SFEMaNS user guide

FL, JLG, CN, DCQ ...

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The installation of the libraries ARPACK and PARPACK is done following these steps:

- Download `arpack96.tar.gz ... ppatch.tar.gz` from the url ...
- Uncompress the archives in the following order,
- In the directory `ARPACK`, replace the file `ARmake.inc` by one of the templates in the subdirectory `ARMAKES`, according to your environment,
- Edit the file `ARmake.inc` to make it compatible with your environment: in particular, the path for the top-level directory of ARPACK, make sure the Fortran compilers (`FC` and `PFC`) are the right ones, and check the path for the command `make`,
- Build the libraries, using `make lib` and `make plib`.

1.1 Mesh generator

1.2 Adaptation to user's environment

x

Chapter 2

Getting started

2.1 First run

Begin with SFEMaNS by creating a directory (e.g. `MY_APPLICATION`) in which you want to create the executable file. Then you need to :

- Copy the content of `$(HOME_SFEMaNS)/TEMPLATE` into `MY_APPLICATION`,
- Edit the file `my_make` to make it compatible with your environment, by specifying the right path to the top level directory of SFEMaNS,
- Edit the file `make.inc` to make it compatible with your environment: in particular, if you have installed ARPACK and PARPACK on your own, you need to use

```
PA_LIB    =  $(HOME_ARPACK)/name_of_your_P_arpack_lib $(HOME_ARPACK)/name_of_your_arpack_lib
```

  in this specific order,
- Create the file `makefile` using the command: `./my_make`,
- Create the executable file `a.exe` using the command: `make a.exe`.

2.2 Type of problem

SFEMaNS can solve three different types of problems, listed below.

2.2.1 Type `nst`

2.2.2 Type `mxw`

2.2.3 Type `mhd`

2.3 Numerical domain

2.4 `data` file

The file `data` contains all the data needed for the computation. The user has to specify the geometry of the domain, the list of conductive and insulating parts, as well as general information for the parallel runs. The `data` file is divided in blocks : one block is mandatory, some others are needed depending on the type of the problem you want to solve, and a couple of blocks are optional. Table 2.1 summarizes the blocks needed for a SFEMaNS run.

2.4.1 Required informations

2.4.2 Type related blocks

2.4.3 Optional blocks

	Basic use			Advanced computations				
Blocks	nst	mxw	mhd	nst with temperature	mxw without ϕ	mhd with temperature	mhd without ϕ	Arpack on $\mathbf{H}$
`GENERAL DATA`	R	R	R	R	R	R	R	R
`Mesh-NAVIER-STOKES`	R	O	R	R	O	R	R	X
`BCs-NAVIER-STOKES`	R	X	R	R	X	R	R	X
`Dynamics-NAVIER-STOKES`	R	X	R	R	X	R	R	X
`LES-NAVIER-STOKES`	R	X	R	R	X	R	R	X
`Solver-velocity-NAVIER-STOKES`	O	X	O	O	X	O	O	X
`Solver-pressure-NAVIER-STOKES`	O	X	O	O	X	O	O	X
`Solver-mass-NAVIER-STOKES`	O	X	O	O	X	O	O	X
`Verbose (diagnostics)`	O	O	O	O	O	O	O	O
`Solver-MAXWELL`	X	O	O	O	O	O	O	O
`H-MAXWELL`	X	R	R	X	R	R	R	R
`Phi-MAXWELL`	X	R	R	X	X	O	X	O
`Verbose-MAXWELL`	X	O	O	X	O	O	O	C
`Phase`	O	X	X	R	X	R	O	X
`Solver-Phase`	O	X	O	O	X	O	O	X
`Post-processing`	O	O	O	O	O	O	O	X
`Periodicity`	O	O	O	O	O	O	O	O
`ARPACK`	X	X	X	X	X	X	X	R
`Visualization`	X	X	X	X	X	X	X	O
`BLOCK`	C	C	C	C	C	C	C	C

Table 2.1: Summary of the blocks in the `data` file (R=Required, O=Optional, X=Useless)

2.5 Boundary conditions (`condlim.f90` file)

2.6 The main program (`main.f90` file)

Chapter 3

Tools

3.1 Backup tools

3.2 Visualization tools

3.3 Variables in SFEMaNS

3.4 Custom variables

Using `read_user_data.f90`, SFEMaNS allows the use of custom variables. All the custom variables have to be declared in the type `user_data_type` and can be used in the `main.f90` file. If needed, the user can read personal data from a file, either by appending the `data` file, or by creating another one. User's data file (e.g. `my_own_data`) has to be read in the `main.f90` with the following

```
CALL read_user_data('my_own_data')
```

After this call, all the variables declared in the type `user_data_type` can be used with the prefix `my_data%`.

Use the template in `read_user_data.f90` to add any number of variables.

Chapter 4

Tests in SFEMaNS

The command

```
./debug_SFEMaNS
```

is used to run 17 different tests. Informations about these cases are listed below.

Chapter 5

Test 1: nst

5.1 Numerical domain and equations to solve

$$\begin{aligned}
\partial_t \mathbf{u} + (\nabla\times\mathbf{u})\times\mathbf{u} - \frac{1}{R_\mathrm{e}}\Delta\mathbf{u} + \nabla p &= \mathbf{f}, \\
\nabla\cdot\mathbf{u} &= 0, \\
\mathbf{u}_{|\Gamma} &= \mathbf{v}, \\
\mathbf{u}_{|t=0} &= \mathbf{u}_0.
\end{aligned}$$

The data are $\mathbf{f}$, $\mathbf{v}$ and $\mathbf{u}_0$. We use $R_\mathrm{e} = 1$.

5.2 Analytical solution

$$\begin{aligned}
u_r(r,\theta,z,t) &= \left(\left(r^2z^3 - 3r^3z^2\right)\cos(\theta) - \left(r^2z^3 + 3r^3z^2\right)\sin(\theta)\right)\cos(t), \\
u_\theta(r,\theta,z,t) &= 3\left(r^3z^2 - r^2z^3\right)(\cos(\theta) + \sin(\theta))\cos(t), \\
u_z(r,\theta,z,t) &= \left(3r^2z^3\cos(\theta) + 5r^2z^3\sin(\theta)\right), \\
p(r,\theta,z,t) &= rz\,(\cos(\theta) + \sin(\theta))\sin(t).
\end{aligned}$$

5.3 Data file

Chapter 6

Test 2: nst + perio

6.1 Numerical domain and equations to solve

$$
\begin{aligned}
\partial_t \mathbf{u} + (\nabla\times\mathbf{u})\times\mathbf{u} - \frac{1}{R_\mathrm{e}}\Delta\mathbf{u} + \nabla p &= \mathbf{f}, \\
\nabla\cdot\mathbf{u} &= 0, \\
\mathbf{u}_{|\Gamma} &= \mathbf{v}, \\
\mathbf{u}_{|t=0} &= \mathbf{u}_0, \\
\mathbf{u} \text{ periodic in the } z\text{-direction.}
\end{aligned}
$$

6.2 Analytical solution

$$
\begin{aligned}
u_r(r,\theta,z,t) &= -r^2\,(1-2\pi r\sin(2\pi z))\sin(\theta)\cos(t), \\
u_\theta(r,\theta,z,t) &= -3r^2\cos(\theta)\cos(t), \\
u_z(r,\theta,z,t) &= r^2\,(4\cos(2\pi z)+1)\sin(\theta)\cos(t), \\
p(r,\theta,z,t) &= r^2\cos(2\pi z)\cos(\theta)\cos(t).
\end{aligned}
$$

6.3 Data file

Chapter 7

Test 3: mxw (with vacuum).

7.1 Numerical domain and equations to solve

$$\partial_t (\mu \mathbf{H}) + \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma} \nabla\times\mathbf{H} \right) - \nabla\times (\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma} \mathbf{j} \right),$$
$$\nabla\cdot(\mu\mathbf{H}) = 0,$$
$$+BC + IC. + \phi$$

7.2 Analytical solution

$$H_r(r,\theta,z,t) = \sum_{m=1}^{3} \frac{1}{m^3} \left(\alpha_m z r^{m-1} m \cos(m\theta) + \beta_m z r^{m-1} m \sin(m\theta)\right) \cos(t),$$
$$H_\theta(r,\theta,z,t) = \sum_{m=1}^{3} \frac{1}{m^3} \left(\beta_m z r^{m-1} m \cos(m\theta) - \alpha_m z r^{m-1} m \sin(m\theta)\right) \cos(t),$$
$$H_z(r,\theta,z,t) = \sum_{m=1}^{3} \frac{1}{m^3} \left(\alpha_m r^m \cos(m\theta) + \beta_m r^m \sin(m\theta)\right) \cos(t),$$
$$\phi(r,\theta,z,t) = \sum_{m=1}^{3} \frac{1}{m^3} \left(\alpha_m z r^m \cos(m\theta) + \beta_m z r^m \sin(m\theta)\right) \cos(t).$$

7.3 Data file

Chapter 8

Test 4: mxw + perio (with vacuum).

8.1 Numerical domain and equations to solve

$$\begin{aligned}
\partial_t\left(\mu\mathbf{H}\right) + \frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\nabla\times\mathbf{H}\right) - \nabla\times\left(\mathbf{u}\times\mu\mathbf{H}\right) &= \frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\mathbf{j}\right),\\
\nabla\cdot(\mu\mathbf{H}) &= 0,\\
+BC + IC + &perio + \phi
\end{aligned}$$

8.2 Analytical solution

$$\begin{aligned}
H_r(r,\theta,z,t) &= \cos(t)\cos(\theta)\cos(2\pi z)\left(\frac{r}{r_0^2} - 2\pi\left(\frac{r}{r_0}\right)^2\left(A + B\frac{r}{r_0}\right)\right),\\
H_\theta(r,\theta,z,t) &= \cos(t)\sin(\theta)\cos(2\pi z)\left(2\pi\left(\frac{r}{r_0}\right)^2 C - 2\frac{r}{r_0^2}\right),\\
H_z(r,\theta,z,t) &= \cos(t)\cos(\theta)\sin(2\pi z)\frac{r}{r_0^2}\left(3A + 4B\frac{r}{r_0} - C\right),\\
\phi(r,\theta,z,t) &= \cos(t)\cos(\theta)\cos(2\pi z)K_1(2\pi r),
\end{aligned}$$

K_1: Bessel function. A, B, C constants.

8.3 Data file

Chapter 9

Test 5: mxw (with vacuum).

9.1 Numerical domain and equations to solve

$$\partial_t\left(\mu\mathbf{H}\right)+\frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\nabla\times\mathbf{H}\right)-\nabla\times\left(\mathbf{u}\times\mu\mathbf{H}\right)=\frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\mathbf{j}\right),$$
$$\nabla\cdot(\mu\mathbf{H})=0,$$
$$+BC+IC.+\phi$$

9.2 Analytical solution

$$H_r(r,\theta,z,t)=\sum_{m=1}^{3}\frac{1}{m^3}\left(\alpha_m z r^{m-1} m\cos(m\theta)+\beta_m z r^{m-1} m\sin(m\theta)\right)\cos(t),$$
$$H_\theta(r,\theta,z,t)=\sum_{m=1}^{3}\frac{1}{m^3}\left(\beta_m z r^{m-1} m\cos(m\theta)-\alpha_m z r^{m-1} m\sin(m\theta)\right)\cos(t),$$
$$H_z(r,\theta,z,t)=\sum_{m=1}^{3}\frac{1}{m^3}\left(\alpha_m r^m\cos(m\theta)+\beta_m r^m\sin(m\theta)\right)\cos(t),$$
$$\phi(r,\theta,z,t)=\sum_{m=1}^{3}\frac{1}{m^3}\left(\alpha_m z r^m\cos(m\theta)+\beta_m z r^m\sin(m\theta)\right)\cos(t).$$

9.3 Data file

Chapter 10

Test 6: mxw (with vacuum).

10.1 Numerical domain and equations to solve

$$\begin{aligned}
\partial_t (\mu\mathbf{H}) + \frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\nabla\times\mathbf{H}\right) - \nabla\times(\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\mathbf{j}\right),\\
\nabla\cdot(\mu\mathbf{H}) = 0,\\
+BC + IC. + \phi
\end{aligned}$$

10.2 Analytical solution

10.3 Data file

Chapter 11

Test 7: mxw (with vacuum).

11.1 Numerical domain and equations to solve

$$\partial_t (\mu \mathbf{H}) + \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma} \nabla\times \mathbf{H} \right) - \nabla\times (\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma} \mathbf{j} \right),$$
$$\nabla\cdot(\mu\mathbf{H}) = 0,$$
$$+BC + IC. + \phi$$

11.2 Analytical solution

11.3 Data file

Chapter 12

Test 8: nst + phase.

12.1 Numerical domain and equations to solve

12.2 Analytical solution

12.3 Data file

Chapter 13

Test 9: nst + phase + perio.

13.1 Numerical domain and equations to solve

13.2 Analytical solution

13.3 Data file

Chapter 14

Test 10: mxw (without vacuum).

14.1 Numerical domain and equations to solve

$$\partial_t(\mu\mathbf{H}) + \frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\nabla\times\mathbf{H}\right) - \nabla\times(\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\mathbf{j}\right),$$
$$\nabla\cdot(\mu\mathbf{H}) = 0,$$
$$+BC + IC.$$

14.2 Analytical solution

$$H_r(r,\theta,z,t) = \sum_{m=1}^{3}\frac{1}{m^3}\left(\alpha_m z r^{m-1} m\cos(m\theta) + \beta_m z r^{m-1} m\sin(m\theta)\right)\cos(t),$$
$$H_\theta(r,\theta,z,t) = \sum_{m=1}^{3}\frac{1}{m^3}\left(\beta_m z r^{m-1} m\cos(m\theta) - \alpha_m z r^{m-1} m\sin(m\theta)\right)\cos(t),$$
$$H_z(r,\theta,z,t) = \sum_{m=1}^{3}\frac{1}{m^3}\left(\alpha_m r^m\cos(m\theta) + \beta_m r^m\sin(m\theta)\right)\cos(t),$$

14.3 Data file

Chapter 15

Test 11: mhd + temperature (without vacuum).

15.1 Numerical domain and equations to solve

15.2 Analytical solution

15.3 Data file

Chapter 16

Test 12: mxw Dirichlet/Neumann (without vacuum).

16.1 Numerical domain and equations to solve

$$
\begin{aligned}
\partial_t (\mu \mathbf{H}) + \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma} \nabla\times \mathbf{H} \right) - \nabla\times (\mathbf{u}\times\mu\mathbf{H}) &= \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma} \mathbf{j} \right), \\
\nabla\cdot (\mu\mathbf{H}) &= 0, \\
&+BC + IC.
\end{aligned}
$$

16.2 Analytical solution

16.3 Data file

Chapter 17

Test 13: mxw Dirichlet/Neumann + perio (without vacuum).

17.1 Numerical domain and equations to solve

$$\partial_t (\mu \mathbf{H}) + \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma} \nabla\times\mathbf{H} \right) - \nabla\times (\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma} \mathbf{j} \right),$$
$$\nabla\cdot(\mu\mathbf{H}) = 0,$$
$$+BC + IC.$$

17.2 Analytical solution

17.3 Data file

Chapter 18

Test 14: mxw + arpack (without vacuum).

18.1 Numerical domain and equations to solve

18.2 Reference results

18.3 Data file

Chapter 19

Test 15: nst (with LES).

19.1 Numerical domain and equations to solve

19.2 Reference results

19.3 Data file

Chapter 20

Test 16: ??.

20.1 Numerical domain and equations to solve

20.2 Reference results

20.3 Data file

Chapter 21

Test 17: mxw with vacuum + variable mu

The purpouse of this case is to test variable permeability with dependence in r and z. There is one conducting region embedded in vaccum.

21.1 Numerical domain and equations to solve

The domain is a cylinder $r \in [0,1]$, $z \in [-1,1]$. The exterior boundary of the vaccum is a sphere of radius 10. Let us recall the kynematic dynamo equations,

$$\begin{cases} \partial_t (\mu \mathbf{H}) + \frac{1}{R_{\mathrm{m}}} \nabla\times \left(\frac{1}{\sigma}\nabla\times\mathbf{H}\right) - \nabla\times(\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_{\mathrm{m}}}\nabla\times\left(\frac{1}{\sigma}\mathbf{j}\right), \\ \nabla\cdot(\mu\mathbf{H}) = 0, \\ +BC + IC. + \phi \end{cases} \tag{21.1.1}$$

21.2 Analytical solution

Let

$$\mathbf{H} = \frac{1}{\mu^c}\nabla\psi, \tag{21.2.1}$$

where $\psi = \psi(r,z)$ and satisfies the Laplace equation in cylindrical coordinates,

$$\partial_{rr}\psi + \frac{1}{r}\partial_r\psi + \partial_{zz}\psi = 0. \tag{21.2.2}$$

If we also set $\mathbf{j} = \nabla\times\mathbf{H}$, $\mathbf{u} = 0$, $\mathbf{E} = \mathbf{0}$ and $\phi(r,\theta,z,t) = \psi(r,z)$, then $\mathbf{H}$, defined as in (21.2.1), satisfies Maxwell equations (21.1.1).

Now, let

$$\mu^c = \mu^c(r,z) = \frac{1}{f(r,z)+1}, \tag{21.2.3}$$

where

$$f(r,z) = b\cdot r^3\cdot(1-r)^3\cdot(z^2-1)^3,$$

and $b \geq 0$ is a parameter which determines the variation of μ^c. Observe that

$$\partial_r f(r,z) = 3b\big(r(1-r)\big)^2(1-2r)(z^2-1)^3, \quad \partial_z f(r,z) = 6bz\big(r(1-r)\big)^3(z^2-1)^2.$$

Moreover, $f(r,z) \leq 0$ for $(r,\theta,z) \in \Omega^c$ and,

$$\sup_{\Omega^c} f(r,z) = f_{\max} = 0, \quad \inf_{\Omega^c} f(r,z) = f_{\min} = -\frac{b}{2^6},$$

then,

$$\mu^c_{\min} = \frac{1}{1+f_{\max}}, \quad \mu^c_{\max} = \frac{1}{1+f_{\min}}, \qquad r_\mu = \frac{\mu_{\max}}{\mu_{\min}} = \frac{\frac{1}{1-\frac{b}{2^6}}}{1}, \quad \text{and} \quad b = 2^6\left(1 - \frac{1}{r_\mu}\right).$$

To get an explicit solution in (21.2.1), equation (21.2.2) is solved using separation of variables, this is, letting $\psi(r,z) = R(r)Z(z)$ we solve the following system of ODEs,

$$\begin{aligned} Z'' - \lambda Z &= 0 \\ R'' + \frac{R'}{r} + \lambda R &= 0, \end{aligned}$$

where λ is any real number. Here we choose $\lambda = 1$, so

$$\psi(r,z) = J_0(r)\cosh(z). \tag{21.2.4}$$

Now, using $J_0'(r) = -J_1(r)$ and $\cosh'(z) = \sinh(z)$ we get,

$$\nabla\psi = \begin{bmatrix} -J_1(r)\cosh(z) \\ 0 \\ J_0(r)\sinh(z) \end{bmatrix} \tag{21.2.5}$$

then by (21.2.1),

$$\mathbf{H}^c = (f(r,z)+1)\begin{bmatrix} -J_1(r)\cosh(z) \\ 0 \\ J_0(r)\sinh(z) \end{bmatrix}, \tag{21.2.6}$$

To get $\nabla \times \mathbf{H}$, we use the identity

$$\nabla \times \left(\frac{1}{\mu^c}\nabla\psi\right) = \nabla\left(\frac{1}{\mu^c}\right) \times \nabla\psi + \frac{1}{\mu^c}\nabla \times \nabla\psi,$$

but $\nabla \times \nabla\psi = 0$. Then using equation (21.2.1),

$$\nabla \times \mathbf{H}^c = \nabla\left(\frac{1}{\mu^c}\right) \times \nabla\psi,$$

and

$$\nabla\frac{1}{\mu^c} = \begin{bmatrix} \partial_r f(r,z) \\ 0 \\ \partial_z f(r,z) \end{bmatrix}; \tag{21.2.7}$$

we obtain,

$$\mathbf{j} = \nabla \times \mathbf{H}^c = \begin{bmatrix} 0 \\ -\partial_r f(r,z) J_0(r)\sinh(z) - \partial_z f(r,z) J_1(r)\cosh(z) \\ 0 \end{bmatrix}. \tag{21.2.8}$$

In summary,

$$\phi(r,\theta,z,t) = J_0(r)\cosh(z). \tag{21.2.9}$$

$$\mathbf{H}^c = \left(2^6\left(1 - \frac{1}{r_\mu}\right) \cdot r^3 \cdot (1-r)^3 \cdot (z^2-1)^3 + 1\right)\begin{bmatrix} -J_1(r)\cosh(z) \\ 0 \\ J_0(r)\sinh(z) \end{bmatrix}, \tag{21.2.10}$$

21.3 Reference results

21.4 Data file