Exam 2 Solutions

1.) For bottom: $d\vec{A} = dx\,dz(-\hat{j})$

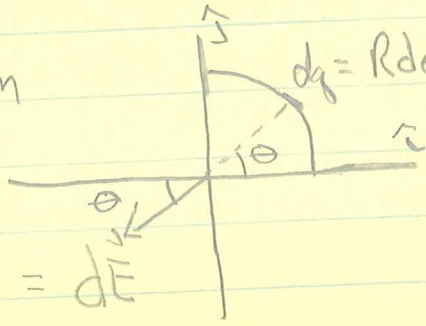
$$\Phi_{bottom} = \int_{bottom} \vec{E}\cdot d\vec{A} = \int (8\hat{i} + 2x\hat{j})\cdot(dx\,dz(-\hat{j}))$$

$$= -\int 2x\,dx\,dz = -\int_2^5 2x\,dx \int_0^3 dz$$

$$= -6\int_2^5 x\,dx = -6\left(\frac{x^2}{2}\right)\Big|_2^5$$

$$= -63\ \frac{Nm^2}{C}$$ Ans. is magnitude.

2.) $\lambda = 5*10^{-9}\ C/m$



$\frac{2\pi R}{4} = 2$ meters

$R = \frac{4}{\pi}$

$dE_x\hat{i} + dE_y\hat{j} = d\vec{E}$

From symmetry we see that $|dE_x| = |dE_y|$ and that the field will be 45° below the $(-\hat{i})$ axis.

$dE_x = -|dE|\cos(\theta)$ ☑

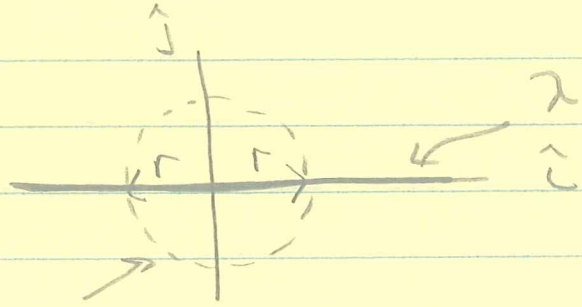
$$\therefore\ E_x = -\int \frac{k\,dq}{R^2}\cos(\theta) = -k\lambda\int_0^{\pi/2} \frac{R\,d\theta\cos\theta}{R^2}$$

$$= -\frac{k\lambda}{R}\int_0^{\pi/2}\cos(\theta)\,d\theta = -\frac{k\lambda}{R}(\sin\theta)\Big|_0^{\pi/2} = -\frac{k}{R}$$

$$= -\frac{k\pi\lambda}{4} = -35.29\ N/C$$

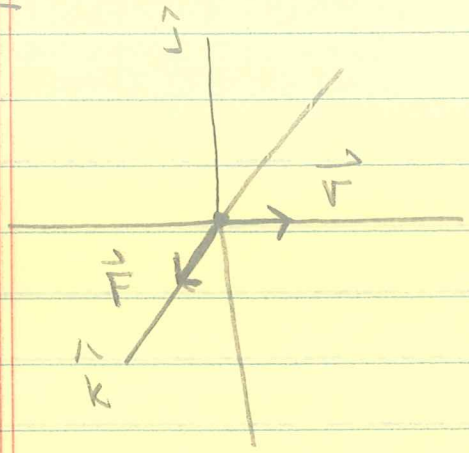
So $\vec{E}_{@\text{origin}} = -35.29\hat{i} - 35.29\hat{j}$

$|\vec{E}_{@\text{origin}}| = 49.9\ N/C$ Answer.

3.) 

Sphere w/ r = 0.05 meters

$\oint \vec{E}\cdot d\vec{A} = \Phi_{sphere} = \frac{q_{enclosed}}{\varepsilon_0} = \frac{(2*r)*\lambda}{\varepsilon_0} = 45.2\ \frac{N\cdot m^2}{C}$ Answer.

4.) 

$\vec{F} = q\vec{v}\times\vec{B}$

∴ $\vec{B}$ must be in the x-y plane.

Since $B_x = 0$, $\vec{B}$ would be in the $+\hat{j}$ direction for a positive charge. $-\hat{j}$ for negative charge.

Answer

5.) $B = \frac{\mu_0 I}{2\pi r} = 5.3*10^{-6}\ T$

6.) $\vec{V} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ 0 & B_y & B_z \end{vmatrix} = +\hat{i}(4B_z - 6B_y)$

$-\hat{j}(2B_z)$

$+\hat{k}(2B_y)$

$\vec{F} = q\vec{V} \times \vec{B}$ (2c)

$\hat{i}$ $F_x = q(4B_z - 6B_y) \Rightarrow 4$

$\hat{j}$ $F_y = q2B_z \Rightarrow -20$

$\hat{k}$ $F_z = q2B_y \Rightarrow 12$

"y-component" is in $\hat{k}$ eqtn.

$\hat{k} \Rightarrow 2qB_y = 12$

$B_y = \frac{12}{4} = 3\ T$

$\leq$ Ans.

7.) $P_{crockpot} = 500 = I_{crockpot}(240)$

$\therefore I_{crockpot} = 2.1A$ $\quad R_{crockpot} = \frac{240}{2.1} = 114\ \Omega$

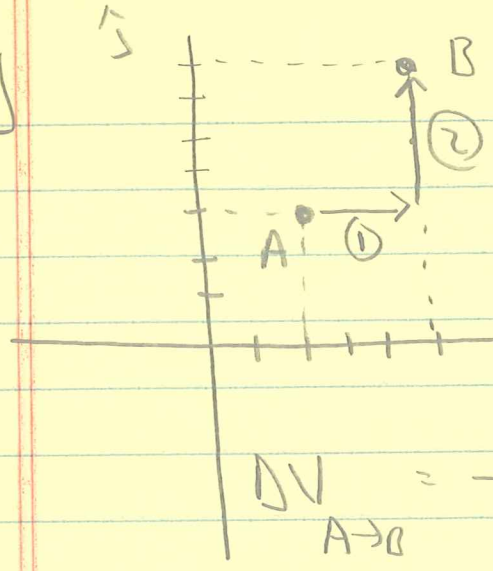
$P_{kettle} = 1000 = I_{kettle}(240)$

$\therefore I_{kettle} = 4.2A$ $\quad R_{kettle} = \frac{240}{4.2} = 57\ \Omega$

$I_{kettle} > I_{crockpot}$

$R_{kettle} < R_{crockpot}$ Ans $\leq$

8.)



$\vec{E} = 4\hat{i} + 3\hat{j}$

$$\Delta V_{A\to B} = -\int \vec{E}\cdot d\vec{s} = -\left[\int_{①} \vec{E}\cdot d\vec{s} + \int_{②} \vec{E}\cdot d\vec{s}\right]$$

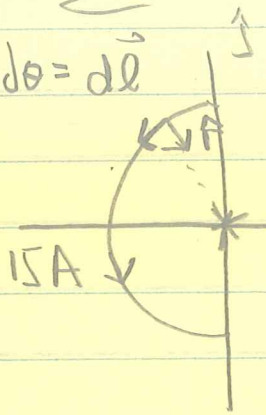
$d\vec{s}$ on ① $= dx\,\hat{i}$; $d\vec{s}$ on ② $= dy\,\hat{j}$

$$\Delta V = -\left[\int_2^5 4\,dx + \int_3^7 3\,dy\right]$$

$$= -\left[4(5-2) + 3(7-3)\right] = \underline{-24 \text{ volts}}$$

Ans.: 24 volts w/ A @ the higher voltage.

9.)



$\pi R = 3$

$\therefore R = 3/\pi$

$$d\vec{B} = \frac{\mu_0 I}{4\pi r^2}\, d\vec{\ell} \times \hat{r}$$

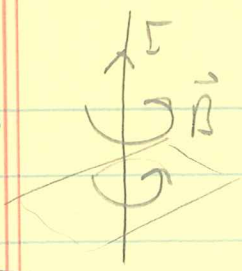
$$dB = \frac{\mu_0 I}{4\pi R^2}\, |d\vec{\ell}|\,|\hat{r}|\sin(90) \quad \boxed{\text{OUT}}$$

$$\therefore B = \frac{\mu_0 I}{4\pi R^2}\int_0^{\pi} R\,d\theta = \frac{\mu_0 I R}{4\pi R^2}\underbrace{\int_0^{\pi} d\theta}_{\pi}$$

$$= \frac{\mu_0 I}{4R} = 4.93\times10^{-6} \text{ tesla} \quad \boxed{\text{OUT}}$$

Answer ⑨

10)



"circles w/ centers on the wire and lie in planes ⊥ to wire."

11)

$$\text{Current density} = \frac{60\ A}{(0.03 \times 0.03)} = 66.7 \times 10^3 \ A/m^2$$

Ampere's Law around dotted square path shown (sides 1.5cm x 1.5cm)

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enclosed}$$

Asked for this!!

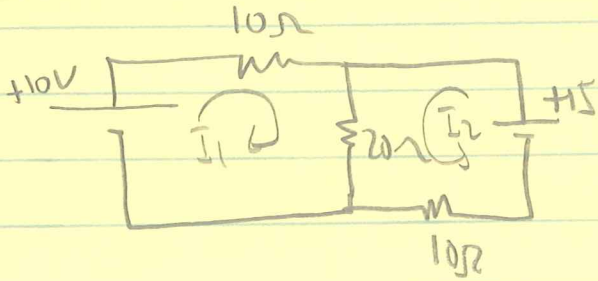
$$I_{enclosed} = (\text{Area}_{enclosed}) * (\text{current density})$$

$$= \mu_0 \left[(0.015 * 0.015) * (66.7 \times 10^3) \right]$$

$$= 1.88 \times 10^{-5}\ T \cdot m$$

Answer.

12)



loop 1: $+10 - 10I_1 - 20I_1 + 20I_2 = 0$

(1) $-30I_1 - 20I_2 = -10$

loop 2: $+15 - 20I_2 - 20I_1 - 10I_2 = 0$

(2) $-20I_1 - 30I_2 = -15$

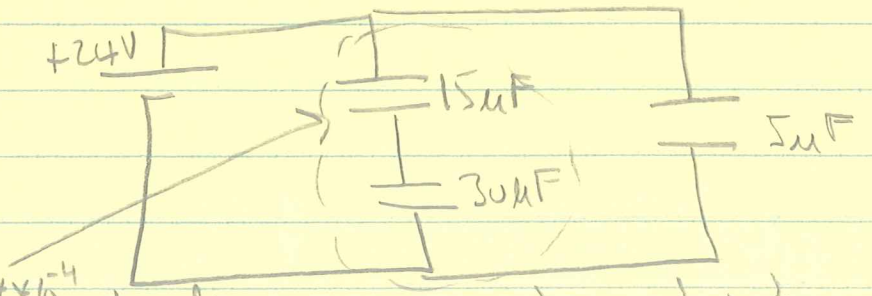
Consider: $2*① - 3*②$

$-40I_2 + 90I_2 = -20 + 45$

$\therefore I_2 = 0.5\,A$

① $\Rightarrow$ $I_1 = \frac{-20I_2 + 10}{30} = 0\,A$ Answer ☺

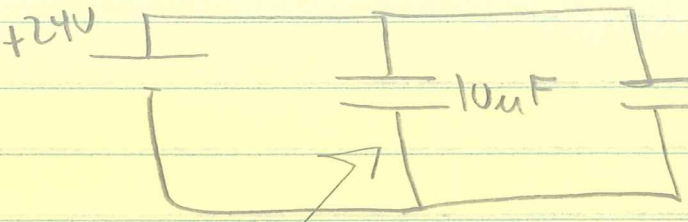
13.)



$V_{15\mu F} = \frac{2.4\times10^{-4}}{15\times10^{-6}} = 16\,volts$ Answer.

$\frac{1}{C} = \frac{1}{15} + \frac{1}{30}$

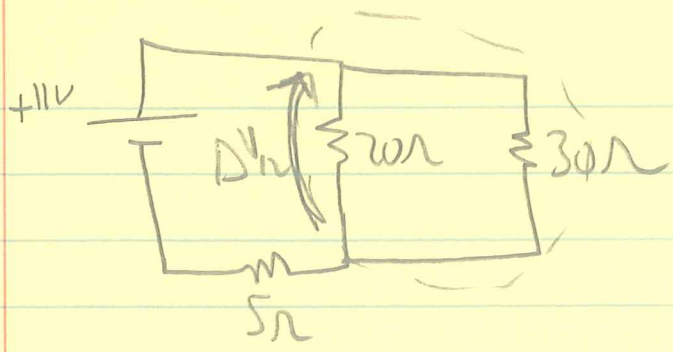
$\therefore C = 10\,\mu F$



don't need to go one ☺

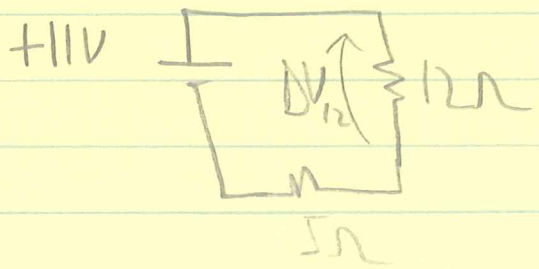
$Q = 10\times10^{-6} * 24$

$Q = 2.4\times10^{-4}\,C$

14) 

$\frac{1}{R} = \frac{1}{20} + \frac{1}{30}$

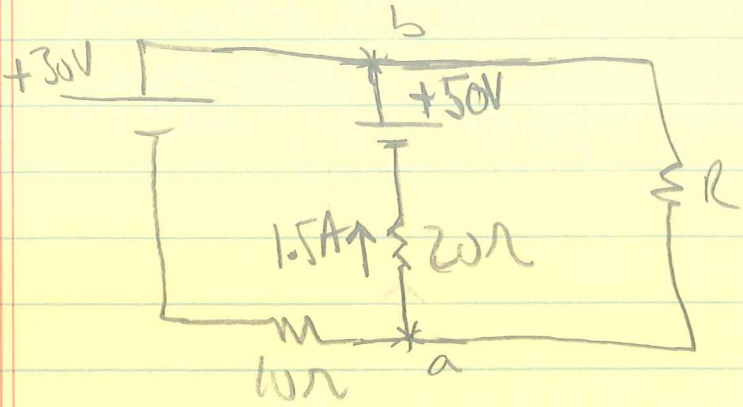
$\therefore R = 12\Omega$



$I = \frac{V}{R} = \frac{11}{12+5} = 0.647A$

$\therefore \Delta V_{12\Omega} = 0.647 * 12 = 7.76$ Volts

Answer (1)

15) 

$\Delta V_{a \to b} = -1.5(20) + 50 = +20$ Volts

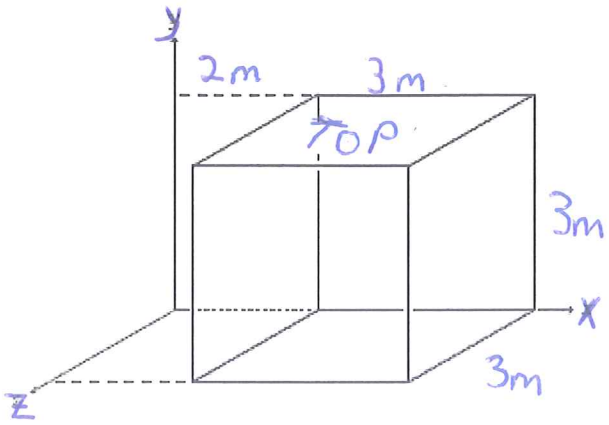
Ans: 20V with the top of R @ the higher voltage

ID: A Name: _____

Exam 2

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. The electric field in the region of space shown is given by $\boldsymbol{E} = (8\mathbf{i} + 2x\mathbf{j})$ N/C where x is in meters. What is the magnitude of the electric flux through the bottom face of the cube shown?



a. 126 N · m²/C
b. 54 N · m²/C
c. 90 N · m²/C
d. 12 N · m²/C
e. 63 N · m²/C

____ 2. A uniformly charged rod (length = 2.0 m, charge per unit length = 5.0 nC/m) is bent to form one quadrant of a circle. What is the magnitude of the electric field at the center of the circle?
NOTES: The radius of the arc is $4/\pi$. Recall that $S = R\theta$.

a. 25 N/C
b. 62 N/C
c. 44 N/C
d. 56 N/C
e. 50 N/C

____ 3. A uniform linear charge with a density of 4.0 nC/m is distributed along the entire x axis. Consider a spherical surface with a radius of 5.0 cm whose center is at the origin. What is the electric flux through this surface?

a. 45 N · m²/C
b. 23 N · m²/C
c. 79 N · m²/C
d. 62 N · m²/C
e. 68 N · m²/C

electron

____ 4. At one instant in time, an ✓ moving in the positive x direction experiences a magnetic force in the positive z direction. If $B_x = 0$ what is the direction of the magnetic field?
HINT: Draw the picture. No determinant is necessary.

a. positive z direction
b. negative y direction
c. negative x direction
d. negative z direction
e. positive y direction

____ 5. A long straight wire carries a current of 40 A. What is the magnitude of the magnetic field 1.5 meters from the axis of the wire?

a. 3.0×10^{-6} T
b. 8.0×10^{-6} T
c. 5.3×10^{-6} T
d. 12×10^{-6} T
e. 7.4×10^{-6} T

____ 6. A 2.0 C charge moves with a velocity of $(2.0\mathbf{i} + 4.0\mathbf{j} + 6.0\mathbf{k})$ m/s and experiences a magnetic force of $(4.0\mathbf{i} - 20\mathbf{j} + 12\mathbf{k})$ N. The x component of the magnetic field is equal to zero. Determine the y component of the magnetic field.
HINT: Using what is given, expand the determinant for the vector cross product. Then look at the vector components of the relevent equation.

a. +3.0 T
b. –3.0 T
c. +5.0 T
d. +6.0 T
e. –5.0 T

____ 7. Appliances are often rated in terms of the power they consume. A cook plugs a 500 watt crockpot and a 1000 watt tea kettle into a 240 V power supply. Treat the crockpot and the kettle as two resistors in parallel, each connected to the power supply. When we compare the two devices, we find that _____.

a. $I_{crockpot} < I_{kettle}$ and $R_{crockpot} > R_{kettle}$.
b. $I_{crockpot} = I_{kettle}$ and $R_{crockpot} = R_{kettle}$.
c. $I_{crockpot} > I_{kettle}$ and $R_{crockpot} < R_{kettle}$.
d. $I_{crockpot} > I_{kettle}$ and $R_{crockpot} > R_{kettle}$.
e. $I_{crockpot} < I_{kettle}$ and $R_{crockpot} < R_{kettle}$.

____ 8. Point A is at x = 2, y =3. Point B is at x = 5, y = 7. The electric field in this region of space is given by $\mathbf{E} = 4\mathbf{i} + 3\mathbf{j}$. All units are SI. What is the magnitude of voltage bewteen these points?

a. 27 V
b. 24 V
c. 33 V
d. 30 V
e. 11 V

____ 9. A segment of wire of total length 3.0 m carries a 15-A current and is formed into a semicircle. To be clear, that is one half of a circle. What is the magnitude of the **magnetic field** at the center of the semicircle? NOTES: The radius is $3/\pi$. Also recall that $S = R\theta$.

a. 4.9 μT
b. 1.0 μT
c. 1.6 μT
d. 15 μT
e. 9.8 μT

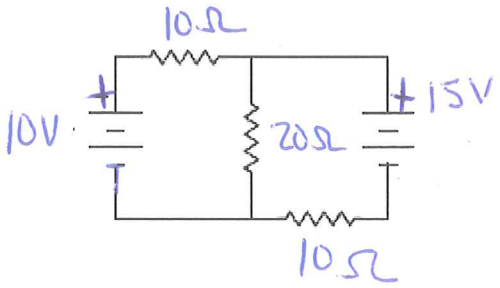
____ 10. By using a compass to measure the magnetic field direction at various points adjacent to a long straight wire, you can show that the wire's magnetic field lines are _____.
Recall that we did this demonstration in class!

a. straight lines in space that are parallel to the wire.
b. straight lines in space that are perpendicular to the wire.
c. straight lines in space that go from one magnetic charge to another.
d. circles that have their centers on the wire and lie in planes perpendicular to the wire.
e. circles that have the wire lying along a diameter of the circle.

____ 11. A long conducting rod with a square cross section (3.0 cm × 3.0 cm) carries a current of 60 A that is uniformly distributed across the cross section. What is the magnitude of the line integral, $\oint \mathbf{B} \cdot d\mathbf{s}$, around a square path (1.5 cm × 1.5 cm) if the path is centered on the center of the rod and lies in a plane perpendicular to the axis of the rod? NOTE: This path defines a square cross section that is contained within the cross section of the rod.

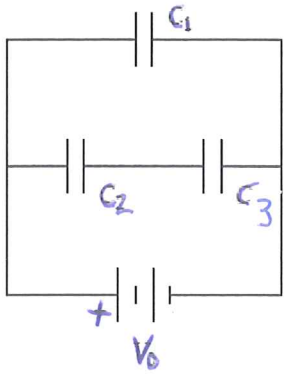
a. 19 μT · m
b. 38 μT · m
c. 14 μT · m
d. 75 μT · m
e. 57 μT · m

____ 12. What is the current delivered by the 10 V battery?



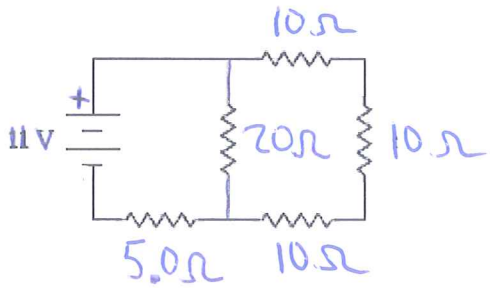
a. 0.75 A
b. 0.00 A
c. 0.25 A
d. 0.50 A
e. 1.00 A

____ 13. What is the voltage across C_2 when $C_1 = 5.0\ \mu F$, $C_2 = 15\ \mu F$, $C_3 = 30\ \mu F$, and $V_0 = 24$ V?



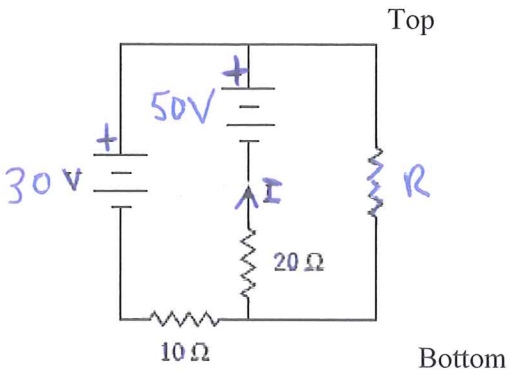
a. 8.0 V
b. 19 V
c. 21 V
d. 16 V
e. 24 V

____ 14. What is the magnitude of the voltage across the 20-Ω resistor?



a. 5.0 V
b. 7.8 V
c. 8.6 V
d. 3.2 V
e. 11 V

____ 15. What is the voltage across the resistor 'R' if the current through the 20Ω resistor is 1.5 amps in the direction shown on the figure?



a. 20 V with the top of R at the higher voltage
b. 20 V with the bottom of R at the higher voltage
c. 80 V with the top of R at the higher voltage
d. 80 V with the bottom of R at the higher voltage
e. There is no way of knowing unless we are given a value for R