

1 Combinatorics

	With Replacement	Without Replacement
Permutations (Ordered)	$P_r(n, k) = n^k$	$P(n, k) = \frac{n!}{(n-k)!}$
Combinations (Unordered)	$C_r(n, k) = \binom{n+k-1}{k} = \frac{(n+k-1)!}{k!(n-1)!}$	$C(n, k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$

2 Distributions

Distribution	Moments	Bernoulli	Binomial	Geometric	Poisson	Exponential	Uniform	Normal
PDF		$p^x(1-p)^{1-x}$	$\binom{n}{x}p^x(1-p)^{n-x}$	$(1-p)^{x-1}p$	$e^{-\lambda} \frac{\lambda^x}{x!}$	$\frac{1}{\lambda} e^{-\frac{x}{\lambda}}$	$\frac{1}{b-a}$	$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
E[X]	$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$	p	np	$\frac{1}{p}$	λ	λ	$\frac{a+b}{2}$	μ
Var[X]	$E[X^2] = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$	$p(1-p)$	$np(1-p)$	$\frac{1-p}{p^2}$	λ	λ^2	$\frac{(b-a)^2}{12}$	σ^2

3 Percentiles; $Q \sim Normal(\mu_Q = 80, \sigma_Q^2 = 9)$; percentile = 97.5

$P(Q \leq q_{0.975}) = 0.975$	(1)
$P(Z \leq \frac{q_{0.975} - 80}{3}) = 0.975$	(2)
$\frac{q_{0.975} - 80}{3} = 1.96$	(3)
$q_{0.975} \approx 85.88$	(4)
So x_Q does not fall within percentile if $x_Q < q_{0.975}$	

4 Hypothesis Testing

H ₀	H _A	Conditions	Test Statistic	Rejection Region	Degrees of freedom
$\mu_1 = \mu_2$	$\mu_1 < \mu_2$ $\mu_1 > \mu_2$ $\mu_1 \neq \mu_2$	Known σ	$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	$Z < -z_\alpha$ $Z > z_\alpha$ $ Z > z_{\alpha/2} $	
$\mu_1 = \mu_2$	$\mu_1 < \mu_2$ $\mu_1 > \mu_2$ $\mu_1 \neq \mu_2$	Unknown but equal σ $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$ $s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$	$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 (\frac{1}{n_1} + \frac{1}{n_2})}}$	$t < -t_{df, \alpha}$ $t > t_{df, \alpha}$ $ t > t_{df, \alpha/2} $	df = $n_1 + n_2 - 2$
$\pi_1 = \pi_2$	$\pi_1 < \pi_2$ $\pi_1 > \pi_2$ $\pi_1 \neq \pi_2$	Z-test for proportions	$Z = \frac{(\hat{\pi}_1 - \hat{\pi}_2) - (\pi_1 - \pi_2)}{\sqrt{\frac{\hat{\pi}_1(1-\hat{\pi}_1)}{n_1} + \frac{\hat{\pi}_2(1-\hat{\pi}_2)}{n_2}}}$	$Z < -z_\alpha$ $Z > z_\alpha$ $ Z > z_{\alpha/2} $	
$\beta_1 = 0$	$\beta_1 \neq 0$	Test for linear relationship	$t = \frac{\hat{\beta}_1 - \beta_1}{\sqrt{\frac{s^2}{S_{xx}}}}$	$t > t_{df, \alpha/2} $	df = $n - 2$

5 Linear Regression Formulas

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$	(5)
$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$	(6)
$S_{xx} = \sum_{i=1}^n (x_i - \bar{x})^2$	(7)
$S_{xy} = \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$	(8)
$SS_{RESID} = \sum_{i=1}^n (y_i - \hat{y})^2$	(9)
Regression Variance : $s^2 = \frac{SS_{RESID}}{n-2}$	(10)

6 Proportion within range; $X \sim Normal(\mu_Q = 1000, \sigma_Q^2 = 625); n = 10$

What proportion of bricks weigh between 950 to 1050 grams?

$$P(950 \leq X \leq 1050) = P\left(\frac{950 - 1000}{25} \leq Z \leq \frac{1050 - 1000}{25}\right) = P(-2 \leq Z \leq 2) = P(Z \leq 2) - P(Z \geq -2) = .97725 - .0228 \approx .95 \quad (11)$$

what is the probability that all of them pass the weight requirement?

$$Y_i = \begin{cases} 1 & 950 \leq X_i \leq 1050 \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

$$P(Y_i = 1) = P(950 \leq X_i \leq 1050) = .95 \quad (13)$$

$$\text{let } Y_i = Y_1 + \dots + Y_{10} = \text{bricks that meet requirement} \quad (14)$$

$$P(Y = 10) = (.95)^{10} \quad (15)$$

What is the probability that exactly 9 out of 10 meet the requirement?

$$P(Y = 9) = \binom{10}{9} \times (.95)^9 \times (1 - .95)^{10-9} \quad (16)$$

What is the probability that at least one fails the requirement?

$$P(Y < 10) = 1 - P(Y = 10) = 1 - (.95)^{10} \quad (17)$$

Suppose it is known that there was exactly one failure. What is the probability that the failure was due to brick number 1?

$$n = 10 \iff \text{answer} = \frac{1}{10} \quad (18)$$

if testing was done sequentially, What is the probability that the first failure is incurred by the fourth brick tested?

$$P(Y_1 = 1, Y_2 = 1, Y_3 = 1, Y_4 = 0) = P(Y_1 = 1) \times \dots \times P(Y_4 = 0) = (.95)^3 \times (.05) \quad (19)$$

What is the probability that the second failure occurs on the 10-th brick. Hint: exactly one failure in first 9 bricks and the 10-th brick is failure.

$$P(\{\text{one failure in 9 bricks}\} \cap \{Y_{10} = 0\}) = \left(\binom{9}{1}\right) \times (.95)^8 \times .05 \times .05 = 9 \times .95^8 \times .05^2 \quad (20)$$

7 Lifetimes; $X \sim Exponential(\lambda_X = 1000), Y \sim Exponential(\lambda_Y = 1200)$

Given industry standard is $\lambda = 400$ derive CDF

$$F_X(x) = \int_0^x f_X(x) dx = \int_0^x \frac{e^{-\frac{t}{\lambda_X}}}{\lambda_X} dt = -e^{-\frac{t}{\lambda_X}} \Big|_0^x \quad (21)$$

What is the proportion of devices produced by Company-X that meets that minimum standard?

$$P(X \geq 400) = 1 - F_X(X = 400) = e^{-\frac{400}{1000}} \approx .67 \quad (22)$$

What is the proportion of devices produced by Company-Y that meets that minimum standard?

$$P(Y \geq 400) = 1 - F_Y(X = 400) = e^{-\frac{400}{1200}} \approx .717 \quad (23)$$

What is the proportion of devices manufactured by Company-X that gives lifetimes between 400 and 1600 hours?

$$P(400 \leq X \leq 1600) = F_X(X = 1600) - F_X(X = 400) = e^{-\frac{400}{1000}} - e^{-\frac{1600}{1000}} \approx .468 \quad (24)$$

Given Company-X makes 0.45/Company-Y makes 0.55 of devices. A device tested passed the minimum standard. Guess which company made that device?

$$T = \text{device that passed} \quad (25)$$

$$C_X = \text{device produced by company X} \quad (26)$$

$$C_Y = \text{device produced by company Y} \quad (27)$$

$$P(C_X) = .45 \quad (28)$$

$$P(C_Y) = .55 \quad (29)$$

$$P(T|C_X) = P(X \geq 400) = .67 \quad (30)$$

$$P(T|C_Y) = P(Y \geq 400) = .717 \quad (31)$$

$$P(T) = P(T|C_X) \times p(C_X) + P(T|C_Y) \times P(C_Y) = (.67)(.45) + (.717)(.55) = .696 \quad (32)$$

$$P(C_X|T) = \frac{P(T|C_X)P(C_X)}{P(T)} = \frac{(.67)(.45)}{.696} = .433 \quad (33)$$

$$P(C_Y|T) = 1 - P(C_X|T) = 1 - .433 = .567 \therefore \text{the device was probably made by Company Y} \quad (34)$$