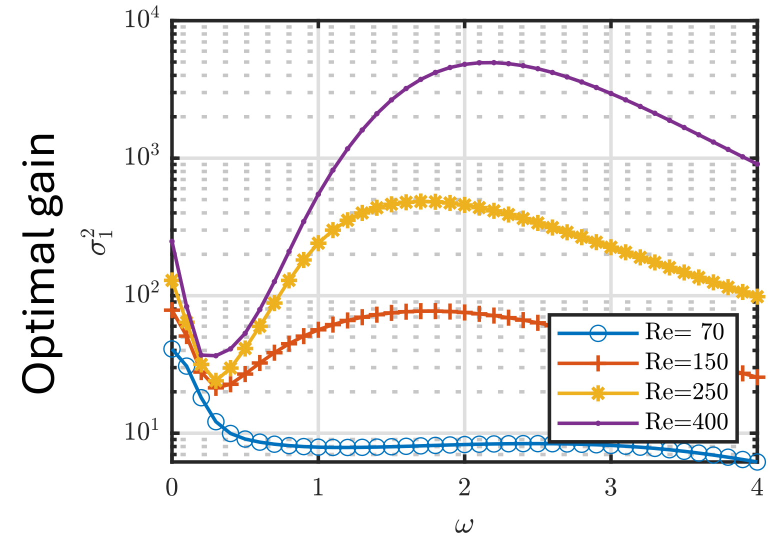


Experimentally seen circular rolls are a forcing response.

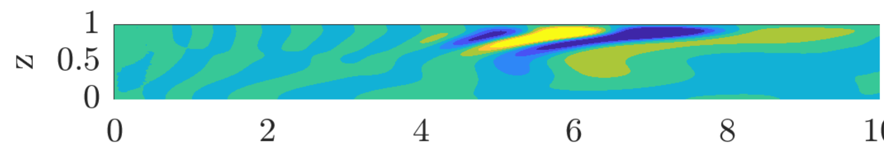
$\Omega(t) = 1 +$ white noise



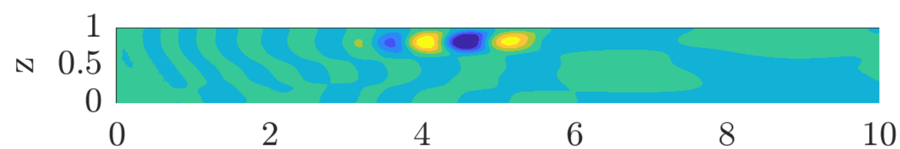
Resolvent analysis (axisymmetric)



f_θ optimal forcing



u_θ optimal response

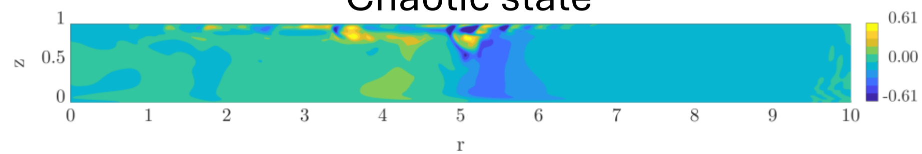


Gesla *et al*. On the origin of circular rolls in rotor-stator flow, JFM 2024.

Self-sustained axisymmetric solutions

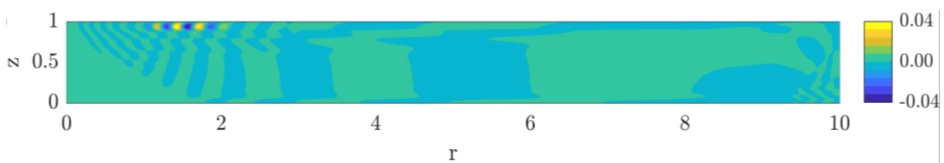
Gesla *et al*. Subcritical axisymmetric solutions in rotor-stator flow, PRF 2024.

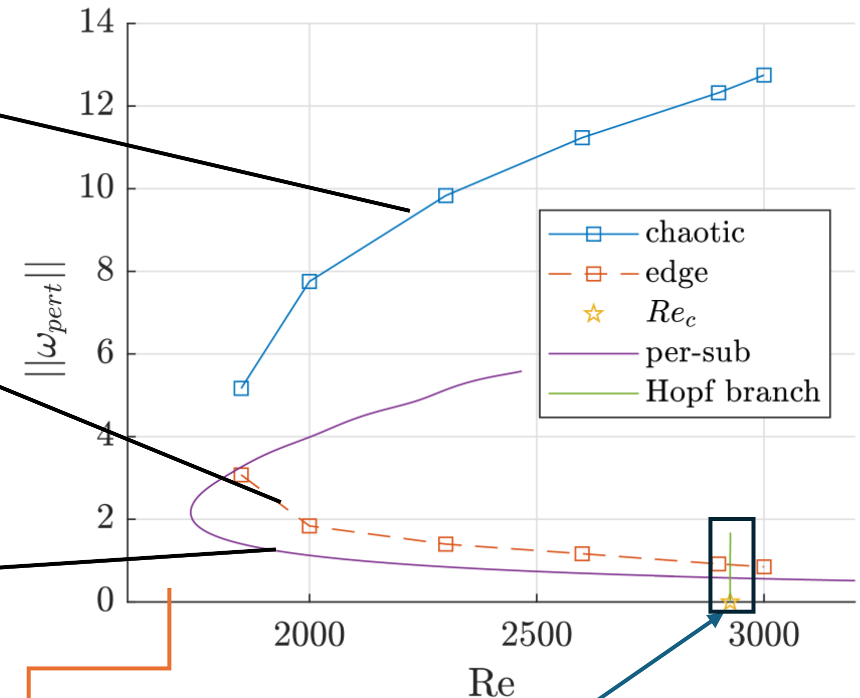
Chaotic state



Biperiodic edge state

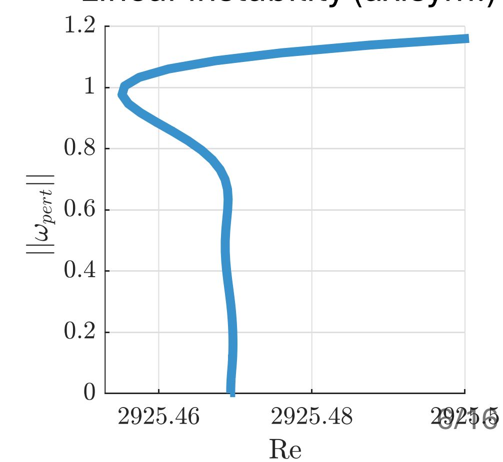
Periodic-subcritical branch





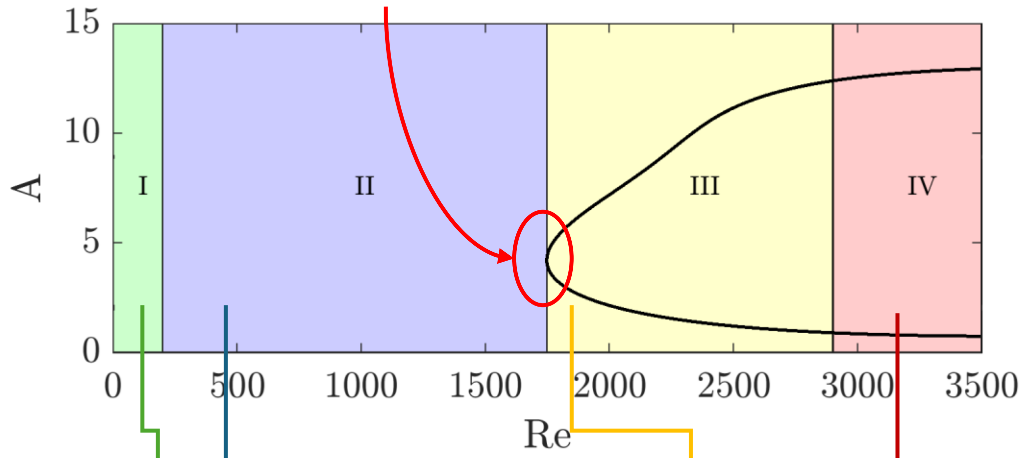
Experimental rolls – $Re \approx 500$

Linear instability (axisym.)



Dynamics of circular rolls

Finite lifetimes



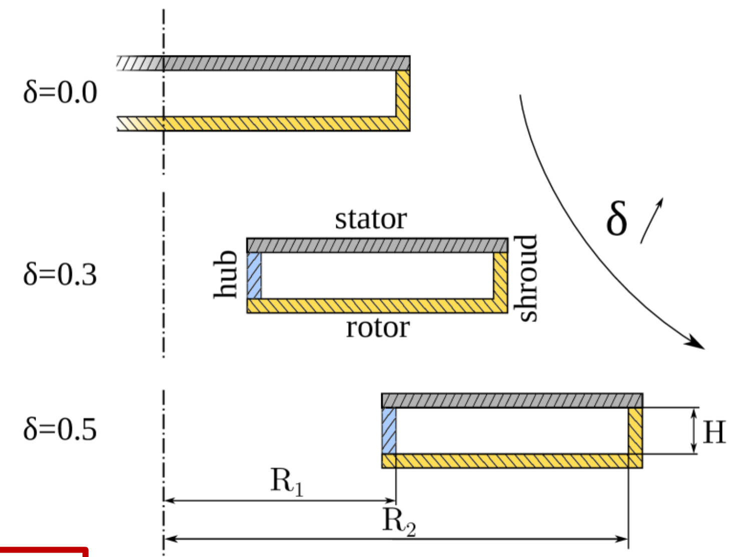
Stable flow, no rolls

Stable flow, circular rolls appear as a forcing respone

Lin. stable flow, self-sustained solutions exist

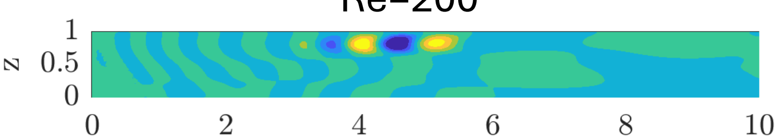
Unstable flow, evolution to chaotic dynamics

Use **homotopy** to explain the origin of the self-sustained states.

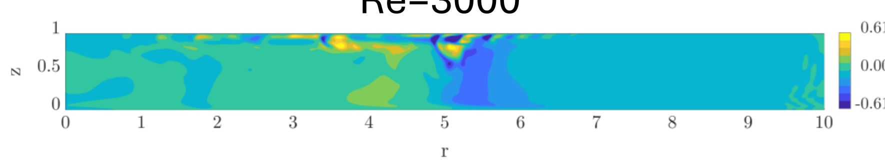


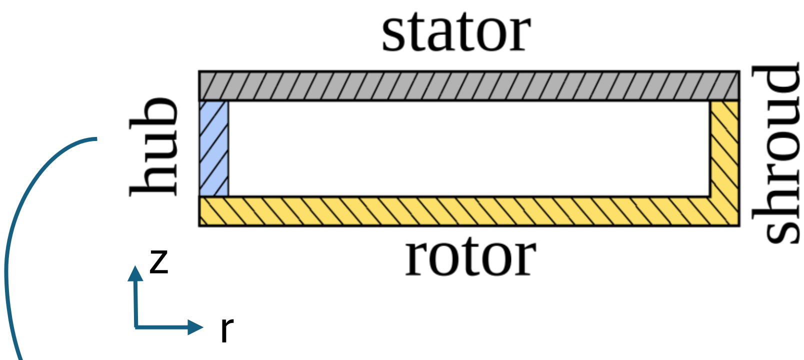
$$\delta = R_1/R_2$$

Re=200



Re=3000





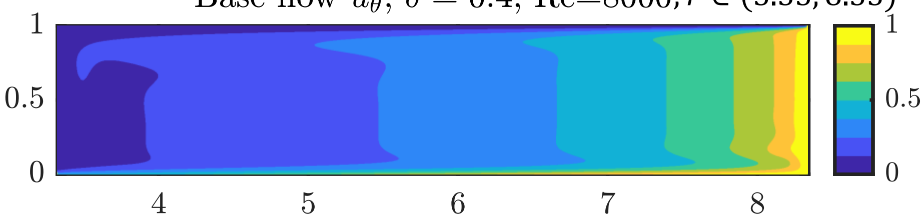
Stress-free on the hub :

$$(u_r\,,\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}\,,\frac{\partial u_z}{\partial r})=(0,0,0)\ \text{at}\ r_1=R_1/H.$$

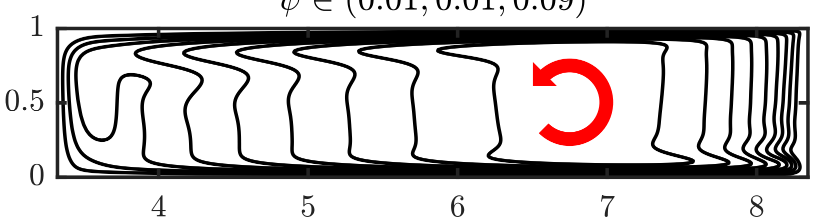
If hub at r=0 (rotor-stator) :

$$(u_r\,,u_\theta\,,\frac{\partial u_z}{\partial r})=(0,0,0).$$

Base flow u_θ, $\delta=0.4$, Re=8000, $r\in(3.33,8.33)$



$\psi\in(0.01,0.01,0.09)$



$$\frac{\partial \mathbf{u}}{\partial t}+\nabla(\mathbf{u}\otimes\mathbf{u})=-\nabla p+\frac{1}{Re}\nabla^2\mathbf{u}$$

$$\nabla\cdot\mathbf{u}=0$$

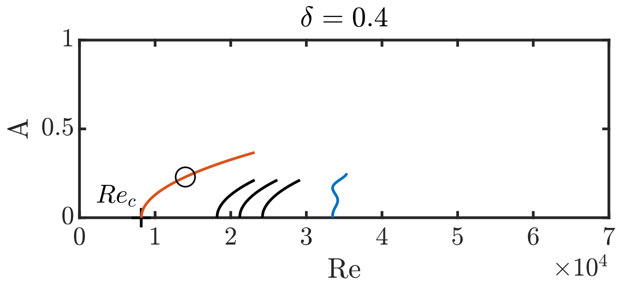
Aspect ratio R/H=5

Boundary conditions :

$\boldsymbol{u}=\mathbf{0}$ at $z=1$	stator,
$\boldsymbol{u}=H\,r/R_2\,\boldsymbol{e}_\theta$ at $z=0$	rotor,
$\boldsymbol{u}=\boldsymbol{e}_\theta$ at $r_2=R_2/H,$	shroud.

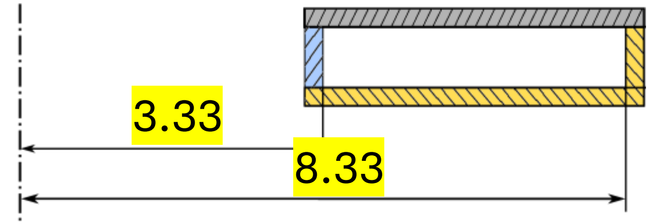
Numerical methods:

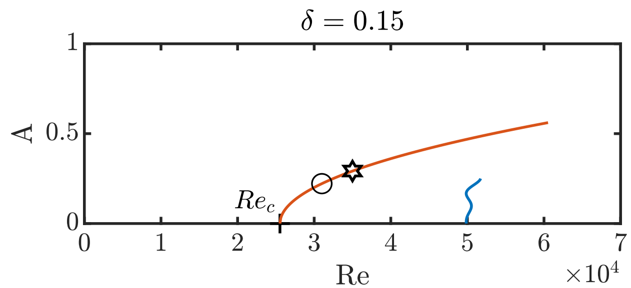
- Finite Volume discretization r-z
- Steady state - Newton method
- Instability - ARPACK
- Time integration with BDF2 scheme



Supercritical transition

∘ – periodic state

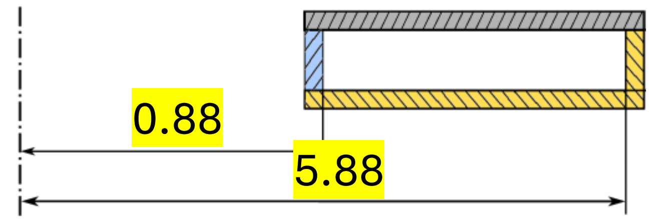


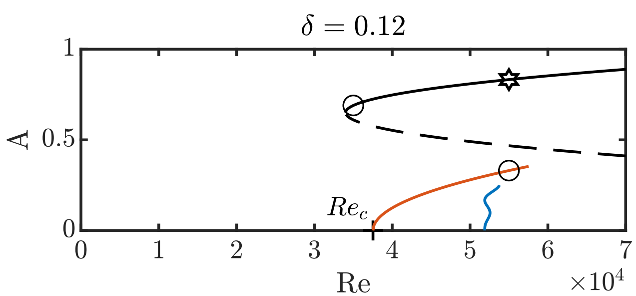


Supercritical transition

∘ – periodic state

✶ - chaotic state

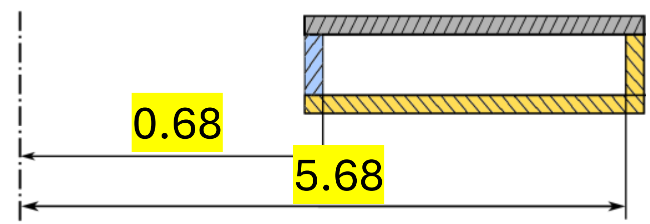


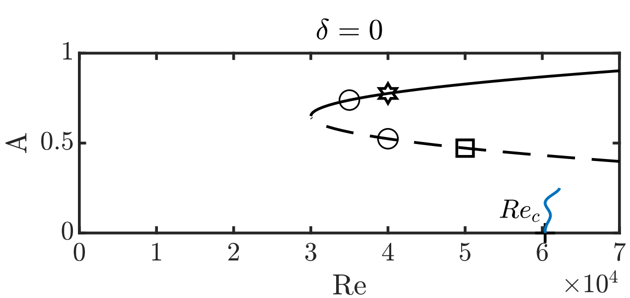


Competition between branches

∘ – periodic state

✶ - chaotic state



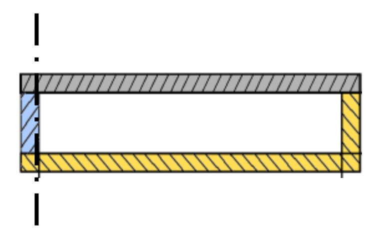


Subcritical transition

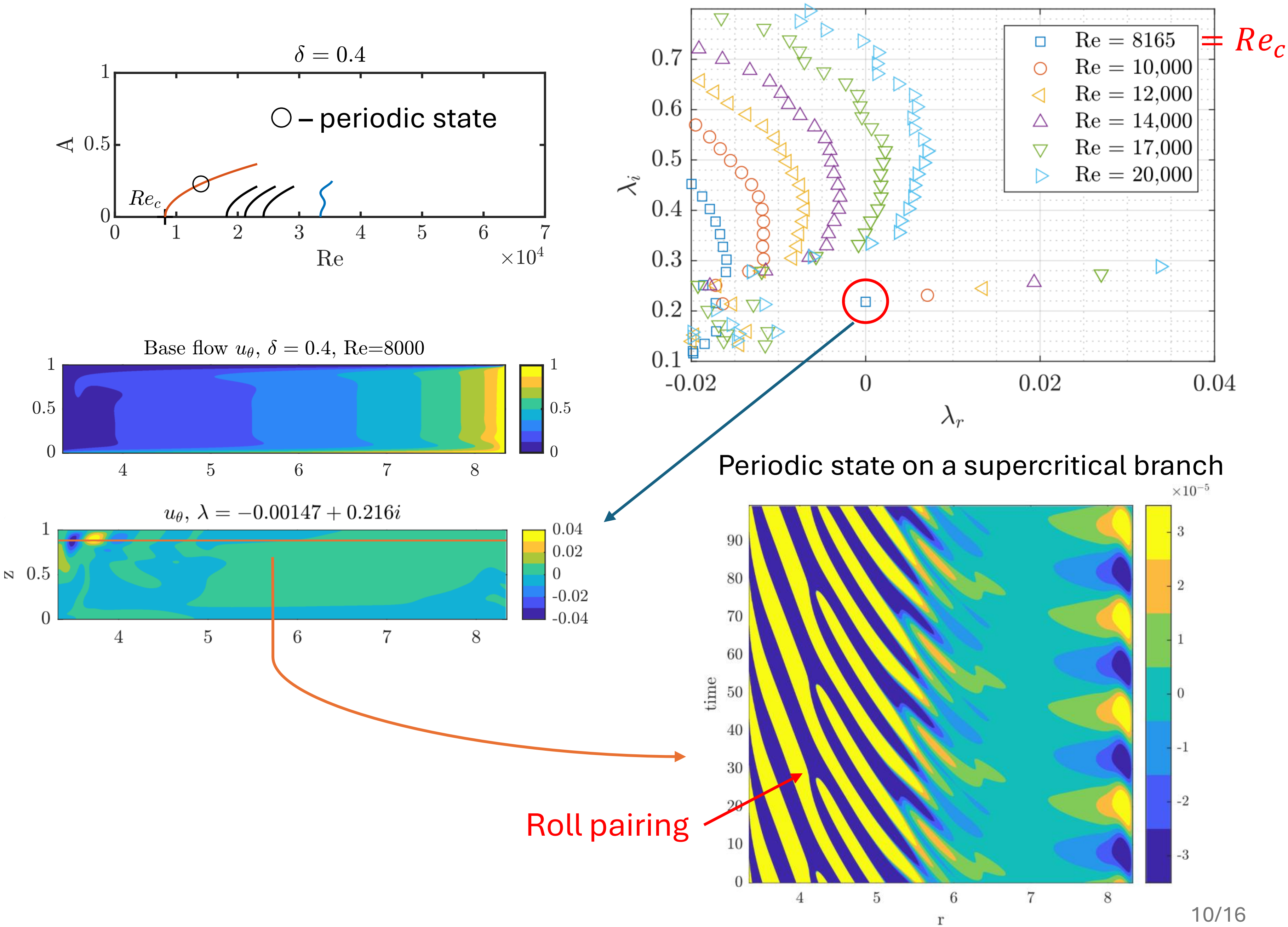
∘ – periodic state

□ - biperiodic state

✶ - chaotic state

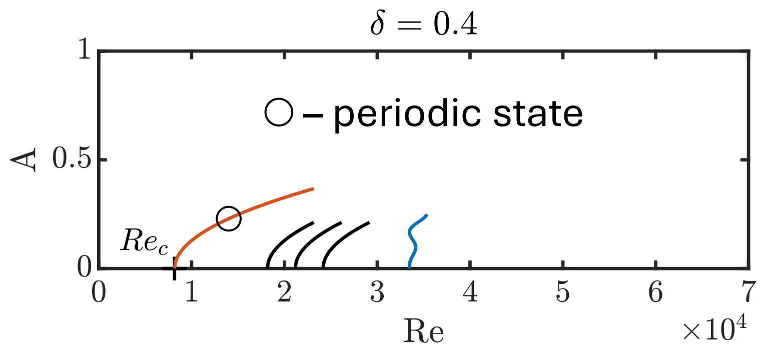


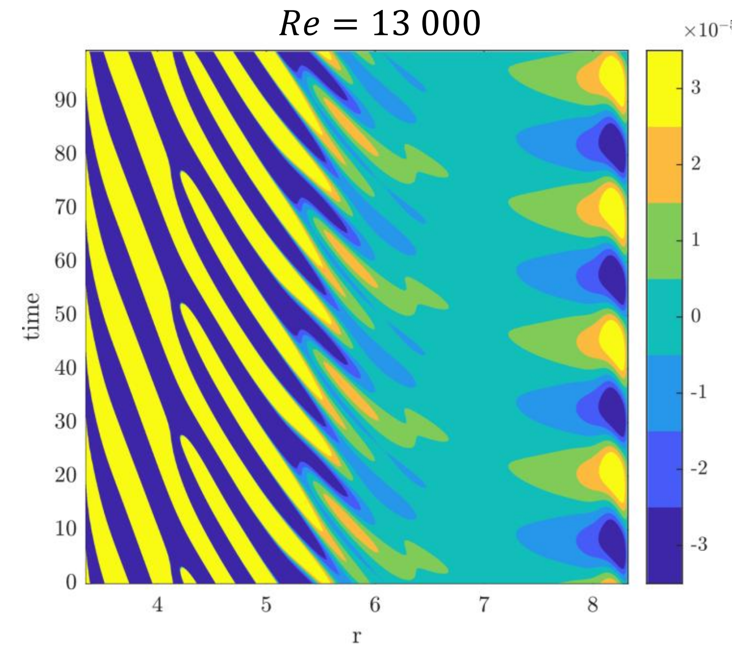
*Here $Re = \Omega R H / \nu$

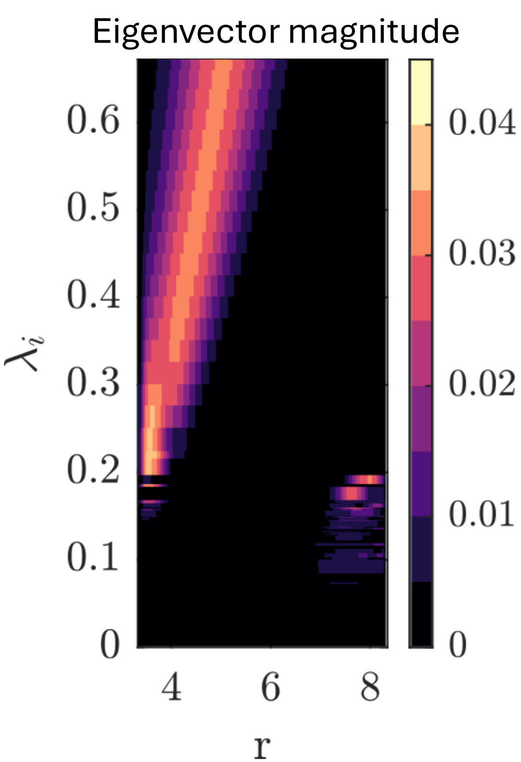


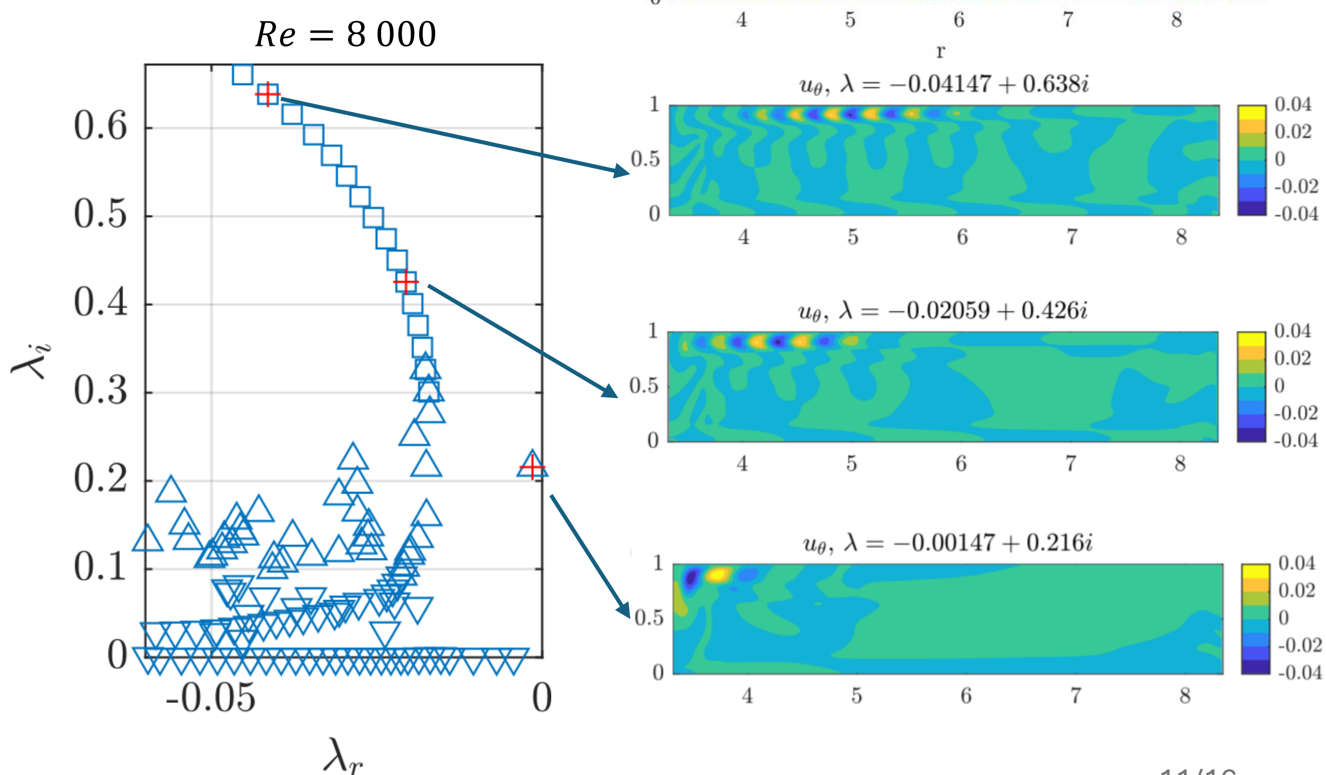
$\delta = 0.4$
○ – periodic state
Re_c
A
Re
$\times 10^4$
λ_i
λ_r
Re = 8165
Re = 10,000
Re = 12,000
Re = 14,000
Re = 17,000
Re = 20,000
$= Re_c$
Base flow u_θ, $\delta = 0.4$, Re=8000
u_θ, $\lambda = -0.00147 + 0.216i$
z
Periodic state on a supercritical branch
$\times 10^{-5}$
time
r
Roll pairing

Pairing scenario

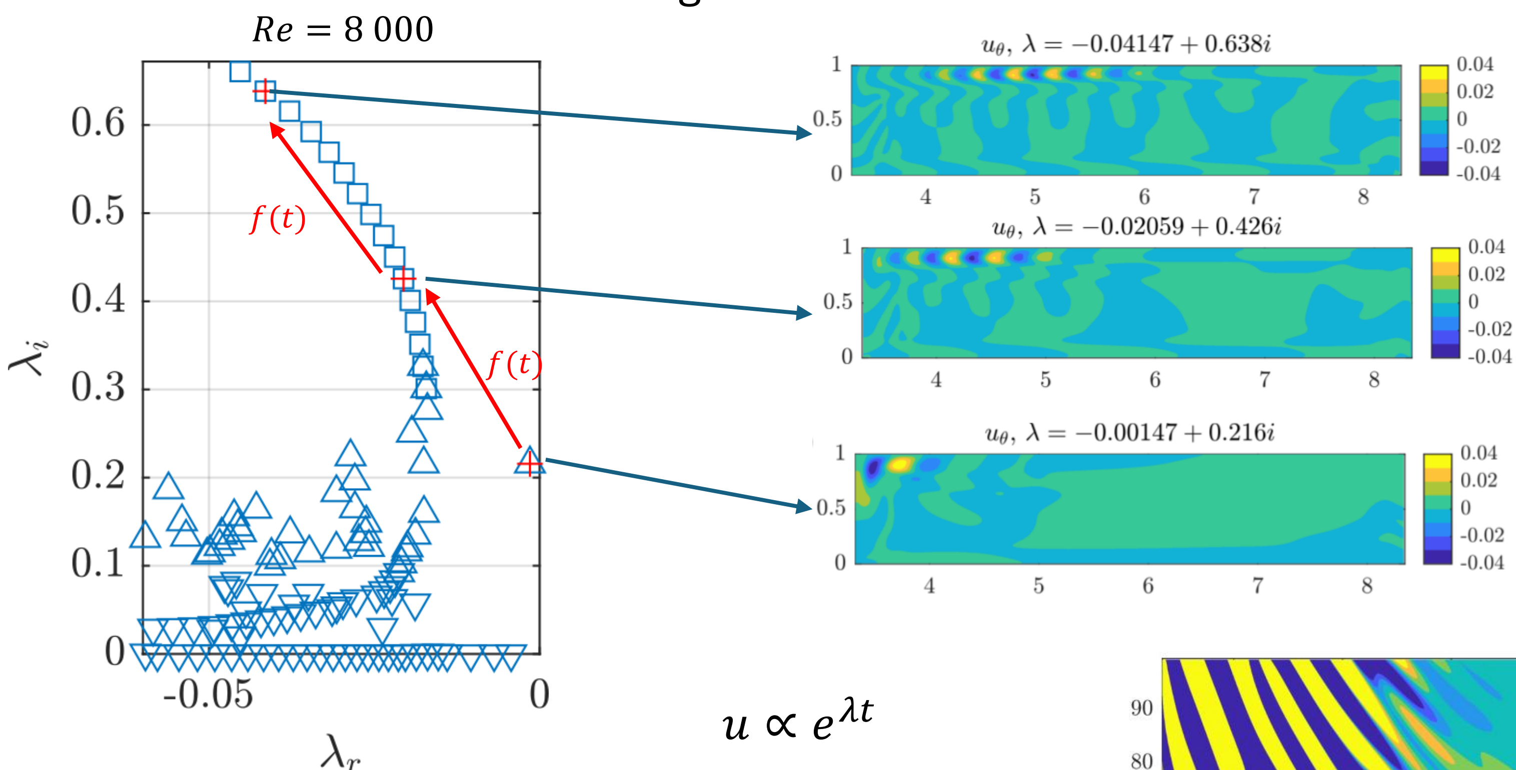




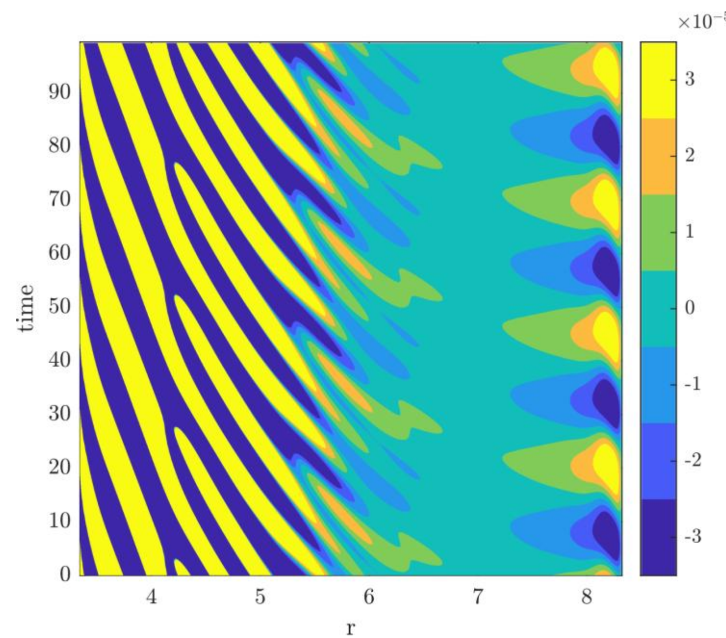




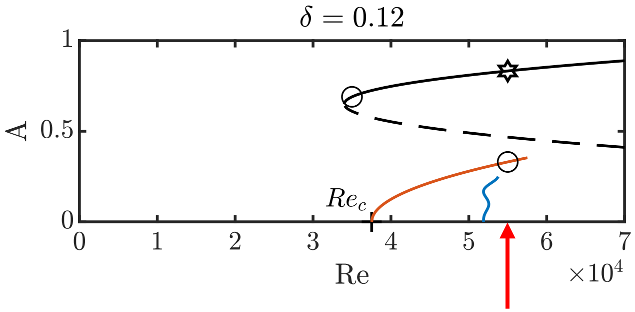
Pairing scenario



$u \propto e^{\lambda t}$



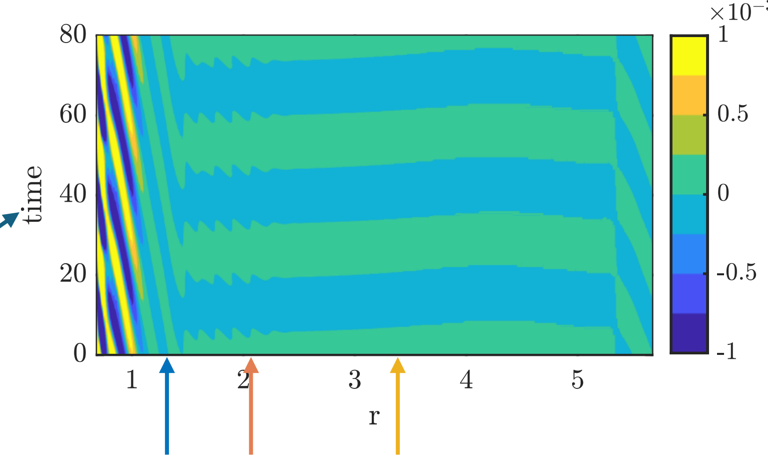
- Interaction of $\lambda_i = 0.216i$ with itself forces the $\lambda_i = 0.426i$
- Interaction of these two forces the $\lambda_i = 0.638i$
- Different spatial support and number of rolls give an illusion of pairing

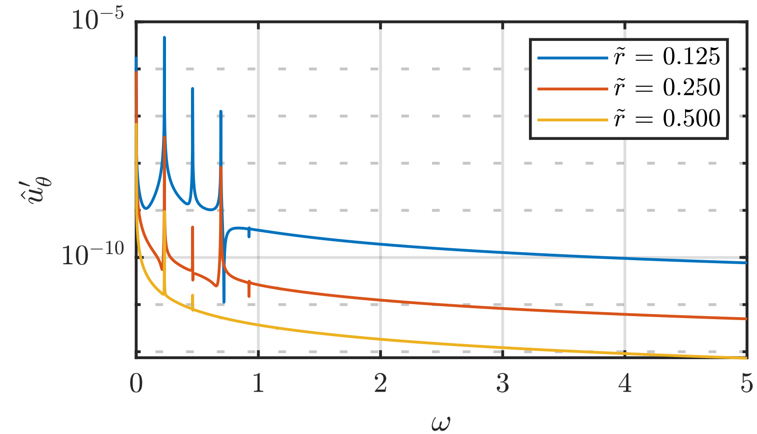


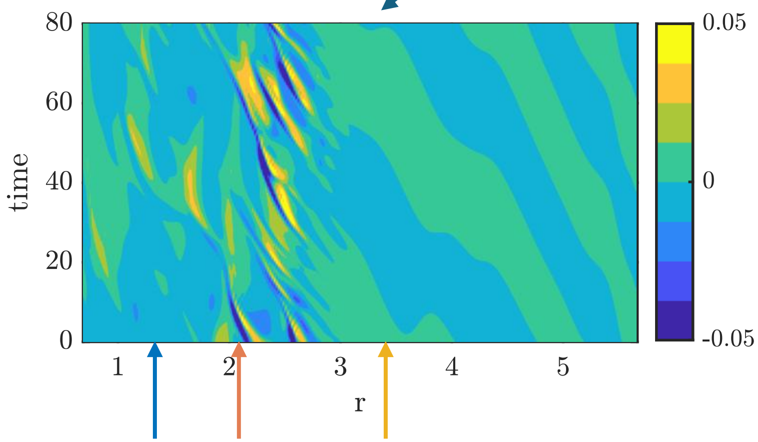
Re = 55 000

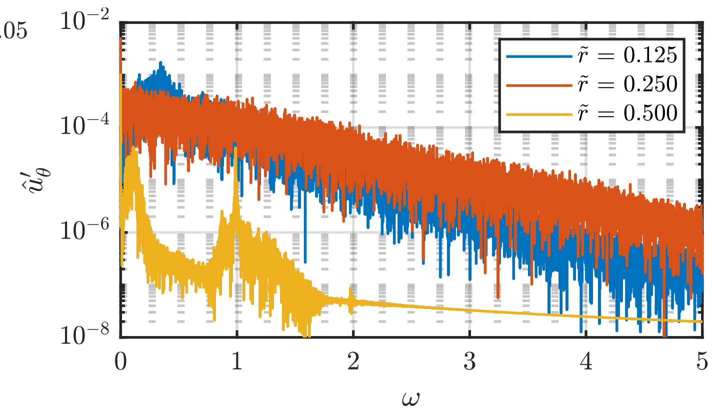
∘ – periodic state

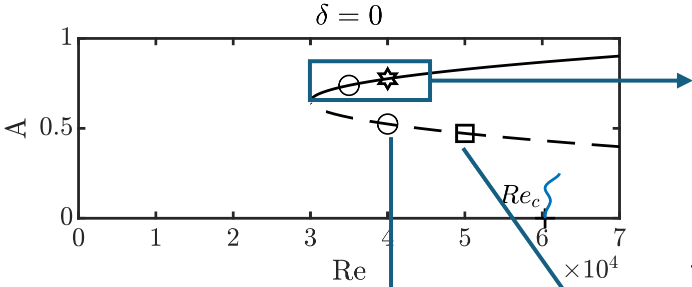
✶ - chaotic state

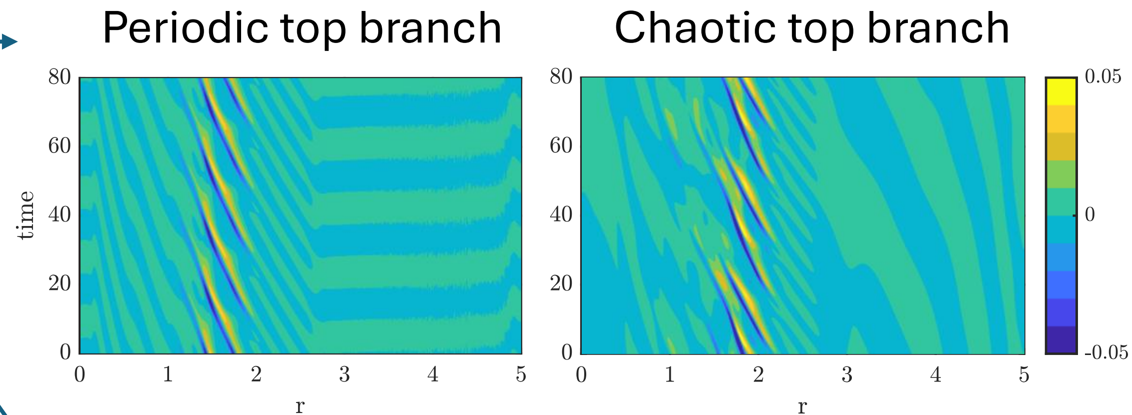










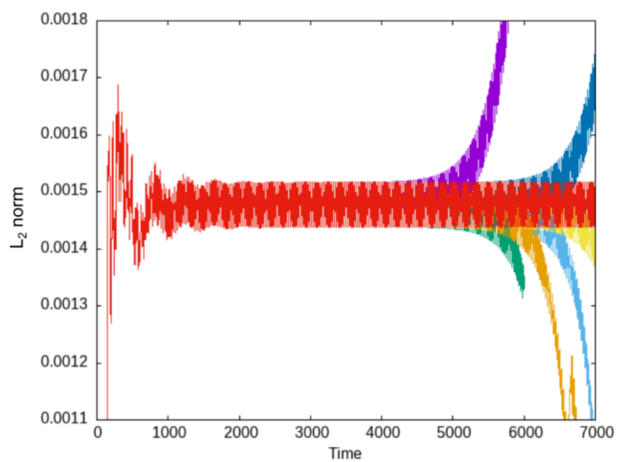


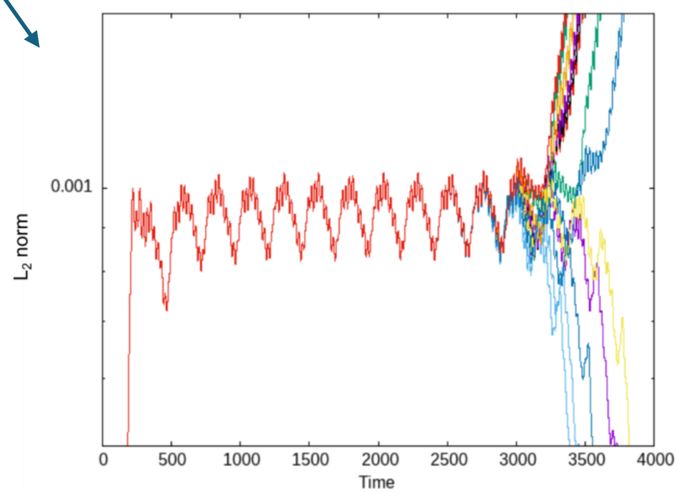
Subcritical transition

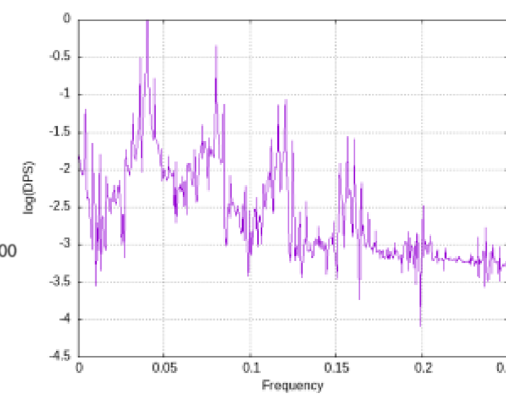
∘ – periodic state

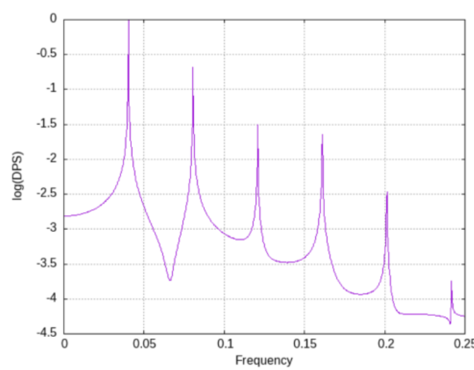
□ - biperiodic state

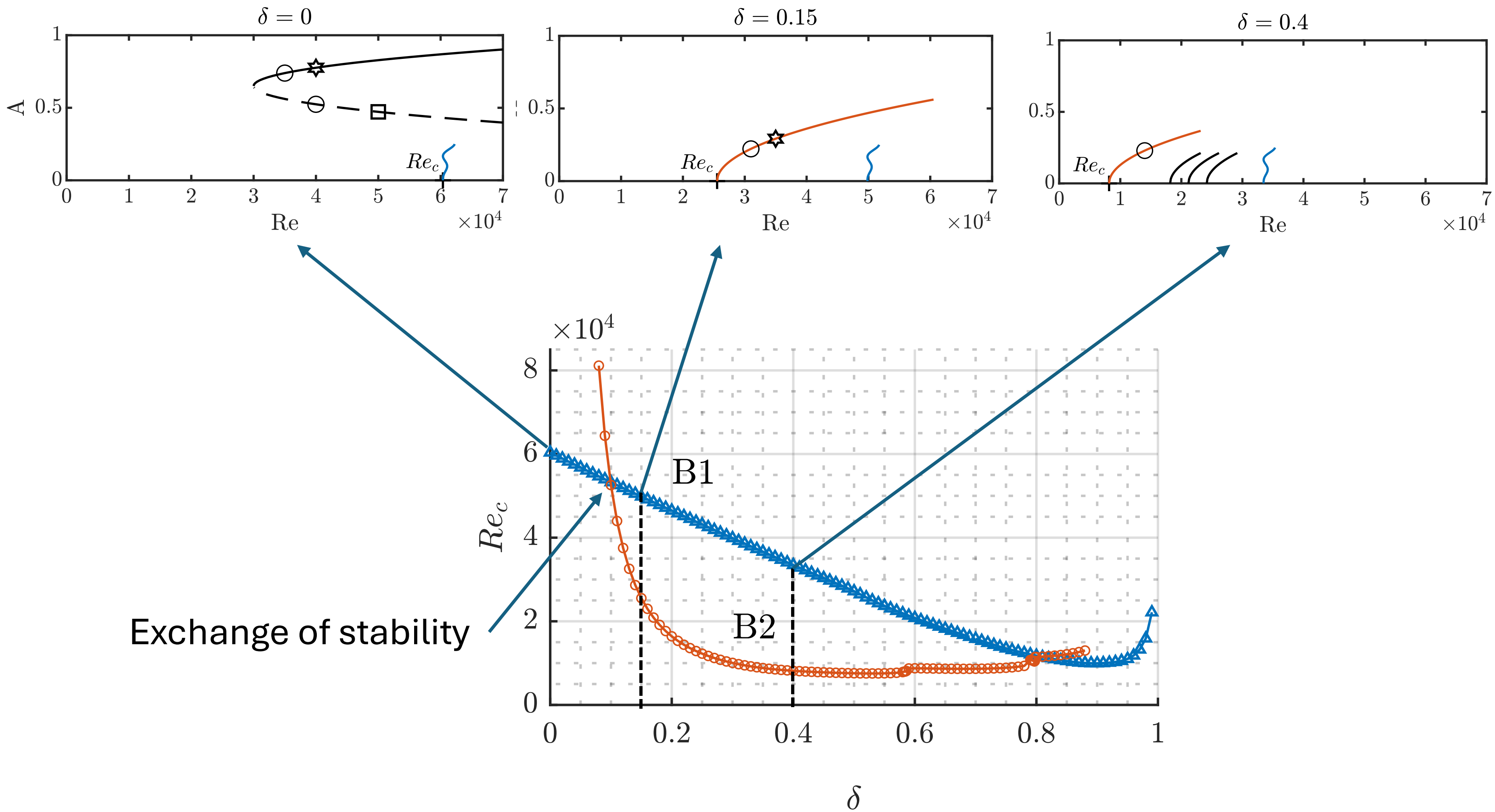
✶ - chaotic state





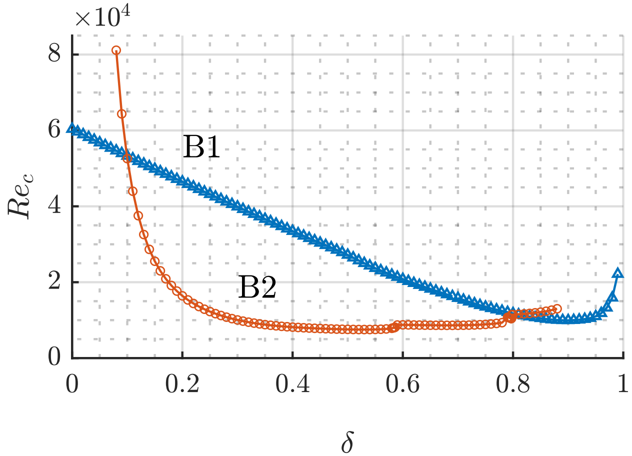




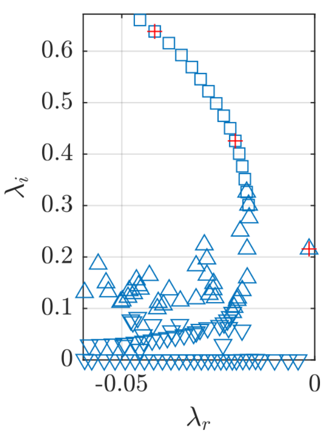


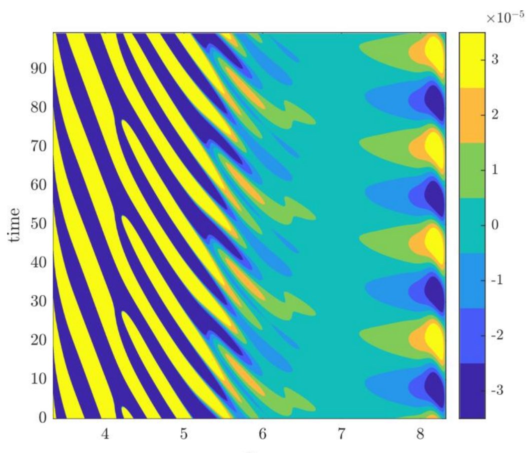
A supercritical branch is replaced by a steep Hopf branch for decreasing δ.
Laminar-turbulent transition becomes subcritical.

Summary:



Stability exchange for a radially displaced cavity.

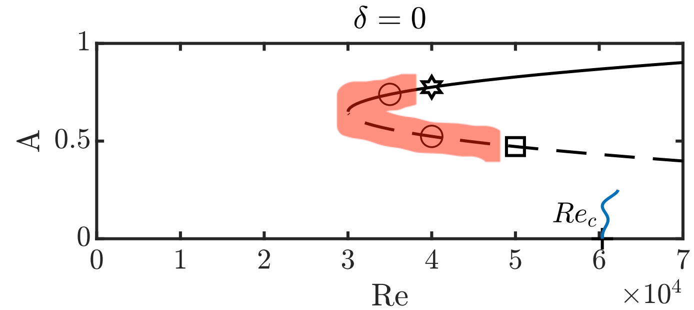




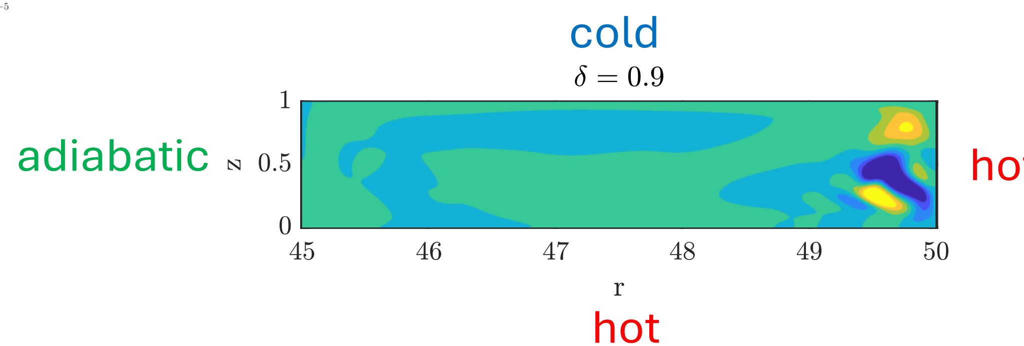
New perspective on the roll pairing.

Gesla *et al*. From annular cavity to rotor-stator flow: nonlinear dynamics of axisymmetric rolls, accepted PRF 2025.

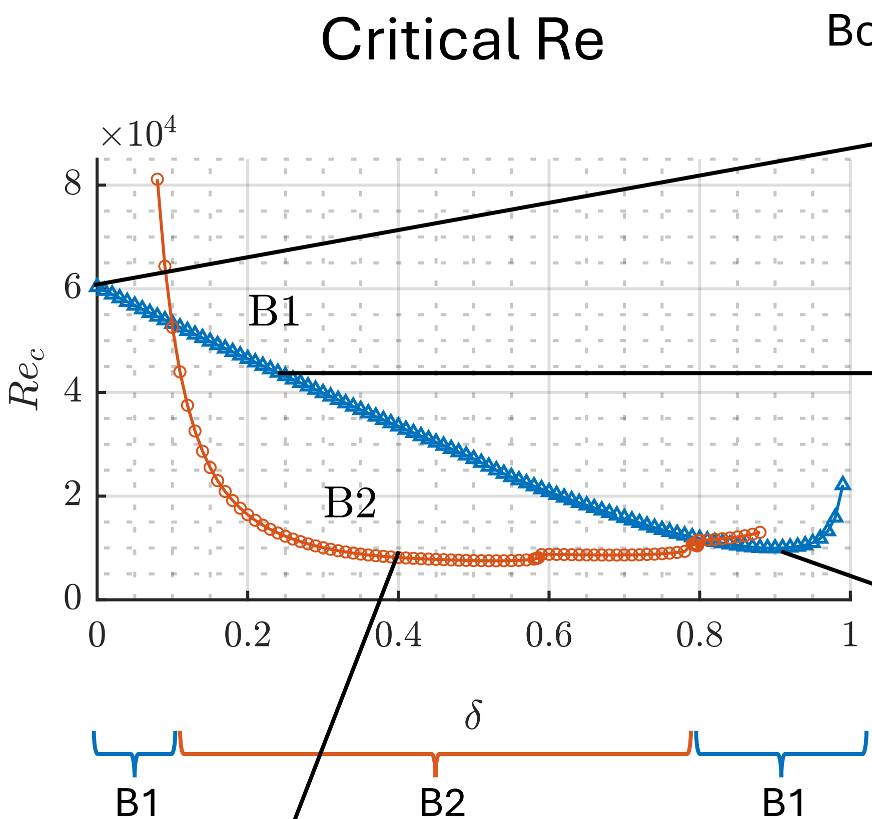
Outlooks:

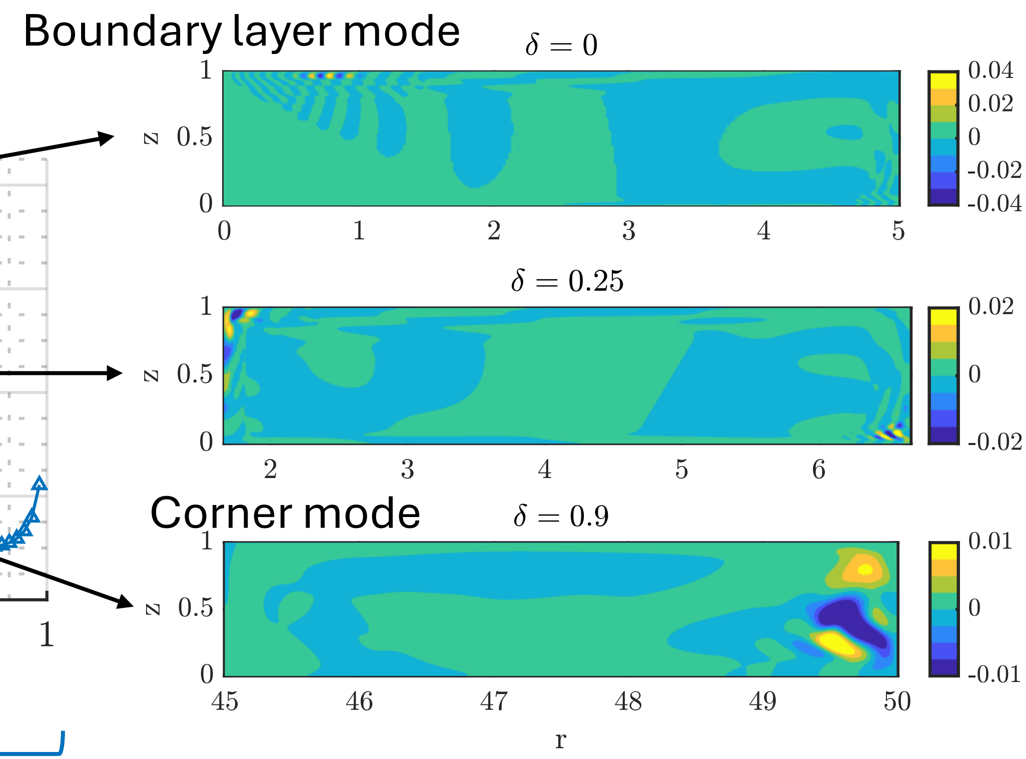


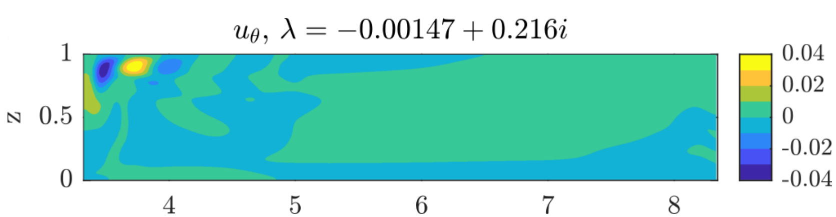
Self-Consistent Model, Harmonic Balance Method, Floquet Analysis on the periodic states identified



$\delta \to 1$ analysis, similarity to a Differentially Heated Cavity for $\mathrm{Pr} = 1$ and $\Delta\Omega \ll \Omega$, comparison of thresholds in Ra and Re



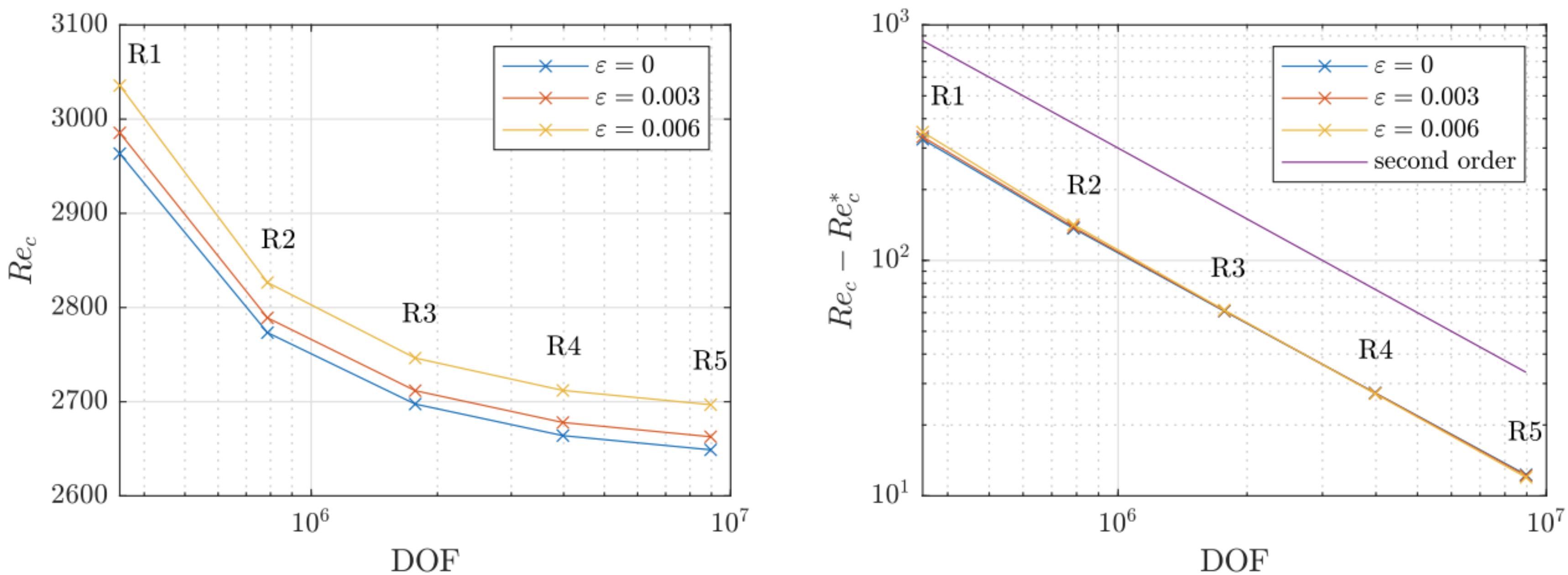




- Branch B1 is responsible for the linear instability at $\delta < 0.1$ and $\delta > 0.8$.
- Eigenmode changes continuously from a boundary layer structure to a corner structure.

Corner singularity

the corner singularity is also considered. This is achieved here by smoothing out the boundary condition at the Bödewadt corner, imposing an exponential velocity profile of the form $u_\theta = r\exp\left(\frac{r-\Gamma}{\varepsilon}\right)$. Two regularisations have been considered : $\varepsilon = 0.003$ and $\varepsilon = 0.006$. The case without any regularization $(u_\theta = 0)$ is referred to as $\varepsilon = 0$ for ease of notation.



resolution	N_r	N_z	type	DOF	$\varepsilon = 0$	$\varepsilon = 0.003$	$\varepsilon = 0.006$
R0	**600**	**160**	uniform	**390 k**	**2925.47**		
R1	683	128	non-uniform	356 k	2963.41	2985.43	3035.61
R2	**1024**	**192**	non-uniform	**796 k**	**2773.3**	2789.01	2826.55
R3	1536	288	non-uniform	1.7 m	2697.48	2711.71	2746.33
R4	2304	432	non-uniform	4.0 m	2663.96	2677.9	2711.97
R5	3456	648	non-uniform	9.0 m	2648.9	2662.8	2696.8
				extrapolation	2636.61	2650.59	2684.81
				order	2.01	2.04	2.09

TABLE V. Critical Reynolds number Re_c depending on the the spatial discretisation. From R1 to R5 the ratio between two consecutive grid resolutions is 1.5 in each direction.