Assignment 3
1D beams: Wing structure

Consider the three-cell wing section, modelled as a beam with the two vertical spars webs and skin depicted in Figure 1a. Only the intermediate cell, highlighted in black, is effective to carry the different structural loads. The Young Modulus of the spar webs and the skin is 85 GPa. The aircraft flies at constant cruise speed maintaining its altitude. In such conditions, the spar must support weight and aerodynamic loads, as depicted in Figure 1b, with the following distribution functions: depicted in Figure 1a

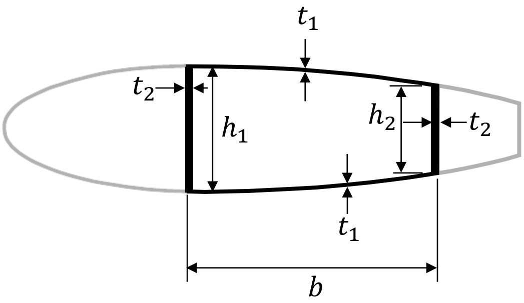
- Mass density distribution:

$$\lambda(x) = \begin{cases} \dfrac{M}{4(L_1+L_2)} + \dfrac{3M}{2L_2^2}(L_1 - x) & \text{for } x < L_1 \\ \dfrac{M}{4(L_1+L_2)} & \text{for } x \geq L_1 \end{cases}$$

- Lift distribution:

$$q(x) = \begin{cases} \ell\left[0.8 - 0.2\cos\left(\dfrac{\pi x}{L_1}\right)\right] & \text{for } x < L_1 \\ \ell\left(1 - \dfrac{x - L_1}{L_2}\right)\left(1 + \dfrac{x - L_1}{L_2}\right) & \text{for } x \geq L_1 \end{cases}$$

with $L_1 = 5$ m, $L_2 = 10$ m, and $M = 35000$ kg. The mass of the engine ($M_e = 2550$ kg) is taken into account as a point load at $x = L_1$, contributing to the total weight. Note: use $g = 9.81$ m/s^2 for the calculations.



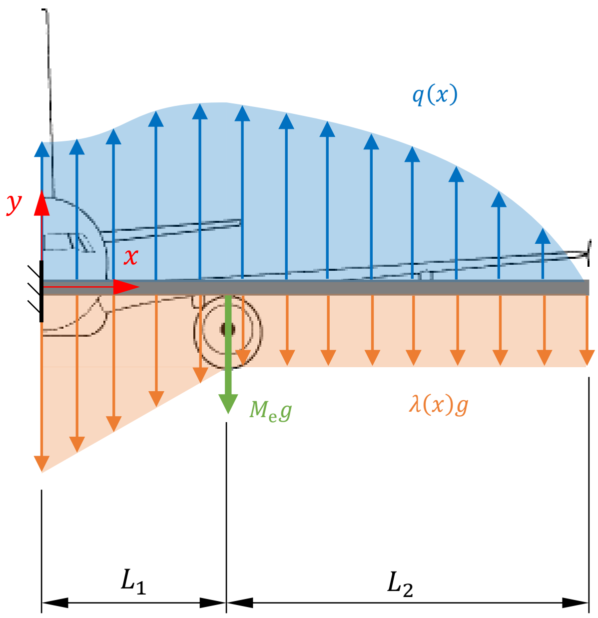


Figure 1. (a) Geometrical parameters of the wing cross section, (b) Load distribution along the wing.

In such scenario, the following is asked:

A) Compute the centroid and the cross-section inertia I_{zz}, for $t_1 = 1.5$ mm, $t_2 = 4$ mm, $h_1 = 500$ mm, $h_2 = 250$ mm, and $b = 775$ mm (see Figure 1a).

B) Compute the parameter ℓ that makes the total lift compensate the total weight.

C) Following the "ALG_1DBEAM" guide, implement a MATLAB® code to numerically compute the following magnitudes' distributions along the spar:

1. Deflection, $u_y(x)$.
2. Section rotation, $\theta_z(x)$.
3. Shear force, $F_y(x)$.
4. Bending moment, $M_z(x)$.

D) Study the convergence of the numerical solution for different element size in the beam's discretization, $h_{\text{el}} = \{5, 2.5, 1.25, 0.625, 0.3125\}$. To do so, compute the relative error of the deflection at the wing tip, $u_y^{(n_{\text{el}})}|_{x=L_1+L_2}$, and compare it to the exact solution, $u_y^*|_{x=L_1+L_2}$ (for this purpose, we can assume the solution for a large number of elements, for instance $h_{\text{el}} = 0.15625$, as the "exact" solution):

$$\epsilon_r(h_{\text{el}}) = \left| \frac{u_y^{(h_{\text{el}})}|_{x=L_1+L_2} - u_y^*|_{x=L_1+L_2}}{u_y^*|_{x=L_1+L_2}} \right|$$

Hint: use a logarithmic scale for the element size axis in the plot.

E) Applying the Von Mises criterion, locate the most solicitated position in the interior of the wing. To do this, take into account the normal and shear stresses distributions generated in each cross section of the wing.

The assignment can be done in groups of maximum 2 people. Only one of the members must submit a compressed ZIP file to Atenea containing the following:

- All the MATLAB® script files.
- A report with the following information:
  - Names of the group members.
  - Requested results for parts A and B.
  - For part C, plots of the deflection, section rotation, shear force and bending moment for <u>different number of elements</u>.
  - For part D, a <u>logarithmic plot</u> of the relative error ϵ_r.
  - For part E, brief description of the procedure and the requested results.