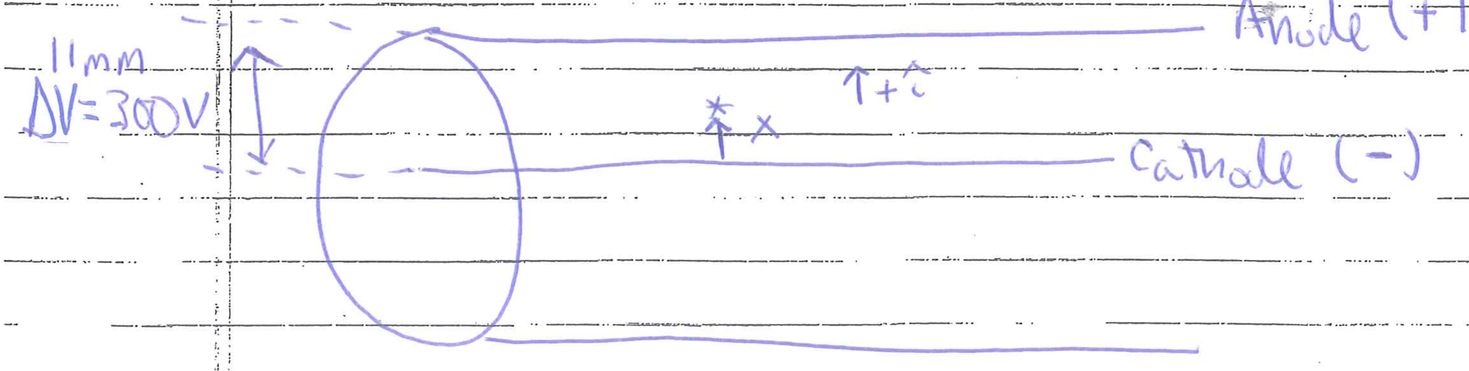
HW.

23.55 Vacuum tube diode



$\boxed{1}$ $V(x) = C x^{4/3}$

Find C
Find E(x)
Find $F_{electron}$ @ ½ way mark

$V(x=0) = 0$
$V(x=0.011) = C(0.011)^{4/3}$

$\Delta V = V_{Anode} - V_{Cathode} = 300 = C(0.011)^{4/3} - 0$

$\therefore C = 1.23 \times 10^5$

$\vec{E}(x) = -\frac{\partial}{\partial x} V(x) = -\frac{4}{3} C x^{1/3} \hat{i}$

$-\frac{d}{dx} V(x)$

E points from Anode to Cathode $\boxed{\checkmark}$

$\vec{F}_{electron}$ @ $x = 0.0055$ $= (-1.6 \times 10^{-19})(-\frac{4}{3} C (0.0055)^{1/3} \hat{i})$

$\boxed{\checkmark}$ does direction make sense?

Book Version of 23.44

Given: $V(x,y,z) = 5x^2y - 8y^2x$

Find $\vec{E}(2,4,0)$.

$\vec{E} = -\nabla V$

$$\vec{E}(x,y,z) = -\frac{\partial}{\partial x}(5x^2y - 8y^2x)\,\hat{i} - \frac{\partial}{\partial y}(5x^2y - 8y^2x)\,\hat{j} - \frac{\partial}{\partial z}(5x^2y - 8y^2x)\,\hat{k}$$

$$-5y\frac{d}{dx}(x^2) + 8y^2\frac{d}{dx}(x)$$

$$-10yx + 8y^2$$

$$\vec{E}(x,y,z) = (-10yx + 8y^2)\,\hat{i} + (-5x^2 + 16yx)\,\hat{j} + 0\,\hat{k}$$

$$\therefore \vec{E}(2,4,0) = (-80 + 128)\,\hat{i} + (-20 + 128)\,\hat{j}$$

$$= 48\,\hat{i} + 108\,\hat{j}$$ Ans

Question: What is the voltage @ the point $(5,5,0)$? Calculate it TWO ways!

$\underbrace{\text{Voltage} \Rightarrow V \Rightarrow \Delta V \Rightarrow \text{Potential difference}}$

Potential Energy ☺

Where is the reference point? $\underbrace{(0,0,0)}$

$V(0,0,0) = 0$

1st WAY: $V(5,5,0) = 5(5)^2 5 - 8(5^2)5 = -375\,V$

Which point is @ higher voltage?

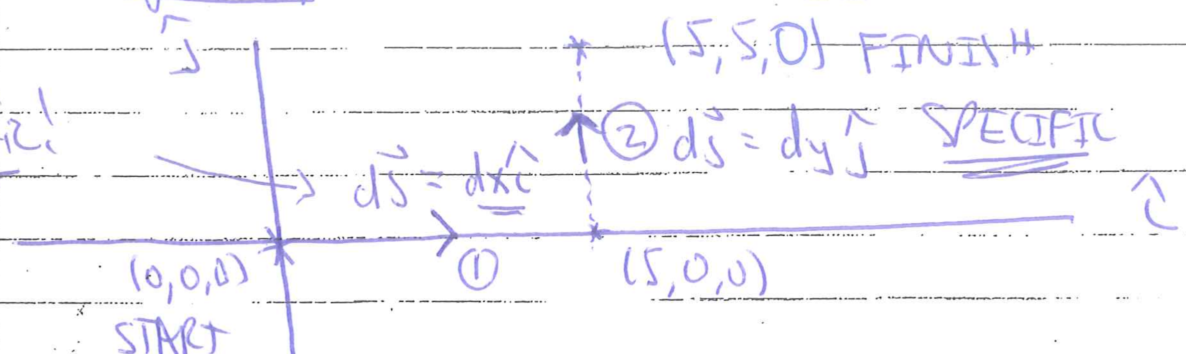
2nd WAY:

$$\Delta V = -\int \vec{E} \cdot d\vec{s} \qquad \Big\| \; \vec{E} = -\nabla V$$

$$\vec{E}(x,y,z) = (-10yx + 8y^2)\hat{i} + (-5x^2 + 16yx)\hat{j}$$

In $\boxed{\text{general}}$: $d\vec{s} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

SPECIFIC!



$$\Delta V = -\int_{①} \vec{E} \cdot d\vec{s} + \left(-\int_{②} \vec{E} \cdot d\vec{s}\right)$$

$$= -\int_{0\;①}^{5} \left[(-10yx + 8y^2)\hat{i} + (-5x^2 + 16yx)\hat{j}\right] \cdot (dx\,\hat{i})$$

$$+\left(-\int_{0\,(2)}^{5}\left[(-10yx+8y^2)\hat{i}+(-5x^2+16yx)\hat{j}\right]\cdot(dy\,\hat{j})\right)$$

$$\Delta V = -\int_{0\,(1)}^{5}(-10\underset{\substack{\uparrow\\0}}{y}x+8\underset{\substack{\uparrow\\0}}{y}^2)\,dx-\int_{0\,(2)}^{5}(-5\underset{\substack{\uparrow\\5}}{x}^2+16y\underset{\substack{\uparrow\\5}}{x})\,dy$$

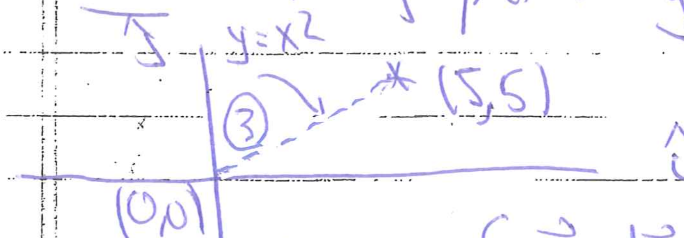
along Path #1

$$= +125\int_0^5 dy - 80\int_0^5 y\,dy$$

$$= +625 - 80\left(\frac{y^2}{2}\right)\Bigg|_0^5 = -375\ \text{V}$$

Answer is independent of the path!

TRY: Along path $y = x$



$$\Delta V = -\int_{(3)}\vec{E}\cdot d\vec{l} = -\int_{(3)}\left[(-10yx+8y^2)\,dx + (-5x^2+16yx)\,dy\right]$$

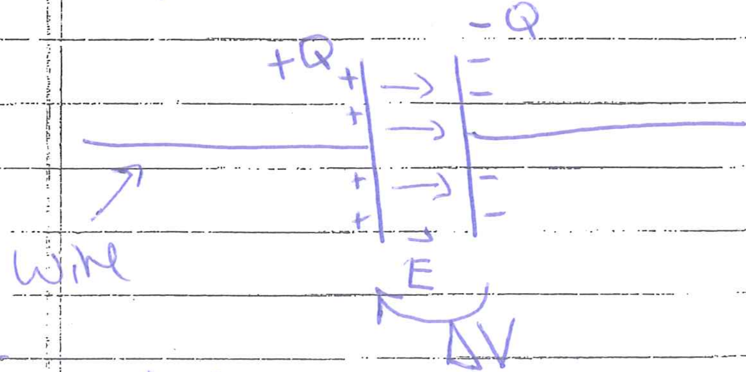
use general form ☆

Choose ONE: $y = x$ $\therefore dy = dx$ || All in y OR All in x || Set limits on y OR set limits on x

If $y = x^3+4$, $dy = 3x^2\,dx$ ☺

Circuits (Ch. 24, Ch. 25, Ch. 26)

Capacitor → a device that stores charge



- Conducting metal plates
- Symmetric
- No charge transport through the device.
- PE is stored in the electric field.

Emperical

$$Q \propto \Delta V$$

$$Q = C \Delta V$$

Numbers are ALWAYS POSITIVE

"capacitance" determined by geometry & construction of device

$$PE_{stored} = \frac{1}{2} C (\Delta V)^2$$

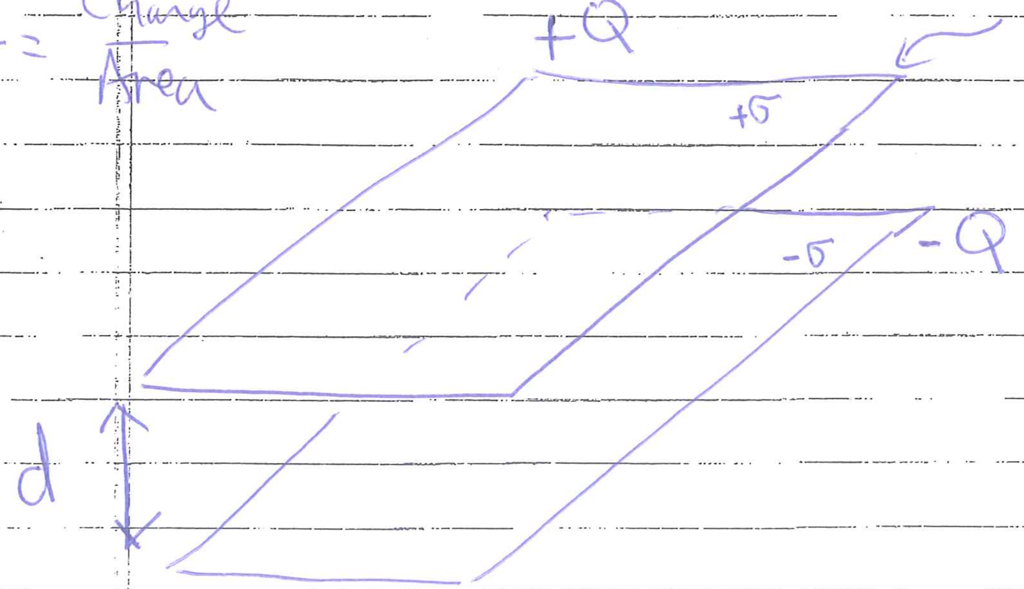
To calculate capacitance of a particular geometry:

- Find an expression for $\vec{E}$ between the plates
- Use $\Delta V = -\int \vec{E} \cdot d\vec{s}$ to find the voltage between plates
- use $C = \frac{Q}{\Delta V}$] Result depends only on geometry

Ex] Parallel plate capacitor.

$\sigma = \frac{\text{Charge}}{\text{Area}}$

Conducting plate
Area $\Rightarrow A$



SIDE VIEW:

$+\sigma$

$d\vec{s}$ $\downarrow$ $\quad \vec{E} = \vec{E}_{+\sigma} + \vec{E}_{-\sigma} = \frac{|\sigma|}{\varepsilon_0}$ ☆

$-\sigma$

$|\vec{E}| = \frac{|\sigma|}{2\varepsilon_0}$

GAUSS

Infinite sheet ☆ Approximation

$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$

$$\Delta V = -\int |\vec{E}||d\vec{s}| \cos\left(\begin{smallmatrix} 0 \\ 180 \end{smallmatrix}\right)$$

$$\Delta V = -\frac{|\sigma|}{\varepsilon_0} \underbrace{\int |d\vec{s}|}_{d} = -\frac{|\sigma| d}{\varepsilon_0}$$

So: $C = \frac{Q}{\Delta V} = \frac{Q}{\sigma d / \varepsilon_0} = \frac{\varepsilon_0 Q}{\sigma d} = \frac{\varepsilon_0 A}{d}$

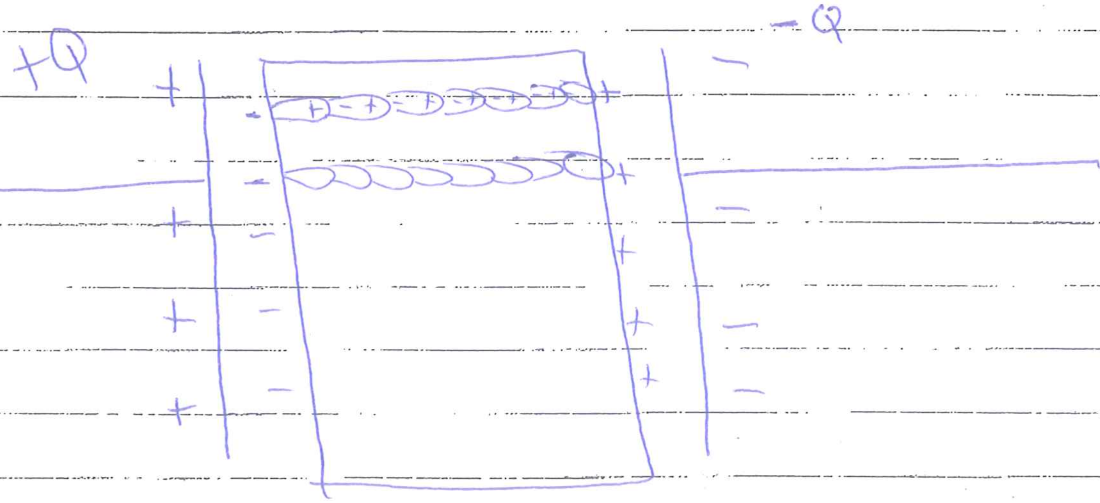
SI. "farads"
(F)

Increase C by adding insulators between plates:

$$C_0 = \frac{\varepsilon_0 A}{d}$$

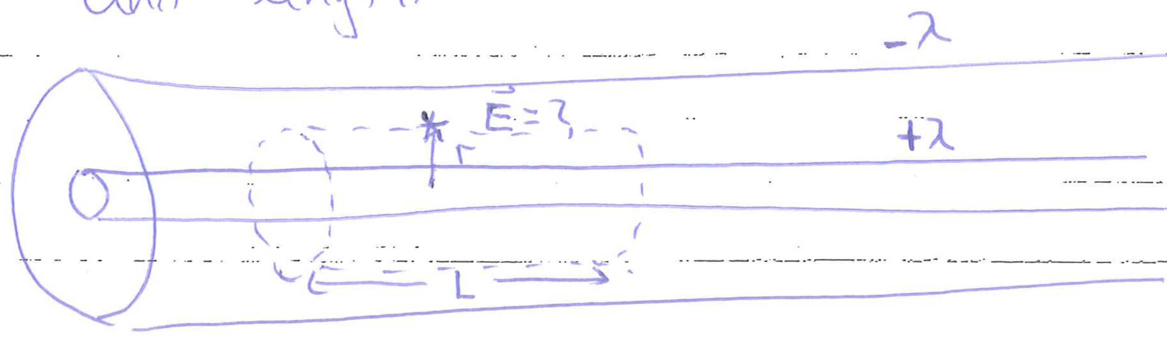
w/insulator, $C_{NEW} = K C_0 = \frac{K \varepsilon_0 A}{d}$

dielectric constant



Polarize molecules in insulator

EX: A coaxial cable has an inner conductor of radius 'a' and an outer conductor of radius 'b'. Find the capacitance per unit length.



Gauss: $$|\vec{E}| = \frac{\frac{\lambda L}{\varepsilon_0}}{2\pi r L} = \frac{\lambda}{2\pi\varepsilon_0 r}$$ radial

L.T.Y.

⋮

$$\Delta V = -\int \vec{E}\cdot d\vec{s} = -\frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

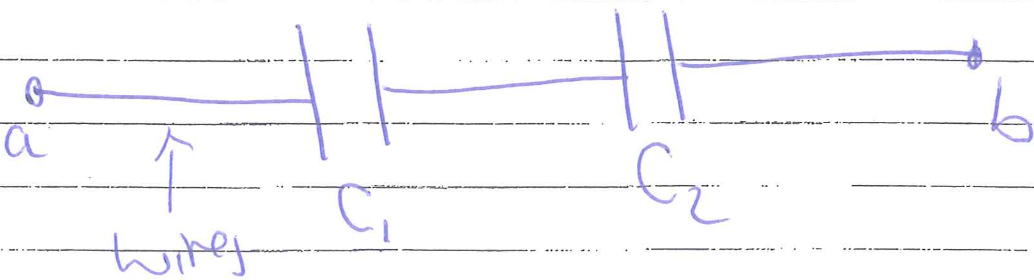
$$\Rightarrow C_{\text{coax cable}} = \frac{Q}{\Delta V} = \frac{\lambda L}{\frac{\lambda \ln\left(\frac{b}{a}\right)}{2\pi\varepsilon_0}} = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$$

$$\left(\frac{C_{\text{coax}}}{L}\right) = \frac{2\pi\varepsilon_0}{\ln(b/a)}$$

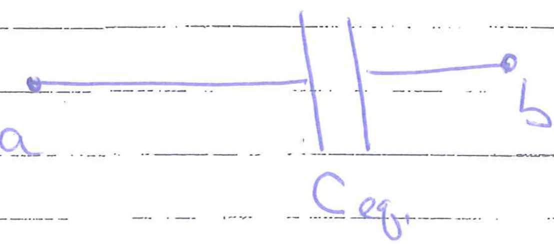
This is example 24.4 on page 789

Practical

Series ⇒ Must pass through both to get from point 'a' to 'b'

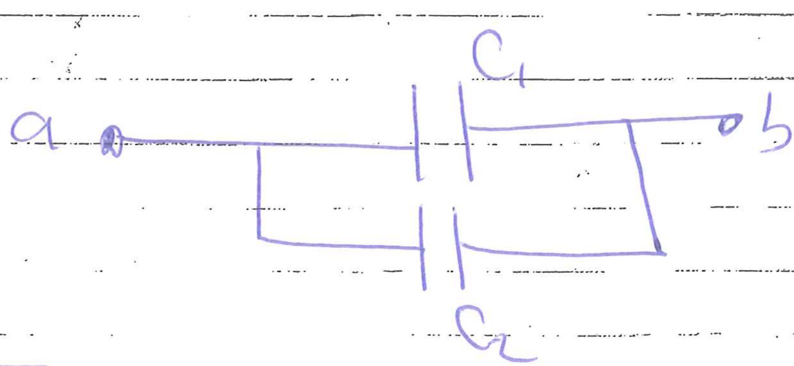


Equivalent Capacitance :



$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

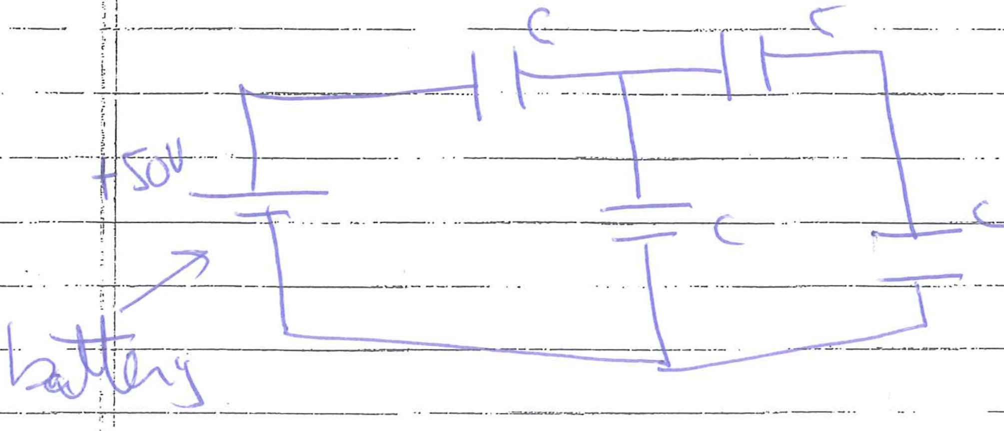
Parallel ⇒ Alternate ways to get from point 'a' to 'b'



$$C_{eq} = C_1 + C_2$$

NOTES: - Capacitors in Series have the same charge.
- Capacitors in parallel have the same voltage

Given an arrangement of capacitors, find the charge an voltage on each.



C = 10uF