From HW) Spherical Capacitor

Inner plate is sphere w/ $r_1 = 0.1$ m
Outer plate is sphere w/ $r_2 = 0.115$ m
$\therefore$ "Separation" $d = 0.015$ m
$Q = 3.3$ nC

a.) Find $|\Delta V|$ between the two plates.

There are TWO ways we can do this.

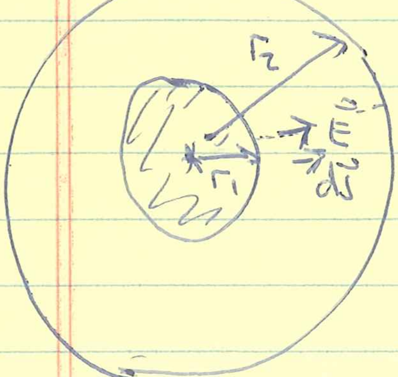
$V_{\substack{pt.\\charge}} \equiv \frac{KQ}{r}$. This set the zero reference point @ $r = \infty$.

We cannot use this for point charges ($r=0$), BUT we don't have a point charge here! We CAN use this to find the voltage on the surface of each conducting sphere (an "equipotential" surface because the conductor is in electrostatic equilibrium).
$\vec{E} = 0$ inside metal

$$V_{\substack{r_1\\sphere}} = \frac{KQ}{r_1} \qquad V_{\substack{r_2\\sphere}} = \frac{KQ}{r_2}$$

$$\therefore \left|\Delta V\right| = \left|V_{\substack{r_2\\sphere}} - V_{\substack{r_1\\sphere}}\right| = \left|KQ\left(\frac{1}{r_2} - \frac{1}{r_1}\right)\right| = 38.7 \text{ volts}$$

The other way is to use Gauss to find $\vec{E}$ between the spheres and then do a path integral.



$$|\vec{E}| = \frac{kQ}{r^2}$$

Can write 'r' and dr $\boxed{\text{or}}$ 'x' and dx. THIS IS $1-\underline{\underline{a}}$

$$|\Delta V_{r_1 r_2}| = \left| - \int_{r_1}^{r_2} \vec{E} \cdot d\vec{s} \right|$$

$|\vec{E}||d\vec{s}|\cos(0)$ $\boxed{\text{OR}}$ $\frac{kQ}{x^2}\hat{i} \cdot dx\hat{i}$

SAME RESULT

$$|\Delta V_{r_1 r_2}| = \left| - \int_{r_1}^{r_2} \frac{kQ}{x^2}\, dx \right|$$

$$= \left| -kQ\left(-\frac{1}{x}\right)\Big|_{r_1}^{r_2} \right|$$

$$= \left| kQ\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \right| = \underline{\underline{38.7 \text{ volts}}}$$

:)

b) Energy stored in capacitor is $\frac{1}{2}C\Delta V^2$. There $\boxed{\text{ARE}}$ equivalent expressions $\boxed{\text{BUT}}$ will you (or I) $\underline{\underline{\text{ever}}}$ remember them??

(c)

If we learn to work from concepts and equations we remember we will be able to do this stuff until the day we die ☺. No worries about being lost in a class two semesters from now!

$$C = \frac{Q}{\Delta V} = \frac{3.3 \times 10^{-9}}{38.7} = 8.527 \times 10^{-11} \text{ farads (F)}$$

$$\therefore \text{Energy} = \frac{1}{2} C V^2 = 6.39 \times 10^{-8} \text{ joules}$$

You could look back @ this calculation and write a general expression for the capacitance of a spherical capacitor.

$$C = \frac{Q}{\Delta V} = \frac{Q}{\underbrace{kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}_{\text{I am writing as a positive \#}}} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

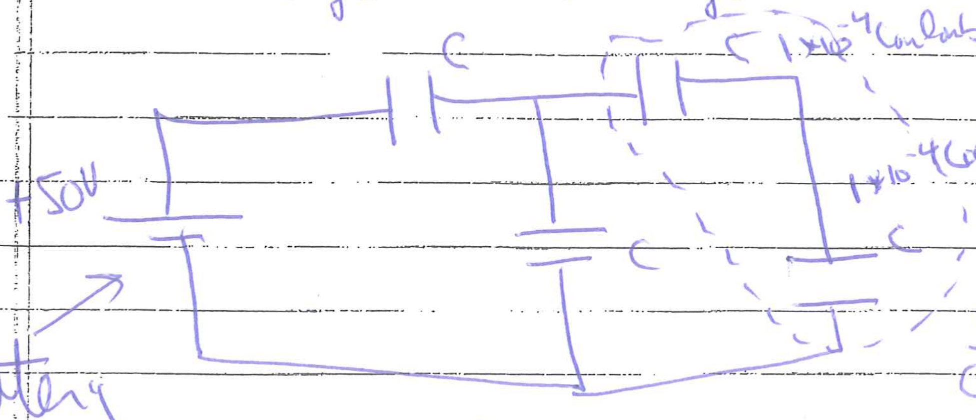
If there were no second plate, would this charged sphere still have a capacitance?

YES! Let $r_2 \to \infty$ and we have

$$C_{sphere} = 4\pi\varepsilon_0 r_1$$

Continued from Monday

Given an arrangement of capacitors, find the charge an voltage on each.



$C = 10\,\mu F$

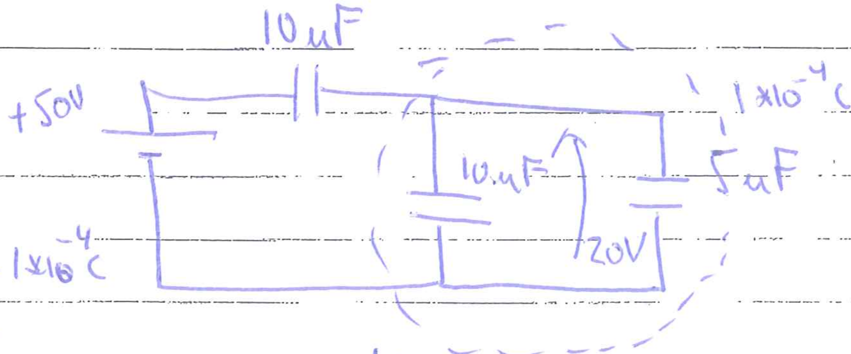
$$\frac{1}{C_{series}} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$$

$\therefore C_{series} = 5\,\mu F$

For $10\,\mu F$

$Q = CV$
$= 10\times10^{-6} \times 20$
$= 2\times10^{-4}\,C$

For $5\,\mu F$ $Q = 1\times10^{-4}\,C$



$C_{parallel} = 10 + 5 = 15\,\mu F$

For $10\,\mu F$

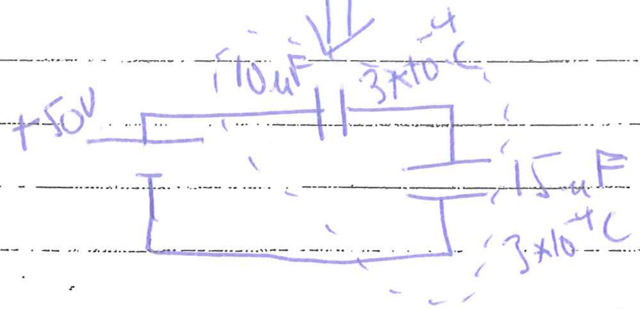
$Q = CV$

$\therefore V_{10\mu F} = \dfrac{3\times10^{-4}\,C}{10\times10^{-6}\,F}$

$= 30$ volts

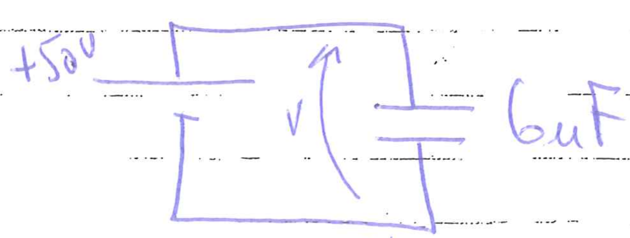
For $15\,\mu F$

$\therefore V_{15\mu F} = 20$ volts



$$\frac{1}{C_{series}} = \frac{1}{10} + \frac{1}{15}$$

$\therefore C_{series} = 6\,\mu F$



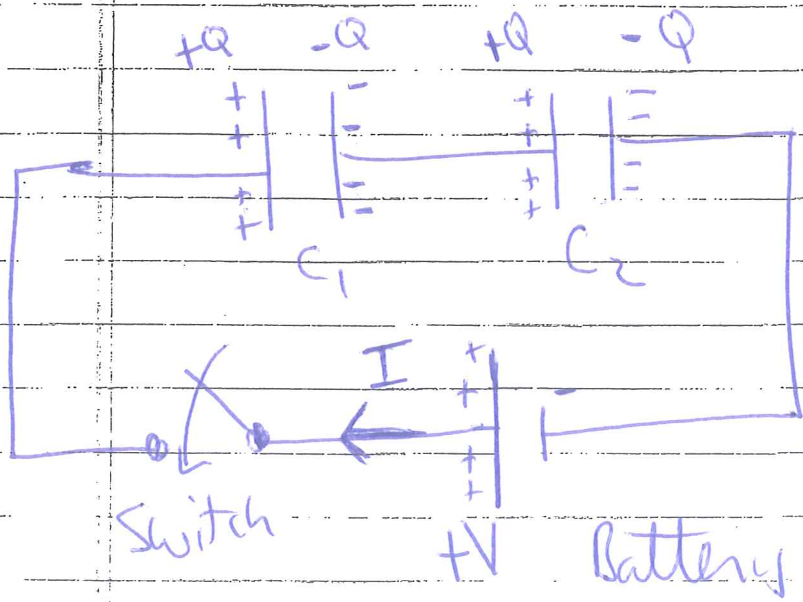
"Equivilent Capacitance"

$V = 50$ volts

$Q = C \times V = 3\times10^{-4}$ coulombs

Rules ⇒ Why?

"Series have the same charge"



$$\text{Current (I)} = \frac{\text{Amount of charge passing by}}{\text{time}}$$ $\frac{\text{Coulombs}}{\text{Second}}$ "ampere" (A) S.I.

ALWAYS assume that Positive charges are moving! "Conventional Current"

☆ EVENTUALLY, Capacitors will fully charge and no current will flow in this circuit.

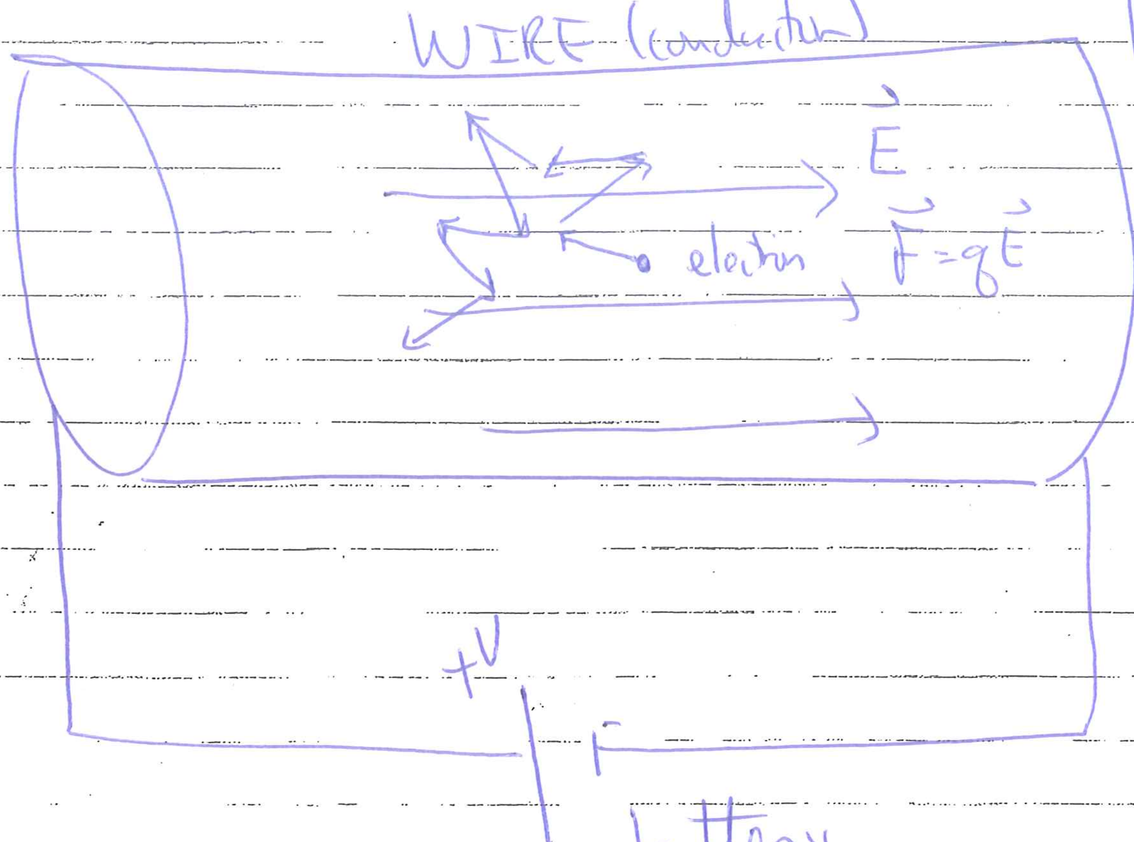
Resistance: A number indicating the difficulty with which charges move
(R) through objects (wires, ...)

"Resitivity" material Property * ρ

Large Resistance ⟹ Very difficult
DEPENDS on Geometry, composition*, and on temperature. S.I. ⟹ "ohm" Ω

$\mho$ "mho"

A model for conduction:



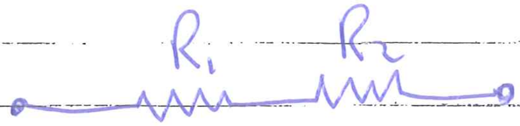
Collisions ⟹ "Ohmic heating"
* Mean free Path
* Drift velocity

DC $\Rightarrow$ direct current $\Rightarrow$ flows in one direction

AC $\Rightarrow$ alternating current

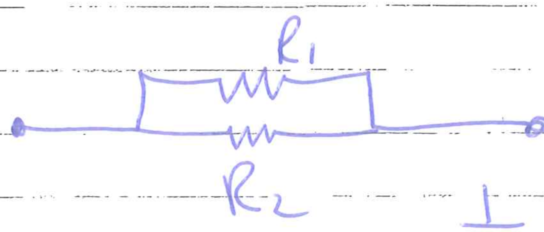
Resistor (light bulb, heater, toaster, ...)

Series



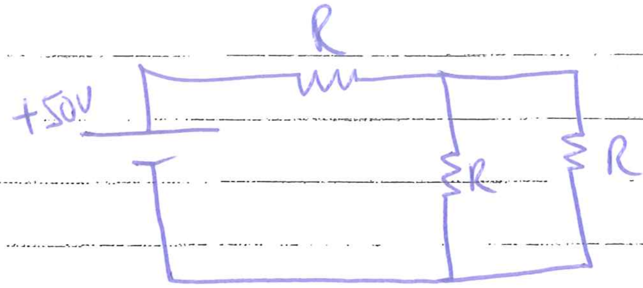
$$R_{series} = R_1 + R_2 + \dots$$

Parallel



$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

EX.)



$R = 10\,\Omega$

Find the current through and voltage across each resistor.

Ohm's Law: $\Delta V = I R$

Solved like previous capacitor problem.

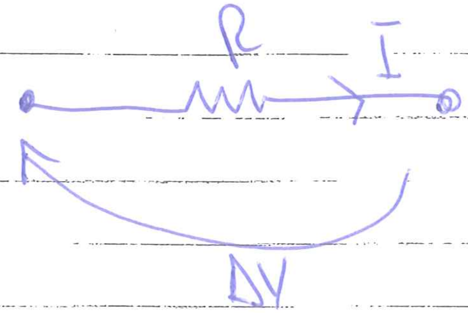
- Find equivalent Resistance (pictures)
- Work 'backwards' using the rules

Rules: Resistors in series have same current. Resistors in parallel have same voltage.

SEE PRACTICE

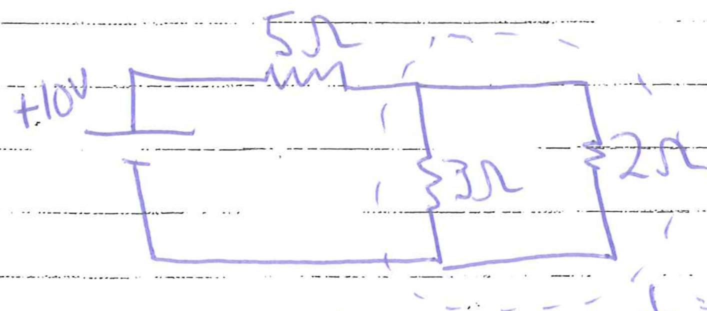
+ Monday HW

Also ohm: Power = $\frac{\text{Energy delivered}}{\text{time}}$ || $\frac{\text{joules}}{\text{sec}}$ "watt" (W)



$$\text{Power (P)} = I^2 R = I\Delta V = \frac{\Delta V^2}{R}$$

EX. Simple DC Circuit



Find the current through the 5Ω resistor

$$\frac{1}{R_{\parallel}} = \frac{1}{3} + \frac{1}{2}$$

$$\therefore R_{\parallel} = 1.2\Omega$$

Series

$$10 = I(5 + 1.2)$$

$$I = 1.6A$$

Power to 5Ω?

Power delivered to circuit by battery?

$P_{5\Omega} = I^2 R = (1.6)^2 5 = \underline{12.8 \text{ watts}}$

$P_{battery} = IV = (1.6)(10) = \underline{16 \text{ watts}}$

Rounding error

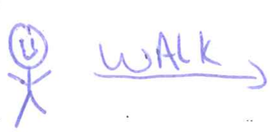
$P_{1.2\Omega} = ?$ $\Rightarrow$ $\underline{3.2 \text{ watts}}$

Model for circuits

- Perfect wires ($R=0$). The wire is an equipotential surface (voltage is constant along the wire). Thus, the P.E. of a charge is constant along the wire. WIRES ⇒ Table tops
  Charges ⇒ Marbles

- Get or give energy when crossing circuit element (ΔV)

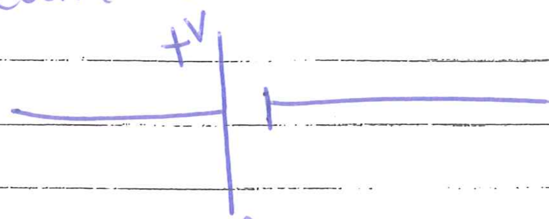
Rules for counting gain/loss:



$\Delta V = -IR$

$\Delta V = +IR$

A battery's job is to maintain a constant voltage across terminals REGARDLESS of current.



Walk → $\Delta V = -V$

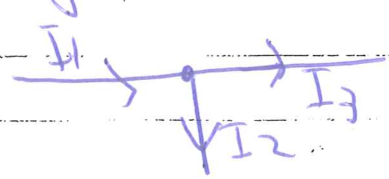
← WALK

$\Delta V = +V$

~1800 Kirchhoff's Loop rule (Ch. 26)
(Voltage law)

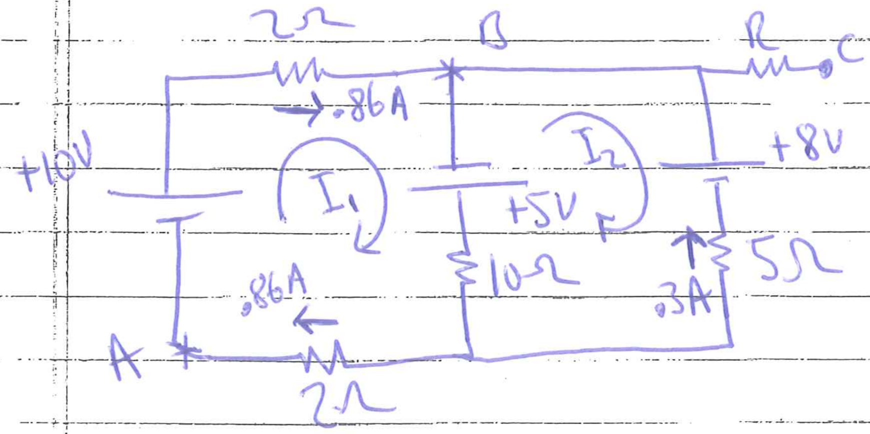
$\Sigma V = 0$ Sum of the voltages around any closed loop in a circuit is ZERO.

Kirchhoff's junction rule



What goes in, comes out.

Find the current through each resistor.



[Model circuit w/ "mesh currents"]

Loop Rule

loop 1

$$+10 \overbrace{-2I_1}^{\Delta V_{2\Omega}} +5 \underbrace{-10I_1 + 10I_2}_{\Delta V_{10\Omega}} -2I_1 = 0$$

$$\boxed{1} \quad -14I_1 + 10I_2 = -15$$

loop 2

$$-8 - 5I_2 \underbrace{-10I_2 + 10I_1}_{\Delta V_{10\Omega}} - 5 = 0$$

$$\boxed{2} \quad +10I_1 - 15I_2 = 13$$

$$\boxed{1} \Rightarrow I_2 = \frac{-15 + 14I_1}{10}$$

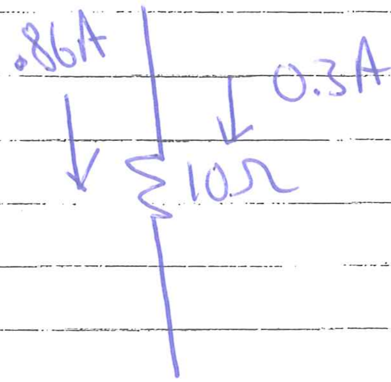
use in $\boxed{2} \Rightarrow 10I_1 - 15\left(\frac{-15 + 14I_1}{10}\right) = 13$

$$-11I_1 + 22.5 = 13$$

$$I_1 = 0.86 \text{ amps}$$

$\therefore I_2 = \frac{-15 + 14 I_1}{10} = -0.3$ amps

For 10Ω :



$\therefore I_{10\Omega} = 1.16A$ down

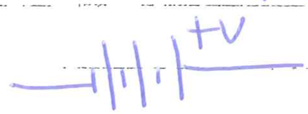
$Power_{10\Omega} = ?$

$\Delta V_{A \to B} = ?$

$\Delta V_{B \to C} = ?$

Electromotive Force (EMF)

↳ Perfect Battery



Real batteries