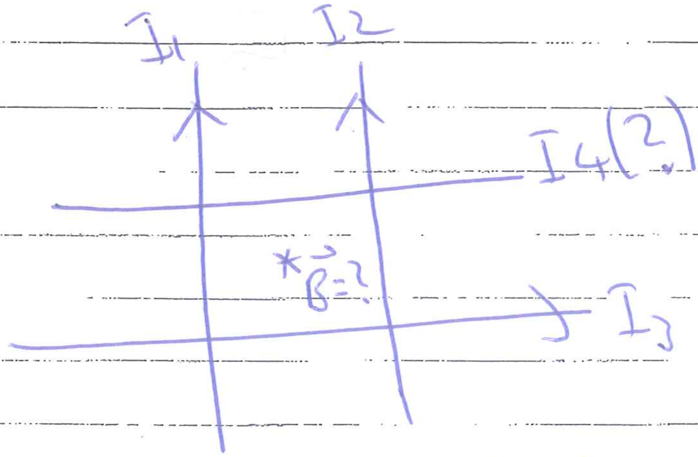
EXAM #2

15 questions
3 of these are on "old" material
!!! Calculus Rich !!! (Also $\vec{v} \times \vec{B}$)

HW Comment on 28.26 :

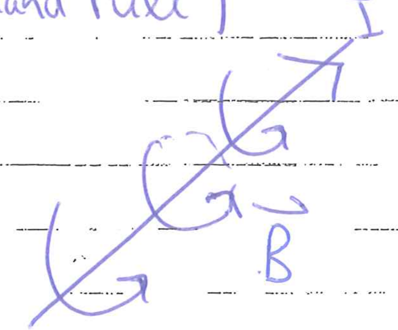


$$|\vec{B}_{wire}| = \frac{\mu_0 I}{2\pi r}$$

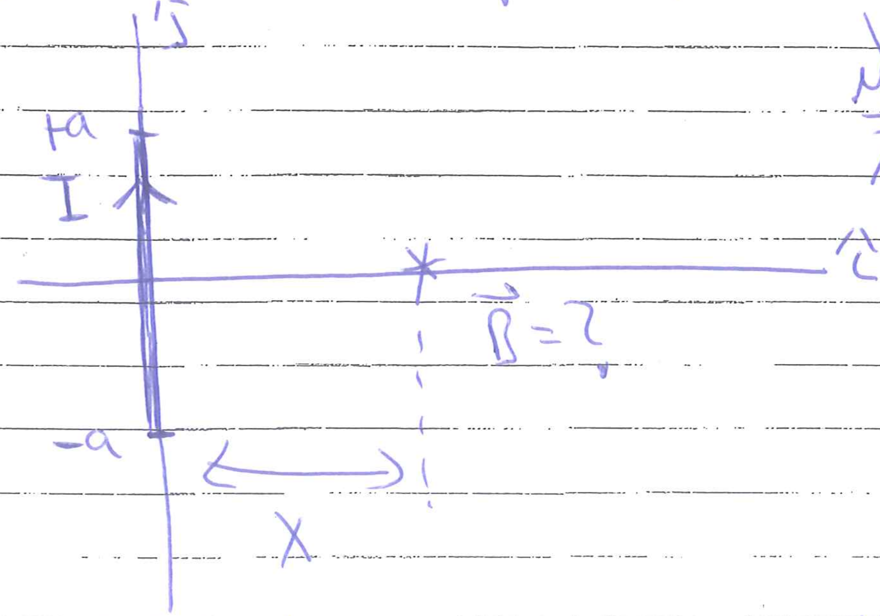
Superposition

How does $\vec{F} = I\vec{\ell} \times \vec{B}$ apply? IT DOES NOT !!

("right hand rule")



EX) A wire of length 2a carries a current I. What is $\vec{B}$ a distance x from its mid point?



$\cancel{\frac{\mu_0 I}{2\pi r}}$ This is a "short wire"

(~1800) Biot-Savart Law

$$d\vec{B} = \frac{\mu_0 I}{4\pi r^2} \, d\vec{l} \times \hat{r}$$

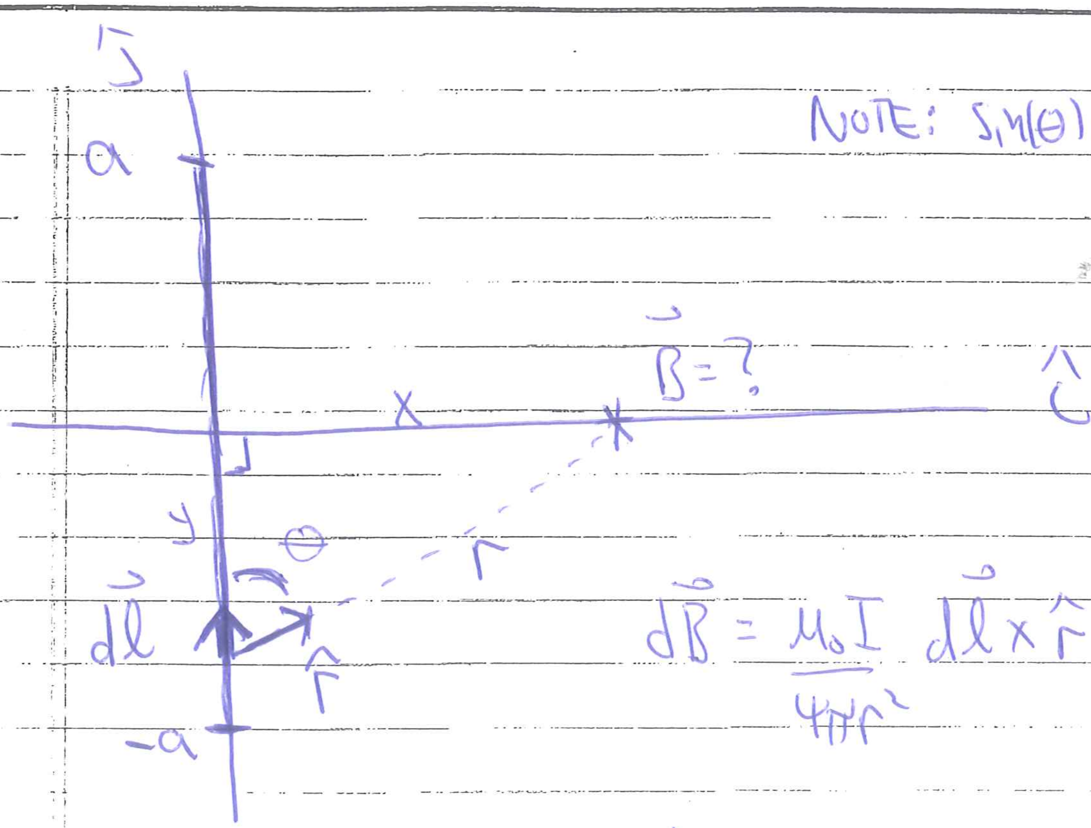
$d\vec{l}$: a small step in direction of I

$\hat{r}$: a unit vector pointing from small piece of current to observation point.

$d\vec{B}$: a small piece of the magnetic field

r: distance from a small piece of current to the observation point

NOTE: $\sin(\theta) = \frac{x}{r} = \frac{x}{\sqrt{y^2+x^2}}$



$$d\vec{B} = \frac{\mu_0 I}{4\pi r^2} \, d\vec{l} \times \hat{r}$$

3 EQTNS! $\hat{i}, \hat{j}, \hat{k}$

NOT for us ☺

$$|d\vec{B}| = \frac{\mu_0 I}{4\pi r^2} |d\vec{l}||\hat{r}|\sin(\theta)$$

IN

$-\hat{k}$ Right handed coordinates

$\hat{i} \times \hat{j} = \hat{k}$

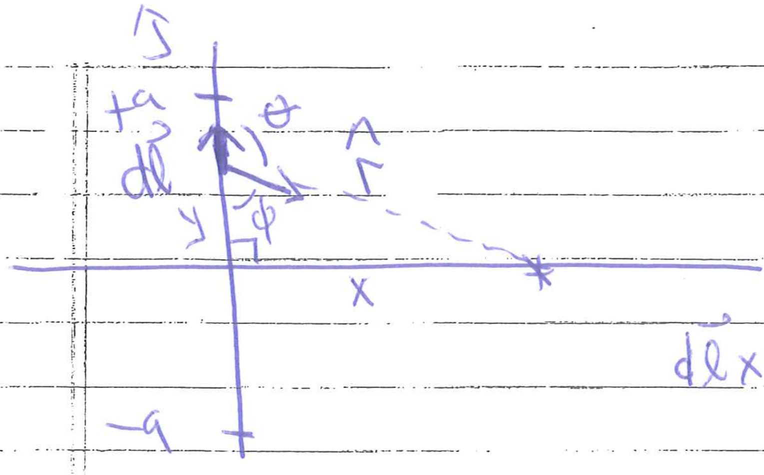
$$\int |d\vec{B}| = \int \frac{\mu_0 I}{4\pi r^2} dy \sin(\theta)$$

$$|\vec{B}| = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dy}{(x^2+y^2)} \left(\frac{x}{(x^2+y^2)^{1/2}} \right)$$

RECALL: x is a constant!!

$$= \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{dy}{(x^2+y^2)^{3/2}}$$

REALLY? Is This valid over The entire range of integration?



$\vec{dl} \times \hat{r} = |\vec{dl}||\vec{r}| \sin(\theta)$

↑ ?

$\sin(\phi) = \frac{x}{r}$ but is this also

$\sin(\theta)$? YES

ii) Trig identity: $\sin(\phi) = \sin(180 - \phi)$

ALSO VERIFY DIRECTION!! ☒

Good! Let's continue.

$$|\vec{B}| = \frac{\mu_0 I x}{4\pi} \underbrace{\int_{-a}^{a} \frac{dy}{(x^2+y^2)^{3/2}}}$$

$$\left[\frac{1}{x^2} \frac{y}{\sqrt{x^2+y^2}}\right]\Bigg|_{-a}^{a}$$ Appendix in Book

$$|\vec{B}| = \frac{\mu_0 I}{4\pi x}\left[\frac{a}{\sqrt{x^2+a^2}} - \frac{(-a)}{\sqrt{x^2+a^2}}\right]$$

$$|\vec{B}| = \frac{\mu_0 I}{2\pi x}\left[\frac{a}{\sqrt{x^2+a^2}}\right](-\hat{k})$$ ii)

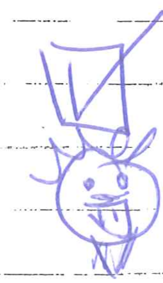
This is DONE in your textbook!

Section 28.3, Pg 926

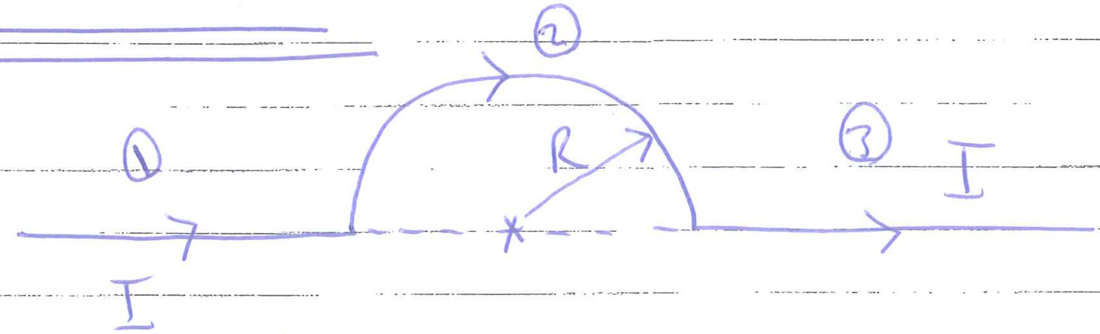
What if this wire were infinite?

$a >> x$

$$|\vec{B}| = \frac{\mu_0 I}{2\pi x}\left[\frac{a}{a\sqrt{\frac{x^2}{a^2}+1}}\right] \Rightarrow \frac{\mu_0 I}{2\pi x}$$



EX)



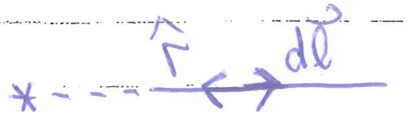
Find $\vec{B}$ @ the center of the semicircle.

$$d\vec{B} = \frac{\mu_0 I}{4\pi r^2}\, d\vec{l} \times \hat{r}$$

1

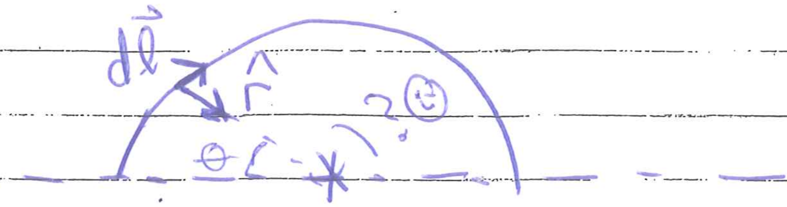
$d\vec{l} \times \hat{r} = |d\vec{l}||\hat{r}|\sin(0) = 0$!!

3



$d\vec{l} \times \hat{r} = |d\vec{l}||\hat{r}|\sin(180) = 0$!!

2



$d\vec{l} \times \hat{r}$ is $\boxed{\text{IN}}$ Over entire range of integration? YES ☑

$$|d\vec{B}| = \frac{\mu_0 I}{4\pi R^2} |d\vec{l}||\hat{r}|\sin(90)$$

Recall
$S = R\theta$
$dS = R\,d\theta$

$$\Rightarrow |\vec{B}| = \int \frac{\mu_0 I}{4\pi R^2} |d\vec{l}|$$

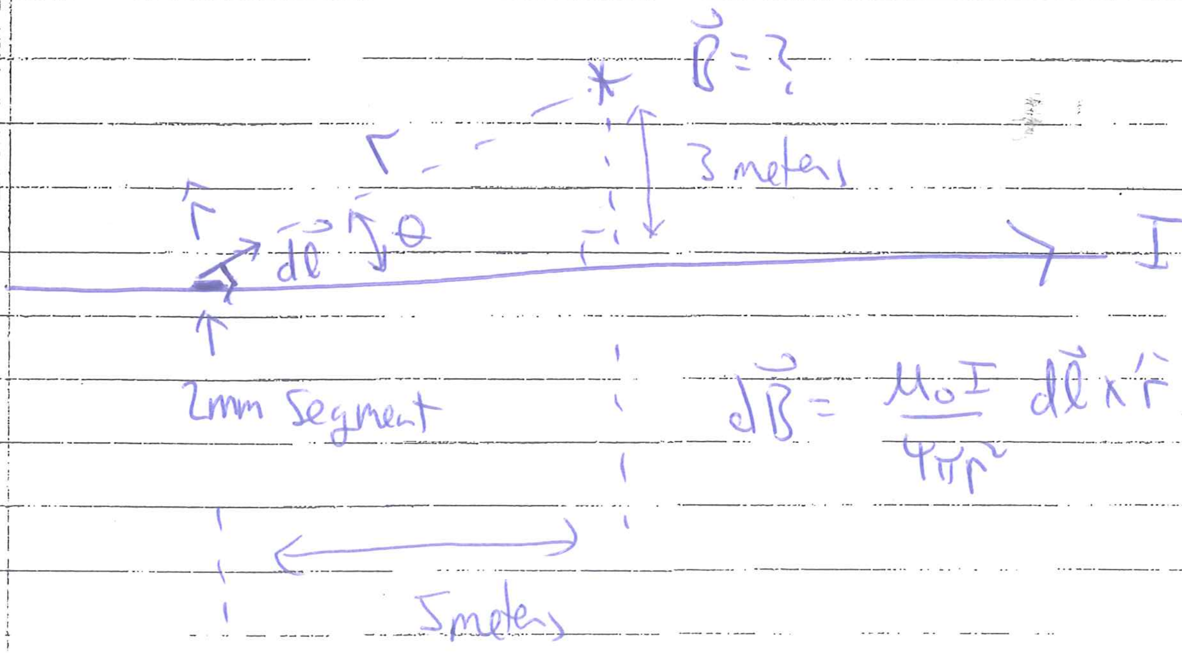
$$= \frac{\mu_0 I}{4\pi R^2} \int_0^{\pi} R\,d\theta$$

$$= \frac{\mu_0 I}{4\pi R} \int_0^{\pi} d\theta = \frac{\mu_0 I}{4R} \ \boxed{\text{IN}}$$

$$\therefore \vec{B}_{@\,\text{center}} = 0 + \frac{\mu_0 I}{4R} + 0 \ \boxed{\text{IN}}$$

Answer

NOTE on some of practice problems



$$d\vec{B} = \frac{\mu_0 I}{4\pi r^2} \vec{dl} \times \hat{r}$$

Approximation $\Rightarrow$ $d\vec{B}$ is the magnetic field @ this location due to the 2mm segment!

$$|d\vec{B}| = \frac{\mu_0 I}{4\pi r^2} |\vec{dl}||\hat{r}|\sin(\theta) \quad \boxed{\text{out}}$$

Answer ☺

Still in Paris ☺

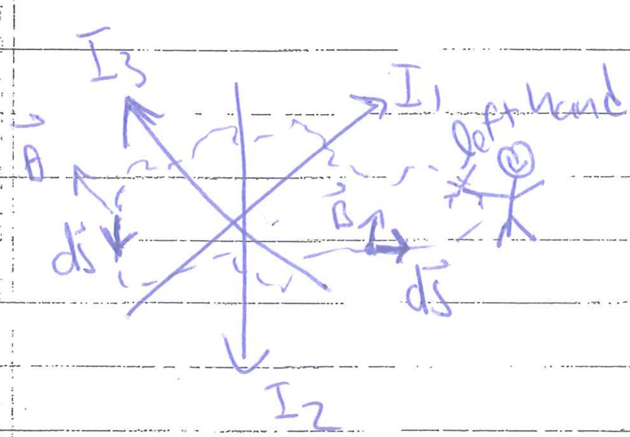
Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enclosed}$$

closed path

$d\vec{l}$

"Path integral"
"line integral"



3-d path

$$\underbrace{\oint_{path} \vec{B} \cdot d\vec{l}}_{??} = \mu_0 (+I_1 - I_2 + I_3)$$

tough to evaluate explicitly

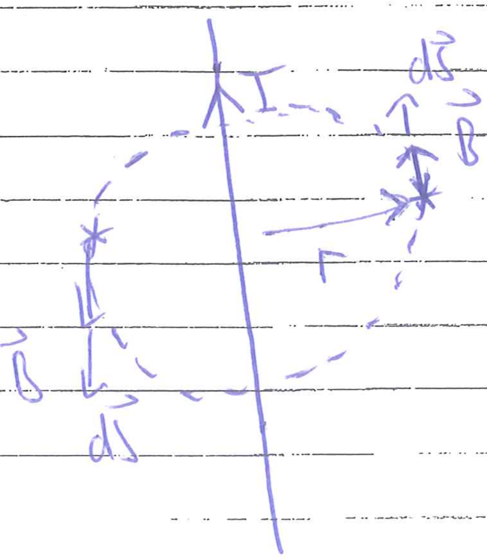
SIGN CONVENTION

We can use this to find $\vec{B}$ when there is SYMMETRY that allows us to Assign the direction of $\vec{B}$ along the entire path!!

EX) Find $\vec{B}$ a distance r from a wire carrying a current I.

$\boxed{B = \frac{\mu_0 I}{2\pi r} \text{ circles}}$

EMPERICAL



"Amperian Path" ⇒ circle of radius r centered on the wire and in plane ⊥ to the wire

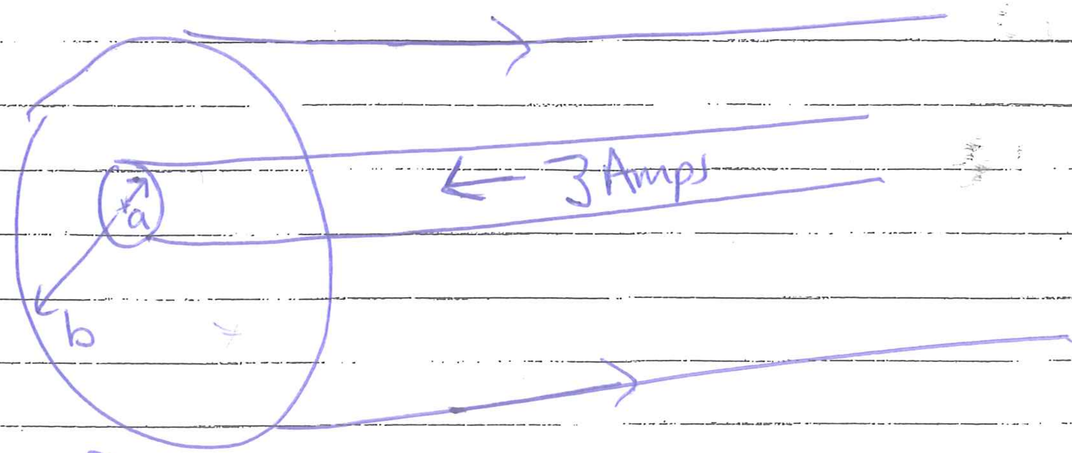
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enclosed}$$

$$\oint |\vec{B}||d\vec{s}|\cos(0) = \mu_0 (+I)$$

$$|\vec{B}| \underbrace{\oint |d\vec{s}|}_{2\pi r \text{ (total distance walked)}} = \mu_0 I$$

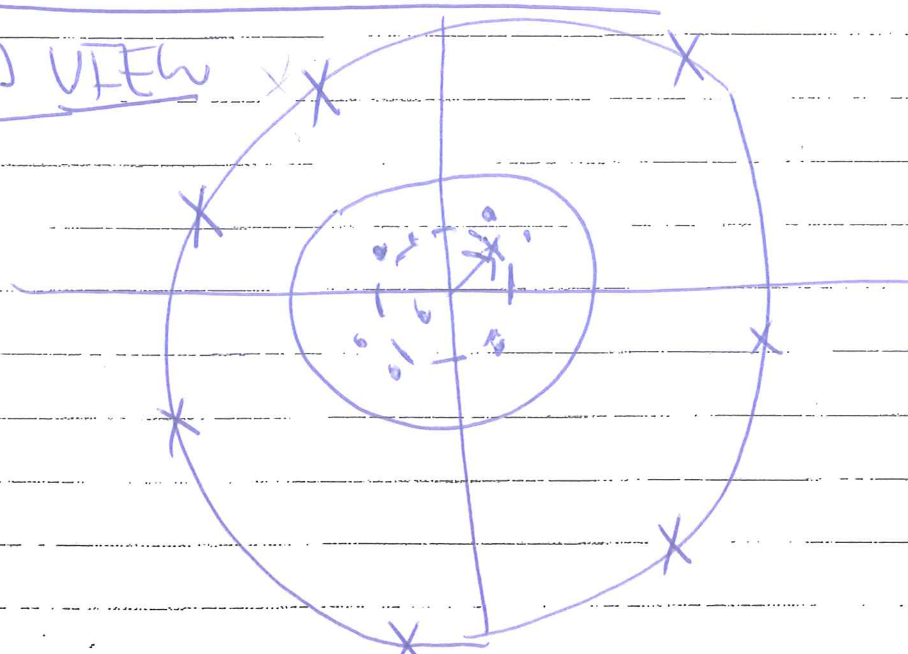
$$\therefore |\vec{B}| = \frac{\mu_0 I}{2\pi r}$$

Ex | A coaxial cable (thin outer conductor)



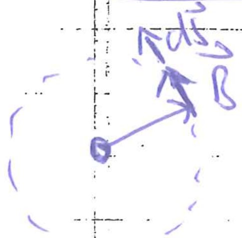
Find $\vec{B}$ everywhere

END VIEW



$r < a$
$a < r < b$
$b < r$

for $r < a$



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enclosed}$$

$$|B| 2\pi r = \mu_0 \underbrace{I_{enclosed}}_{\text{Some fraction of 3 amps}}$$

Aside: Current density for inner conductor $= \dfrac{\text{Current}}{\text{Area}} = \dfrac{3}{\pi a^2}$

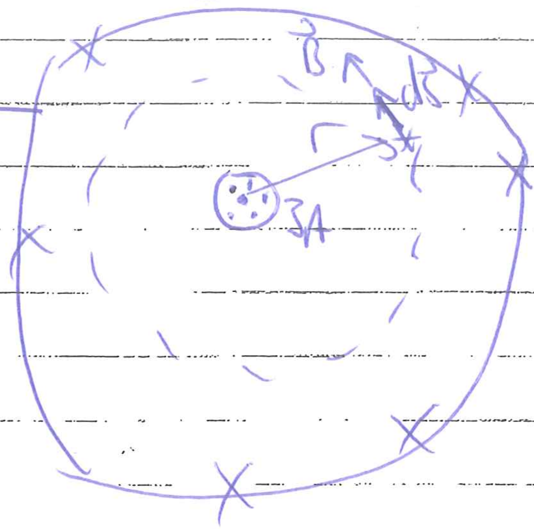
$\therefore$ $I_{enclosed} = \left(\begin{matrix}\text{current}\\ \text{density}\end{matrix}\right) * \left(\text{Area enclosed}\right)$

$$= \frac{3}{\pi a^2}\left(\pi r^2\right) = \frac{3r^2}{a^2}$$

So: $|\vec{B}|2\pi r = \mu_0 \dfrac{3r^2}{a^2}$

$$|\vec{B}| = \frac{3\mu_0 r}{2\pi a^2} \quad \text{for } r < a$$

for $a < r < b$

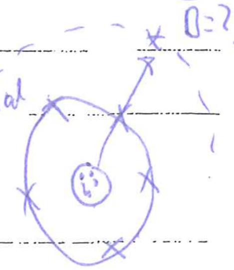


$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$

$|\vec{B}|2\pi r = \mu_0(3)$

$|\vec{B}| = \dfrac{3\mu_0}{2\pi r}$

$0 = I_{enclosed}$



for $r > b$ $\quad |\vec{B}| = 0$ why?

END EXAM 2 CONTENT