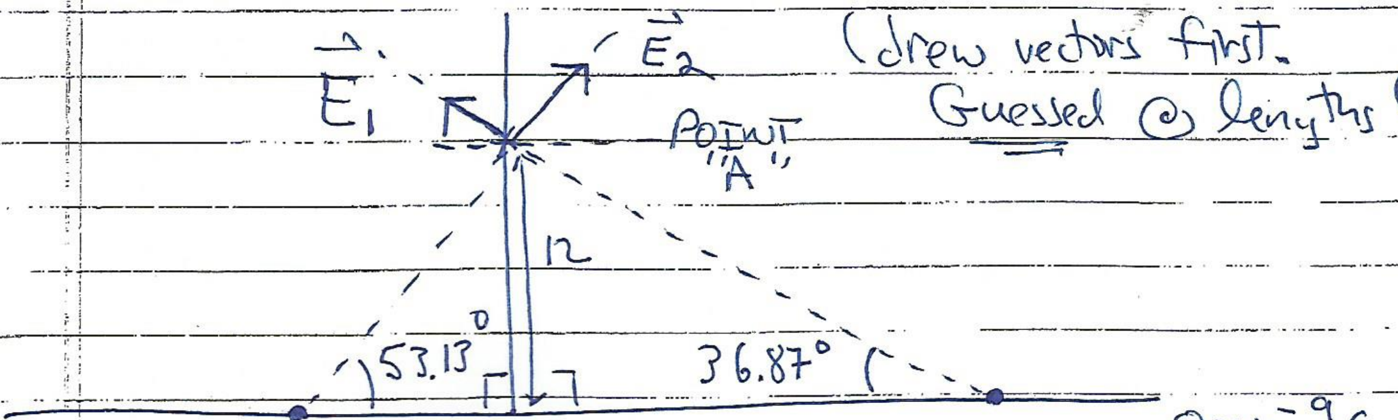
From HW

"Electric field due to multiple point charges"

(drew vectors first. Guessed @ lengths ☺)



$q_2 = 6 \times 10^{-9}\,C$ $(-9, 0)$

$q_1 = 8 \times 10^{-9}\,C$ $(16, 0)$

At "A"

$$|\vec{E}_2| = \frac{k|q_2|}{r_2^2} = \frac{8.99\times10^{9}(6\times10^{-9})}{(9^2+12^2)} = 0.240\ \tfrac{N}{C}$$

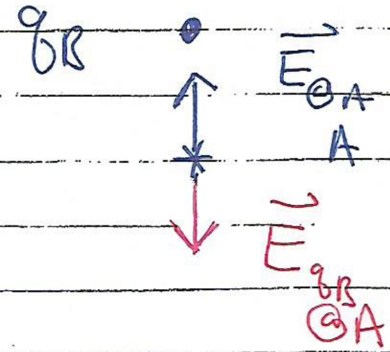
$$\therefore \vec{E}_2 = +0.240\cos(53.13)\,\hat{i} + 0.240\sin(53.13)\,\hat{k}$$
$$= +0.144\,\hat{i} + .192\,\hat{j}$$

$$|\vec{E}_1| = \frac{k|q_1|}{r_1^2} = \frac{8.99\times10^{9}(8\times10^{-9})}{(16^2+12^2)} = 0.180\ N/C$$

$$\therefore \vec{E}_1 = -0.18\cos(36.87)\,\hat{i} + 0.18\sin(36.87)\,\hat{j}$$
$$= -0.144\,\hat{i} + 0.108\,\hat{j}$$

So We have $\vec{E}_{@A} = \vec{E}_1 + \vec{E}_2 = \underline{\underline{0.300\,\hat{j}}}$ Answer

Now we consider placing a charge 3 meters above point "A" so that its $\vec{E}$ will exactly cancel the one we just found



So q_B must be POSITIVE and

$$|\vec{E}_{q_B @A}| = |\vec{E}_{@A}|$$

$$\frac{k|q_B|}{3^2} = 0.300$$

$$\underline{\underline{q_B = 3 \times 10^{-10}\ C}}$$ Answer

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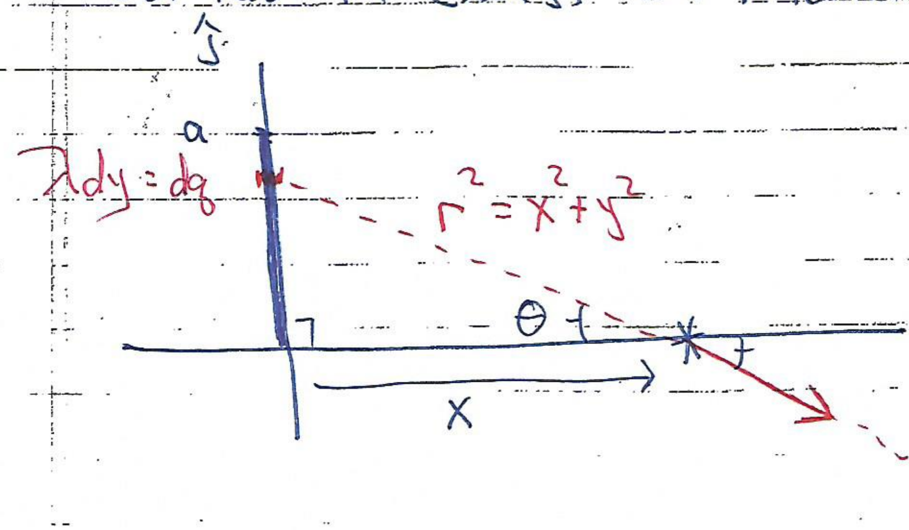
21.82) First, we are finding $\vec{E}$ due to the line charge at a point along the $+\hat{i}$ axis. This is the "TOP HALF" of the problem I worked in class on Monday !! ☺

$$\lambda = Q/a$$

$$|d\vec{E}| = \frac{k|dq|}{r^2}$$



$$d\vec{E} = dE_x\,\hat{i} + dE_y\,\hat{j}$$

$$dE_x = |d\vec{E}|\cos\theta = \frac{k\lambda\,dy}{(x^2+y^2)}\left[\frac{x}{(x^2+y^2)^{1/2}}\right]$$

REALLY? ☑ yes (see Monday class notes)

$$\int dE_x = k\lambda x \underbrace{\int_0^a \frac{dy}{(x^2+y^2)^{3/2}}}_{\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]\Big|_0^a}$$

Appendix in back of text

$$E_x = \frac{k\lambda x a}{x^2\sqrt{x^2+a^2}} = \frac{kQ}{x\sqrt{x^2+a^2}}$$

$$dE_y = -|d\vec{E}|\sin\theta = -\frac{k\lambda\,dy}{(x^2+y^2)}\left[\frac{y}{(x^2+y^2)^{1/2}}\right]$$

REALLY? ☑

$$\int dE_y = -k\lambda \underbrace{\int_0^a \frac{y\,dy}{(x^2+y^2)^{3/2}}}_{\left(-\frac{1}{\sqrt{x^2+y^2}}\right)\Big|_0^a}$$

Appendix

$$\left(-\frac{1}{\sqrt{x^2+a^2}} + \frac{1}{x}\right)$$

$$E_y = -\frac{kQ}{a}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+a^2}}\right)$$

We can now write the electric field @ any location along the $+\hat{i}$ axis:

$$\vec{E}_{@x} = \frac{kQ}{x\sqrt{x^2+a^2}}\,\hat{i} - \frac{kQ}{a}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+a^2}}\right)\hat{j}$$

Now suppose you place a charge $(-q)$ on the x-axis @ location "x". It experiences a force due to this electric field:

$$\vec{F} = (-q)\,\vec{E}_{@x}$$

$$= \frac{-KqQ}{x\sqrt{x^2+a^2}}\,\hat{i} + \frac{KqQ}{a}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+a^2}}\right)\hat{j}$$

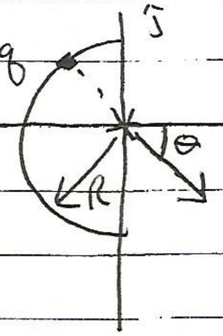
☺

You SHOULD have been able to march through this problem if you understood the example we worked in class on Monday ☺

Take a look @ the practice set and make sure you can do arcs of charge (hint given @ end of Monday class).

EX.] A charged rod w/ uniformily distributed charge +Q is bent into a semicircle of radius R. Find $\vec{E}$ @ the center.

$\lambda R d\theta = \lambda dS = dq$



(HINT: $S = R\theta$, $dS = R\,d\theta$)

$d\vec{E} = dE_x \hat{i} + \cancel{dE_y \hat{j}}$ → 0 Symmetry

$\lambda = \frac{Q}{\left(\frac{2\pi R}{2}\right)} = \frac{Q}{\pi R}$

$$dE_x = |d\vec{E}|\cos\theta = \frac{k\,dq}{R^2}\cos\theta = \frac{k\lambda R\cos\theta\,d\theta}{R^2}$$

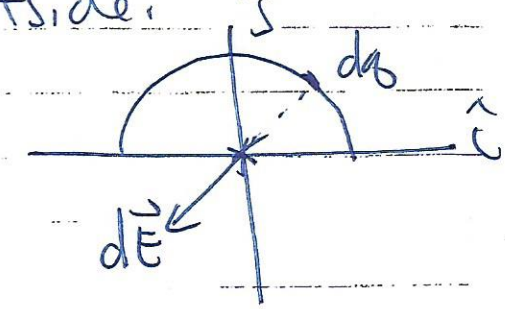
REALLY?? dE_x is Always "+" $\boxed{\checkmark}$

$$\int dE_x = \int_{-\pi/2}^{\pi/2} \frac{k\lambda\cos\theta\,d\theta}{R} = \frac{k\lambda}{R}\Big[\sin(\theta)\Big]\Big|_{-\pi/2}^{\pi/2}$$

$$E_x = \frac{k\lambda}{R}\underbrace{\left[1-(-1)\right]}_{2} = \frac{2kQ}{\pi R^2}$$

$$\therefore \vec{E}_{@\,\text{origin}} = \frac{2kQ}{\pi R^2}\hat{i} + 0\hat{j}$$

Aside:



Recall from Monday: $\Phi_E = \int \vec{E} \cdot d\vec{A}$

(page 7, Monday)

2b

~~FYI:~~ $\cancel{d\vec{A}_{right} = + dy\,dz\,\hat{i}}$

Gauss (1777-1855)

"Gauss' Laws": Equiv. to Coulomb (1785)

BUT in new paradigm (fields) $\vec{E}$

$$\underbrace{\oint \vec{E}\cdot d\vec{A}}_{\Phi_E} = \frac{q_{enclosed}}{\varepsilon_0}$$

Faraday ~ 1844 *

* closed surface

$K = \frac{1}{4\pi\varepsilon_0}$

Formulated 1835 *

Published 1867 ☺

* e.g. Surface of a sphere

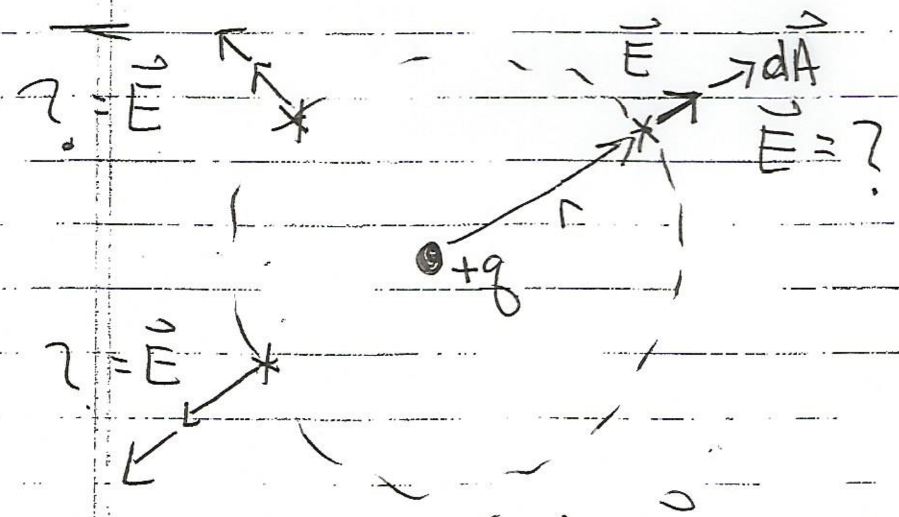
A statement of the relation between charge and the "disturbance" it creates.

Useful:
- To find $\vec{E}$ for symmetrical charge distributions
- To say something about the net charge enclosed by a surface.

EX.] Find $\vec{E}$ a distance r from a point charge +q.

Steps:

- Chose an imaginary, closed surface over which we will calculate the electric flux. Make sure that surface contains the point @ which we want to know the electric field
- Assign the direction of $\vec{E}$ @ every point on surface based on the symmetry of the charge.
- Apply Gauss' Law.



Imagine a sphere of radius 'r' w/ q @ center

"Gaussian Surface"

$$\oint \vec{E} \cdot d\hat{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

ε_0 ← permittivity of free space 8.854×10^{-12}

$\oint |\vec{E}||d\hat{A}|\cos(\theta)$

over sphere

$$\cancel{|\vec{E}|} \oint |d\vec{A}| \cancel{\cos(\theta)}$$

Symmetry allows This!

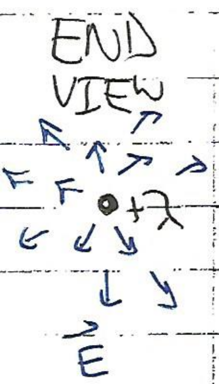
$$\cancel{|\vec{E}|\cos(0)} \underbrace{\oint |d\vec{A}|}_{4\pi r^2}$$

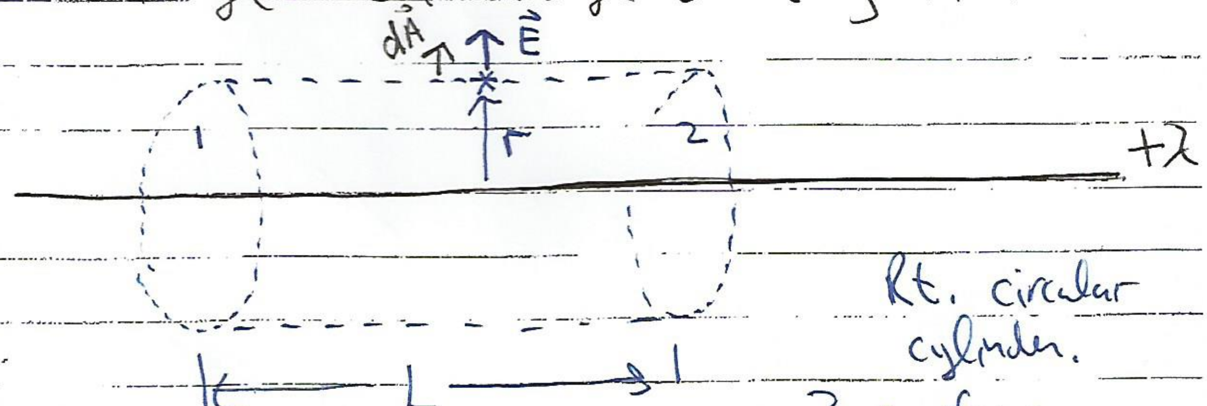
Surface area of sphere!!

$$\therefore \cancel{|\vec{E}|(1)4\pi r^2} = \frac{+q_0}{\varepsilon_0}$$

$$\therefore |\vec{E}| = \frac{+q_0}{4\pi\varepsilon_0 r^2} \qquad \boxed{\frac{kq_0}{r^2}}$$

Ex.] Find $\vec{E}$ a distance r from an infinite line charge w/ charge density $+\lambda$.





Rt. circular cylinder.

3 surfaces

ENDS $\Rightarrow$ 1, 2

"Roll" $\Rightarrow$ 3

$$\oint_{\text{cylinder}} \vec{E}\cdot d\vec{A}$$

$$\int_{\text{left end}} \vec{E}\cdot d\vec{A} + \int_{\text{Roll}} \vec{E}\cdot d\vec{A} + \int_{\text{right end}} \vec{E}\cdot d\vec{A}$$

$\vec{E}$ $d\vec{A}$

$$\int |\vec{E}||d\vec{A}|\cos(90)$$

$d\vec{A}$ $\vec{E}$

!! 0

DITTO!

GAUSS

$$0 + \int_{Roll} |\vec{E}||d\vec{A}|\cos(0) + 0 = \frac{q_{enclosed}}{\varepsilon_0}$$

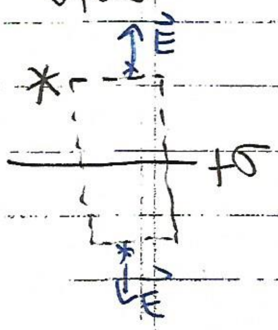
Symmetry

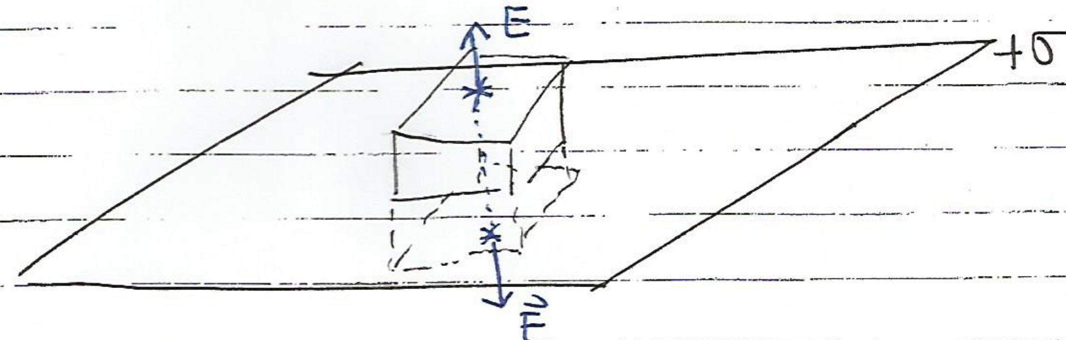
$$|\vec{E}|(1)\underbrace{\int_{Roll} |d\vec{A}|}_{(2\pi r)L} = \frac{\lambda L}{\varepsilon_0}$$

$$|\vec{E}| = \frac{\lambda L}{2\pi r L \varepsilon_0} = \frac{\lambda}{2\pi \varepsilon_0 r}$$

Ex.) Find $\vec{E}$ a distance 'd' above an infinite sheet of charge w/ density $+\sigma$

Edge View





(sides of length 'b')

* Imagine a cube w/ charged sheet cutting in half.

$\oint_{cube} \vec{E}\cdot d\vec{A} \Rightarrow$ only non-zero Terms are from TOP and BOTTOM ($\vec{E}$ and $d\vec{A}$ same direction)

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc.}}{\varepsilon_0}$$

$$|\vec{E}|b^2 + |\vec{E}|b^2 = \frac{\sigma b^2}{\varepsilon_0}$$

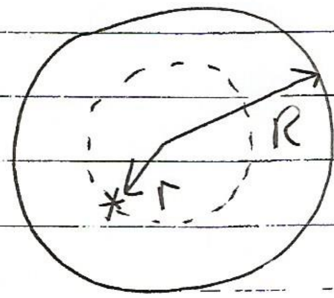
$$|\vec{E}| = \frac{\sigma}{2\varepsilon_0}$$

What Gauss can tell us about conductors.

Assume "electrostatic equilibrium". *
(charges are not moving)

EX] A conducting sphere of radius R has a total charge of +Q. Find $\vec{E}$ everywhere.

+Q



for r < R

[Imagine a spherical surface of radius r

[$\vec{E}$ must be zero (because problem demands *)

* $\vec{E} \equiv 0$ inside the physical conductor

$\Leftarrow \vec{F} = q\vec{E}$

↑ NOT MOVING

$$\oint \cancel{\vec{E}} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

$\hookrightarrow 0$

Hmmm... ∴ $q_{enclosed}$ must be zero.

Any "excess" charge must reside on the surface.

for r > R

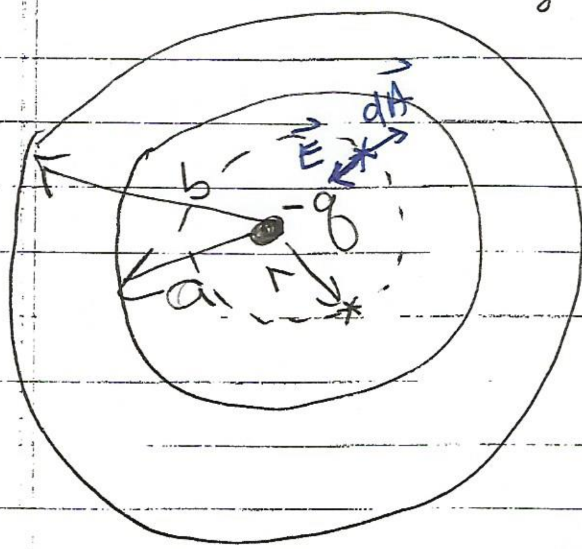
looks like the point charge ☺

Aside: If sphere is charged insulator ... charge distributed throughout. Still have spherical symmetry. Charge outside Gaussian surface does not matter ☺

EX] A spherical conducting shell in electrostatic equilibrium w/ net charge +Q. Inner radius 'a', outer radius 'b'. A charge (-q) is placed in the center cavity.

Find $\vec{E}$ everywhere.

Find the charge densities on the surfaces.



for $r < a$

[spherical surface
radius r

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc.}}{\varepsilon_0}$$

$$|\vec{E}| 4\pi r^2 (-1) = \frac{-q}{\varepsilon_0}$$

$$|\vec{E}| = \frac{q}{4\pi\varepsilon_0 r^2}$$ ☺