Week 4 Problem Set Artur Savchii

1. $\frac{dy}{dx} = \frac{7x^2}{y^3}$ $y(3) = 2$

$\int 7x^2 dx = \int y^3 dy$

$\frac{7x^3}{3} + C = \frac{y^4}{4}$

$y^4 = \frac{28x^3}{3} + C$

$y = \pm\sqrt[4]{\frac{28x^3}{3} + C}$

$y(3) = \sqrt[4]{28 \cdot 9 + C} = 2$

$252 + C = 16$

$C = -236$

$$y = \pm\sqrt[4]{\frac{28x^3}{3} - 236}$$

2. $\frac{dy}{dx} = \frac{e^x}{y^2}$ $y(0) = 1$

$\int y^2 dy = \int e^x dx$

$\frac{y^3}{3} = e^x + C$

$y^3 = 3e^x + C$

$y = \sqrt[3]{3e^x + C}$

$y(0) = \sqrt[3]{3e^0 + C} = 1$

$\sqrt[3]{3 + C} = 1$

$3 + C = 1$

$C = -2$

$$y = \sqrt[3]{3e^x - 2}$$

3. $\frac{dy}{dx} = \frac{\sin(x)}{\cos(y)}$ $y(0) = \frac{3\pi}{2}$

$\int \cos(y) dy = \int \sin(x) dx$

$\sin(y) = -\cos(x) + C$

$y = \arcsin(C - \cos(x))$

$y(0) = \arcsin(C - \cos(0)) = \frac{3\pi}{2}$

$\arcsin(C - 1) = \frac{3\pi}{2}$

$C - 1 = -1$

$C = 0$

$$y = \arcsin(-\cos(x))$$

4. $P(t) = P_0 e^{rt}$

$P(0) = 4000$

$P(0) = P_0 e^{r\cdot 0} = 4000$

$P_0 = 4000$

$P(3) = 6500$

$P(3) = 4000 e^{3r} = 6500$

$e^{3r} = 1625$

$e^{3r} = e^{\ln(1.625)}$

$3r = \ln(1.625)$

$3r = 0.4855$

$r = 0.1619$

$P(t) = 4000 e^{0.1619t}$

$P(10) = 4000 e^{1.619} = 20192$

5. $h = 0.25$ $\quad y(1) = ?$ $\quad y' = y - x$ $\quad y(0) = 2$

x	y
0	2
0.25	2.5
0.5	3.0625
0.75	3.7031
1	4.4414

$y_n = y_{n-1} + h y'(x_{n-1}, y_{n-1})$

$= 2 + 0.25(2-0) = 2.5$

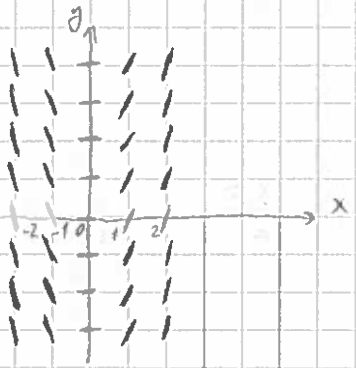
$2.5 + 0.25(2.5 - 0.25) = 3.0625$

$3.0625 + 0.25(3.0625 - 0.5) = 3.7031$

$3.7031 + 0.25(3.7031 - 0.75) =$

$y(1) \approx 4.4414$

6. $\frac{dy}{dx} = 2x$



7. $\frac{dx}{dt} = k(1000 - x)x$

$\int \frac{dx}{(1000-x)x} = \int k\, dt$

$\frac{A}{1000-x} + \frac{B}{x} = \frac{1}{(1000-x)x}$

$Ax + 1000B - Bx = 1$

$\begin{cases} A - B = 0 \\ 1000B = 1 \end{cases}$ $\quad A = B = 0.001$

$$\int \frac{1}{1000-x}dx + \int \frac{1}{x}dx = k\int dt$$

$$0.001(\ln(x) - \ln(1000-x)) = kt + C$$

$$0.001\left(\ln\frac{x}{1000-x}\right) = kt + C$$

$$0.001\left(\ln\frac{1000-x}{x}\right) = -kt + C$$

$$\ln\frac{1000-x}{x} = -1000kt + C$$

$$\frac{1000-x}{x} = e^{(-1000kt+C)} = e^{C}\cdot e^{-1000kt} = Ae^{-1000kt}$$

$$\frac{1000}{x} - 1 = Ae^{-1000kt}$$

$$\frac{1000}{x} = Ae^{-1000kt} + 1$$

$$x = \frac{1000}{Ae^{-1000kt}+1}$$

$x(4) = 50$

$x(0) = 1$

$$\frac{1000}{Ae^{-4000k}+1} = 50$$

$$\frac{1000}{Ae^{0}+1} = 1$$

$$\Rightarrow Ae^{0}+1 = 1000$$

$$A = 999$$

$$Ae^{-4000k} + 1 = 20$$

$$999e^{-4000k} = 19$$

$$e^{-4000k} = 0.019$$

$$-4000k = \ln(0.019)$$

$$-4000k = -3.9633$$

$$k = 0.000991$$

$$x(t) = \frac{1000}{999e^{-0.991t}+1}$$

$$x(6) = \frac{1000}{999e^{-0.991\cdot 6}+1} = 275.5$$

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