

## Coulomb's Law Tutorial

### Learning Goal:

To understand how to calculate forces between charged particles, particularly the dependence on the sign of the charges and the distance between them.

Coulomb's law describes the force that two charged particles exert on each other (by Newton's third law, those two forces must be equal and opposite). The force  $\vec{F}_{21}$  exerted *by* particle 2 (with charge  $q_2$ ) *on* particle 1 (with charge  $q_1$ ) is proportional to the charge of each particle and inversely proportional to the square of the distance  $r$  between them:

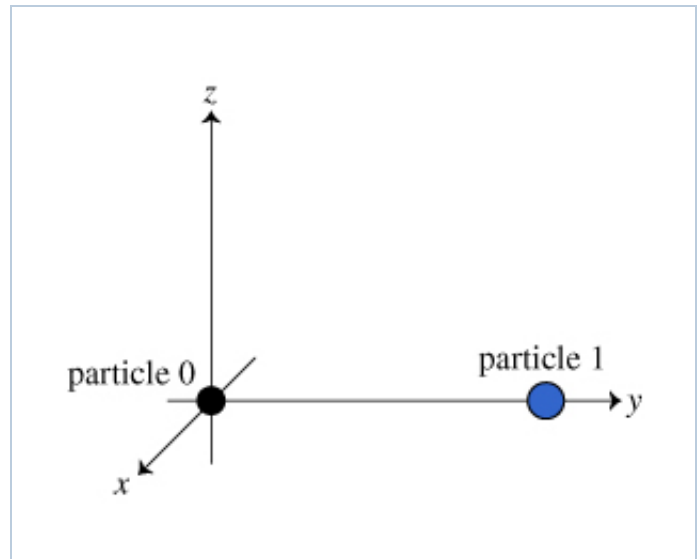
$$\vec{F}_{21} = \frac{k q_2 q_1}{r^2} \hat{r}_{21},$$

where  $k = \frac{1}{4\pi\epsilon_0}$  and  $\hat{r}_{21}$  is the unit vector pointing *from* particle 2 *to* particle 1. The force vector will be parallel or antiparallel to the direction of  $\hat{r}_{21}$ , parallel if the product  $q_1 q_2 > 0$  and antiparallel if  $q_1 q_2 < 0$ ; the force is *attractive* if the charges are of opposite sign and *repulsive* if the charges are of the same sign.

### Part A

Consider two positively charged particles, one of charge  $q_0$  (particle 0) fixed at the origin, and another of charge  $q_1$  (particle 1) fixed on the  $y$ -axis at  $(0, d_1, 0)$ . What is the net force  $\vec{F}$  on particle 0 *due to* particle 1?

**Express your answer (a vector) using any or all of  $k$ ,  $q_0$ ,  $q_1$ ,  $d_1$ ,  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$ .**



ANSWER:

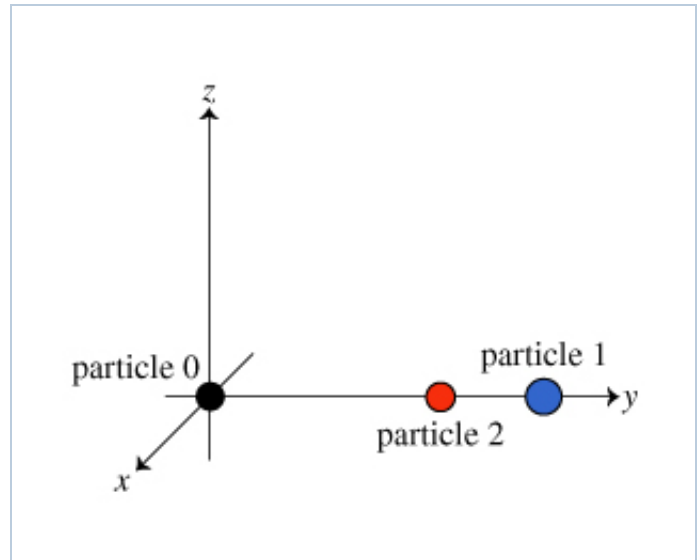
$$\vec{F} = -k \frac{|q_0 q_1|}{d_1^2} \hat{j}$$

**Correct**

### Part B

Now add a third, negatively charged, particle, whose charge is  $-q_2$  (particle 2). Particle 2 fixed on the  $y$ -axis at position  $(0, d_2, 0)$ . What is the new net force on particle 0, *from* particle 1 and particle 2?

**Express your answer (a vector) using any or all of  $k, q_0, q_1, q_2, d_1, d_2, \hat{i}, \hat{j},$  and  $\hat{k}$ .**



ANSWER:

$$\vec{F} = -k \frac{|q_0 q_1|}{d_1^2} \hat{j} + k \frac{|q_0 q_2|}{d_2^2} \hat{j}$$

**Correct**

### Part C

Particle 0 experiences a repulsion *from* particle 1 and an attraction *toward* particle 2. For certain values of  $d_1$  and  $d_2$ , the repulsion and attraction should balance each other, resulting in no net force on particle 0?

**Express your answer in terms of any or all of the following variables:  $k, q_0, q_1, q_2$ .**

ANSWER:

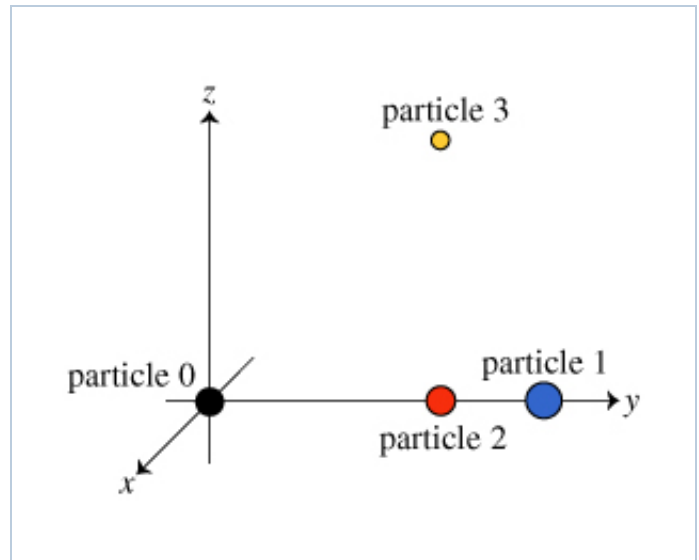
$$d_1/d_2 = \frac{\sqrt{q_1}}{\sqrt{q_2}}$$

**Correct**

### Part D

Now add a fourth charged particle, particle 3, with positive charge  $q_3$ , fixed in the  $yz$ -plane at  $(0, d_2, d_2)$ . What is the net force  $\vec{F}$  on particle 0 due *solely* to this charge?

**Express your answer (a vector) using  $k$ ,  $q_0$ ,  $q_3$ ,  $d_2$ ,  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$ . Include only the force caused by particle 3.**



### Hint 1. Find the magnitude of force from particle 3

What is the magnitude of the force on particle 0 from particle 3, fixed at  $(0, d_2, d_2)$ ?

**Express your answer using  $k$ ,  $q_0$ ,  $q_3$ ,  $d_2$ .**

### Hint 1. Distance to particle 3

Use the Pythagorean theorem to find the straight line distance between the origin and  $(0, d_2, d_2)$ .

ANSWER:

$$F_3 = \frac{kq_0q_3}{2d_2^2}$$

### Hint 2. Vector components

The force vector points from  $q_3$  to  $q_0$ . Because  $q_3$  is symmetrically located between the  $y$ -axis and the  $z$ -axis, the angle between  $\hat{r}_{30}$ , the unit vector pointing *from* particle 3 *to* particle 0, and the  $y$ -axis is  $\pi/4$  radians. You have already calculated the magnitude of the vector above. Now break up the force vector into its  $y$  and  $z$  components.

ANSWER:

$$\vec{F} = -k \frac{|q_0q_3|}{(\sqrt{2}d_2)^2} \frac{\sqrt{2}}{2} \hat{j} - k \frac{|q_0q_3|}{(\sqrt{2}d_2)^2} \frac{\sqrt{2}}{2} \hat{k}$$

**Correct**