# Problem Set 5

1. Use the definition of convergence for a sequence to show that the following sequences converge to the given limit.

   a) $\lim \frac{1}{(6n^2+1)} = 0$

   b) $\lim \left(\frac{3n+1}{2n+5}\right) = \frac{3}{2}$

   c) $\lim \left(\frac{2}{\sqrt{n+3}}\right) = 0$

2. Suppose that for a given $\epsilon > 0$, we find an "$N$" value such that for a sequence $(a_n)$, $|a_n - a| < \epsilon$ $\forall n \geq N$.

   a) Will a larger $N$ also work for the same $\epsilon > 0$ or will a smaller $N$ work? Why?

   b) Will our $N$ value work for a smaller $\epsilon_2$? What about a larger $\epsilon_2$? Why?

3. Create a definition - similar to one we are already using - for a sequence that "converges to $\infty$".

   a) Does $(\sqrt{n})$ converge by your definition?

   b) What about $(n(-1)^n)$?

   c) What about $(1, 0, 2, 0, 3, 0, 4, 0, \ldots)$?

4. Defn: For $A \subseteq \mathbb{R}$:

   i) $(a_n)$ is eventually in $A$ if $\exists$ $N \in \mathbb{N}$ such that $a_n \in A$ $\forall$ $n \geq N$.

   ii) $(a_n)$ is frequently in $A$ if $\forall$ $N \in \mathbb{N}$, there exist $n \geq N$ such that $a_n \in A$.

   a) Which definition is "stronger"?

   b) Suppose an infinite number of terms of a sequence $(a_n)$ are equal to 2. Is $(x_n)$ eventually in $(1.9, 2.1)$? What about frequently?