Willkommen in der guten Stube :D

Aufgabe

Seien $s, t, u, v > 0$ positive reelle Zahlen. Man zeige die Gültigkeit der folgenden Abschätzung:

$$\frac{1}{s+t+u} + \frac{1}{s+t+v} + \frac{1}{s+u+v} + \frac{1}{t+u+v} > \frac{4}{s+t+u+v}.$$

Beweis

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$$\frac{1}{s+t+u} + \frac{1}{s+t+v} + \frac{1}{s+u+v} + \frac{1}{t+u+v}$$

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$$\frac{1}{s+t+u} + \frac{1}{s+t+v} + \frac{1}{s+u+v} + \frac{1}{t+u+v}$$
$$= \frac{1}{s+t+u+v}\left(\frac{s+t+u+v}{s+t+u} + \frac{s+t+u+v}{s+t+v} + \frac{s+t+u+v}{s+u+v} + \frac{s+t+u+v}{t+u+v}\right)$$

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$$\frac{1}{s+t+u} + \frac{1}{s+t+v} + \frac{1}{s+u+v} + \frac{1}{t+u+v}$$
$$= \frac{1}{s+t+u+v}\left(\frac{s+t+u+v}{s+t+u} + \frac{s+t+u+v}{s+t+v} + \frac{s+t+u+v}{s+u+v} + \frac{s+t+u+v}{t+u+v}\right)$$
$$= \frac{1}{s+t+u+v}\left(\frac{s+t+u}{s+t+u} + \frac{v}{s+t+u} + \frac{s+t+u+v}{s+t+v} + \frac{s+t+u+v}{s+u+v} + \frac{s+t+u+v}{t+u+v}\right)$$

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Seien $s, t, u, v > 0$ positive reelle Zahlen. Dann gilt:

$$\begin{aligned}
&\frac{1}{s+t+u} + \frac{1}{s+t+v} + \frac{1}{s+u+v} + \frac{1}{t+u+v} \\
&= \frac{1}{s+t+u+v}\left(\frac{s+t+u+v}{s+t+u} + \frac{s+t+u+v}{s+t+v} + \frac{s+t+u+v}{s+u+v} + \frac{s+t+u+v}{t+u+v}\right) \\
&= \frac{1}{s+t+u+v}\left(\frac{s+t+u}{s+t+u} + \frac{v}{s+t+u} + \frac{s+t+u+v}{s+t+v} + \frac{s+t+u+v}{s+u+v} + \frac{s+t+u+v}{t+u+v}\right) \\
&= \frac{1}{s+t+u+v}\left(1 + \frac{v}{s+t+u} + \frac{s+t+u+v}{s+t+v} + \frac{s+t+u+v}{s+u+v} + \frac{s+t+u+v}{t+u+v}\right)
\end{aligned}$$

Beweis

Seien $s, t, u, v > 0$ positive reelle Zahlen. Dann gilt:

$$\frac{1}{s+t+u} + \frac{1}{s+t+v} + \frac{1}{s+u+v} + \frac{1}{t+u+v}$$
$$= \frac{1}{s+t+u+v}\left(\frac{s+t+u+v}{s+t+u} + \frac{s+t+u+v}{s+t+v} + \frac{s+t+u+v}{s+u+v} + \frac{s+t+u+v}{t+u+v}\right)$$
$$= \frac{1}{s+t+u+v}\left(\frac{s+t+u}{s+t+u} + \frac{v}{s+t+u} + \frac{s+t+u+v}{s+t+v} + \frac{s+t+u+v}{s+u+v} + \frac{s+t+u+v}{t+u+v}\right)$$
$$= \frac{1}{s+t+u+v}\left(1 + \frac{v}{s+t+u} + \frac{s+t+u+v}{s+t+v} + \frac{s+t+u+v}{s+u+v} + \frac{s+t+u+v}{t+u+v}\right)$$
$$= \frac{1}{s+t+u+v}\left(1 + \frac{v}{s+t+u} + 1 + \frac{u}{s+t+v} + 1 + \frac{t}{s+u+v} + 1 + \frac{s}{t+u+v}\right)$$

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$$\begin{aligned}
&\frac{1}{s+t+u} + \frac{1}{s+t+v} + \frac{1}{s+u+v} + \frac{1}{t+u+v} \\
&= \frac{1}{s+t+u+v}\left(\frac{s+t+u+v}{s+t+u} + \frac{s+t+u+v}{s+t+v} + \frac{s+t+u+v}{s+u+v} + \frac{s+t+u+v}{t+u+v}\right) \\
&= \frac{1}{s+t+u+v}\left(\frac{s+t+u}{s+t+u} + \frac{v}{s+t+u} + \frac{s+t+u+v}{s+t+v} + \frac{s+t+u+v}{s+u+v} + \frac{s+t+u+v}{t+u+v}\right) \\
&= \frac{1}{s+t+u+v}\left(1 + \frac{v}{s+t+u} + \frac{s+t+u+v}{s+t+v} + \frac{s+t+u+v}{s+u+v} + \frac{s+t+u+v}{t+u+v}\right) \\
&= \frac{1}{s+t+u+v}\left(1 + \frac{v}{s+t+u} + 1 + \frac{u}{s+t+v} + 1 + \frac{t}{s+u+v} + 1 + \frac{s}{t+u+v}\right) \\
&= \frac{1}{s+t+u+v}\left(4 + \frac{v}{s+t+u} + \frac{u}{s+t+v} + \frac{t}{s+u+v} + \frac{s}{t+u+v}\right)
\end{aligned}$$

Beweis

Seien $s, t, u, v > 0$ positive reelle Zahlen. Dann gilt:

$$
\begin{aligned}
&\frac{1}{s+t+u}+\frac{1}{s+t+v}+\frac{1}{s+u+v}+\frac{1}{t+u+v}\\
&=\frac{1}{s+t+u+v}\left(\frac{s+t+u+v}{s+t+u}+\frac{s+t+u+v}{s+t+v}+\frac{s+t+u+v}{s+u+v}+\frac{s+t+u+v}{t+u+v}\right)\\
&=\frac{1}{s+t+u+v}\left(\frac{s+t+u}{s+t+u}+\frac{v}{s+t+u}+\frac{s+t+u+v}{s+t+v}+\frac{s+t+u+v}{s+u+v}+\frac{s+t+u+v}{t+u+v}\right)\\
&=\frac{1}{s+t+u+v}\left(1+\frac{v}{s+t+u}+\frac{s+t+u+v}{s+t+v}+\frac{s+t+u+v}{s+u+v}+\frac{s+t+u+v}{t+u+v}\right)\\
&=\frac{1}{s+t+u+v}\left(1+\frac{v}{s+t+u}+1+\frac{u}{s+t+v}+1+\frac{t}{s+u+v}+1+\frac{s}{t+u+v}\right)\\
&=\frac{1}{s+t+u+v}\left(4+\frac{v}{s+t+u}+\frac{u}{s+t+v}+\frac{t}{s+u+v}+\frac{s}{t+u+v}\right)\\
&>\frac{4}{s+t+u+v}.
\end{aligned}
$$