Laboratorio de Microeconomía I

Tarea 3

Centro de Investigación y Docencia Económicas
Maestría en Economía
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Definition 3.B.6 (Mas-Colell et al.): A monotone preference relation $\succsim$ on $X = \mathbb{R}^L_+$ is *homothetic* if all indifference sets are related by proportional expansion along rays; that is, if $x \sim y$, then $\alpha x \sim \alpha y$ for any $\alpha \geq 0$.

Definition 3.B.7 (Mas-Colell et al.): The preference relation $\succsim$ on $X = (-\infty, \infty) \times \mathbb{R}^{L-1}_+$ is *quasilinear* with respect to commodity 1 (called, in this case, the *numeraire commodity*) if

(i) All the indifference sets are parallel displacements of each other along the axis of commodity 1. That is, if $x \sim y$, then $(x + \alpha e_1) \sim (y + \alpha e_1)$ for $e_1 = (1, 0, \ldots, 0)$ and any $\alpha \in \mathbb{R}$.

(ii) Good 1 is desirable; that is, $x + \alpha e_1 \succ x$ for all x and $\alpha > 0$.

Proposition 3.C.1 (Mas-Colell et al.): Suppose that the rational preference relation $\succsim$ on X is continuous. Then there is a continuous utility function $u(x)$ that represents $\succsim$.

Exercises

Ex. 1.6 Cite a credible example where the preferences of an 'ordinary consumer' would be unlikely to satisfy the axiom of convexity.

Ex. 1.7 Prove that under Axiom 5', the set $\succsim (\mathbf{x^0})$ is a convex set for any $x^0 \in X$.

Ex. 1.8 Sketch a map of indifference sets that are all parallel, negatively sloped straight lines, with preference increasing north-easterly. We know that preferences such as these satisfy Axioms 1, 2, 3, and 4. Prove that they also satisfy Axiom 5'. Prove that they do not satisfy Axiom 5.

Ex. 1.9 Sketch a map of indifference sets that are all parallel right angles that 'kink' on the line $x_1 = x_2$. If preference increases north-easterly, these preferences will satisfy Axioms 1, 2, 3, and 4'. Prove that they also satisfy Axiom 5'. Do they satisfy Axiom 4? Do they satisfy Axiom 5?

Ex. 1.10 Sketch a set of preferences that satisfy Axioms 1, 2, 3, and 4, whose indifference sets are convex to the origin in some places and contain 'linear segments' in others. Prove that preferences such as these are consistent with Axiom 5', but violate Axiom 5.

Ex. 3.C.5 Establish the following two results:

(a) A continuous $\succsim$ is homothetic if and only if it admits a utility function $u(x)$ that is homogeneous of degree one; i.e., $u(\alpha x) = \alpha u(x)$ for all $\alpha > 0$.

(b) A continuous $\succsim$ on $(-\infty, \infty) \times \mathbb{R}_+^{L-1}$ is quasilinear with respect to the first commodity if and only if it admits a utility function $u(x)$ of the form

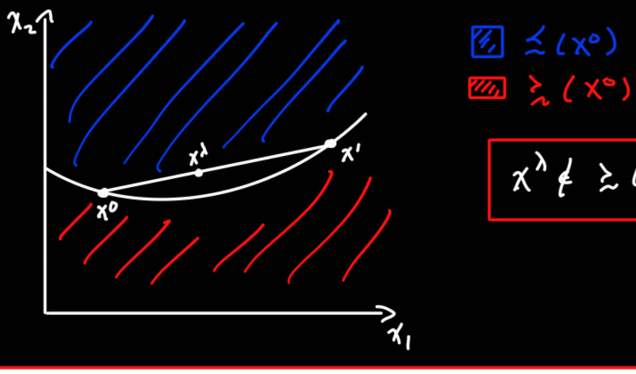
$$u(x) = x_1 + \phi(x_2, \cdots, x_L).$$

[Hint: The existence of some continuous utility representation is guaranteed by Proposition 3.C.1.]

Ex (Opcional). 3.C.2 Show that if $u(\cdot)$ is a continuous utility function representing $\succsim$, then $\succsim$ is continuous.

Ex. 1.6 Cite a credible example where the preferences of an 'ordinary consumer' would be unlikely to satisfy the axiom of convexity.

Preferencias donde x_1 es un bien y x_2 es un mal "numerario".



$x^\lambda \notin \succsim(x^0) \therefore \succsim(x^0)$ no es convexo

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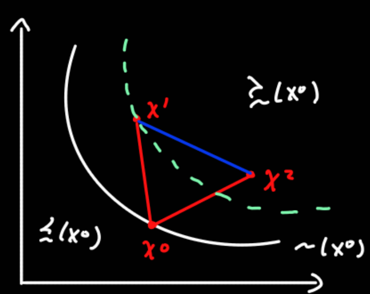
Ex. 1.7 Prove that under Axiom 5', the set $\succsim(\mathbf{x^0})$ is a convex set for any $x^0 \in X$.

Axioma 5': Si $x^1 \succsim x^0$, entonces $\lambda x^1 + (1-\lambda)x^0 \succsim x^0 \ \forall \lambda \in [0,1]$.

Idea:



Demostración:

Sean $x^0 \in X$ y $x^1, x^2 \in \succsim(x^0)$. Entonces, $x^1 \succsim x^0$ & $x^2 \succsim x^0$.

Supongamos que se cumple el Axioma 5', i.e.:

$\lambda x^1 + (1-\lambda)x^0 \succsim x^0$ & $\lambda x^2 + (1-\lambda)x^0 \succsim x^0 \quad \forall \lambda \in [0,1]$.

Queremos demostrar que $\lambda x^1 + (1-\lambda)x^2 \succsim x^0, \forall t \in [0,1]$

Por completitud, sabemos que $x^1 \succsim x^2$ o $x^2 \succsim x^1$ o ambos. Entonces:

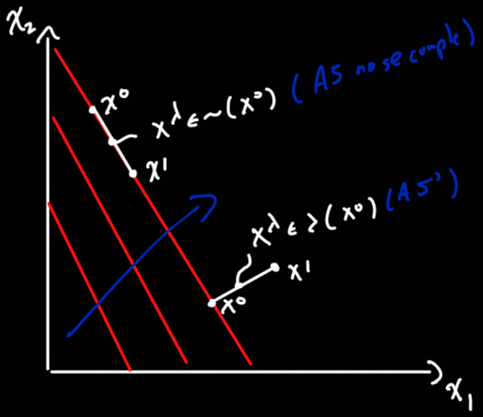
- Si $x^1 \succsim x^2$, por el axioma 5' $\lambda x^1 + (1-\lambda)x^2 \succsim x^2 \ \forall \lambda \in [0,1]$. Por transitividad de $\succsim$, esto implica que $\lambda x^1 + (1-\lambda)x^2 \succsim x^0$ porque $x^2 \succsim x^0$.
- Si $x^2 \succsim x^1$, por el axioma 5' $\lambda x^2 + (1-\lambda)x^1 \succsim x^1 \ \forall \lambda \in [0,1]$. Por transitividad de $\succsim$, esto implica que $\lambda x^2 + (1-\lambda)x^1 \succsim x^0$ porque $x^1 \succsim x^0$.
- Si $x^1 \sim x^2$, por los casos anteriores $\lambda x^1 + (1-\lambda)x^2 \succsim x^0$ & $\lambda x^2 + (1-\lambda)x^1 \succsim x^0 \ \forall \lambda \in [0,1]$.

Por lo tanto, $\forall \lambda \in [0,1]$ $\lambda x^1 + (1-\lambda)x^2 \succsim x^0$ o $\lambda x^2 + (1-\lambda)x^1 \succsim x^0$ o ambas, lo cual implica que $\succsim(x^0)$ es un conjunto convexo.

Ex. 1.8 Sketch a map of indifference sets that are all parallel, negatively sloped straight lines, with preference increasing north-easterly. We know that preferences such as these satisfy Axioms 1, 2, 3, and 4. Prove that they also satisfy Axiom 5'. Prove that they do not satisfy Axiom 5.

Axioma 5': Si $x' \succsim x^0$, entonces $\lambda x' + (1-\lambda) x^0 \succsim x^0 \quad \forall \lambda \in [0,1]$.

Axioma 5 : Si $x' \neq x^0$ y $x' \succsim x^0$, entonces $\lambda x' + (1-\lambda) x^0 \succ x^0 \quad \forall \lambda \in [0,1]$.



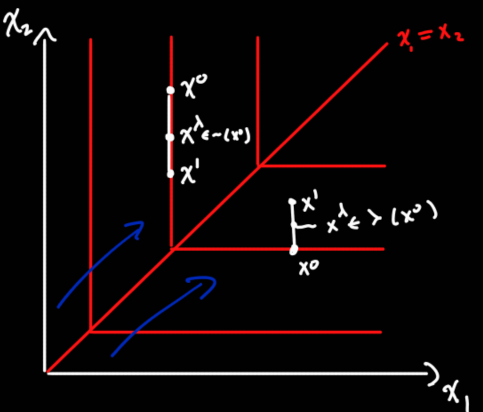
Demostración: (Contraejemplo para Axioma 5)

Sea $x^0 \in X$ y $x' \in \sim(x^0)$ con $x' \neq x^0$. Entonces $x' \succsim x^0$, $x' \neq x^0$.

Queremos demostrar que $\lambda x' + (1-\lambda) x^0 \succsim x^0 \quad \forall \lambda \in [0,1]$, pero $\neg(\lambda x' + (1-\lambda) x^0 \succ x^0)$.

Por el tipo de curvas de indiferencia, vemos que el segmento de recta que conecta a x' con x^0 pertenece a la recta de indiferencia. Todos los puntos en este segmento son: $x^\lambda = \lambda x' + (1-\lambda) x^0 \quad \forall \lambda \in [0,1]$. Es decir, $x^\lambda \in \sim(x^0) \quad \forall \lambda \in [0,1]$, lo cual implica que $x^\lambda \succsim x^0$ & $x^0 \succsim x^\lambda$. Entonces, $x^\lambda \succsim x^0$ pero $\neg(x^\lambda \succ x^0)$ lo que demuestra que $\succsim$ no es estrictamente convexa.

Ex. 1.9 Sketch a map of indifference sets that are all parallel right angles that 'kink' on the line $x_1 = x_2$. If preference increases north-easterly, these preferences will satisfy Axioms 1, 2, 3, and 4'. Prove that they also satisfy Axiom 5'. Do they satisfy Axiom 4? Do they satisfy Axiom 5?



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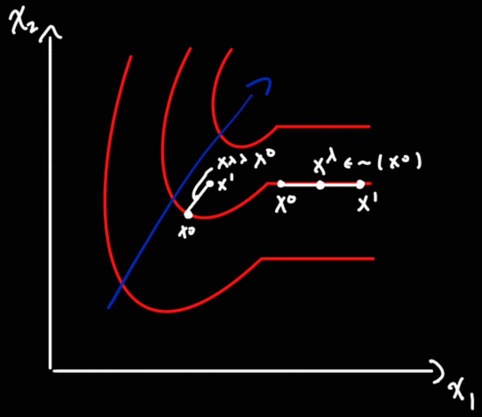
Por otro lado, vemos que si $x' \geq x^0$, entonces $x' \succsim x^0$, mientras que si $x' \gg 0$, entonces $x' \succ x^0$, por lo que $\succsim$ cumple con el Axioma 4.

Ex. 1.10 Sketch a set of preferences that satisfy Axioms 1, 2, 3, and 4, whose indifference sets are convex to the origin in some places and contain 'linear segments' in others. Prove that preferences such as these are consistent with Axiom 5', but violate Axiom 5.



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