# SFEMaNS user guide

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# Contents

# Chapter 1

# General presentation

## 1.1 Introduction

SFEMaNS is a code to solve the MHD equations in axi-symmetric domains.

### 1.1.1 SFEMaNS credit

The current maintainers of SFEMaNS are Jean-Luc Guermond, Francky Luddens, and Caroline Nore.

This project has evolved through the PhD or Master theses of the following people:

- Loïc Cappanera (PhD, 2015)
- Daniel Castanon (PhD, 2015)
- Raphaël Laguerre (PhD, Dec 2006)
- Francky Luddens (PhD, Dec 2012)
- Remi Ménard (Master, Nov 2013)
- Adolfo Ribeiro (PhD, Dec 2010)

## 1.2 System of equations

### 1.2.1 Time-dependent problems

SFEMaNS solves the time-dependent MHD equations assuming that the flow is incompressible.

$$
\begin{aligned}
\partial_t \mathbf{u} + (\nabla\times\mathbf{u})\times\mathbf{u} - \frac{1}{R_\mathrm{e}}\Delta\mathbf{u} + \nabla p + 2\boldsymbol{\Omega}\times\mathbf{u} &= (\nabla\times\mathbf{H})\times\mathbf{H} + \mathbf{f}(T),\\
\nabla\cdot\mathbf{u} &= 0,\\
\partial_t T + \mathbf{u}\cdot\nabla t - \kappa\Delta T &= g(T),\\
\mathbf{u}_{|\Gamma} &= \mathbf{v},\\
\mathbf{u}_{|t=0} &= \mathbf{u}_0.
\end{aligned}
$$

$$
\begin{aligned}
\partial_t\left(\mu\mathbf{H}\right) + \frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\nabla\times\mathbf{H}\right) - \nabla\times\left(\mathbf{u}\times\mu\mathbf{H}\right) &= \frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\mathbf{j}\right),\\
\nabla\cdot(\mu\mathbf{H}) &= 0,\\
+BC + IC. + \phi &
\end{aligned}
$$

## 1.3 General features of SFEMaNS

# Chapter 2

# Installing SFEMaNS

## 2.1 How to obtain SFEMaNS?

## 2.2 External tools

SFEMaNS uses PETSc. The graph partitioning is done with parmetis. The linear algebra is handled with hypre and MUMPS. We recommend to download these softwares with PETSc.

## 2.3 Installing PETSc, MUMPS, HYPRE, etc.

Install PETSc from `http://www.mcs.anl.gov/petsc/`

1. Be sure to download the latest version of PETSc. For instance use the following command:
   `wget http://ftp.mcs.anl.gov/pub/petsc/release-snapshots/petsc-3.4.2.tar.gz`

2. Untar the archive and move it in the directory that best suits your needs.

3. Go to the PETSc directory and set the environment variables. For csh/tcsh use
   ```
   setenv PETSC_DIR $PWD
   setenv PETSC_ARCH linux-whatever-you-like
   ```
   for bash use
   ```
   export PETSC_DIR=$PWD
   export PETSC_ARCH=linux-whatever-you-like
   ```

4. Specify variable on command line to configure
   ```
   ./configure --download-f-blas-lapack=1 --with-shared-libraries=1 --download-hypre=1
   --download-mumps=1 --download-spooles=1 --download-scalapack=1
   --download-metis=1 --download-parmetis=1 --download-blacs=1
   --with-dynamic-loading=1 --with-debugging=0 --with-x=0
   ```

5. Start the make. Do as recommended by PETSc or type
   `make all`

6. Finish installation with
   `make test`

### 2.3.1 ARPACK library

The installation of the libraries ARPACK and PARPACK is done following these steps:

- Download `arpack96.tar.gz parpack96.tar.gz patch.tar.gz ppatch.tar.gz`
  `wget http://www.caam.rice.edu/software/ARPACK/SRC/arpack96.tar.gz`
  `wget http://www.caam.rice.edu/software/ARPACK/SRC/parpack96.tar.gz`
  `wget http://www.caam.rice.edu/software/ARPACK/SRC/patch.tar.gz`
  `wget http://www.caam.rice.edu/software/ARPACK/SRC/ppatch.tar.gz`
- Uncompress the archive `arpack96.tar.gz` first, then uncompress the others.
- Be sure to read the README before anything.
- If you are using a VENDOR SUPPLIED VERSION of MPI, you must replace the mpif.h in the following directories `ARPACK/PARPACK/SRC/MPI/mpif.h` `ARPACK/PARPACK/UTIL/MPI/mpif.h` `ARPACK/PARPACK/EXAMPLES/MPI/mp` with the one for the native implementation.
- To avoid possible problems with the function `etime`, edit `UTIL/second.f` and replace the lines

  ```
  23        REAL              ETIME
  24        EXTERNAL          ETIME
  ```

  by

  ```
  23 *      REAL               ETIME
  24        EXTERNAL REAL      ETIME
  ```

- In the directory `ARPACK`, replace the file `ARmake.inc` by one of the templates in the subdirectory `ARMAKES`, according to your environment,
- Edit the file `ARmake.inc` to make it compatible with your environment: in particular, the path for the top-level directory of ARPACK, make sure the Fortran compilers (`FC` and `PFC`) are the right ones, and check the path for the command `make` (use `which make` to determine the path `make`).
- Build the libraries, using `make lib` and `make plib`.

## 2.4 Mesh generator

## 2.5 Adaptation to user's environment

x

# Chapter 3

# Getting started

## 3.1 First run

Begin with SFEMaNS by creating a directory (e.g. `MY_APPLICATION`) in which you want to create the executable file. Then you need to :

- Copy the content of `$(HOME_\sfemans)/TEMPLATE` into `MY_APPLICATION`,
- Edit the file `my_make` to make it compatible with your environment, by specifying the right path to the top level directory of SFEMaNS,
- Edit the file `make.inc` to make it compatible with your environment: in particular, if you have installed ARPACK and PARPACK on your own, you need to use

  ```
  PA_LIB = $(HOME_ARPACK)/name_of_your_P_arpack_lib
   $(HOME_ARPACK)/name_of_your_arpack_lib
  ```

  in this specific order,
- Create the file `makefile` using the command: `./my_make`,
- Create the executable file `a.exe` using the command: `make a.exe`.

## 3.2 Type of problem

SFEMaNS can solve three different types of problems, listed below.

### 3.2.1 Type `nst`

### 3.2.2 Type `mxw`

### 3.2.3 Type `mhd`

## 3.3 Numerical domain

## 3.4 `data` file

The file `data` contains all the data needed for the computation. The user has to specify the geometry of the domain, the list of conductive and insulating parts, as well as general information for the parallel runs. The `data` file is divided in blocks : one block is mandatory, some others are needed depending on the type of the problem you want to solve, and a couple of blocks are optional. Table 3.1 summarizes the blocks needed for a SFEMaNS run.

### 3.4.1 Required informations

### 3.4.2 Type related blocks

### 3.4.3 Optional blocks

## 3.5 Boundary conditions (`condlim.f90` file)

## 3.6 The main program (`main.f90` file)

| | Basic use | | | Advanced computations | | | | |
|---|---|---|---|---|---|---|---|---|
| Blocks | nst | mxw | mhd | nst with temperature | mxw without $\phi$ | mhd with temperature | mhd without $\phi$ | Arpack on $\mathbf{H}$ |
| `GENERAL DATA` | R | R | R | R | R | R | R | R |
| `Mesh-NAVIER-STOKES` | R | O | R | R | O | R | R | X |
| `BCs-NAVIER-STOKES` | R | X | R | R | X | R | R | X |
| `Dynamics-NAVIER-STOKES` | R | X | R | R | X | R | R | X |
| `LES-NAVIER-STOKES` | R | X | R | R | X | R | R | X |
| `Solver-velocity-NAVIER-STOKES` | O | X | O | O | X | O | O | X |
| `Solver-pressure-NAVIER-STOKES` | O | X | O | O | X | O | O | X |
| `Solver-mass-NAVIER-STOKES` | O | X | O | O | X | O | O | X |
| `Verbose (diagnostics)` | O | O | O | O | O | O | O | O |
| `Solver-MAXWELL` | X | O | O | O | O | O | O | O |
| `H-MAXWELL` | X | R | R | X | R | R | R | R |
| `Phi-MAXWELL` | X | R | R | X | X | O | X | O |
| `Verbose-MAXWELL` | X | O | O | X | O | O | O | C |
| `Phase` | O | X | X | R | X | R | O | X |
| `Solver-Phase` | O | X | O | O | X | O | O | X |
| `Post-processing` | O | O | O | O | O | O | O | X |
| `Periodicity` | O | O | O | O | O | O | O | O |
| `ARPACK` | X | X | X | X | X | X | X | R |
| `Visualization` | X | X | X | X | X | X | X | O |
| `BLOCK` | C | C | C | C | C | C | C | C |

Table 3.1: Summary of the blocks in the `data` file (R=Required, O=Optional, X=Useless)

# Chapter 4

# Tools

## 4.1 Backup tools

## 4.2 Visualization tools

## 4.3 Variables in SFEMaNS

## 4.4 Custom variables

Using `read_user_data.f90`, SFEMaNS allows the use of custom variables. All the custom variables have to be declared in the type `user_data_type` and can be used in the `main.f90` file. If needed, the user can read personal data from a file, either by appending the `data` file, or by creating another one. User's data file (e.g. `my_own_data`) has to be read in the `main.f90` with the following

```
CALL read_user_data('my_own_data')
```

After this call, all the variables declared in the type `user_data_type` can be used with the prefix `my_data%`.

Use the template in `read_user_data.f90` to add any number of variables.

# Chapter 5

# Tests in SFEMaNS

The command

`./debug_\sfemans`

is used to run 20 different tests. Informations about these cases are listed below.

# Chapter 6

# Test 1: nst

## 6.1 Numerical domain and equations to solve

$$
\begin{aligned}
\partial_t \mathbf{u} + (\nabla\times\mathbf{u})\times\mathbf{u} - \frac{1}{R_\mathrm{e}}\Delta\mathbf{u} + \nabla p &= \mathbf{f}, \\
\nabla\cdot\mathbf{u} &= 0, \\
\mathbf{u}_{|\Gamma} &= \mathbf{v}, \\
\mathbf{u}_{|t=0} &= \mathbf{u}_0.
\end{aligned}
$$

The data are $\mathbf{f}$, $\mathbf{v}$ and $\mathbf{u}_0$. We use $R_\mathrm{e} = 1$.

## 6.2 Analytical solution

$$
\begin{aligned}
u_r(r,\theta,z,t) &= \left(\left(r^2z^3 - 3r^3z^2\right)\cos(\theta) - \left(r^2z^3 + 3r^3z^2\right)\sin(\theta)\right)\cos(t), \\
u_\theta(r,\theta,z,t) &= 3\left(r^3z^2 - r^2z^3\right)(\cos(\theta)+\sin(\theta))\cos(t), \\
u_z(r,\theta,z,t) &= \left(3r^2z^3\cos(\theta) + 5r^2z^3\sin(\theta)\right), \\
p(r,\theta,z,t) &= rz\,(\cos(\theta)+\sin(\theta))\sin(t).
\end{aligned}
$$

## 6.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'Mesh_10_form.FEM'
===Number of processors in meridian section
2
===Number of processors in Fourier space
```

```
3
===Number of Fourier modes
3
===Select Fourier modes? (true/false)
.t.
===List of Fourier modes (if select_mode=.TRUE.)
0 1 2
===Problem type: (nst, mxw, mhd)
'nst'
===Restart on velocity (true/false)
.f.
===Time step and number of time iterations
.01d0, 100

============================================
              Mesh-NAVIER-STOKES
============================================
===Number of subdomains in Navier-Stokes mesh
1
===List of subdomains for Navier-Stokes mesh
1

============================================
              BCs-NAVIER-STOKES
============================================
===How many boundary pieces for Dirichlet BCs on uradial?
3
===List of boundary pieces for Dirichlet BCs on uradial
5 2 4
===How many boundary pieces for Dirichlet BCs on utheta?
3
===List of boundary pieces for Dirichlet BCs on utheta
5 2 4
===How many boundary pieces for Dirichlet BCs on uzaxis?
3
===List of boundary pieces for Dirichlet BCs on uzaxis
5 2 4
===How many boundary pieces for Dirichlet BCs on pressure?
0
===List of boundary pieces for Dirichlet BCs on pressure
0


============================================
              Dynamics-NAVIER-STOKES
```

```
============================================
===Reynolds number
1.d0
xx===Is there a precession term (true/false)?
xx.f.

============================================
              LES-NAVIER-STOKES
============================================
xx===Use LES? (true/false)
xx.f.
xx===Coefficients for LES
xx
xx
xx
xx

============================================
              Solver-velocity-NAVIER-STOKES
============================================
===Maximum number of iterations for velocity solver
100
===Relative tolerance for velocity solver
1.d-6
===Absolute tolerance for velocity solver
1.d-10
===Velocity solver verbose? (true/false)
.f.
===Solver type for velocity (FGMRES, CG, ...)
GMRES
===Preconditionner type for velocity solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-pressure-NAVIER-STOKES
============================================
===Maximum number of iterations for pressure solver
100
===Relative tolerance for pressure solver
1.d-6
===Absolute tolerance for pressure solver
1.d-10
===Pressure solver verbose? (true/false)
.f.
===Solver type for pressure (FGMRES, CG, ...)
```

```
GMRES
===Preconditionner type for pressure solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-mass-NAVIER-STOKES
============================================
===Maximum number of iterations for mass matrix solver
100
===Relative tolerance for mass matrix solver
1.d-6
===Absolute tolerance for mass matrix solver
1.d-10
===Mass matrix solver verbose? (true/false)
.f.
===Solver type for mass matrix (FGMRES, CG, ...)
CG
===Preconditionner type for mass matrix solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Verbose (diagnostics)
============================================
===Verbose timing? (true/false)
.t.
===Verbose divergence? (true/false)
.t.
===Verbose CFL? (true/false)
.t.

============================================
(Mesh_10_form.FEM)
===Reference results
1.7131800369932059E-005  L2 error on velocity
6.9806907549809004E-004  H1 error on velocity
3.3197801481577309E-003  L2 norm of divergence
4.6075938712700633E-004  L2 error on pressure
```

# Chapter 7

# Test 2: nst + perio

## 7.1 Numerical domain and equations to solve

$$
\begin{aligned}
\partial_t \mathbf{u} + (\nabla\times\mathbf{u})\times\mathbf{u} - \frac{1}{R_\mathrm{e}}\Delta\mathbf{u} + \nabla p &= \mathbf{f},\\
\nabla\cdot\mathbf{u} &= 0,\\
\mathbf{u}_{|\Gamma} &= \mathbf{v},\\
\mathbf{u}_{|t=0} &= \mathbf{u}_0,\\
\mathbf{u} \text{ periodic in the } z\text{-direction.}
\end{aligned}
$$

## 7.2 Analytical solution

$$
\begin{aligned}
u_r(r,\theta,z,t) &= -r^2\,(1-2\pi r\sin(2\pi z))\sin(\theta)\cos(t),\\
u_\theta(r,\theta,z,t) &= -3r^2\cos(\theta)\cos(t),\\
u_z(r,\theta,z,t) &= r^2\,(4\cos(2\pi z)+1)\sin(\theta)\cos(t),\\
p(r,\theta,z,t) &= r^2\cos(2\pi z)\cos(\theta)\cos(t).
\end{aligned}
$$

## 7.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'Mesh_10_form.FEM'
===Number of processors in meridian section
2
===Number of processors in Fourier space
```

```
3
===Number of Fourier modes
3
===Select Fourier modes? (true/false)
.t.
===List of Fourier modes (if select_mode=.TRUE.)
0 1 2
===Problem type: (nst, mxw, mhd)
'nst'
===Restart on velocity (true/false)
.f.
===Time step and number of time iterations
.01d0, 100

============================================
              Periodicity
============================================
===How many pieces of periodic boundary?
1
===Indices of periodic boundaries and corresponding vectors
4 2 .0 1.

============================================
              Mesh-NAVIER-STOKES
============================================
===Number of subdomains in Navier-Stokes mesh
1
===List of subdomains for Navier-Stokes mesh
1

============================================
              BCs-NAVIER-STOKES
============================================
===How many boundary pieces for Dirichlet BCs on uradial?
1
===List of boundary pieces for Dirichlet BCs on uradial
5
===How many boundary pieces for Dirichlet BCs on utheta?
1
===List of boundary pieces for Dirichlet BCs on utheta
5
===How many boundary pieces for Dirichlet BCs on uzaxis?
1
===List of boundary pieces for Dirichlet BCs on uzaxis
5
```

```
===How many boundary pieces for Dirichlet BCs on pressure?
0
===List of boundary pieces for Dirichlet BCs on pressure
0

============================================
              Dynamics-NAVIER-STOKES
============================================
===Reynolds number
1.d0
xx===Is there a precession term (true/false)?
xx.f.

============================================
              LES-NAVIER-STOKES
============================================
xx===Use LES? (true/false)
xx.f.
xx===Coefficients for LES
xx
xx
xx
xx

============================================
              Solver-velocity-NAVIER-STOKES
============================================
===Maximum number of iterations for velocity solver
100
===Relative tolerance for velocity solver
1.d-6
===Absolute tolerance for velocity solver
1.d-10
===Velocity solver verbose? (true/false)
.f.
===Solver type for velocity (FGMRES, CG, ...)
GMRES
===Preconditionner type for velocity solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-pressure-NAVIER-STOKES
============================================
===Maximum number of iterations for pressure solver
100
```

```
===Relative tolerance for pressure solver
1.d-6
===Absolute tolerance for pressure solver
1.d-10
===Pressure solver verbose? (true/false)
.f.
===Solver type for pressure (FGMRES, CG, ...)
GMRES
===Preconditionner type for pressure solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-mass-NAVIER-STOKES
============================================
===Maximum number of iterations for mass matrix solver
100
===Relative tolerance for mass matrix solver
1.d-6
===Absolute tolerance for mass matrix solver
1.d-10
===Mass matrix solver verbose? (true/false)
.f.
===Solver type for mass matrix (FGMRES, CG, ...)
CG
===Preconditionner type for mass matrix solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
     (Mesh_10_form.FEM)
===Reference results
7.8210055452531603E-005  L2 error on velocity
3.0488888603617285E-003  H1 error on velocity
2.1302874069012830E-002  L2 norm of divergence
2.1987096627546854E-003  L2 error on pressure
```

# Chapter 8

# Test 3: mxw (with vacuum).

## 8.1 Numerical domain and equations to solve

$$
\begin{aligned}
\partial_t (\mu \mathbf{H}) + \frac{1}{R_\mathrm{m}} \nabla\times \left( \frac{1}{\sigma} \nabla\times \mathbf{H} \right) - \nabla\times (\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_\mathrm{m}} \nabla\times \left( \frac{1}{\sigma} \mathbf{j} \right), \\
\nabla\cdot(\mu\mathbf{H}) = 0, \\
+BC + IC. + \phi
\end{aligned}
$$

## 8.2 Analytical solution

$$
\begin{aligned}
H_r(r,\theta,z,t) &= \sum_{m=1}^{3} \frac{1}{m^3} \big(\alpha_m z r^{m-1} m \cos(m\theta) + \beta_m z r^{m-1} m \sin(m\theta)\big) \cos(t), \\
H_\theta(r,\theta,z,t) &= \sum_{m=1}^{3} \frac{1}{m^3} \big(\beta_m z r^{m-1} m \cos(m\theta) - \alpha_m z r^{m-1} m \sin(m\theta)\big) \cos(t), \\
H_z(r,\theta,z,t) &= \sum_{m=1}^{3} \frac{1}{m^3} \left(\alpha_m r^{m} \cos(m\theta) + \beta_m r^{m} \sin(m\theta)\right) \cos(t), \\
\phi(r,\theta,z,t) &= \sum_{m=1}^{3} \frac{1}{m^3} \left(\alpha_m z r^{m} \cos(m\theta) + \beta_m z r^{m} \sin(m\theta)\right) \cos(t).
\end{aligned}
$$

## 8.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'Mesh_10_form.FEM'
```

```
===Number of processors in meridian section
2
===Number of processors in Fourier space
3
===Number of Fourier modes
3
===Select Fourier modes? (true/false)
.t.
===List of Fourier modes (if select_mode=.TRUE.)
1 2 3
===Problem type: (nst, mxw, mhd)
'mxw'
===Restart on velocity (true/false)
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
1.d-2, 100

============================================
              H-MAXWELL
============================================
===Solve Maxwell with H (true) or B (false)?
.t.
===Number of subdomains in magnetic field (H) mesh
1
===List of subdomains for magnetic field (H) mesh
1
===Number of interfaces in H mesh
0
===List of interfaces in H mesh
xx
===Number of Dirichlet sides for Hxn
2
===List of Dirichlet sides for Hxn
2 4
===Permeability in the conductive part (1:nb_dom_H)
1.d0
===Conductivity in the conductive part (1:nb_dom_H)
1.d0
===Type of finite element for magnetic field
2
===Magnetic Reynolds number
1.d0
===Stabilization coefficient (divergence)
```

```
1.d0
===Stabilization coefficient for Dirichlet H and/or interface H/H
1.d0

============================================
              Phi-MAXWELL
============================================
===Number of subdomains in magnetic potential (phi) mesh
1
===List of subdomains for magnetic potential (phi) mesh
2
===How many boundary pieces for Dirichlet BCs on phi?
3
===List of boundary pieces for Dirichlet BCs on phi
2 3 4
===Number of interfaces between H and phi
1
===List of interfaces between H and phi
5
===Permeability in vacuum
1.d0
===Type of finite element for scalar potential
2
===Stabilization coefficient (interface H/phi)
1.d0

============================================
              Solver-MAXWELL
============================================
===Maximum number of iterations for Maxwell solver
100
===Relative tolerance for Maxwell solver
1.d-6
===Absolute tolerance for Maxwell solver
1.d-10
===Maxwell solver verbose? (true/false)
.f.
===Solver type for Maxwell (FGMRES, CG, ...)
GMRES
===Preconditionner type for Maxwell solver (HYPRE, JACOBI, MUMPS...)
MUMPS


============================================
Mesh_10_form.FEM, P2P2
```

```
===Reference results
1.6738636391940218E-005 L2 norm on Hn
4.3868117926651355E-005 L2 norm of Curl(Hn)
2.4447250060399849E-004 L2 norm of Div(mu Hn)
1.3034153680010757E-005 H1 norm on phin
```

# Chapter 9

# Test 4: mxw + perio (with vacuum).

## 9.1 Numerical domain and equations to solve

$$\partial_t (\mu \mathbf{H}) + \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma}\nabla\times\mathbf{H}\right) - \nabla\times(\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\mathbf{j}\right),$$
$$\nabla\cdot(\mu\mathbf{H}) = 0,$$
$$+BC + IC + perio + \phi$$

## 9.2 Analytical solution

$$H_r(r,\theta,z,t) = \cos(t)\cos(\theta)\cos(2\pi z)\left(\frac{r}{r_0^2} - 2\pi\left(\frac{r}{r_0}\right)^2\left(A + B\frac{r}{r_0}\right)\right),$$
$$H_\theta(r,\theta,z,t) = \cos(t)\sin(\theta)\cos(2\pi z)\left(2\pi\left(\frac{r}{r_0}\right)^2 C - 2\frac{r}{r_0^2}\right),$$
$$H_z(r,\theta,z,t) = \cos(t)\cos(\theta)\sin(2\pi z)\frac{r}{r_0^2}\left(3A + 4B\frac{r}{r_0} - C\right),$$
$$\phi(r,\theta,z,t) = \cos(t)\cos(\theta)\cos(2\pi z)K_1(2\pi r),$$

$K_1$: Bessel function. $A, B, C$ constants.

## 9.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'Mesh_10_form.FEM'
```

```
===Number of processors in meridian section
2
===Number of processors in Fourier space
1
===Number of Fourier modes
1
===Select Fourier modes? (true/false)
.t.
===List of Fourier modes (if select_mode=.TRUE.)
1
===Problem type: (nst, mxw, mhd)
'mxw'
===Restart on velocity (true/false)
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
.01d0, 100

============================================
              Periodicity
============================================
===How many pieces of periodic boundary?
1
===Indices of periodic boundaries and corresponding vectors
4 2 .0 1.

============================================
              H-MAXWELL
============================================
===Number of subdomains in magnetic field (H) mesh
1
===List of subdomains for magnetic field (H) mesh
1
===Number of interfaces in H mesh
0
===List of interfaces in H mesh
xx
===Number of Dirichlet sides for Hxn
0
===List of Dirichlet sides for Hxn
xx
===Permeability in the conductive part (1:nb_dom_H)
1.d0
===Conductivity in the conductive part (1:nb_dom_H)
```

```
1.d0
===Type of finite element for magnetic field
1
===Magnetic Reynolds number
1.d0
===Stabilization coefficient (divergence)
1.d0
===Stabilization coefficient for Dirichlet H and/or interface H/H
1.d0

=============================================
              Phi-MAXWELL
=============================================
===Number of subdomains in magnetic potential (phi) mesh
1
===List of subdomains for magnetic potential (phi) mesh
2
===How many boundary pieces for Dirichlet BCs on phi?
1
===List of boundary pieces for Dirichlet BCs on phi
3
===Number of interfaces between H and phi
1
===List of interfaces between H and phi
5
===Permeability in vacuum
1.d0
===Type of finite element for scalar potential
2
===Stabilization coefficient (interface H/phi)
1.d0

=============================================
              Solver-MAXWELL
=============================================
===Maximum number of iterations for Maxwell solver
100
===Relative tolerance for Maxwell solver
1.d-6
===Absolute tolerance for Maxwell solver
1.d-10
===Maxwell solver verbose? (true/false)
.f.
===Solver type for Maxwell (FGMRES, CG, ...)
GMRES
```

```
===Preconditionner type for Maxwell solver (HYPRE, JACOBI, MUMPS...)
MUMPS


============================================
Mesh_10_form.FEM, P2P2
===Reference results
0.1168311779995666 L2 norm on Hn
0.1584136476033097 L2 norm of Curl(Hn)
0.2731997096907101 L2 norm of Div(mu Hn)
0.2367241234384680 H1 norm on phin
```

# Chapter 10

# Test 5: mxw (with vacuum).

## 10.1 Numerical domain and equations to solve

$$\partial_t (\mu \mathbf{H}) + \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma}\nabla\times\mathbf{H}\right) - \nabla\times(\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\mathbf{j}\right),$$
$$\nabla\cdot(\mu\mathbf{H}) = 0,$$
$$+BC + IC. + \phi$$

## 10.2 Analytical solution

$$H_r(r,\theta,z,t) = \sum_{m=1}^{3} \frac{1}{m^3}\left(\alpha_m z r^{m-1} m\cos(m\theta) + \beta_m z r^{m-1} m \sin(m\theta)\right)\cos(t),$$
$$H_\theta(r,\theta,z,t) = \sum_{m=1}^{3} \frac{1}{m^3}\left(\beta_m z r^{m-1} m\cos(m\theta) - \alpha_m z r^{m-1} m \sin(m\theta)\right)\cos(t),$$
$$H_z(r,\theta,z,t) = \sum_{m=1}^{3} \frac{1}{m^3}\left(\alpha_m r^{m}\cos(m\theta) + \beta_m r^{m} \sin(m\theta)\right)\cos(t),$$
$$\phi(r,\theta,z,t) = \sum_{m=1}^{3} \frac{1}{m^3}\left(\alpha_m z r^{m}\cos(m\theta) + \beta_m z r^{m} \sin(m\theta)\right)\cos(t).$$

## 10.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'CYL10_TCM_PERIO_form.FEM'
```

```
===Number of processors in meridian section
2
===Number of processors in Fourier space
2
===Number of Fourier modes
2
===Select Fourier modes? (true/false)
.t.
===List of Fourier modes (if select_mode=.TRUE.)
1 2
===Problem type: (nst, mxw, mhd)
'mxw'
===Restart on velocity (true/false)
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
.01d0, 100

============================================
              H-MAXWELL
============================================
===Number of subdomains in magnetic field (H) mesh
2
===List of subdomains for magnetic field (H) mesh
1 2
===Number of interfaces in H mesh
1
===List of interfaces in H mesh
5
===Number of Dirichlet sides for Hxn
2
===List of Dirichlet sides for Hxn
2 4
===Permeability in the conductive part (1:nb_dom_H)
1.d0 1.d0
===Conductivity in the conductive part (1:nb_dom_H)
1.d0 1.d0
===Type of finite element for magnetic field
1
===Magnetic Reynolds number
1.d0
===Stabilization coefficient (divergence)
1.d0
===Stabilization coefficient for Dirichlet H and/or interface H/H
```

```
1.d0

============================================
              Phi-MAXWELL
============================================
===Number of subdomains in magnetic potential (phi) mesh
1
===List of subdomains for magnetic potential (phi) mesh
3
===How many boundary pieces for Dirichlet BCs on phi?
3
===List of boundary pieces for Dirichlet BCs on phi
2 3 4
===Number of interfaces between H and phi
1
===List of interfaces between H and phi
6
===Permeability in vacuum
1.d0
===Type of finite element for scalar potential
2
===Stabilization coefficient (interface H/phi)
1.d0

============================================
              Solver-MAXWELL
============================================
===Maximum number of iterations for Maxwell solver
100
===Relative tolerance for Maxwell solver
1.d-6
===Absolute tolerance for Maxwell solver
1.d-10
===Maxwell solver verbose? (true/false)
.f.
===Solver type for Maxwell (FGMRES, CG, ...)
GMRES
===Preconditionner type for Maxwell solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Verbose (diagnostics)
============================================
===Verbose timing? (true/false)
.t.
```

```
===Verbose divergence? (true/false)
.t.

============================================
CYL10_TCM_PERIO_form.FEM, P1P2
===Reference results
1.8939829880987270E-004 L2 norm on Hn
5.9103702421915737E-004 L2 norm of Curl(Hn)
3.2277143425528082E-003 L2 norm of Div(mu Hn)
5.2986805694968567E-006 H1 norm on phin
```

# Chapter 11

# Test 6: mxw (with vacuum).

## 11.1 Numerical domain and equations to solve

$$\partial_t (\mu \mathbf{H}) + \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma}\nabla\times\mathbf{H}\right) - \nabla\times(\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\mathbf{j}\right),$$
$$\nabla\cdot(\mu\mathbf{H}) = 0,$$
$$+BC + IC. + \phi$$

## 11.2 Analytical solution

## 11.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'sphere_0.05_form.FEM'
===Number of processors in meridian section
2
===Number of processors in Fourier space
1
===Number of Fourier modes
1
===Select Fourier modes? (true/false)
.f.
xx===List of Fourier modes (if select_mode=.TRUE.)
xx
===Problem type: (nst, mxw, mhd)
'mxw'
===Restart on velocity (true/false)
```

```
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
1.d10, 1
===Frequence to create plots
1

============================================
              H-MAXWELL
============================================
===Solve Maxwell with H (true) or B (false)?
.t.
===Number of subdomains in magnetic field (H) mesh
2
===List of subdomains for magnetic field (H) mesh
1 2
===How many boundary pieces for Dirichlet BCs on phi?
1
===List of boundary pieces for Dirichlet BCs on phi
4
===Number of interfaces in H mesh
1
===List of interfaces in H mesh
2
===Number of Dirichlet sides for Hxn
0
xx===List of Dirichlet sides for Hxn
xx
===Permeability in the conductive part (1:nb_dom_H)
1.d0 2.d0
===Conductivity in the conductive part (1:nb_dom_H)
1.d0 1.d0
===Type of finite element for magnetic field
1
===Magnetic Reynolds number
1.d0
===Stabilization coefficient (divergence)
1.d0
===Stabilization coefficient for Dirichlet H and/or interface H/H
1.d0

============================================
              Phi-MAXWELL
============================================
```

```
===Number of subdomains in magnetic potential (phi) mesh
1
===List of subdomains for magnetic potential (phi) mesh
3
===Number of interfaces between H and phi
1
===List of interfaces between H and phi
3
===Permeability in vacuum
1.d0
===Type of finite element for scalar potential
2
===Stabilization coefficient (interface H/phi)
1.d0

============================================
              Solver-MAXWELL
============================================
===Maximum number of iterations for Maxwell solver
100
===Relative tolerance for Maxwell solver
1.d-6
===Absolute tolerance for Maxwell solver
1.d-10
===Maxwell solver verbose? (true/false)
.f.
===Solver type for Maxwell (FGMRES, CG, ...)
GMRES
===Preconditionner type for Maxwell solver (HYPRE, JACOBI, MUMPS...)
MUMPS


============================================
sphere_0.05_form.FEM, P1P2
===Reference results
1.0944795959400701E-002 L2 norm on Hn
1.4070271111780259E-002 L2 norm of Curl(Hn)
9.3273252124008943E-002 L2 norm of Div(mu Hn)
3.7538161006761745E-004 H1 norm on phin
```

# Chapter 12

# Test 7: mxw (with vacuum).

## 12.1 Numerical domain and equations to solve

$$\partial_t (\mu \mathbf{H}) + \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma}\nabla\times\mathbf{H}\right) - \nabla\times(\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_\mathrm{m}} \nabla\times\left(\frac{1}{\sigma}\mathbf{j}\right),$$
$$\nabla\cdot(\mu\mathbf{H}) = 0,$$
$$+BC + IC. + \phi$$

## 12.2 Analytical solution

## 12.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'sphere_0.05_form.FEM'
===Number of processors in meridian section
1
===Number of processors in Fourier space
2
===Number of Fourier modes
4
===Select Fourier modes? (true/false)
.f.
xx===List of Fourier modes (if select_mode=.TRUE.)
xx
===Problem type: (nst, mxw, mhd)
'mxw'
===Restart on velocity (true/false)
```

```
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
1.d10, 1

============================================
              H-MAXWELL
============================================
===Solve Maxwell with H (true) or B (false)?
.t.
===Number of subdomains in magnetic field (H) mesh
1
===List of subdomains for magnetic field (H) mesh
2
===Number of interfaces in H mesh
0
xx===List of interfaces in H mesh
xx
===Number of Dirichlet sides for Hxn
0
xx===List of Dirichlet sides for Hxn
xx
===Permeability in the conductive part (1:nb_dom_H)
2.d0
===Conductivity in the conductive part (1:nb_dom_H)
1.d0
===Type of finite element for magnetic field
2
===Magnetic Reynolds number
1.d0
===Stabilization coefficient (divergence)
1.d0
===Stabilization coefficient for Dirichlet H and/or interface H/H
1.d0

============================================
              Phi-MAXWELL
============================================
===Number of subdomains in magnetic potential (phi) mesh
2
===List of subdomains for magnetic potential (phi) mesh
1 3
===How many boundary pieces for Dirichlet BCs on phi?
1
```

```
===List of boundary pieces for Dirichlet BCs on phi
4
===Number of interfaces between H and phi
2
===List of interfaces between H and phi
2 3
===Permeability in vacuum
1.d0
===Type of finite element for scalar potential
2
===Stabilization coefficient (interface H/phi)
1.d0

============================================
              Solver-MAXWELL
============================================
===Maximum number of iterations for Maxwell solver
100
===Relative tolerance for Maxwell solver
1.d-6
===Absolute tolerance for Maxwell solver
1.d-10
===Maxwell solver verbose? (true/false)
.f.
===Solver type for Maxwell (FGMRES, CG, ...)
GMRES
===Preconditionner type for Maxwell solver (HYPRE, JACOBI, MUMPS...)
MUMPS


============================================
sphere_0.05_form.FEM, P2P2
===Reference results
3.53294251486503371E-004 L2 norm on Hn
1.37965492616509418E-003 L2 norm of Curl(Hn)
7.87576179071500931E-003 L2 norm of Div(mu Hn)
1.36116445279202273E-005 H1 norm on phin
```

# Chapter 13

# Test 8: nst + phase.

## 13.1 Numerical domain and equations to solve

$$\begin{aligned}
\partial_t \mathbf{u} + (\nabla\times\mathbf{u})\times\mathbf{u} - \frac{1}{R_\mathrm{e}}\Delta\mathbf{u} + \nabla p &= \mathbf{f} + gT\mathbf{e}_z,\\
\nabla\cdot\mathbf{u} &= 0,\\
\partial_t T + \nabla\cdot(\mathbf{u}T) - \kappa\Delta T &= f_T\\
\mathbf{u}_{|\Gamma} &= \mathbf{v}, \quad T_{|\Gamma} = T_a,\\
\mathbf{u}_{|t=0} &= \mathbf{u}_0, \quad T_{|t=0} = T_0.
\end{aligned}$$

## 13.2 Analytical solution

$$u_r = r^3\cos(2\pi z)\sin(t) \tag{13.2.1}$$

$$u_\theta = r^2 z\sin(t) \tag{13.2.2}$$

$$u_z = -\frac{4r^2}{2\pi}\sin(2\pi z)\sin(t) \tag{13.2.3}$$

$$p = 0 \tag{13.2.4}$$

$$T = (r^2 z + r^2 z^2(\cos(\theta) + 2\sin(2\theta))\cos(t) \tag{13.2.5}$$

## 13.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'Mesh_10_form.FEM'
===Number of processors in meridian section
```

```
1
===Number of processors in Fourier space
3
===Number of Fourier modes
3
===Select Fourier modes? (true/false)
.t.
===List of Fourier modes (if select_mode=.TRUE.)
0 1 2
===Problem type: (nst, mxw, mhd)
'nst'
===Restart on velocity (true/false)
.f.
===Time step and number of time iterations
.02d0, 50

============================================
              Mesh-NAVIER-STOKES
============================================
===Number of subdomains in Navier-Stokes mesh
1
===List of subdomains for Navier-Stokes mesh
1

============================================
              BCs-NAVIER-STOKES
============================================
===How many boundary pieces for Dirichlet BCs on uradial?
3
===List of boundary pieces for Dirichlet BCs on uradial
5 2 4
===How many boundary pieces for Dirichlet BCs on utheta?
3
===List of boundary pieces for Dirichlet BCs on utheta
5 2 4
===How many boundary pieces for Dirichlet BCs on uzaxis?
3
===List of boundary pieces for Dirichlet BCs on uzaxis
5 2 4
===How many boundary pieces for Dirichlet BCs on pressure?
0
===List of boundary pieces for Dirichlet BCs on pressure
0
```

```
============================================
              Dynamics-NAVIER-STOKES
============================================
===Reynolds number
1.d0
xx===Is there a precession term (true/false)?
xx.f.

============================================
              LES-NAVIER-STOKES
============================================
xx===Use LES? (true/false)
xx.f.
xx===Coefficients for LES
xx
xx
xx
xx

============================================
              Solver-velocity-NAVIER-STOKES
============================================
===Maximum number of iterations for velocity solver
100
===Relative tolerance for velocity solver
1.d-6
===Absolute tolerance for velocity solver
1.d-10
===Velocity solver verbose? (true/false)
.f.
===Solver type for velocity (FGMRES, CG, ...)
GMRES
===Preconditionner type for velocity solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-pressure-NAVIER-STOKES
============================================
===Maximum number of iterations for pressure solver
100
===Relative tolerance for pressure solver
1.d-6
===Absolute tolerance for pressure solver
1.d-10
===Pressure solver verbose? (true/false)
```

```
.f.
===Solver type for pressure (FGMRES, CG, ...)
GMRES
===Preconditionner type for pressure solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-mass-NAVIER-STOKES
============================================
===Maximum number of iterations for mass matrix solver
100
===Relative tolerance for mass matrix solver
1.d-6
===Absolute tolerance for mass matrix solver
1.d-10
===Mass matrix solver verbose? (true/false)
.f.
===Solver type for mass matrix (FGMRES, CG, ...)
CG
===Preconditionner type for mass matrix solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Phase
============================================
===Is there a temperature field?
.t.
===Non-dimensional gravity coefficient
10.d0
===Diffusivity coefficient for temperature
1.d-1
===How many boundary pieces for Dirichlet BCs on temperature?
3
===List of boundary pieces for Dirichlet BCs on temperature
5 2 4

============================================
              Solver-Phase
============================================
===Maximum number of iterations for temperature solver
100
===Relative tolerance for temperature solver
1.d-6
===Absolute tolerance for temperature solver
1.d-10
```

```
===Temperature solver verbose? (true/false)
.f.
===Solver type for temperature (FGMRES, CG, ...)
GMRES
===Preconditionner type for temperature solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Verbose (diagnostics)
============================================
===Verbose timing? (true/false)
.t.
===Verbose divergence? (true/false)
.t.
===Verbose CFL? (true/false)
.t.

============================================
(Mesh_10_form.FEM)
===Reference results
2.4124425032285686E-006  L2 error on temperature
1.3518523432452288E-004  H1 error on temperature
5.8036107239867471E-003  L2 norm of divergence
1.4649860286322502E-003  L2 error on pressure
```

# Chapter 14

# Test 9: nst + phase + perio.

## 14.1 Numerical domain and equations to solve

## 14.2 Analytical solution

## 14.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'Mesh_10_form.FEM'
===Do we read metis partition? (true/false)
.t.
===Number of processors in meridian section
2
===Number of processors in Fourier space
3
===Number of Fourier modes
3
===Select Fourier modes? (true/false)
.t.
===List of Fourier modes (if select_mode=.TRUE.)
0 1 2
===Problem type: (nst, mxw, mhd)
'nst'
===Restart on velocity (true/false)
.f.
===Time step and number of time iterations
.02d0, 50
```

```
============================================
              Periodicity
============================================
===How many pieces of periodic boundary?
1
===Indices of periodic boundaries and corresponding vectors
4 2 .0 1.

============================================
              Mesh-NAVIER-STOKES
============================================
===Number of subdomains in Navier-Stokes mesh
1
===List of subdomains for Navier-Stokes mesh
1

============================================
              BCs-NAVIER-STOKES
============================================
===How many boundary pieces for Dirichlet BCs on uradial?
1
===List of boundary pieces for Dirichlet BCs on uradial
5
===How many boundary pieces for Dirichlet BCs on utheta?
1
===List of boundary pieces for Dirichlet BCs on utheta
5
===How many boundary pieces for Dirichlet BCs on uzaxis?
1
===List of boundary pieces for Dirichlet BCs on uzaxis
5
===How many boundary pieces for Dirichlet BCs on pressure?
0
===List of boundary pieces for Dirichlet BCs on pressure
0


============================================
              Dynamics-NAVIER-STOKES
============================================
===Reynolds number
1.d0

============================================
              Solver-velocity-NAVIER-STOKES
```

```
============================================
===Maximum number of iterations for velocity solver
100
===Relative tolerance for velocity solver
1.d-6
===Absolute tolerance for velocity solver
1.d-10
===Velocity solver verbose? (true/false)
.f.
===Solver type for velocity (FGMRES, CG, ...)
GMRES
===Preconditionner type for velocity solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-pressure-NAVIER-STOKES
============================================
===Maximum number of iterations for pressure solver
100
===Relative tolerance for pressure solver
1.d-6
===Absolute tolerance for pressure solver
1.d-10
===Pressure solver verbose? (true/false)
.f.
===Solver type for pressure (FGMRES, CG, ...)
GMRES
===Preconditionner type for pressure solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-mass-NAVIER-STOKES
============================================
===Maximum number of iterations for mass matrix solver
100
===Relative tolerance for mass matrix solver
1.d-6
===Absolute tolerance for mass matrix solver
1.d-10
===Mass matrix solver verbose? (true/false)
.f.
===Solver type for mass matrix (FGMRES, CG, ...)
CG
===Preconditionner type for mass matrix solver (HYPRE, JACOBI, MUMPS...)
MUMPS
```

```
============================================
              Phase
============================================
===Is there a temperature field?
.t.
===Non-dimensional gravity coefficient
10.d0
===Diffusivity coefficient for temperature
1.d-1
===How many boundary pieces for Dirichlet BCs on temperature?
1
===List of boundary pieces for Dirichlet BCs on temperature
5

============================================
              Solver-Phase
============================================
===Maximum number of iterations for temperature solver
100
===Relative tolerance for temperature solver
1.d-6
===Absolute tolerance for temperature solver
1.d-10
===Temperature solver verbose? (true/false)
.f.
===Solver type for temperature (FGMRES, CG, ...)
GMRES
===Preconditionner type for temperature solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Verbose (diagnostics)
============================================
===Verbose timing? (true/false)
.t.
===Verbose divergence? (true/false)
.t.
===Verbose CFL? (true/false)
.t.

============================================
(Mesh_10_form.FEM)
===Reference results
1.8592485266281552E-005  L2 error on temperature
```

```
1.2120578337607992E-003  H1 error on temperature
5.8107738968646312E-003  L2 norm of divergence
1.3878902797707896E-003  L2 error on pressure
```

# Chapter 15

# Test 10: mxw (without vacuum).

## 15.1 Numerical domain and equations to solve

$$\partial_t (\mu \mathbf{H}) + \frac{1}{R_{\mathrm{m}}} \nabla\times \left( \frac{1}{\sigma} \nabla\times \mathbf{H} \right) - \nabla\times (\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_{\mathrm{m}}} \nabla\times \left( \frac{1}{\sigma} \mathbf{j} \right),$$
$$\nabla\cdot(\mu \mathbf{H}) = 0,$$
$$+BC + IC.$$

## 15.2 Analytical solution

$$H_r(r,\theta,z,t) = \sum_{m=1}^{3} \frac{1}{m^3} \left( \alpha_m z r^{m-1} m \cos(m\theta) + \beta_m z r^{m-1} m \sin(m\theta) \right) \cos(t),$$
$$H_\theta(r,\theta,z,t) = \sum_{m=1}^{3} \frac{1}{m^3} \left( \beta_m z r^{m-1} m \cos(m\theta) - \alpha_m z r^{m-1} m \sin(m\theta) \right) \cos(t),$$
$$H_z(r,\theta,z,t) = \sum_{m=1}^{3} \frac{1}{m^3} \left( \alpha_m r^{m} \cos(m\theta) + \beta_m r^{m} \sin(m\theta) \right) \cos(t),$$

## 15.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'Mesh_10_form.FEM'
===Number of processors in meridian section
```

```
2
===Number of processors in Fourier space
3
===Number of Fourier modes
3
===Select Fourier modes? (true/false)
.t.
===List of Fourier modes (if select_mode=.TRUE.)
1 2 3
===Problem type: (nst, mxw, mhd)
'mxw'
===Restart on velocity (true/false)
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
.01d0, 100

============================================
              H-MAXWELL
============================================
===Number of subdomains in magnetic field (H) mesh
1
===List of subdomains for magnetic field (H) mesh
1
===Number of interfaces in H mesh
0
===List of interfaces in H mesh
xx
===Number of Dirichlet sides for Hxn
3
===List of Dirichlet sides for Hxn
2 4 5
===Permeability in the conductive part (1:nb_dom_H)
1.d0
===Conductivity in the conductive part (1:nb_dom_H)
1.d0
===Type of finite element for magnetic field
2
===Magnetic Reynolds number
1.d0
===Stabilization coefficient (divergence)
1.d0
===Stabilization coefficient for Dirichlet H and/or interface H/H
1.d0
```

```
============================================
              Phi-MAXWELL
============================================
===Number of subdomains in magnetic potential (phi) mesh
0

============================================
              Solver-MAXWELL
============================================
===Maximum number of iterations for Maxwell solver
100
===Relative tolerance for Maxwell solver
1.d-6
===Absolute tolerance for Maxwell solver
1.d-10
===Maxwell solver verbose? (true/false)
.f.
===Solver type for Maxwell (FGMRES, CG, ...)
GMRES
===Preconditionner type for Maxwell solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Verbose-MAXWELL
============================================
===Verbose divergence? (true/false)
.t.

============================================
Mesh_10_form.FEM, P2P2
===Reference results
3.3416755313793619E-006 L2 norm on Hn
3.5052220977933145E-005 L2 norm of Curl(Hn)
1.0497038966528700E-004 L2 norm of Div(mu Hn)
1.d0                    No scalar potential
```

# Chapter 16

# Test 11: mhd + temperature (without vacuum).

## 16.1 Numerical domain and equations to solve

## 16.2 Analytical solution

## 16.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'Mesh_BENCH1_20.FEM'
===Number of processors in meridian section
2
===Number of processors in Fourier space
3
===Number of Fourier modes
3
===Select Fourier modes? (true/false)
.t.
===List of Fourier modes (if select_mode=.TRUE.)
0 4 8
===Problem type: (nst, mxw, mhd)
'mhd'
===Restart on velocity (true/false)
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
```

```
2d-2, 20

============================================
              Mesh-NAVIER-STOKES
============================================
===Number of subdomains in Navier-Stokes mesh
1
===List of subdomains for Navier-Stokes mesh
1

============================================
              BCs-NAVIER-STOKES
============================================
===How many boundary pieces for Dirichlet BCs on uradial?
2
===List of boundary pieces for Dirichlet BCs on uradial
2 4
===How many boundary pieces for Dirichlet BCs on utheta?
2
===List of boundary pieces for Dirichlet BCs on utheta
2 4
===How many boundary pieces for Dirichlet BCs on uzaxis?
2
===List of boundary pieces for Dirichlet BCs on uzaxis
2 4
===How many boundary pieces for Dirichlet BCs on pressure?
0
===List of boundary pieces for Dirichlet BCs on pressure
0


============================================
              Dynamics-NAVIER-STOKES
============================================
===Reynolds number
1000d0
===Is there a precession term (true/false)?
.t.
===Precession rate
1.d0
===Precession angle over pi
0.d0

============================================
              Solver-velocity-NAVIER-STOKES
```

```
============================================
===Maximum number of iterations for velocity solver
100
===Relative tolerance for velocity solver
1.d-6
===Absolute tolerance for velocity solver
1.d-10
===Velocity solver verbose? (true/false)
.f.
===Solver type for velocity (FGMRES, CG, ...)
GMRES
===Preconditionner type for velocity solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-pressure-NAVIER-STOKES
============================================
===Maximum number of iterations for pressure solver
100
===Relative tolerance for pressure solver
1.d-6
===Absolute tolerance for pressure solver
1.d-10
===Pressure solver verbose? (true/false)
.f.
===Solver type for pressure (FGMRES, CG, ...)
GMRES
===Preconditionner type for pressure solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-mass-NAVIER-STOKES
============================================
===Maximum number of iterations for mass matrix solver
100
===Relative tolerance for mass matrix solver
1.d-6
===Absolute tolerance for mass matrix solver
1.d-10
===Mass matrix solver verbose? (true/false)
.f.
===Solver type for mass matrix (FGMRES, CG, ...)
CG
===Preconditionner type for mass matrix solver (HYPRE, JACOBI, MUMPS...)
MUMPS
```

```
============================================
              Phase
============================================
===Is there a temperature field?
.t.
===Non-dimensional gravity coefficient
0.065
===Diffusivity coefficient for temperature
1.d-3
===How many boundary pieces for Dirichlet BCs on temperature?
2
===List of boundary pieces for Dirichlet BCs on temperature
2 4

============================================
              Solver-Phase
============================================
===Maximum number of iterations for temperature solver
100
===Relative tolerance for temperature solver
1.d-6
===Absolute tolerance for temperature solver
1.d-10
===Temperature solver verbose? (true/false)
.f.
===Solver type for temperature (FGMRES, CG, ...)
GMRES
===Preconditionner type for temperature solver (HYPRE, JACOBI, MUMPS...)
MUMPS


============================================
              H-MAXWELL
============================================
===Number of subdomains in magnetic field (H) mesh
1
===List of subdomains for magnetic field (H) mesh
1
===Number of interfaces in H mesh
0
xx===List of interfaces in H mesh
xx
===Number of Dirichlet sides for Hxn
2
```

```
===List of Dirichlet sides for Hxn
2 4
===Permeability in the conductive part (1:nb_dom_H)
1.d0
===Conductivity in the conductive part (1:nb_dom_H)
1.d0
===Type of finite element for magnetic field
2
===Magnetic Reynolds number
5000d0
===Stabilization coefficient (divergence)
1.d0
===Stabilization coefficient for Dirichlet H and/or interface H/H
100.d0

============================================
              Phi-MAXWELL
============================================
===Number of subdomains in magnetic potential (phi) mesh
0

============================================
              Solver-MAXWELL
============================================
===Maximum number of iterations for Maxwell solver
100
===Relative tolerance for Maxwell solver
1.d-6
===Absolute tolerance for Maxwell solver
1.d-10
===Maxwell solver verbose? (true/false)
.f.
===Solver type for Maxwell (FGMRES, CG, ...)
GMRES
===Preconditionner type for Maxwell solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Verbose-MAXWELL
============================================
===Verbose divergence? (true/false)
.t.

============================================
Mesh_BENCH1_20.FEM, P2
```

```
===Reference results
0.1454813724834997      H1 norm of velocity
0.1602088337636311      L2 norm of magnetic field
1.4789649733694142E-002 L2 norm of pressure
1.105345331385413       L2 norm of temperature
```

# Chapter 17

# Test 12: mxw Dirichlet/Neumann (without vacuum).

## 17.1 Numerical domain and equations to solve

$$\begin{aligned} \partial_t (\mu \mathbf{H}) + \frac{1}{R_\mathrm{m}} \nabla\times \left( \frac{1}{\sigma} \nabla\times \mathbf{H} \right) - \nabla\times (\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_\mathrm{m}} \nabla\times \left( \frac{1}{\sigma} \mathbf{j} \right), \\ \nabla\cdot(\mu\mathbf{H}) = 0, \\ +BC + IC. \end{aligned}$$

## 17.2 Analytical solution

## 17.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'Mesh_10_form.FEM'
===Number of processors in meridian section
2
===Number of processors in Fourier space
2
===Number of Fourier modes
2
===Select Fourier modes? (true/false)
.t.
===List of Fourier modes (if select_mode=.TRUE.)
0 1
```

```
===Problem type: (nst, mxw, mhd)
'mxw'
===Restart on velocity (true/false)
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
1.d0, 1
===Frequence to create plots
100

============================================
              H-MAXWELL
============================================
===Number of subdomains in magnetic field (H) mesh
1
===List of subdomains for magnetic field (H) mesh
1
===Number of interfaces in H mesh
0
xx===List of interfaces in H mesh
xx
===Number of Dirichlet sides for Hxn
2
===List of Dirichlet sides for Hxn
2 4
===Permeability in the conductive part (1:nb_dom_H)
1.d0
===Conductivity in the conductive part (1:nb_dom_H)
1.d0
===Type of finite element for magnetic field
1
===Magnetic Reynolds number
1.d0
===Stabilization coefficient (divergence)
1.d0
===Stabilization coefficient for Dirichlet H and/or interface H/H
1.d0

============================================
              Phi-MAXWELL
============================================


============================================
```

```
              Solver-MAXWELL
============================================
===Maximum number of iterations for Maxwell solver
100
===Relative tolerance for Maxwell solver
1.d-6
===Absolute tolerance for Maxwell solver
1.d-10
===Maxwell solver verbose? (true/false)
.f.
===Solver type for Maxwell (FGMRES, CG, ...)
GMRES
===Preconditionner type for Maxwell solver (HYPRE, JACOBI, MUMPS...)
MUMPS
HYPRE

============================================
Mesh_10_form.FEM, P1
===Reference results
4.7955119283070420E-003 L2 norm of error on Hn
1.1250512979221913E-002 L2 norm of error on Curl(Hn)
4.7920805854326572E-002 L2 norm of error on Div(mu Hn)
1.                      Dummy ref
```

# Chapter 18

# Test 13: mxw Dirichlet/Neumann + perio (without vacuum).

## 18.1 Numerical domain and equations to solve

$$
\begin{aligned}
\partial_t (\mu \mathbf{H}) + \frac{1}{R_\mathrm{m}} \nabla\times \left( \frac{1}{\sigma} \nabla\times \mathbf{H} \right) - \nabla\times (\mathbf{u}\times\mu\mathbf{H}) &= \frac{1}{R_\mathrm{m}} \nabla\times \left( \frac{1}{\sigma} \mathbf{j} \right), \\
\nabla\cdot(\mu\mathbf{H}) &= 0, \\
&+BC + IC.
\end{aligned}
$$

## 18.2 Analytical solution

## 18.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'Mesh_20_form.FEM'
===Number of processors in meridian section
2
===Number of processors in Fourier space
3
===Number of Fourier modes
3
===Select Fourier modes? (true/false)
.t.
===List of Fourier modes (if select_mode=.TRUE.)
0 1 2
```

```
===Problem type: (nst, mxw, mhd)
'mhd'
===Restart on velocity (true/false)
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
2d-2, 10

============================================
              Post-processing
============================================
===Frequence to create plots
100
===Frequence to write energies
100
===Check numerical stability (true/false)
.t.
===Verbose timing? (true/false)
.t.

============================================
              Periodicity
============================================
===How many pieces of periodic boundary?
1
===Indices of periodic boundaries and corresponding vectors
4 2 .0 1.

============================================
              Mesh-NAVIER-STOKES
============================================
===Number of subdomains in Navier-Stokes mesh
1
===List of subdomains for Navier-Stokes mesh
1

============================================
              BCs-NAVIER-STOKES
============================================
===How many boundary pieces for Dirichlet BCs on uradial?
1
===List of boundary pieces for Dirichlet BCs on uradial
5
===How many boundary pieces for Dirichlet BCs on utheta?
```

```
1
===List of boundary pieces for Dirichlet BCs on utheta
5
===How many boundary pieces for Dirichlet BCs on uzaxis?
1
===List of boundary pieces for Dirichlet BCs on uzaxis
5
===How many boundary pieces for Dirichlet BCs on pressure?
0
===List of boundary pieces for Dirichlet BCs on pressure
0

============================================
              Dynamics-NAVIER-STOKES
============================================
===Reynolds number
1.d1

============================================
              Solver-velocity-NAVIER-STOKES
============================================
===Maximum number of iterations for velocity solver
100
===Relative tolerance for velocity solver
1.d-6
===Absolute tolerance for velocity solver
1.d-10
===Velocity solver verbose? (true/false)
.f.
===Solver type for velocity (FGMRES, CG, ...)
GMRES
===Preconditionner type for velocity solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-pressure-NAVIER-STOKES
============================================
===Maximum number of iterations for pressure solver
100
===Relative tolerance for pressure solver
1.d-6
===Absolute tolerance for pressure solver
1.d-10
===Pressure solver verbose? (true/false)
.f.
```

```
===Solver type for pressure (FGMRES, CG, ...)
GMRES
===Preconditionner type for pressure solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-mass-NAVIER-STOKES
============================================
===Maximum number of iterations for mass matrix solver
100
===Relative tolerance for mass matrix solver
1.d-6
===Absolute tolerance for mass matrix solver
1.d-10
===Mass matrix solver verbose? (true/false)
.f.
===Solver type for mass matrix (FGMRES, CG, ...)
CG
===Preconditionner type for mass matrix solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              H-MAXWELL
============================================
===Number of subdomains in magnetic field (H) mesh
1
===List of subdomains for magnetic field (H) mesh
1
===Number of interfaces in H mesh
0
===List of interfaces in H mesh
xx
===Number of Dirichlet sides for Hxn
0
===List of Dirichlet sides for Hxn
0
===Permeability in the conductive part (1:nb_dom_H)
1.d0
===Conductivity in the conductive part (1:nb_dom_H)
1.d0
===Type of finite element for magnetic field
2
===Magnetic Reynolds number
1.d0
===Stabilization coefficient (divergence)
```

```
1.d0
===Stabilization coefficient for Dirichlet H and/or interface H/H
1.d0

============================================
              Solver-MAXWELL
============================================
===Maximum number of iterations for Maxwell solver
100
===Relative tolerance for Maxwell solver
1.d-6
===Absolute tolerance for Maxwell solver
1.d-10
===Maxwell solver verbose? (true/false)
.f.
===Solver type for Maxwell (FGMRES, CG, ...)
GMRES
===Preconditionner type for Maxwell solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Verbose-MAXWELL
============================================
===Verbose divergence? (true/false)
.t.

============================================
Mesh_BENCH1_20.FEM, P2
===Reference results
3.5080113209672602E-002      H1 norm on velocity
3.7955516735117460E-006      L2 norm of div(Hn)
0.8862355585360645           L2 norm of Hn
2.3155175182237630E-003      L2 norm of pressure
```

# Chapter 19

# Test 14: mxw + arpack (without vacuum).

## 19.1 Numerical domain and equations to solve

## 19.2 Reference results

## 19.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'VKS_MND_form_10.FEM'
===Number of processors in meridian section
1
===Number of processors in Fourier space
2
===Number of Fourier modes
2
===Select Fourier modes? (true/false)
.f.
===List of Fourier modes (if select_mode=.TRUE.)
1 2 3
===Problem type: (nst, mxw, mhd)
'mxw'
===Restart on velocity (true/false)
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
```

```
1.d-2, 1

============================================
              Verbose (diagnostics)
============================================
===Verbose timing? (true/false)
.t.

============================================
              ARPACK
============================================
===Do we use Arpack?
.t.
===Number of eigenvalues to compute
5
===Maximum number of Arpack iterations
3000
===Tolerance for Arpack
1.d-3
===Which eigenvalues ('LM', 'SM', 'SR', 'LR' 'LI', 'SI')
'LR'

============================================
              Visualization
============================================
===Create 2D vtu files for Arpack? (true/false)
.t.

============================================
              H-MAXWELL
============================================
===Number of subdomains in magnetic field (H) mesh
1
===List of subdomains for magnetic field (H) mesh
1
===Number of interfaces in H mesh
0
===List of interfaces in H mesh
xx
===Number of Dirichlet sides for Hxn
3
===List of Dirichlet sides for Hxn
2 4 3
===Permeability in the conductive part (1:nb_dom_H)
1.d0
```

```
===Conductivity in the conductive part (1:nb_dom_H)
1.d0
===Type of finite element for magnetic field
2
===Magnetic Reynolds number
80.d0
===Stabilization coefficient (divergence)
1.d0
===Stabilization coefficient for Dirichlet H and/or interface H/H
1.d0


============================================
              Solver-MAXWELL
============================================
===Maximum number of iterations for Maxwell solver
100
===Relative tolerance for Maxwell solver
1.d-6
===Absolute tolerance for Maxwell solver
1.d-10
===Maxwell solver verbose? (true/false)
.f.
===Solver type for Maxwell (FGMRES, CG, ...)
GMRES
===Preconditionner type for Maxwell solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
Quick test with VKS_MND_form_10.FEM, dt=1.d-2, tol=1.d-3, 5 eigenvalues
===Reference results
-4.9931432394524113E-002 Real part eigenvalue 1, mode 0
7.5216697219226050E-003  |div(Bn)|_L2/|Bn|_H1, eigenvalue 1, mode 0
4.7582245496955054E-003  Real part eigenvalue 1, mode 1
1.3812815769560133E-002  |div(Bn)|_L2/|Bn|_H1, eigenvalue 1, mode 1

Accurate test with VKS_MND_form_10.FEM, dt=1.d-2, tol=1.d-6, 3 eigenvalues
-4.9942724762923045E-002  Real part eigenvalue 1, mode 0
7.5217880247633283E-003   |div(Bn)|_L2/|Bn|_H1, eigenvalue 1, mode 0
4.6811968540408653E-003   Real part eigenvalue 1, mode 1
1.3812375800102094E-002   |div(Bn)|_L2/|Bn|_H1, eigenvalue 1, mode 1
```

# Chapter 20

# Test 15: nst (with LES).

## 20.1 Numerical domain and equations to solve

## 20.2 Reference results

## 20.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'RECT10_BENCHMARK_CONVECTION_LES.FEM'
===Number of processors in meridian section
1
===Number of processors in Fourier space
3
===Number of Fourier modes
3
===Select Fourier modes? (true/false)
.f.
===List of Fourier modes (if select_mode=.TRUE.)
1 2 3
===Problem type: (nst, mxw, mhd)
'nst'
===Restart on velocity (true/false)
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
5.d-2, 10
```

```
============================================
              Verbose (diagnostics)
============================================
===Verbose timing? (true/false)
.t.
===Verbose divergence? (true/false)
.t.
===Verbose CFL? (true/false)
.t.

============================================
              Mesh-NAVIER-STOKES
============================================
===Number of subdomains in Navier-Stokes mesh
1
===List of subdomains for Navier-Stokes mesh
1

============================================
              BCs-NAVIER-STOKES
============================================
===How many boundary pieces for Dirichlet BCs on uradial?
2
===List of boundary pieces for Dirichlet BCs on uradial
2 3
===How many boundary pieces for Dirichlet BCs on utheta?
2
===List of boundary pieces for Dirichlet BCs on utheta
2 3
===How many boundary pieces for Dirichlet BCs on uzaxis?
2
===List of boundary pieces for Dirichlet BCs on uzaxis
2 3
===How many boundary pieces for Dirichlet BCs on pressure?
0
===List of boundary pieces for Dirichlet BCs on pressure
0


============================================
              Dynamics-NAVIER-STOKES
============================================
===Reynolds number
50.d0
```

```
============================================
              LES-NAVIER-STOKES
============================================
===Use LES? (true/false)
.t.
===Coefficients for LES
0.03d0
0.2d0
0.d0
0.8d0

============================================
              Solver-velocity-NAVIER-STOKES
============================================
===Maximum number of iterations for velocity solver
100
===Relative tolerance for velocity solver
1.d-6
===Absolute tolerance for velocity solver
1.d-10
===Velocity solver verbose? (true/false)
.f.
===Solver type for velocity (FGMRES, CG, ...)
GMRES
===Preconditionner type for velocity solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-pressure-NAVIER-STOKES
============================================
===Maximum number of iterations for pressure solver
100
===Relative tolerance for pressure solver
1.d-6
===Absolute tolerance for pressure solver
1.d-10
===Pressure solver verbose? (true/false)
.f.
===Solver type for pressure (FGMRES, CG, ...)
GMRES
===Preconditionner type for pressure solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Solver-mass-NAVIER-STOKES
```

```
============================================
===Maximum number of iterations for mass matrix solver
100
===Relative tolerance for mass matrix solver
1.d-6
===Absolute tolerance for mass matrix solver
1.d-10
===Mass matrix solver verbose? (true/false)
.f.
===Solver type for mass matrix (FGMRES, CG, ...)
CG
===Preconditionner type for mass matrix solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
              Phase
============================================
===Is there a temperature field?
.t.
===Non-dimensional gravity coefficient
50.d0
===Diffusivity coefficient for temperature
1.d0
===How many boundary pieces for Dirichlet BCs on temperature?
1
===List of boundary pieces for Dirichlet BCs on temperature
2

============================================
              Solver-Phase
============================================
===Maximum number of iterations for temperature solver
100
===Relative tolerance for temperature solver
1.d-6
===Absolute tolerance for temperature solver
1.d-10
===Temperature solver verbose? (true/false)
.f.
===Solver type for temperature (FGMRES, CG, ...)
GMRES
===Preconditionner type for temperature solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
```

```
RECT10_BENCHMARK_CONVECTION_LES.FEM, dt=5.d-2, it_max=10
===Reference results
5.07341792236525100E-004    !e_c_u_0
0.11398438949353928         !e_c_u_1
9.23763427659176073E-002    !e_c_u_2
7.34819988015688319E-002    !||div(un)||_L2/|un|_H1
```

# Chapter 21

# Test 16: ??.

## 21.1 Numerical domain and equations to solve

## 21.2 Reference results

## 21.3 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'ELL_b0p8_40_form.FEM'
===Number of processors in meridian section
2
===Number of processors in Fourier space
4
===Number of Fourier modes
8
===Select Fourier modes? (true/false)
.f.
===List of Fourier modes (if select_mode=.TRUE.)
0 4 8
===Problem type: (nst, mxw, mhd)
'nst'
===Restart on velocity (true/false)
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
1d-1, 20
===Do we read metis partition? (true/false)
```

```
.f.
===Which partitioning: "old" or "new"?
'new'
===Frequency to write restart file
2000
===Frequency to write energies
20
===Frequency to create plots
2000
===Just postprocessing without computing? (true/false)
.f.

============================================
              Mesh-NAVIER-STOKES
============================================
===Number of subdomains in Navier-Stokes mesh
1
===List of subdomains for Navier-Stokes mesh
1

============================================
              BCs-NAVIER-STOKES
============================================
===How many boundary pieces for Dirichlet BCs on uradial?
0
===List of boundary pieces for Dirichlet BCs on uradial
1 4 5 6
===How many boundary pieces for Dirichlet BCs on utheta?
0
===List of boundary pieces for Dirichlet BCs on utheta
1 4 5 6
===How many boundary pieces for Dirichlet BCs on uzaxis?
0
===List of boundary pieces for Dirichlet BCs on uzaxis
1 4 5 6
===How many boundary pieces for Dirichlet BCs on pressure?
0
===List of boundary pieces for Dirichlet BCs on pressure
0


============================================
              Dynamics-NAVIER-STOKES
============================================
===Reynolds number
```

```
42.d0
===Is there a precession term (true/false)?
.t.
===Precession rate
0.25d0
===Precession angle over pi
0.5d0

============================================
              Stress BC
============================================
===Stress boundary conditions? (true/false)
.t.
===stab_bdy_ns
1.d0

============================================
              Solver-velocity-NAVIER-STOKES
============================================
===Maximum number of iterations for velocity solver
100
===Relative tolerance for velocity solver
1.d-6
===Absolute tolerance for velocity solver
1.d-10
===Velocity solver verbose? (true/false)
.f.
===Solver type for velocity (FGMRES, CG, ...)
GMRES
===Preconditionner type for velocity solver (HYPRE, JACOBI, MUMPS...)
MUMPS
HYPRE

============================================
              Solver-pressure-NAVIER-STOKES
============================================
===Maximum number of iterations for pressure solver
100
===Relative tolerance for pressure solver
1.d-6
===Absolute tolerance for pressure solver
1.d-10
===Pressure solver verbose? (true/false)
.f.
===Solver type for pressure (FGMRES, CG, ...)
```

```
GMRES
===Preconditionner type for pressure solver (HYPRE, JACOBI, MUMPS...)
MUMPS
HYPRE

============================================
              Solver-mass-NAVIER-STOKES
============================================
===Maximum number of iterations for mass matrix solver
100
===Relative tolerance for mass matrix solver
1.d-6
===Absolute tolerance for mass matrix solver
1.d-10
===Mass matrix solver verbose? (true/false)
.f.
===Solver type for mass matrix (FGMRES, CG, ...)
CG
===Preconditionner type for mass matrix solver (HYPRE, JACOBI, MUMPS...)
MUMPS
HYPRE

============================================
              Verbose-MAXWELL
============================================
===Verbose divergence? (true/false)
.t.
===Verbose timing? (true/false)
.t.
===Verbose CFL? (true/false)
.t.

============================================
ELL_b0p8_40_form.FEM, dt=1d-1, it_max=20
===Reference results
6.67819858929439170E-003 !Total kinetic energy at t=2
9.60802004529626424E-004 !Mx
4.88028990370793364E-002 !My
0.12185004841513863  !Mz
```

# Chapter 22

# Test 17: mxw with vacuum + variable mu

The purpouse of this case is to test variable permeability with dependence in $r$ and $z$. There is one conducting region embedded in vaccum.

## 22.1 Numerical domain and equations to solve

The domain is a cylinder $r \in [0,1]$, $z \in [-1,1]$. The exterior boundary of the vaccum is a sphere of radius 10 centered at 0. Let us recall the kynematic dynamo equations,

$$\begin{cases} \partial_t (\mu \mathbf{H}) + \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma}\nabla\times\mathbf{H}\right) - \nabla\times(\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_\mathrm{m}}\nabla\times\left(\frac{1}{\sigma}\mathbf{j}\right), \\ \nabla\cdot(\mu\mathbf{H}) = 0, \\ +BC + IC. + \phi \end{cases} \tag{22.1.1}$$

## 22.2 Analytical solution

Let

$$\mathbf{H} = \frac{1}{\mu^c}\nabla\psi, \tag{22.2.1}$$

where $\psi = \psi(r,z)$ and satisfies the Laplace equation in cylindrical coordinates,

$$\partial_{rr}\psi + \frac{1}{r}\partial_r\psi + \partial_{zz}\psi = 0. \tag{22.2.2}$$

If we also set $\mathbf{j} = \nabla\times\mathbf{H}$, $\mathbf{u} = 0$, $\mathbf{E} = \mathbf{0}$ and $\phi(r,\theta,z,t) = \psi(r,z)$, then $\mathbf{H}$, defined as in (22.2.1), satisfies Maxwell equations (23.1.1).

Now, let

$$\mu^c = \mu^c(r,z) = \frac{1}{f(r,z)+1}, \tag{22.2.3}$$

where

$$f(r,z) = b \cdot r^3 \cdot (1-r)^3 \cdot (z^2-1)^3,$$

and $b \geq 0$ is a parameter which determines the variation of $\mu^c$. Observe that

$$\partial_r f(r,z) = 3b\big(r(1-r)\big)^2(1-2r)(z^2-1)^3, \quad \partial_z f(r,z) = 6bz\big(r(1-r)\big)^3(z^2-1)^2.$$

Moreover, $f(r,z) \leq 0$ for $(r,\theta,z) \in \Omega^c$ and,

$$\sup_{\Omega^c} f(r,z) = f_{\max} = 0, \quad \inf_{\Omega^c} f(r,z) = f_{\min} = -\frac{b}{2^6},$$

then,

$$\mu^c_{\min} = \frac{1}{1+f_{\max}}, \quad \mu^c_{\max} = \frac{1}{1+f_{\min}}, \qquad r_\mu = \frac{\mu_{\max}}{\mu_{\min}} = \frac{\frac{1}{1-\frac{b}{2^6}}}{1}, \quad \text{and} \quad b = 2^6\left(1-\frac{1}{r_\mu}\right).$$

To get an explicit solution in (22.2.1), equation (22.2.2) is solved using separation of variables, this is, letting $\psi(r,z) = R(r)Z(z)$ we solve the following system of ODEs,

$$\begin{aligned} Z'' - \lambda Z &= 0 \\ R'' + \frac{R'}{r} + \lambda R &= 0, \end{aligned}$$

where $\lambda$ is any real number. Here we choose $\lambda = 1$, so

$$\psi(r,z) = J_0(r)\cosh(z). \tag{22.2.4}$$

Now, using $J_0'(r) = -J_1(r)$ and $\cosh'(z) = \sinh(z)$ we get,

$$\nabla\psi = \begin{bmatrix} -J_1(r)\cosh(z) \\ 0 \\ J_0(r)\sinh(z) \end{bmatrix} \tag{22.2.5}$$

then by (22.2.1),

$$\mathbf{H}^c = (f(r,z)+1)\begin{bmatrix} -J_1(r)\cosh(z) \\ 0 \\ J_0(r)\sinh(z) \end{bmatrix}, \tag{22.2.6}$$

To get $\nabla \times \mathbf{H}$, we use the identity

$$\nabla \times \left(\frac{1}{\mu^c}\nabla\psi\right) = \nabla\left(\frac{1}{\mu^c}\right) \times \nabla\psi + \frac{1}{\mu^c}\nabla \times \nabla\psi,$$

but $\nabla \times \nabla\psi = 0$. Then using equation (22.2.1),

$$\nabla \times \mathbf{H}^c = \nabla\left(\frac{1}{\mu^c}\right) \times \nabla\psi,$$

and

$$\nabla \frac{1}{\mu^c} = \begin{bmatrix} \partial_r f(r,z) \\ 0 \\ \partial_z f(r,z) \end{bmatrix}; \tag{22.2.7}$$

we obtain,

$$\mathbf{j} = \nabla \times \mathbf{H}^c = \begin{bmatrix} 0 \\ -\partial_r f(r,z) J_0(r)\sinh(z) - \partial_z f(r,z) J_1(r)\cosh(z) \\ 0 \end{bmatrix}. \tag{22.2.8}$$

In summary,

$$\phi(r,\theta,z,t) = J_0(r)\cosh(z). \tag{22.2.9}$$

$$\mathbf{H}^c = \left(2^6\left(1-\frac{1}{r_\mu}\right)\cdot r^3 \cdot (1-r)^3 \cdot (z^2-1)^3 + 1\right) \begin{bmatrix} -J_1(r)\cosh(z) \\ 0 \\ J_0(r)\sinh(z) \end{bmatrix}, \tag{22.2.10}$$

## 22.3 Reference results

## 22.4 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'mesh_17_0.1.FEM'
===Number of processors in meridian section
1
===Number of processors in Fourier space
1
===Number of Fourier modes
1
===Select Fourier modes? (true/false)
.t.
===List of Fourier modes (if select_mode=.TRUE.)
0
===Problem type: (nst, mxw, mhd)
'mxw'
===Restart on velocity (true/false)
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
1d40, 1
```

```
============================================
              H-MAXWELL
============================================
===Solve Maxwell with H (true) or B (false)?
.f.
===Number of subdomains in magnetic field (H) mesh
1
===List of subdomains for magnetic field (H) mesh
1
===Number of interfaces in H mesh
0
===Number of Dirichlet sides for Hxn
0
===List of Dirichlet sides for Hxn
2
===Stabilization coefficient for Dirichlet H and/or interface H/H
1.d0
===Is permeability defined analytically (true/false)?
.t.
===Permeability in the conductive part (1:nb_dom_H)
1.d0
===Conductivity in the conductive part (1:nb_dom_H)
1.d0
===Type of finite element for magnetic field
2
===Magnetic Reynolds number
1.d0
===Stabilization coefficient (divergence)
1.d1

============================================
              Phi-MAXWELL
============================================
===Number of subdomains in magnetic potential (phi) mesh
1
===List of subdomains for magnetic potential (phi) mesh
2
===How many boundary pieces for Dirichlet BCs on phi?
1
===List of boundary pieces for Dirichlet BCs on phi
3
===Number of interfaces between H and phi
1
===List of interfaces between H and phi
2
```

```
===Permeability in vacuum
1
===Type of finite element for scalar potential
2
===Stabilization coefficient (interface H/phi)
1.d0

============================================
Solver-MAXWELL
============================================
===Maximum number of iterations for Maxwell solver
100
===Relative tolerance for Maxwell solver
1.d-6
===Absolute tolerance for Maxwell solver
1.d-10
===Maxwell solver verbose? (true/false)
.f.
===Solver type for Maxwell (FGMRES, CG, ...)
GMRES
===Preconditionner type for Maxwell solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
mesh_17_0.1.FEM, dt=1d40, it_max=1
===Reference results
5.86043413986104319E-003 !L2 norm of div(mu (H-Hexact))
6.73618675647699467E-003 !L2 norm of curl(H-Hexact)
5.70731229113286258E-003 !L2 norm of H-Hexact
1.58622538657745486E-003 !L2 norm of phi-phiexact
```

# Chapter 23

# Test 18: mxw with variable mu and no vaccum

The purpouse of this case is to test variable permeability with dependence in $r$ and $z$. There are two conducting regions with discontinuous magentic permeability.

## 23.1 Numerical domain and equations to solve

The domain is a cylinder $r \in [0, 2]$, $z \in [\frac{1}{4}, 1]$. Let us recall the kynematic dynamo equations,

$$\begin{cases} \partial_t (\mu\mathbf{H}) + \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma}\nabla\times\mathbf{H}\right) - \nabla\times (\mathbf{u}\times\mu\mathbf{H}) = \frac{1}{R_\mathrm{m}} \nabla\times \left(\frac{1}{\sigma}\mathbf{j}\right), \\ \nabla\cdot(\mu\mathbf{H}) = 0, \\ +BC + IC. \end{cases} \tag{23.1.1}$$

## 23.2 Analytical solution

We first set $\mathbf{u} = 0$, $\mathbf{E} = 0$, and $\Omega_v = \emptyset$. In order to define $\Omega^c$, let $z_0 > 0$ be a positive parameter (SFEMaNS sets $z_0 = \frac{1}{4}$ in this test), then we set $\Omega^c = \Omega^c_1 \cup \Omega^c_2$, where

$$\Omega^c_1 = \{(r, \theta, z) \in \mathbb{R}^3 : (r, \theta, z) \in [0, 1] \times [0, 2\pi) \times [z_0, 1]\},$$

and

$$\Omega^c_2 = \{(r, \theta, z) \in \mathbb{R}^3 : (r, \theta, z) \in [1, 2] \times [0, 2\pi) \times [z_0, 1]\}.$$

In order to construct a magnetic field with a jump in $\mu$ on $\Sigma_\mu$, let us define first,

$$\mathbf{H}^c = \begin{bmatrix} H_r \\ 0 \\ H_z \end{bmatrix},$$

where,

$$H_r = \begin{cases} H_{1,r} = f(r)g(z) & \text{in} \quad \Omega_1^c, \\ H_{2,r} & \text{in} \quad \Omega_2^c \end{cases}. \tag{23.2.1}$$

It is assumed that $f(r)$ and $g(z)$ are given functions. Moreover, we also assume $\mu_1$ is given and $\mu_1 = \mu_1(r) > 0$. Thus, the unknowns to solve are $\mu_2, H_{2,r}$, and $H_z$. Observe that $H_z$ must be continuous, since $\mathbf{H}^c$ must be continuous along the tangential direction over $\Sigma_\mu$. We also need $\mu_2 > 0$. Now, the divergence constraint establishes,

$$\nabla\cdot(\mu\mathbf{H}) = 0 \quad \text{in} \quad \Omega^c,$$

which gives,

$$0 = \begin{cases} \partial_r(\mu_1 H_{1,r}) + \frac{\mu_1 H_{1,r}}{r} + \partial_z(\mu_1 H_z) & \text{in} \quad \Omega_1^c, \\ \partial_r(\mu_2 H_{1,r}) + \frac{\mu_2 H_{2,r}}{r} + \partial_z(\mu_2 H_z) & \text{in} \quad \Omega_2^c \end{cases}. \tag{23.2.2}$$

Using $\mu_1 = \mu_1(r)$ and $H_{1,r} = f(r)g(z)$, we can solve for $\partial_z H_z$ using the first equation of (23.2.2),

$$\partial_z H_z = -\frac{1}{\mu_1(r)} \left( \partial_r(\mu_1(r)f(r)g(z)) + \frac{\mu_1(r)f(r)g(z)}{r} \right)$$

thus,

$$H_z = -\frac{1}{\mu_1(r)} \left( \partial_r(\mu_1(r)f(r)) + \frac{\mu_1(r)f(r)}{r} \right) \int g(z)dz + \psi_1(r), \tag{23.2.3}$$

where $\psi_1(r)$ is a parameter function. Now, since each right side of (23.2.2) must be equal to zero, we get,

$$\partial_r(\mu_1 H_{1,r}) + \frac{\mu_1 H_{1,r}}{r} + \partial_z(\mu_1 H_z) = \partial_r(\mu_2 H_{1,r}) + \frac{\mu_2 H_{2,r}}{r} + \partial_z(\mu_2 H_z), \tag{23.2.4}$$

but the continuity condition along the normal on $\Sigma_\mu$ establishes,

$$\mu_2 H_{2,r} = \mu_1 H_{1,r} = \mu_1(r)f(r)g(r), \tag{23.2.5}$$

so plug-in this equality into (23.2.4) gives,

$$\partial_r(\mu_1 H_{1,r}) + \frac{\mu_1 H_{1,r}}{r} + \partial_z(\mu_1 H_z) = \partial_r(\mu_1 H_{1,r}) + \frac{\mu_1 H_{1,r}}{r} + \partial_z(\mu_2 H_z),$$

therefore,

$$\partial_z(\mu_1 H_z) = \partial_z(\mu_2 H_z).$$

Integrating this last equation with respect to $z$ and solving for $\mu_2$, we obtain,

$$\mu_2 = \mu_1 + \frac{\psi_2(r)}{H_z}, \tag{23.2.6}$$

where $\psi_2(r)$ is a parameter function and sets the amount of the jump in $\mu$ on $\Sigma_\mu$. In particular, if $\psi_2(r)$=0 then $\mu_1 = \mu_2$. Finally, from (23.2.5) we compute,

$$H_{2,r} = \frac{\mu_1(r)f(r)g(z)}{\mu_2}. \tag{23.2.7}$$

**Remark.** Care must be taken about how to choose $f(r)$, $g(z)$, $\mu(r)$, $\psi_1(r)$ and $\psi_2(r)$ such that $H_z \neq 0$ and $\mu_2 > 0$.

Here we choose,

$$f(r) = r, \quad g(z) = z, \quad \mu_1(r) = 1 + r, \quad \psi_2(r) = 0, \quad \text{and} \qquad \psi_2(r) = -\lambda_\mu,$$

where $\lambda_\mu \in \mathbb{R}$ and $\lambda_\mu \geq 0$. Now, using (23.2.7), (23.2.3) and (23.2.4) we compute,

$$\begin{aligned} H_{1,r} &= rz, \\ H_{2,r} &= \frac{rz^3(3r+2)}{3z^2r + 2z^2 + 2\lambda_\mu}, \\ H_z &= -\frac{1}{2}\frac{z^2(3r+2)}{1+r}. \\ \mu_2 &= 1 + r + \frac{2\lambda_\mu(1+r)}{z^2(3r+2)} \end{aligned}$$

Finally, using $\mathbf{j} = \nabla \times \mathbf{H}^c$ we get,

$$\mathbf{j} = \begin{bmatrix} 0 \\ j_\theta \\ 0 \end{bmatrix}, \tag{23.2.8}$$

where,

$$j_\theta = \begin{cases} j_{1,\theta} & \text{in} \quad \Omega_1^c \\ j_{2,\theta} & \text{in} \quad \Omega_2^c \end{cases},$$

$$j_{1,\theta} = r + \frac{3}{2}\frac{z^2}{1+r} - \frac{1}{2}\frac{z^2(3r+2)}{(1+r)^2},$$

and,

$$j_{2,\theta} = \frac{rz^2(3r+2)(3z^2r + 2z^2 + 6\lambda_\mu)}{(3z^2r + 2z^2 + 2\lambda_\mu)^2} + \frac{3}{2}\frac{z^2}{1+r} - \frac{1}{2}\frac{z^2(3r+2)}{(1+r)^2}.$$

## 23.3 Reference results

## 23.4 Data file

```
===Is mesh file formatted (true/false)?
.t.
===Directory and name of mesh file
'.' 'mesh_18_0.05.FEM'
===Number of processors in meridian section
1
===Number of processors in Fourier space
```

```
1
===Number of Fourier modes
1
===Select Fourier modes? (true/false)
.t.
===List of Fourier modes (if select_mode=.TRUE.)
0
===Problem type: (nst, mxw, mhd)
'mxw'
===Restart on velocity (true/false)
.f.
===Restart on magnetic field (true/false)
.f.
===Time step and number of time iterations
1d40, 1

============================================
              H-MAXWELL
============================================
===Number of subdomains in magnetic field (H) mesh
2
===List of subdomains for magnetic field (H) mesh
1 2
===Number of interfaces in H mesh
1
===List of interfaces in H mesh
5
===Number of Dirichlet sides for Hxn
3
===List of Dirichlet sides for Hxn
2 3 4
===Stabilization coefficient for Dirichlet H and/or interface H/H
1.d0
===Is permeability defined analytically (true/false)?
.t.
===Conductivity in the conductive part (1:nb_dom_H)
1.d0 1.d0
===Type of finite element for magnetic field
2
===Magnetic Reynolds number
1.d0
===Stabilization coefficient (divergence)
1.d0

============================================
```

```
Solver-MAXWELL
============================================
===Maximum number of iterations for Maxwell solver
100
===Relative tolerance for Maxwell solver
1.d-6
===Absolute tolerance for Maxwell solver
1.d-10
===Maxwell solver verbose? (true/false)
.f.
===Solver type for Maxwell (FGMRES, CG, ...)
GMRES
===Preconditionner type for Maxwell solver (HYPRE, JACOBI, MUMPS...)
MUMPS

============================================
mesh_18_0.05.FEM, dt=1d40, it_max=1
===Reference results
1.13234091486652422E-004 !L2 norm of div(mu (H-Hexact))
4.31157595700767786E-004 !L2 norm of curl(H-Hexact)
1.67240082121627564E-004 !L2 norm of H-Hexact
2.2038729295399340       !L2 norm of H
```