Heat equation in a 2D trapezoid

Cartesian case

Equation : $\Delta u = 0$

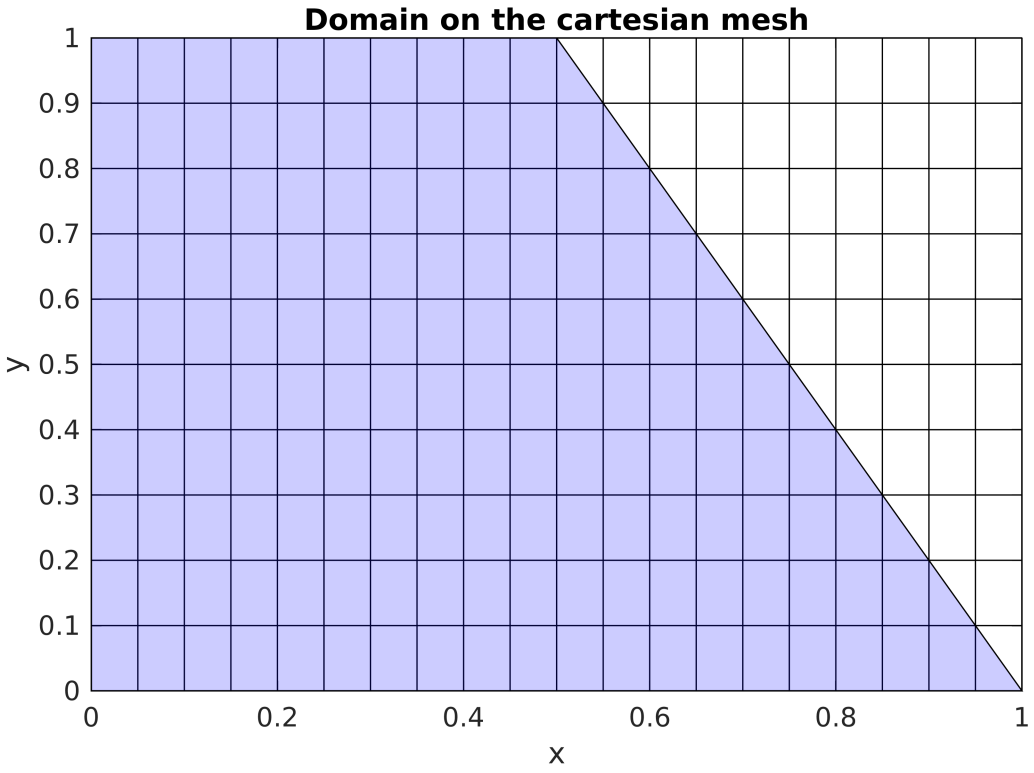
Domain : $0 < x < 1, 0 < y < 2(1-x)$

Boundary cond. : $u = 1, y = 2(1-x); u = 0$ otherwise

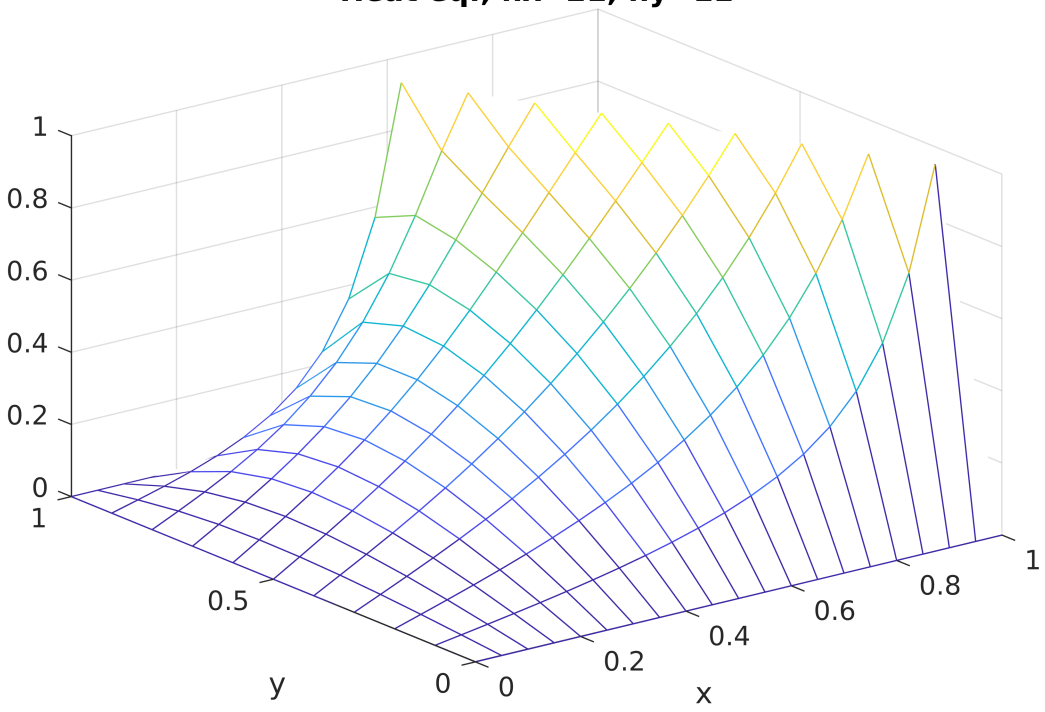
```
nx=21;
main_head_2d_trapz_cartesian;
```

```
Elapsed time is 0.000671 seconds.
```

```
a=load("dataCart-21.mat");

[X,Y]=meshgrid(a.x,a.y);
plot(X,Y,'k'); hold on;
plot(X',Y','k');
fill([0 1 0.5 0 0],[0 0 1 1 0],'blue','FaceAlpha',0.2);
title("Domain on the cartesian mesh");
xlabel('x'); ylabel('y');
```



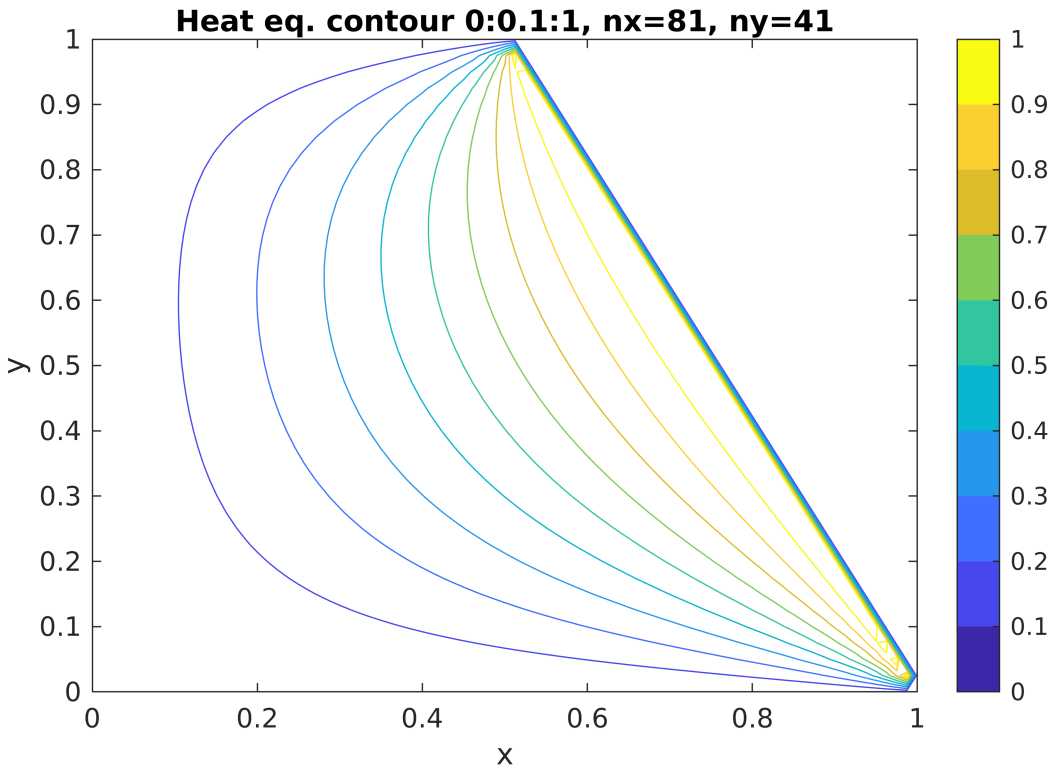

```
clf;
mesh(a.x,a.y,a.uPhys);
xlabel('x'); ylabel('y'); title("Heat eq., nx="+num2str(nx)+", ny="+num2str(ny));
```




```
nx=81;
main_head_2d_trapz_cartesian;
```

```
Elapsed time is 0.005866 seconds.
```

```
a=load("dataCart-81.mat");
contour(a.x,a.y,a.uPhys,[0:0.1:1]); colorbar(); colormap(parula(10))
xlabel('x'); ylabel('y'); title("Heat eq. contour 0:0.1:1, nx="+num2str(nx)+", ny="+num
```



Modified coordinate

Transform : $(x, y) \to (\theta, z)$

$x = (2 - z)\tan(\theta),$

$y = z$

Equation :

$$0 = \frac{\partial^2 u}{\partial \theta^2}\left[\frac{\cos(\theta)^2}{(L-z)}\right]^2 - \frac{\partial u}{\partial \theta}\frac{\cos(\theta)^2}{(L-z)}\frac{2\sin(\theta)\cos(\theta)}{(L-z)} + \frac{\partial^2 u}{\partial z^2} + \frac{\partial u}{\partial \theta}\frac{\sin(\theta)\cos(\theta)}{(L-z)^2} + \frac{\partial^2 u}{\partial \theta \partial z}\frac{2\sin(\theta)\cos(\theta)}{(L-z)} + \frac{\partial u}{\partial \theta}\frac{\cos(2\theta)\sin(\theta)\cos(\theta)}{(L-z)^2} + \frac{\partial^2 u}{\partial \theta^2}\left[\frac{\sin(\theta)\cos(\theta)}{(L-z)}\right]^2$$

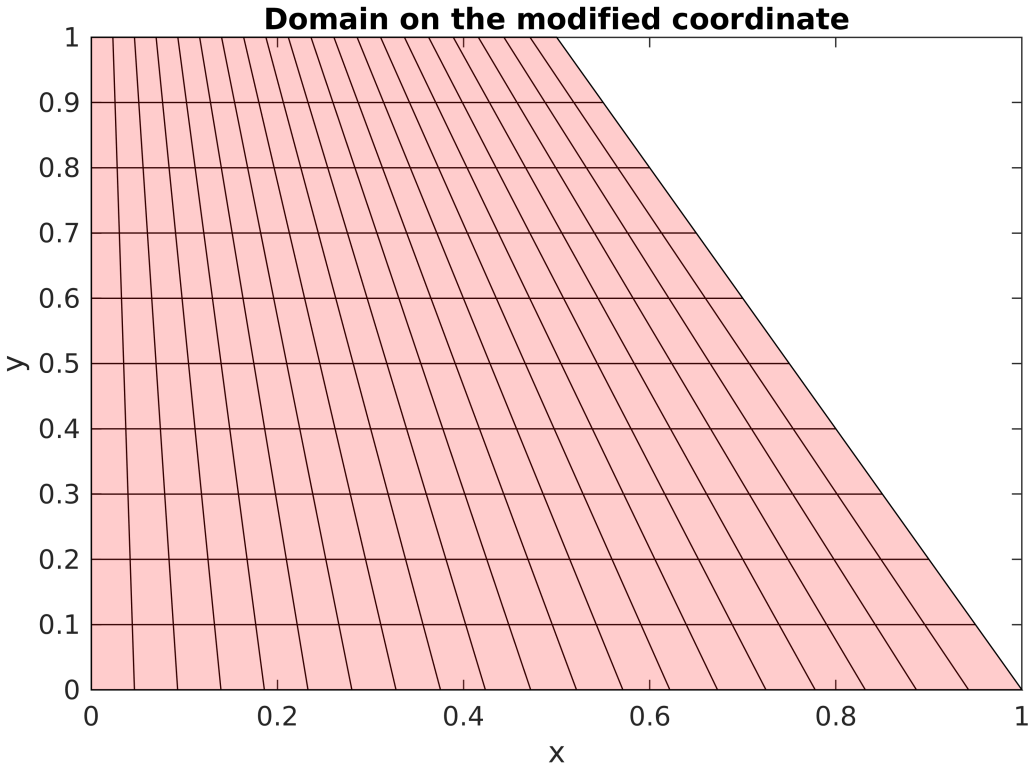
Domain : $0 < z < 1, 0 < \theta < \tan^{-1}(0.5)$

Boundary cond. : $u = 1, \theta = \tan^{-1}(0.5);\ u = 0$ otherwise

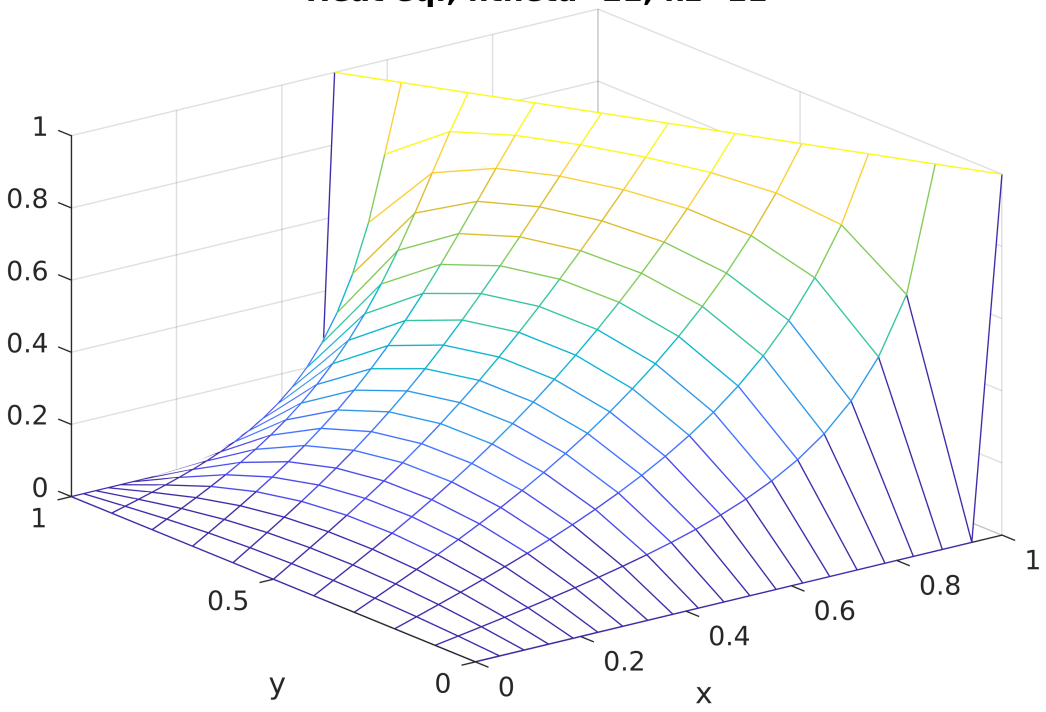
```
nx=21;
diff_2d_mod;
```

```
Elapsed time is 0.000622 seconds.
```

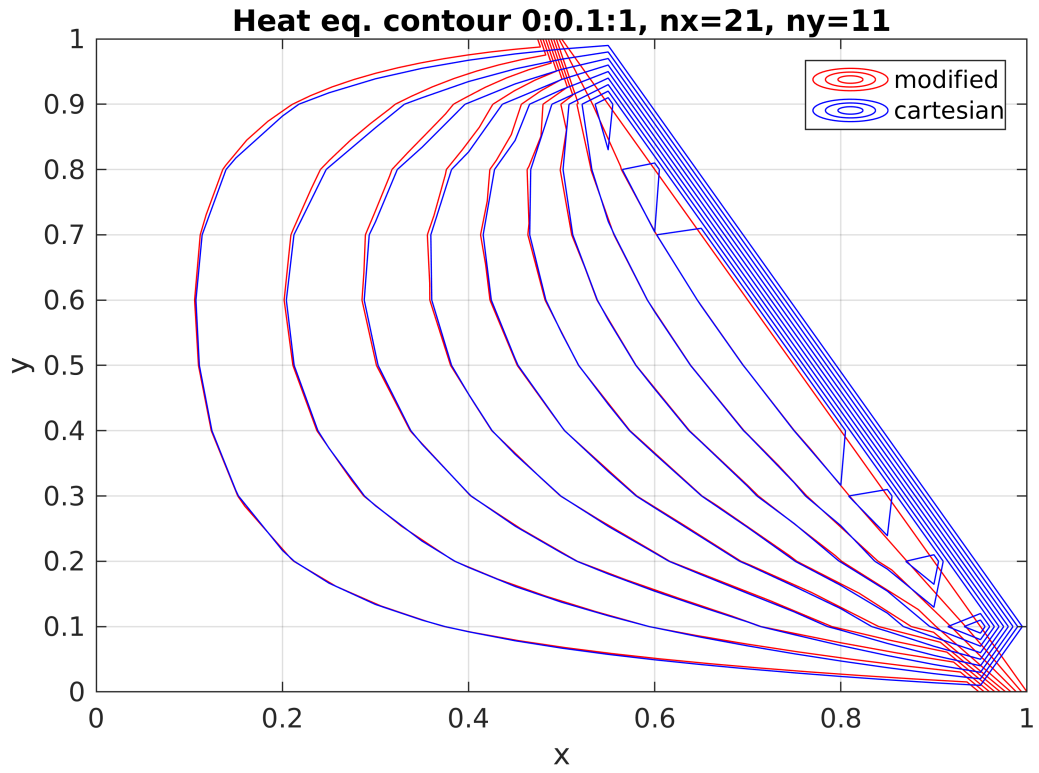
```
a=load("dataMod-21.mat");

X=a.X; Y=a.Y;
plot(X,Y,'k'); hold on;
plot(X',Y','k');
fill([0 1 0.5 0 0],[0 0 1 1 0],'red','FaceAlpha',0.2);
title("Domain on the modified coordinate");
xlabel('x'); ylabel('y');
```




```
clf;
mesh(a.X,a.Y,a.uPhys);
xlabel('x'); ylabel('y'); title("Heat eq., ntheta="+num2str(nx)+", nz="+num2str(ny));
```



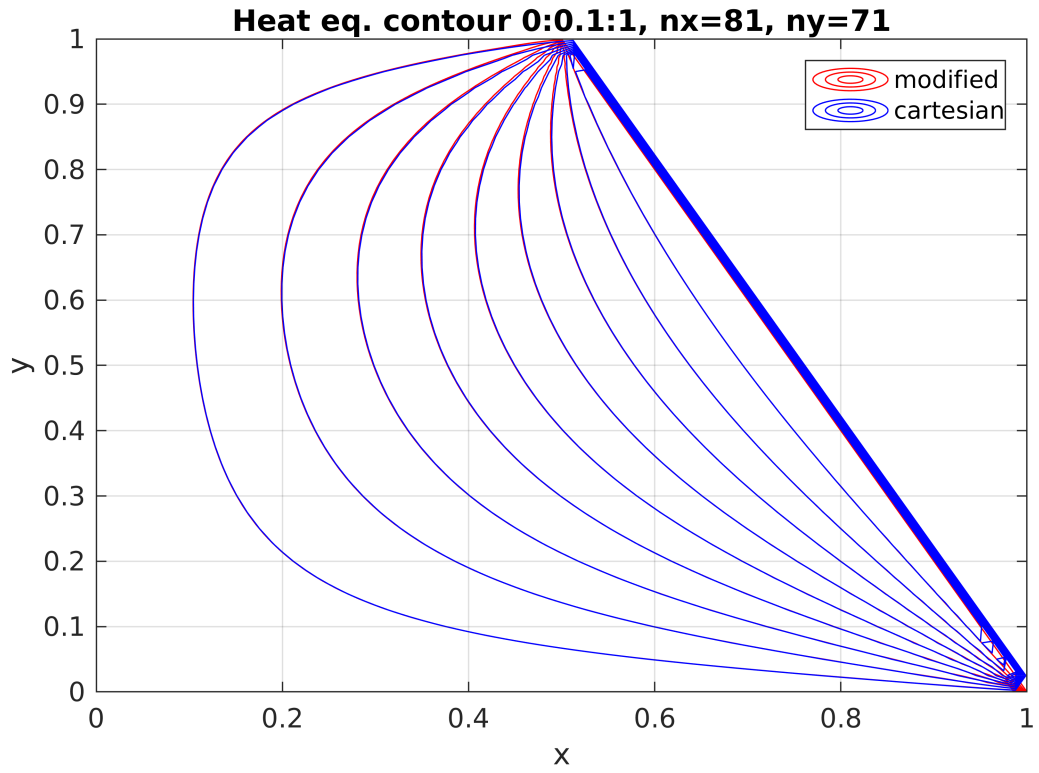

```
contour(a.X,a.Y,a.uPhys,[0:0.1:1],'r');
xlabel('x'); ylabel('y'); title("Heat eq. contour 0:0.1:1, nx="+num2str(nx)+", ny='+num
grid on; hold on;
a=load("dataCart-21.mat");
contour(a.x,a.y,a.uPhys,[0:0.1:1],'b');
legend("modified",'cartesian',"Location","northeast");
```




```
clf;
nx=81;
diff_2d_mod;
```

```
Elapsed time is 0.024575 seconds.
```

```
a=load("dataMod-81.mat");


contour(a.X,a.Y,a.uPhys,[0:0.1:1],'r');
xlabel('x'); ylabel('y'); title("Heat eq. contour 0:0.1:1, nx="+num2str(nx)+", ny="+num
grid on; hold on;
a=load("dataCart-81.mat");
contour(a.x,a.y,a.uPhys,[0:0.1:1],'b');
legend("modified",'cartesian',"Location","northeast");
```



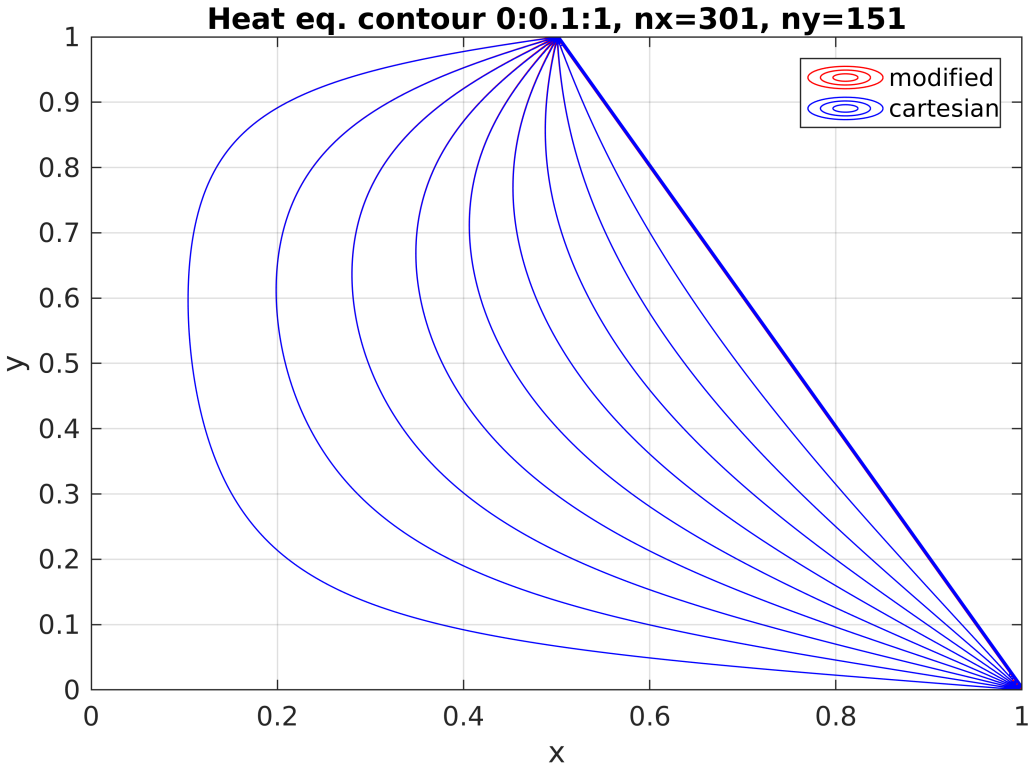

```
clf;
nx=301;
diff_2d_mod;
```

```
Elapsed time is 0.422537 seconds.
```

```
main_head_2d_trapz_cartesian;
```

```
Elapsed time is 0.115084 seconds.
```

```
a=load("dataMod-301.mat");

contour(a.X,a.Y,a.uPhys,[0:0.1:1],'r');
xlabel('x'); ylabel('y'); title("Heat eq. contour 0:0.1:1, nx="+num2str(nx)+", ny="+num
grid on; hold on;

a=load("dataCart-301.mat");

contour(a.x,a.y,a.uPhys,[0:0.1:1],'b');
legend("modified",'cartesian',"Location","northeast");
```


Heat eq. contour 0:0.1:1, nx=301, ny=151
modified
cartesian
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1