Problem Set 2 (6.7/7) Artur Savchii

(1) $T_0 \approx 170$ (from the graph) ✓

$T_m \approx 80$ (from the graph: over the time, when ✓ $T = T_m$, $\frac{dT}{dt} = 0 \Rightarrow$ horizontal line, no change)

$$\frac{dT}{dt} = k(T - T_m)$$

$$\int \frac{1}{T - T_m}\, dT = \int k\, dt$$

$$\ln(T - T_m) = kt + c$$

$$T - T_m = e^{kt+c} = Ce^{kt}$$

$$T(t) = Ce^{kt} + T_m$$

$$T(0) = C + T_m \Rightarrow C = T(0) - T_m = 170 - 80 = 90$$

$$k = \frac{\ln\left(\frac{T - T_m}{C}\right)}{t}$$

$T(15) = 100$ (a point from the graph)

$$k = \frac{\ln\left(\frac{100 - 80}{90}\right)}{15} \approx -0.1 \checkmark$$

(2) From the graph we can see that $T_m(t)$ is a negative cos function shifted 80 units up, horizontally streched by $\frac{\pi}{12}$, and veritically by 30.

$$T_m(t) = -30\cos\left(\frac{t\pi}{12}\right) + 80$$

The body temperature rate of change is proportional to the difference of the ~~body~~ ambient temperature and the body temperature

$$\frac{dT}{dt} = k\left(-30\cos\left(\frac{t\pi}{12}\right) + 80 - T\right), \text{ where } k > 0 \checkmark$$

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$$\frac{dA}{dt} = \text{salt pumped into} - \text{salt pumped out}$$

salt pumped into = rate · proportion of salt =

$= 3 \cdot 0 = 0$

salt pumped out = rate · proportion of salt =

$= 3 \cdot \frac{A}{300} = \frac{A}{100}$

$$\frac{dA}{dt} = -\frac{A}{100}$$

✓

$$\int \frac{1}{A}\, dA = -\int \frac{1}{100}\, dt$$

$\ln|A| = -\frac{t}{100} + C$ , A is nonnegative

$$A = e^{-\frac{t}{100} + C} = Ce^{-\frac{t}{100}}$$

$A(0) = 50$

$$A(0) = Ce^{-\frac{0}{100}} = C = 50$$

$$A(t) = 50e^{-\frac{t}{100}}$$

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salt pumped into = rate · proportion of salt =

$= 3 \cdot 0.24 = 0.72$

salt pumped out $= 2 \cdot \frac{A}{300 + (3t - 2t)} = \frac{2A}{300 + t}$

$$\frac{dA}{dt} = \overset{6}{0.\cancel{72}} - \frac{2A}{300+t}$$

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$$\frac{dA}{dt} = r_{in} \cdot C_{in} - r_{out} \cdot \frac{A}{N_0 + (r_{in} - r_{out})t}$$

✓