Faraday/Lenz Examples (from text)

29.8) Given a coil w/ 200 turns
Area of 1 turn is $12\ cm^2$.
Coil is rotated from max. Φ_B position to min. Φ_B in a time of 0.04 s.

$|\vec{B}| = 6 \times 10^{-5}\ T$ uniform, constant

Find ΔV induced in coil.

Faraday $|\Delta V| = \left| -\frac{d\Phi_B}{dt} \right|$ Assign direction of current w/ Lenz ☺

$$\Phi_{B_{max}} = \int \vec{B} \cdot d\vec{A} = \int |\vec{B}||d\vec{A}|\cos(0) = |\vec{B}| \underbrace{\int |d\vec{A}|}_{\text{effective Area of the coils}}$$

$$A_{effective} = \underbrace{\left(12\ cm^2 \times \frac{1\ m}{100\ cm} \times \frac{1\ m}{100\ cm} \right)}_{\text{one coil}} \times 200$$

$\therefore \Phi_{B_{Max}} = 1.44 \times 10^{-5}\ T \cdot m^2$

$\Phi_{B_{min}} = 0 \quad (\vec{B} \perp d\vec{A})$

To get an Average value:

$$|\Delta V| = \left| -\frac{\Delta \Phi_B}{\Delta t} \right| = \frac{1.44\times10^{-5}}{0.04} = 3.6\times10^{-4}\,V$$

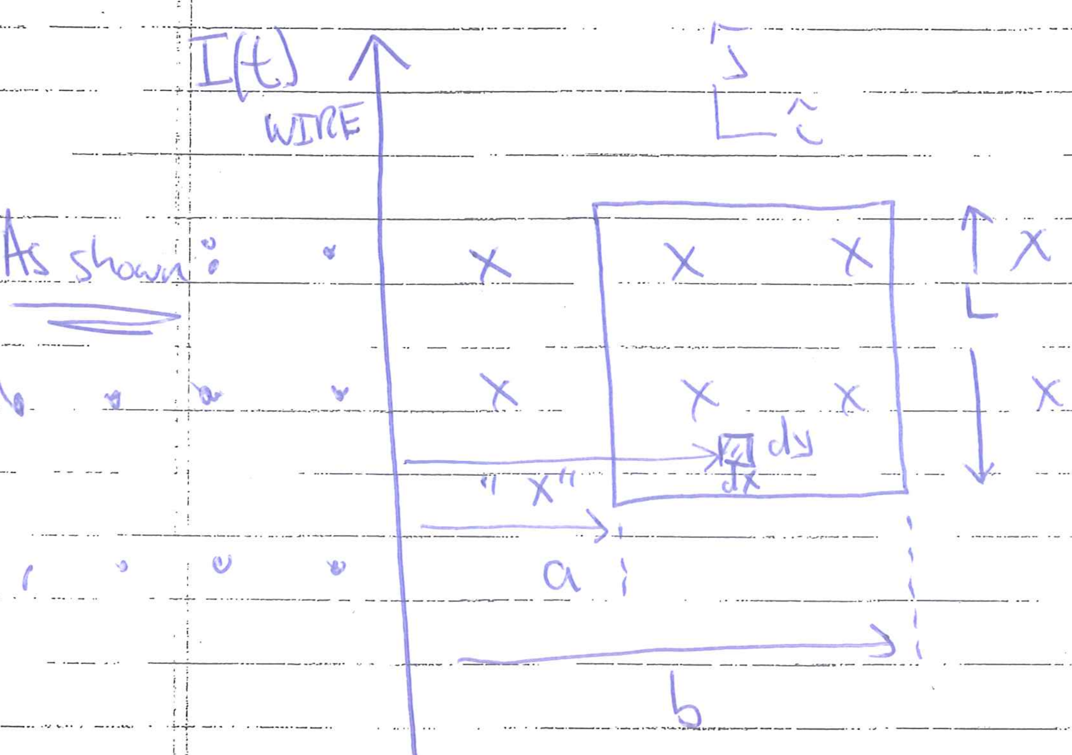
Answer

29.1) Sort of ☺

If the loop has resistance R, Find $I(t)_{Loop}$.

$$\Delta V_{loop} = -\frac{d\Phi_{Loop}}{dt}$$

As shown:



$$\Phi_{B_{Loop}} = \int \vec{B}(t)\cdot d\vec{A} = \int |\vec{B}||d\vec{A}|\cos(0)$$

over the area of loop

INTO PAGE (+)

$$= \int \frac{\mu_0 I_{wire}}{2\pi x}\,dx\,dy = \frac{\mu_0 I_{wire}}{2\pi}\int_a^b \frac{dx}{x}\int_0^L dy$$

$$\Phi_{B_{Loop}} = \frac{\mu_0 L}{2\pi} I_{wire}(t)\ln(b/a) = \frac{\mu_0 L \ln\left(\frac{b}{a}\right)}{2\pi} I_{wire}(t)$$

1

$$\Delta V_{Loop} = -\frac{d\Phi_{B_{Loop}}}{dt} = -\frac{\mu_0 L \ln\left(\frac{b}{a}\right)}{2\pi}\left[\frac{d}{dt} I_{wire}(t)\right]$$

ANSWER 1

GIVEN $I_{wire}(t) = 50\cos(32t)$ amps

$\therefore \Delta V_{Loop} = I_{Loop} R$

$I_{Loop} = \frac{\Delta V_{Loop}}{R} = \boxed{\checkmark}$

ALL HERE

Scotland, 1865, James Clerk Maxwell
Theory of E+M.

include a displacement current term in Ampère's law.

Maxwell's theory of electromagnetism is embodied in four equations that we today call **Maxwell's equations.** These are

"del" operator

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0} \qquad \text{Gauss's law} \qquad \nabla \cdot \vec{E} = \frac{1}{\varepsilon_0}\rho$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{Gauss's law for magnetism} \qquad \nabla \cdot \vec{B} = 0$$

$$* \quad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_{\text{m}}}{dt} \qquad \text{Faraday's law} \qquad \nabla \times \vec{E} = -\frac{\partial B}{\partial t}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} + \boxed{\epsilon_0 \mu_0 \frac{d\Phi_{\text{e}}}{dt}} \qquad \text{Ampère-Maxwell law} \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

You've seen all of these equations earlier in this chapter. It was Maxwell who first wrote them in a consistent mathematical form similar to this. (Not exactly the same because our present-day vector notation wasn't developed until the 1890s, but Maxwell's versions were mathematically equivalent.) Neither Gauss nor Faraday nor Ampère would recognize these equations, but Maxwell had succeeded in capturing their physical ideas in a concise mathematical form.

Maxwell's claim is that these four equations are a *complete* description of electric and magnetic fields. They tell us how fields are created by charges and currents, and also how fields can be induced by the changing of other fields. We need one more equation for completeness, an equation that tells us how matter responds to electromagnetic fields. The general force equation

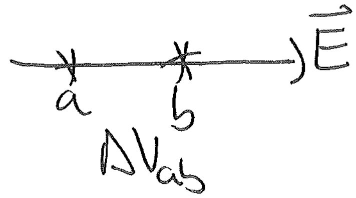
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \qquad \text{(Lorentz force law)}$$

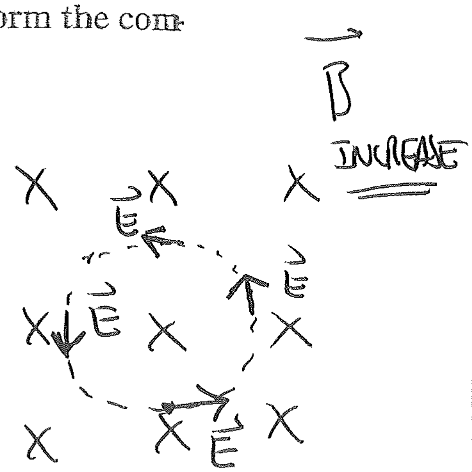
"displacement current"
Fig 29.32 pg 973

is known as the *Lorentz force law.* Maxwell's equations for the fields, together with the Lorentz force law to tell us how matter responds to the fields, form the complete theory of electromagnetism.

$* \quad \Delta V = -\int \vec{E} \cdot d\vec{s}$ line integral

non-coulombic





With the "displacement current" term Maxwell was able to seperate the differential equations in $\vec{E}$ and $\vec{B}$!!

Ch. 32

Six eqtns of the form:

A WAVE EQTN

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}$$

1-d WAVE EQTN

$$\frac{\partial^2 f(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f(x,t)}{\partial t^2}$$

$f(x,t)$ describes a wave travelling in the +/- x direction

v = speed of wave

Hmm...

$$\frac{1}{v^2} = \varepsilon_0 \mu_0$$

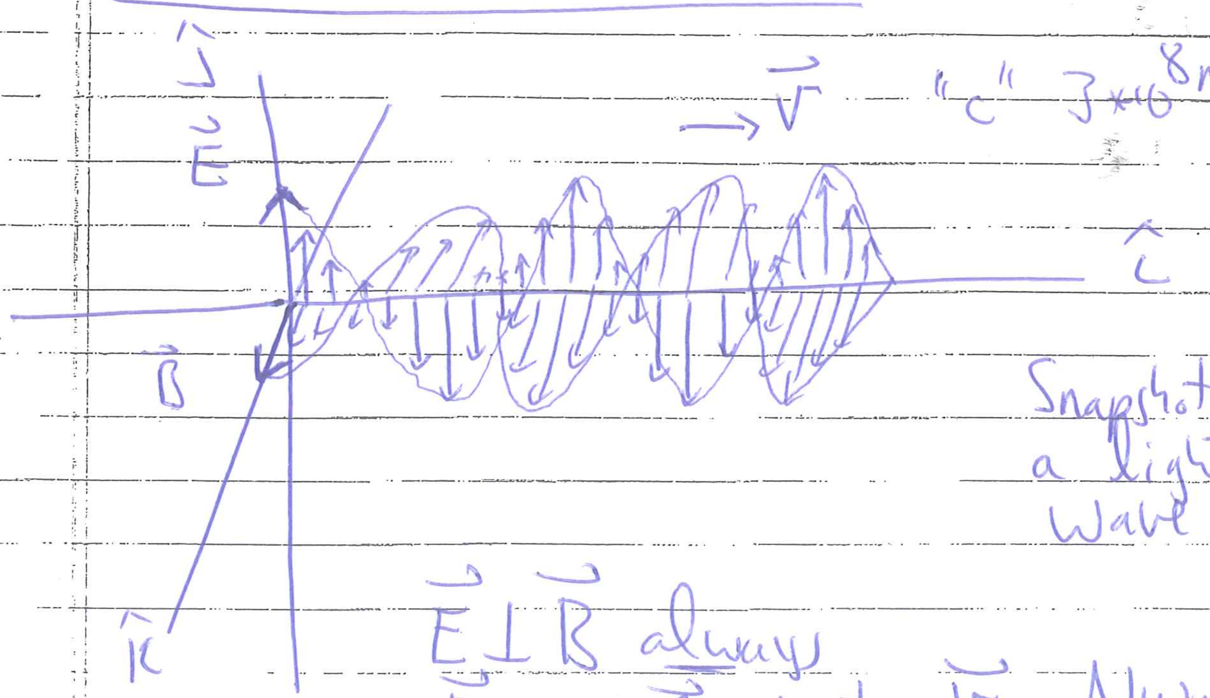
"c"

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 * 10^8 \ m/s$$

EUREKA

Light is an E-M WAVE!

Sinusoidal Plane E-M wave



"c" 3×10^{8} m/s

Snapshot of a light wave.

$\vec{E} \perp \vec{B}$ always
$\vec{E}$ and $\vec{B}$ $\perp$ to $\vec{v}$ Always

☆ $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$

"PHYSLETS"

Poynting vector
Direction is direction of propagation

Recall waves from Chapter 15

Wavelength (λ)
frequency (f)

$\boxed{\lambda * f = v}$

1^{-d} wave function (general form)

$y(x,t) = A \, {}^{\sin}_{\cos}(kx \pm \omega t + \phi)$

phase angle

Amplitude wave # angular freq

$K = \frac{2\pi}{\lambda}$

$\omega = 2\pi f$

All on formula sheet !!

$Kx \pm \omega t \Rightarrow$ "−" was wave travelling in $\pm X$ direction

$Ky + \omega t \Rightarrow$ travelling in the $-y$ direction

Write eqtns describing the wave @ the top of page 5 in notes.

direction of propagation $\Rightarrow +\hat{i}$

"Snapshot" is @ $t=0$

@ $x=0$ and @ $t=0$, $E = +E_{MAX}$, $B = +B_{MAX}$

Answer:

$\vec{E}(x,t) = E_{max} \cos(Kx - \omega t)\hat{j}$ ☑

OR

$E_{max} \sin(Kx - \omega t + \pi/2)$ ☑

$\vec{B}(x,t) = -B_{max} \cos(Kx - \omega t)\hat{k}$ ☑

Recall from Sound (Ch. 16)

$$\text{Intensity} = \frac{\text{Power}}{\text{Area}} \quad \Big\| \quad \frac{\text{watts}}{m^2}$$

$\uparrow$

$\frac{\text{power transported}}{\text{time}}$

EX.] A wave source generates a spherical wave carrying a power of 60 watts. (e.g. speaker, lightbulb)

a) Find intensity 5 meters away.

$$I = \frac{P}{4\pi r^2} = \frac{60}{4\pi(5)^2} = \underline{0.19\ W/m^2}$$

surface area of sphere

Poynting Vector gives us Intensity!!!

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

Formula sheet

$$\text{Intensity} = S_{AVERAGE} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c}$$

$$= \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}\,E_{max}^2 = \ldots$$

ALWAYS TRUE: $c = \frac{E}{B} = \frac{E_{max}}{B_{max}}$