Problem Set 1 (6/6) Artur

① $\frac{dP}{dt} = P(1-P)$

$P = \frac{ce^t}{1+ce^t}$

$\frac{dP}{dt} = \frac{ce^t(1+ce^t) - (ce^t)^2}{(1+ce^t)^2} = \frac{ce^t}{(1+ce^t)^2}$

$P(1-P) = \frac{ce^t}{1+ce^t} \cdot \left(1 - \frac{ce^t}{1+ce^t}\right) = \frac{ce^t}{1+ce^t} \cdot \frac{1+ce^t-ce^t}{1+ce^t} =$

$= \frac{ce^t}{(1+ce^t)^2}$ $\frac{dP}{dt} = P(1-P)$

Yes, it is a family of solutions ✓

② $y = e^{mx}$

a) $y' + 2y = 0$

$me^{mx} + 2e^{mx} = 0$

$e^{mx}(m+2) = 0$

$m = -2$ $y = e^{-2x}$ ✓

b) $y'' - 5y' + 6y = 0$

$m^2e^{mx} - 5me^{mx} + 6e^{mx} = 0$

$e^{mx}(m^2 - 5m + 6) = 0$

$m^2 - 5m + 6 = 0$ $y = e^{2x}, y = e^{3x}$

$m = 2, m = 3$ ✓

③ $y = x^m$

a) $xy'' + 2y' = 0$

$x \cdot m(m-1)x^{(m-2)} + 2mx^{(m-1)} = 0$

$m(m-1)x^{m-1} + 2mx^{m-1} = 0$ ✓

$mx^{m-1}(m+1) = 0$

$m = 0, \quad m = -1$ $y = x^0 = 1$ $y = x^{-1}$

b) $x^2 \cdot m(m-1)x^{m-2} - 7x \cdot m x^{m-1} + 15x^m = 0$

$m(m-1) \cdot x^m - 7m x^m + 15x^m = 0$

$x^m(m(m-1) - 7m + 15) = 0$

$m^2 - 8m + 15 = 0$

$m = 5 \quad \checkmark \quad m = 3$ $\checkmark$ $y = x^5$, $y = x^3$

(4)

Let $N(t)$ be radioactive material left after time

The amount $N$ is going to decay exponentially

$N(t) = Ce^{-kt}$

$N(0) = C$

Half time is 1600 years: $N(1600) = \frac{C}{2}$

$N(1600) = Ce^{K \cdot 1600} = \frac{C}{2}$

$e^{1600K} = \frac{1}{2}$

$-1600K = \ln\frac{1}{2}$

$1600K = \ln 2$

$K = \frac{\ln 2}{1600}$

$N(t) = N_0 \cdot e^{-\frac{\ln 2}{1600}t}$

$\frac{dN}{dt} = -N_0 \frac{\ln 2}{1600} \cdot e^{-\frac{\ln 2}{1600}t}$; $A = \frac{\ln 2}{1600}$

$\frac{dN}{dt} = -N_0 A e^{-At}$

$\checkmark$