1 Combinatorics

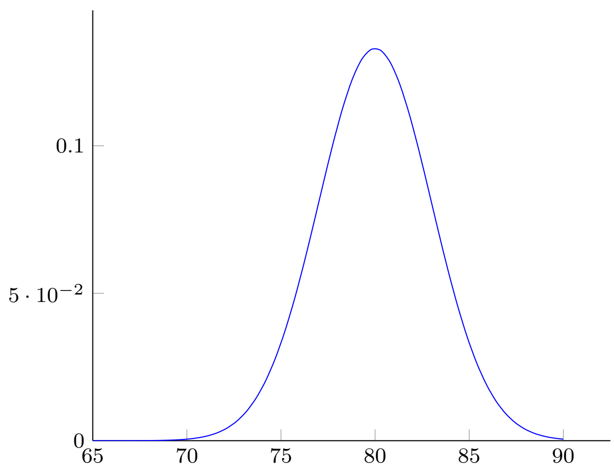
	With Replacement	Without Replacement
Permutations (Ordered)	$P_r(n,k) = n^k$	$P(n,k) = \frac{n!}{(n-k)!}$
Combinations (Unordered)	$C_r(n,k) = \binom{n+k-1}{k} = \frac{(n+k-1)!}{k!(n-1)!}$	$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$

2 Distributions

Distribution	Moments	Bernoulli	Binomial	Geometric	Poisson	Exponential	Uniform	Normal
PDF		$p^x(1-p)^{1-x}$	$\binom{n}{x}p^x(1-p)^{n-x}$	$(1-p)^{x-1}p$	$\mathrm{e}^{-\lambda}\frac{\lambda^x}{x!}$	$\frac{1}{\lambda}\mathrm{e}^{\frac{-x}{\lambda}}$	$\frac{1}{b-a}$	$\frac{1}{\sqrt{2\pi\sigma^2}}\mathrm{e}^{\frac{-(x-\mu)^2}{2\sigma^2}}$
E[X]	$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$	p	np	$\frac{1}{p}$	λ	λ	$\frac{a+b}{2}$	μ
Var[X]	$E[X^2] = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$	$p(1-p)$	$np(1-p)$	$\frac{1-p}{p^2}$	λ	λ^2	$\frac{(b-a)^2}{12}$	σ^2

3 Percentiles; $Q \sim Normal(\mu_Q = 80, \sigma_Q^2 = 9)$; percentile = 97.5

$$P(Q \le q_0.975) = 0.975 \tag{1}$$



(2)

$$P(Z \le \frac{q_0.975 - 80}{3}) = 0.975 \tag{3}$$

$$\frac{q_0.975 - 80}{3} = 1.96 \tag{4}$$

$$q_0.975 \approx 85.88 \tag{5}$$

So x_Q does not fall within percentile if $x_Q < q_0.975$

4 Proportion within range; $X \sim Normal(\mu_Q = 1000, \sigma_Q^2 = 625); n = 10$

What proportion of bricks weigh between 950 to 1050 grams?

$$P(950 \leq X \leq 1050) = P(\frac{950-1000}{25} \leq Z \leq \frac{1050-1000}{25}) = P(-2 \leq Z \leq 2) = P(-2 \leq Z) \times P(Z \leq 2) = .97725 \times .97725 \approx .95 \tag{6}$$

what is the probability that all of them pass the weight requirement?

$$Y_i = \begin{cases} 1 & 950 \leq X_i \leq 1050 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

$$P(Y_i = 1) = P(950 \leq X_i \leq 1050) = .95 \tag{8}$$

$$\text{let } Y_i = Y_1 + ... + Y_10 = \text{bricks that meet requirement} \tag{9}$$

$$P(Y = 10) = (.95)^{10} \tag{10}$$

What is the probability that exactly 9 out of 10 meet the requirement?

$$P(Y = 9) = \binom{10}{9} \times (.95)^9 \times (1 - .95)^{10-9} \tag{11}$$

What is the probability that at least one fails the requirement?

$$P(Y < 10) = 1 - P(Y = 10) = 1 - (.95)^{10} \tag{12}$$

Suppose it is known that there was exactly one failure. What is the probability that the failure was due to brick number 1?

$$n = 10 \iff answer = \frac{1}{10} \tag{13}$$

if testing was done sequentially, What is the probability that the first failure is incurred by the fourth brick tested?

$$P(Y_1 = 1, Y_2 = 1, Y_3 = 1, Y_4 = 0) = P(Y_1 = 1) \times ... \times P(Y_4 = 0) = (.95)^3 \times (.05) \tag{14}$$

What is the probability that the second failure occurs on the 10-th brick. Hint: exactly one failure in first 9 bricks and the 10-th brick is failure.

$$P(\{\text{one failure in 9 bricks}\} \cap \{Y_{10} = 0\}) = (\binom{9}{1} \times (.95)^8 \times .05) \times .05 = 9 \times .95^8 \times .05^2 \tag{15}$$

5 Lifetimes; $X \sim Exponential(\lambda_X = 1000), Y \sim Exponential(\lambda_Y = 1200)$

Given industry standard is $\lambda = 400$ derive CDF

$$F_X(x) = \int_0^x f_X(x)dx = \int_0^x \frac{e^{\frac{-t}{\lambda_X}}}{\lambda_X} dt = -e^{\frac{-t}{\lambda_X}} \Big|_0^x \tag{16}$$

What is the proportion of devices produced by Company-X that meets that minimum standard?

$$P(X \geq 400) = 1 - F_X(X = 400) = e^{\frac{-400}{1000}} \approx .67 \tag{17}$$

What is the proportion of devices produced by Company-Y that meets that minimum standard?

$$P(Y \geq 400) = 1 - F_Y(X = 400) = e^{\frac{-400}{1200}} \approx .717 \tag{18}$$

What is the proportion of devices manufactured by Company-X that gives lifetimes between 400 and 1600 hours?

$$P(400 \leq X \leq 1600) = F_X(X = 1600) - F_X(X = 400) = e^{\frac{-400}{1000}} - e^{\frac{-1600}{1000}} \approx .468 \tag{19}$$

Given Company-X makes 0.45/Company-Y makes 0.55 of devices. A device tested passed the minimum standard. Guess which company made that devic

$$T = \text{device that passed} \tag{20}$$

$$C_X = \text{device produced by company X} \tag{21}$$

$$C_Y = \text{device produced by company Y} \tag{22}$$

$$P(C_X) = .45 \tag{23}$$

$$P(C_Y) = .55 \tag{24}$$

$$P(T|C_X) = P(X \geq 400) = .67 \tag{25}$$

$$P(T|C_Y) = P(Y \geq 400) = .717 \tag{26}$$

$$P(T) = P(T|C_X) \times p(C_X) + P(T|C_Y) \times P(C_Y) = (.67)(.45) + (.717)(.55) = .696 \tag{27}$$

$$P(C_X|T) = \frac{P(T|C_X)P(C_X)}{P(T)} = \frac{(.67)(.45)}{.696} = .433 \tag{28}$$

$$P(C_Y|T) = 1 - P(C_X|T) = 1 - .433 = .567 \therefore \text{the device was probably made by Company Y} \tag{29}$$

6 Hypothesis Testing

H_0	H_A	Conditions	Test Statistic	Rejection Region	Degrees of freedom
$\mu_1 = \mu_2$	$\mu_1 < \mu_2$ $\mu_1 > \mu_2$ $\mu_1 \neq \mu_2$	Known σ	$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	$Z < -z_\alpha$ $Z > z_\alpha$ $\lvert Z \rvert > \lvert z_{\alpha/2} \rvert$	
$\mu_1 = \mu_2$	$\mu_1 < \mu_2$ $\mu_1 > \mu_2$ $\mu_1 \neq \mu_2$	Unknown but equal σ $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$ $s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$	$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p{}^2(\frac{1}{n_1} + \frac{1}{n_2})}}$	$t < -t_{df,\alpha}$ $t > t_{df,\alpha}$ $\lvert t \rvert > \lvert t_{df,\alpha/2} \rvert$	$df = n_1 + n_2 - 2$
$\pi_1 = \pi_2$	$\pi_1 < \pi_2$ $\pi_1 > \pi_2$ $\pi_1 \neq \pi_2$	Z-test for proportions	$Z = \frac{(\hat{\pi}_1 - \hat{\pi}_2) - (\pi_1 - \pi_2)}{\sqrt{\frac{\hat{\pi}_1(1-\hat{\pi}_1)}{n_1} + \frac{\hat{\pi}_2(1-\hat{\pi}_2)}{n_2}}}$	$Z < -z_\alpha$ $Z > z_\alpha$ $\lvert Z \rvert > \lvert z_{\alpha/2} \rvert$	
$\beta_1 = 0$	$\beta_1 \neq 0$	Test for linear relationship	$t = \frac{\hat{\beta}_1 - \beta_1}{\sqrt{\frac{s^2}{S_{xx}}}}$	$t > \lvert t_{df,\alpha/2} \rvert$	$df = n - 2$

7 Linear Regression Formulas

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \tag{30}$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} \tag{31}$$

$$S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2 \tag{32}$$

$$S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \tag{33}$$

$$SS_{RESID} = \sum_{i=1}^{n} (y_i - \hat{y}) \tag{34}$$

$$Regression Variance : s^2 = \frac{SS_{RESID}}{n-2} \tag{35}$$

$$Normal Equations : \begin{Bmatrix} \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0 \\ \sum_{i=1}^{n} x_i (y_i - \beta_0 - \beta_1 x_i) = 0 \end{Bmatrix} \tag{36}$$