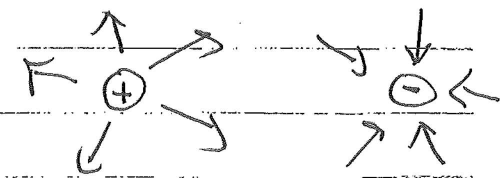
Find the electric field due to continuous charge distributions.

Charge density: 1-d $\lambda = \frac{\text{Charge}}{\text{length}}$

2-d $\sigma = \frac{\text{charge}}{\text{Area}}$

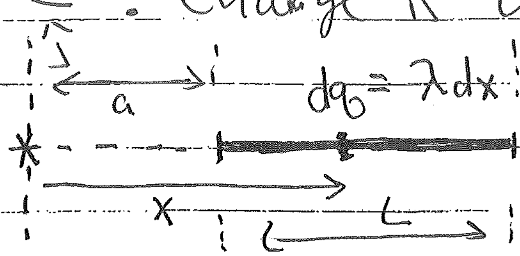
3-d $\rho = \frac{\text{charge}}{\text{Volume.}}$

Recall: $\left|\vec{E}_{\substack{pt\\ \text{charge}}}\right| = \frac{k|q|}{r^2}$



Ex. Find $\vec{E}$ a distance 'a' from the end of a line of charge w/ total charge $+Q$ and length L. Charge is uniformly distributed.

$\frac{k|dq|}{r^2} = |d\vec{E}|$



$\hat{\imath}$ $\lambda = \frac{Q}{L}$

Write expression for the small part of the electric field ($d\vec{E}$) produced by a small piece of charge dq.

- Choose coordinate system (that sets up integrals you can do!)
- Write vector expression for $d\vec{E}$
- Integrate (count contributions from all charge.)

$$d\vec{E} = \frac{k|dq'|}{x^2} = -\frac{k\lambda dx}{\cancel{x^2}}\ \hat{i}$$

!! !! Assigned from Symmetry

$$\Rightarrow \int |d\vec{E}| = \int_a^{(a+L)} \frac{k\lambda dx}{x^2}$$

$$|\vec{E}| = k\lambda \int_a^{(a+L)} \frac{dx}{x^2} = k\lambda\left[-\frac{1}{x}\right]\Bigg|_a^{a+L}$$

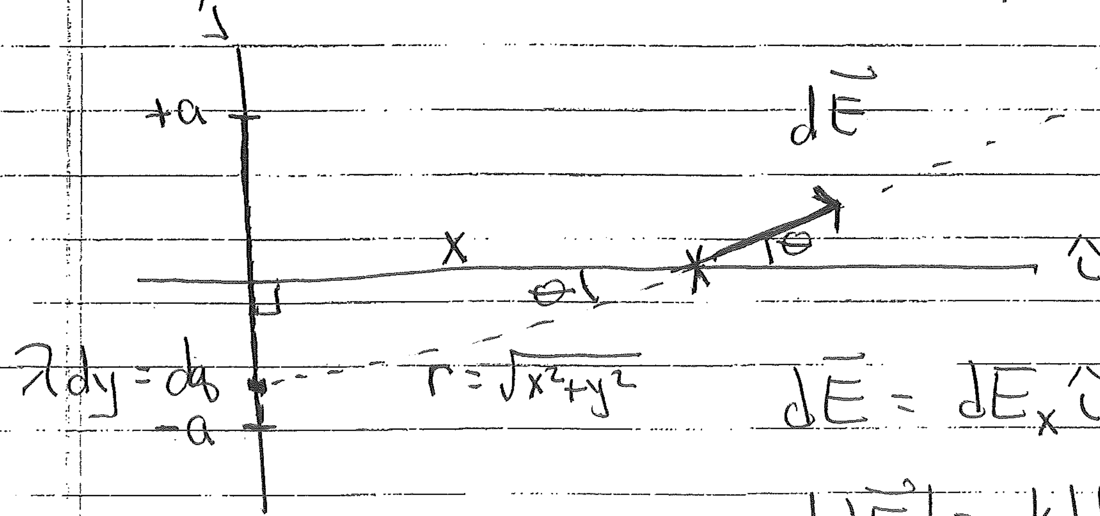
$$= k\lambda\left[-\frac{1}{a+L} + \frac{1}{a}\right]$$

$$= \frac{kQ}{L}\left[\frac{1}{a} - \frac{1}{a+L}\right]$$

$$\therefore \vec{E} = -\frac{kQ}{L}\left[\frac{1}{a} - \frac{1}{a+L}\right]\hat{i}$$

L.T.Y. Show that if $a >> L$, $\vec{E} \approx -\frac{kQ}{a^2}\hat{i}$

Ex. 21.10) A line of charge w/ length '2a' and uniform charge density $+\lambda$. Find $\vec{E}$ a distance 'x' from its midpoint.



$$d\vec{E} = dE_x\,\hat{i} + dE_y\,\hat{j}$$

$$|d\vec{E}| = \frac{k|dq|}{r^2}$$

$$dE_y = |d\vec{E}|\sin\theta$$

$$= \frac{k\lambda\,dy}{(x^2+y^2)}\underbrace{\sin\theta}_{\left(\frac{y}{r}\right)} = \frac{k\lambda\,y\,dy}{(x^2+y^2)^{3/2}}$$

~~REALLY ???~~

Check over the range of integration (charge distribution)

If y is "−", then dE_y must be "+"

If y is "+", then dE_y must be "−"

$$\therefore\ dE_y = -\frac{k\lambda\,y\,dy}{(x^2+y^2)^{3/2}}$$

☑

$$\int dE_y = -\int_{-a}^{a} \frac{k\lambda y\,dy}{(x^2+y^2)^{3/2}}$$

$$E_y = -k\lambda \underbrace{\int_{-a}^{a} \frac{y\,dy}{(x^2+y^2)^{3/2}}}_{\text{Appendix A}}$$

$$\left[-\frac{1}{\sqrt{y^2+x^2}}\right]\Bigg|_{-a}^{a}$$

$$E_y = -k\lambda\left[-\frac{1}{\sqrt{a^2+x^2}} - \left(-\frac{1}{\sqrt{a^2+x^2}}\right)\right] = \underline{\underline{0}}$$

$$dE_x = |d\vec{E}|\cos\theta = \frac{k\,dq}{r^2}\underbrace{\cos(\theta)}_{\frac{x}{r} \,=\, \frac{x}{(x^2+y^2)^{1/2}}} = \frac{k\lambda\,dy\,x}{(x^2+y^2)^{3/2}}$$

$$dE_x = \frac{k\lambda x\,dy}{(x^2+y^2)^{3/2}}$$

REALLY???

$$\int dE_x = \int_{-a}^{a} \frac{k\lambda x\,dy}{(x^2+y^2)^{3/2}}$$

$$E_x = k\lambda x \underbrace{\int_{-a}^{a} \frac{dy}{(x^2+y^2)^{3/2}}}_{\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}}$$

$$= k\lambda x \left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]\Bigg|_{-a}^{a}$$

$$= \frac{k\lambda x}{x^2}\left(\frac{2a}{\sqrt{x^2+a^2}}\right)$$

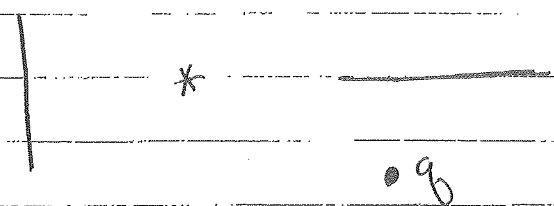
$$\therefore \quad \vec{E}_{@x} = \frac{2k\lambda a}{x\sqrt{x^2+a^2}}\,\hat{\imath} + 0\,\hat{\jmath}$$

Answer ☺

L.T.Y. Show that this becomes

$$\vec{E}_{@x} = \underbrace{\frac{k(2a\lambda)}{x^2}}_{\text{point charge!!}} \quad \text{if } x \gg a$$

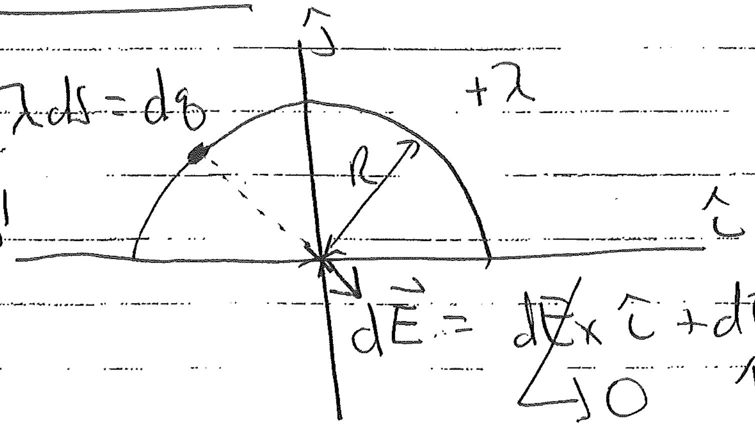
Be aware: Superposition



H.W. HINT $s = R\theta$

$\lambda R\, d\theta = \lambda\, ds = dq$

$|d\vec{E}| = \frac{k|dq|}{R^2}$



$\lambda = \frac{+Q}{\pi R}$

$d\vec{E} = \underbrace{dE_x}_{0}\,\hat{\imath} + dE_y\,\hat{\jmath}$

End ch. 21

Ch. 22

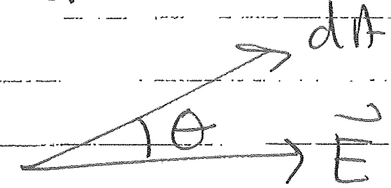
Electric Flux: A count of electric field lines passing through a surface area
(Φ_E)

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

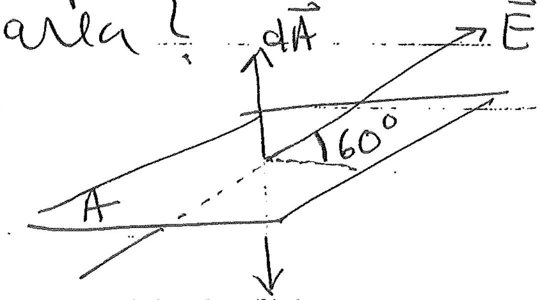
$d\vec{A}$ direction is $\perp$ to the surface

$\int$ ↑ a counting over the surface area.

$$\vec{E} \cdot d\hat{A} = |\vec{E}||d\hat{A}|\cos(\theta)$$



EX) A sheet of paper lies in the x-y plane an has an area $A = 5\ m^2$.
All of the surrounding space has an electric field that make an angle of $60°$ w.r.t. the x-y plane, has a magnitude of $150\ N/C$, and has a $+\hat{k}$ component. What is Φ_E through the area?

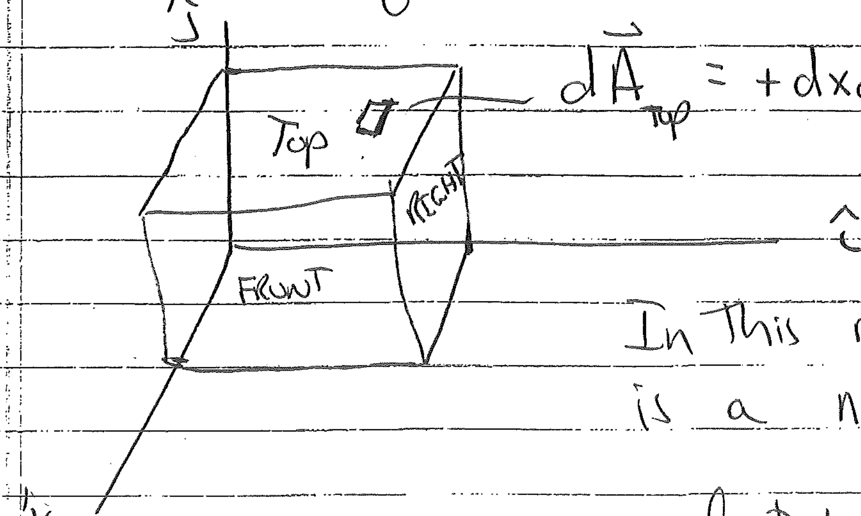


$$\Phi_E = \int \vec{E} \cdot d\hat{A} = \int |\vec{E}||d\hat{A}|\cos(30°)$$

$$= |\vec{E}|\cos(30)\underbrace{\int |d\vec{A}|}_{A}$$

$$= EA\cos(30) = 649.5\ \frac{Nm^2}{C}$$

Ex.) A cube w/sides of length 'a' has one corner @ the origin and sits in the 1st quadrant.



$d\vec{A}_{TOP} = +dxdz\,\hat{j}$

In this region of space there is a nonuniform, constant electric field.

$$\vec{E}(x,y,z) = 4zx\,\hat{i} + 8y^2z\,\hat{j} + 7y\,\hat{k}$$

Find Φ_E through the top face of the cube.

$$\Phi_{E_{TOP}} = \int_{TOP} \vec{E}\cdot d\vec{A} = \int_{TOP} \left[(4zx\,\hat{i} + 8y^2z\,\hat{j} + 7y\,\hat{k})\cdot dxdz\,\hat{j}\right]$$

$$\boxed{\begin{array}{l} \hat{i}\cdot\hat{i} = |\hat{i}||\hat{i}|\cos(0) = 1*1*1 = 1 \\ \hat{i}\cdot\hat{j} = |\hat{i}||\hat{j}|\cos(90) = 1*1*0 = 0 \end{array}}$$

$$= \int_{TOP} 8y^2z\,dx\,dz = 8a^2 \iint z\,dxdz$$

'a' anywhere on top surface

$$= 8a^2 \int_0^a dx \int_0^a z\,dz$$

$$= 8a^3 \left(\frac{z^2}{2}\right)\Big|_0^a = \frac{8a^5}{2} = +4a^5\ \frac{Nm^2}{C}$$

+ In direction of $d\vec{A}$

FYI: $d\vec{A}_{\text{Right}} = +\,dy\,dz\,\hat{i}$

Gauss (1777-1855)

"Gauss' Law": Equiv. to Coulomb (1785)

BUT in new paradigm (fields) $\vec{E}$

$$\underbrace{\oint \vec{E}\cdot d\vec{A}}_{\Phi_E} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

closed surface

$K = \frac{1}{4\pi\varepsilon_0}$

Faraday $\approx$ 1844 *

Formulated 1835 *

Published 1867 ☺