Questions:

1. The differential equation:

   $\frac{dP}{dt} = P(a - bP)$

   is a well-known population model with a and b positive constants.

   a) What is the "carrying capacity" of the system? I.e., what is the maximum stable population? How do you determine that?

   b) What is the growth rate? I.e., a small population will grow in a manner very close to an exponential function. What is the "r" value of that exponential function?

   c) Suppose that a population is modelled by the differential equation:

   $\frac{dP}{dt} = P(aP - b)$

   instead (a and b are still positive constants). Discuss what happens to the population P as time t increases.

2. The autonomous differential equation:

   $m\frac{dv}{dt} = mg - kv$

   where k is a positive constant of proportionality and g is the acceleration due to gravity, is a model for the velocity v of a body of mass m that is falling under the influence of gravity. Because the term $-kv$ represents air resistance, the velocity of a body falling from a great height does not increase without bound as time t increases.

   a) Use a phase portrait of the differential equation to find the terminal velocity of the body. Be sure to explain your reasoning.

   b) Find the terminal velocity of the body if air resistance is proportional to v^2.