Artur

Questions:

1. The right-circular conical tank shown in Figure 1.31 loses water out of a circular hole at it's bottom. Determine a differential equation for the height of the water h at a time t. The radius of the hole is 2 inches $g = 32\frac{ft}{s^2}$,and the friction/contraction factor is c= 0.6

$V = \frac{1}{3}\pi r^2 h \Rightarrow h = \frac{3V}{\pi r^2}$

$\frac{dh}{dt} = \frac{3}{\pi r^2} \cdot \frac{dV}{dt} = \frac{75}{4\pi h^2} \cdot \frac{dV}{dt}$

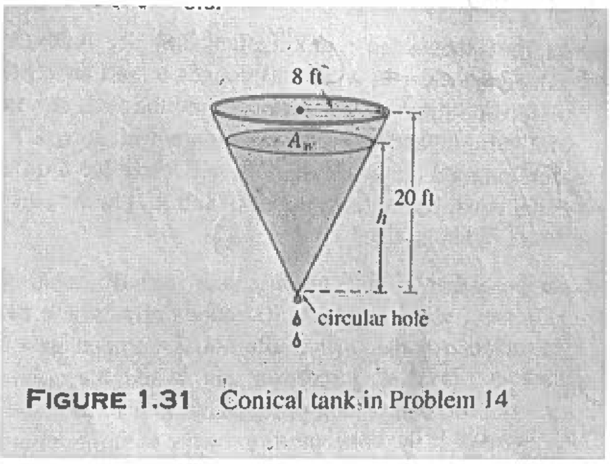

FIGURE 1.31 Conical tank in Problem 14

$\frac{r}{h} = \frac{8}{20}$ $r = \frac{2}{5}h$

$\frac{dV}{dt} = -cA_h\sqrt{2gh}$

$\frac{dh}{dt} = \frac{-75cA_h\sqrt{2gh}}{4\pi h^2} = -2.5h^{-\frac{3}{2}}$

$\int h^{\frac{3}{2}} dh = \int -2.5 dt$

$\frac{2h^{\frac{5}{2}}}{5} = -2.5t + C_1$

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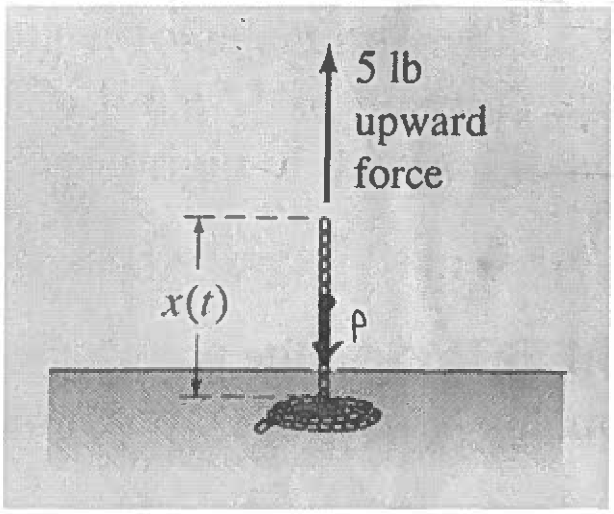
$h^{\frac{5}{2}} = -6.25t + C_1$

$h(t) = (-6.25t + C_1)^{\frac{2}{5}}$

2. A uniform 10-foot-long chain is coiled loosely on the ground. As shown below, one end of the chain is pulled vertically upward by means of a constant force of 5lbs. The chain weighs 1lb/ft. Determine a differential equation for the height h(x) of the end above ground level at a time t.



$F = 5 - P$

$P = x \cdot \rho = x \cdot 1 = x$ – weigh of the end above the

$F = 5 - x$

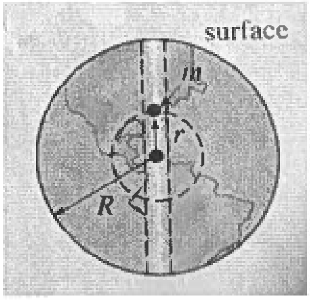
$ma = 5 - x$

$a = \frac{5-x}{m}$ $m = \frac{P}{g} = \frac{x}{g}$

$a = \frac{g(5-x)}{x}$

$$\boxed{\frac{d^2x}{dt^2} = \frac{32(5-x)}{x}}$$ ✓

3. Suppose a hole is drilled through the center of the earth and a bowling ball of mass m is dropped into the hole, as shown below. Construct a mathematical model that describes the motion og the ball. At time t, let r denote the distance from the center of the earth to the mass m, M denote the mass of the earth, M_r denote the mass of that portion of the earth within a sphere of radius r, and δ denote the constant density of the earth.



$F = G\frac{m_1 m_2}{r^2}$ $\quad F = G\frac{m M_r}{r^2}$

$M_r = V_r \cdot \delta = \frac{4}{3}\pi r^3 \cdot \delta$

$F = G\frac{4\pi m \delta r}{3}$; $\quad ma = G\frac{4\pi m \delta r}{3}$

$$\boxed{\frac{d^2 r}{dt^2} = -G\frac{4\pi\delta}{3}\cdot r}$$

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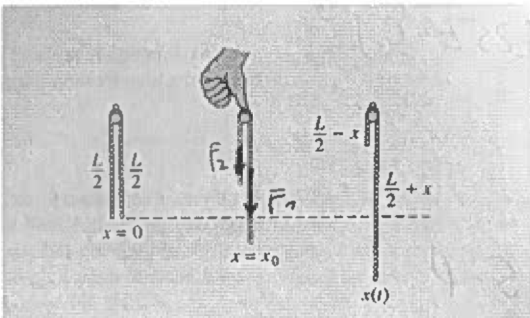
4. Suppose a uniform chain of length L with weight density ρ is draped over a friction-less peg as shown in the diagram, and is displaced by a length of x_0. Create a differential equation that models the displacement of the chain at a time t.



(a) equilibrium (b) chain held until $t = 0$ (c) motion for $t > 0$

$F = F_1 - F_2$

$F = \rho l_1 - \rho l_2 = \rho(l_1 - l_2)$

$F = \rho\left(\frac{L}{2} + x - \left(\frac{L}{2} - x\right)\right) = 2\rho x$

$ma = 2\rho x$

$a = \frac{2\rho x}{m}$ $\quad // m = \frac{\rho}{g} = \frac{L\rho}{g}$

$$\boxed{\frac{d^2 x}{dt} = \frac{2g}{L}\cdot x}$$

✓