Lab 1: Standing Waves on Strings

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Superposition and Standing Waves

- Standing waves occur when two identical travelling waves (travelling in opposite directions) interfere.
- In the lab, the mechanical oscillator will create a travelling wave that will travel down the string and reflect at the opposite end of the string. When this wave reflects, it will travel in opposite direction and interfere with another wave that the mechanical oscillator generates.
- We define the two travelling waves as follows:

$$y_1(x,t) = -A\cos(kx+\omega t)\ (incident\ wave\ travelling\ to\ the\ left)$$
$$y_2(x,t) = A\cos(kx-\omega t)\ (incident\ wave\ travelling\ to\ the\ right)$$

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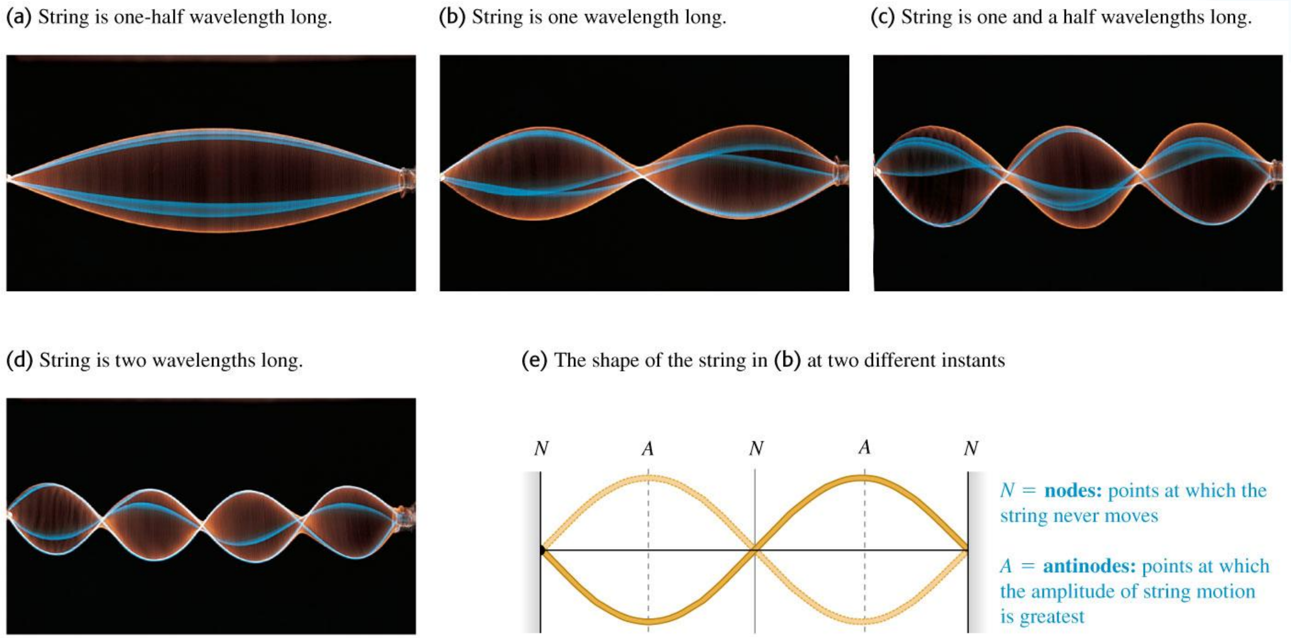
Superposition and Standing Waves

- The superposition of the two travelling waves will result in a standing wave as follows.

$$\begin{aligned} y(x,t) &= y_1(x,t) + y_2(x,t) \\ &= A[-\cos(kx+\omega t) + \cos(kx-\omega t)] \\ &= A[(-\cos(kx)\cos(\omega t) + \sin(kx)\sin(\omega t)) + (\cos(kx)\cos(\omega t) + \sin(kx)\sin(\omega t))] \\ &\boxed{= 2A\sin(kx)\sin(\omega t)} \end{aligned}$$

For a chosen point along the string, the wave does not move, but instead oscillates up and down in time. The position of the wave is independent of time.

Characteristics of Standing Waves



- Points that experience ZERO displacement are called nodes. (Labeled N, in the figure).
- Points that experience MAXIMUM displacement are called anti-nodes. (Labeled A, in the figure).
  - The distance between successive nodes or anti-nodes is half of the wavelength ($\frac{\lambda}{2}$).

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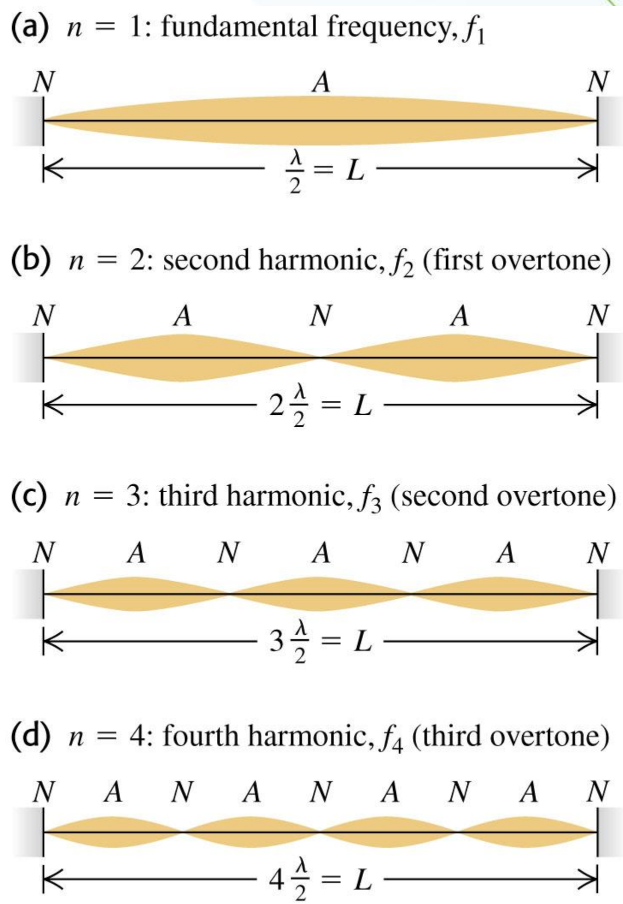
Harmonics

- For a string with a standing wave, both ends are fixed in position (they are nodes).
- Harmonics or “modes” are the frequencies/wavelengths that can exist as a standing wave on the string.
- For a string of length L fixed at both ends, an integer number of half-wavelengths must exist. (Shown in the figure).
- Since there can only exist an integer number of wavelengths on the string, the wavelength can be found from: $L = n\frac{\lambda}{2}$
  - Where L is the length of the string, n is the order number or “harmonic”, and λ is the wavelength.



Result

$$\boxed{\lambda_n = \frac{2L}{n}}$$

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Harmonics and Frequency

- The allowed frequencies or "harmonics" can be found from using the following relationship between the velocity of the travelling waves, their frequency, and the wavelength.

- Which can be rearranged to find the frequency:

$$f = \frac{v}{\lambda}$$

  – This a linear relation between frequency and wavelength. Remember that a linear equation is one that looks like:

$$y = mx + b$$

We can find the equation for the harmonic frequencies using: $v = \lambda f$

$$\boxed{f_n = n\frac{v}{2L}}$$

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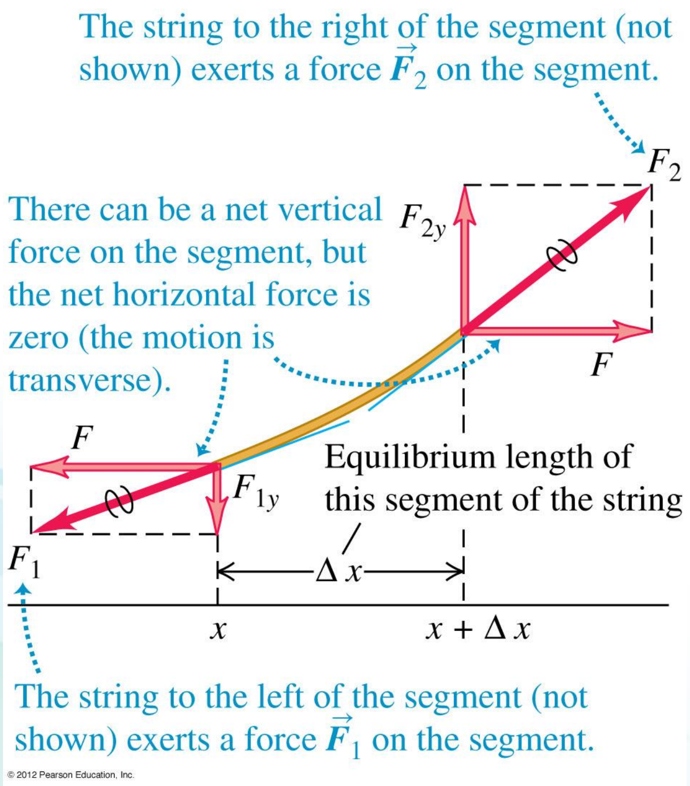
Speed of A Wave on a String (1)

- How can we find the speed of a wave on a string?
  - Consider the figure below. The string experiences a force on each end while the segment in the middle is at equilibrium.



- The ratio of F_{1y} and F is equal to the slope of the string at the point x.
- The ratio of F_{2y} and F is equal to the slope of the string at the point (x+ Δx).

$$\frac{F_{1y}}{F} = -\left(\frac{\delta y}{\delta x}\right)_x$$

$$\frac{F_{2y}}{F} = -\left(\frac{\delta y}{\delta x}\right)_{x+\Delta x}$$

The mass of the segment Δx is equal to $m = \mu \Delta x$ where μ is the mass per unit length of the string.

Speed of A Wave on a String (2)

- Using these two relations, we can find the net force in the y-direction on the string:

$$\frac{F_{1y}}{F} = -\left(\frac{\delta y}{\delta x}\right)_x$$

$$\frac{F_{2y}}{F} = -\left(\frac{\delta y}{\delta x}\right)_{x+\Delta x}$$

$$F_y = F_{1y} + F_{2y} = F\left[\left(\frac{\delta y}{\delta x}\right)_{x+\Delta x} - \left(\frac{\delta y}{\delta x}\right)_x\right]$$

$$F_y = m\frac{\delta^2 y}{\delta t^2} = \mu \Delta x \frac{\delta^2 y}{\delta t^2}$$

$$F\left[\left(\frac{\delta y}{\delta x}\right)_{x+\Delta x} - \left(\frac{\delta y}{\delta x}\right)_x\right] = \mu \Delta x \frac{\delta^2 y}{\delta t^2}$$

Rearranging Gives:

$$\frac{\left(\frac{\delta y}{\delta x}\right)_{x+\Delta x} - \left(\frac{\delta y}{\delta x}\right)_x}{\Delta x} = \frac{\mu}{F}\frac{\delta^2 y}{\delta t^2}$$

Take the limit of each side as Δx goes to zero. When you do this, the left hand side is the definition of a derivative of the function: $\frac{\delta y}{\delta x}$

We can therefore rewrite the equation as:

$$\boxed{\frac{\delta^2 y}{\delta x^2} = \frac{\mu}{F}\frac{\delta^2 y}{\delta t^2}}$$

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Wave Equation

- The relation we have just found is actually the wave equation:

$$\boxed{\frac{\delta^2 y}{\delta x^2} = \frac{1}{v^2}\frac{\delta^2 y}{\delta t^2}}$$

$$\boxed{\frac{\delta^2 y}{\delta x^2} = \frac{\mu}{F}\frac{\delta^2 y}{\delta t^2}}$$

Comparing the two equations we can see that there is a direct relationship between the force on the string, mass per unit length and the speed of the wave. The theoretical speed of the wave is given by:

$$v = \sqrt{\frac{F}{\mu}}$$

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