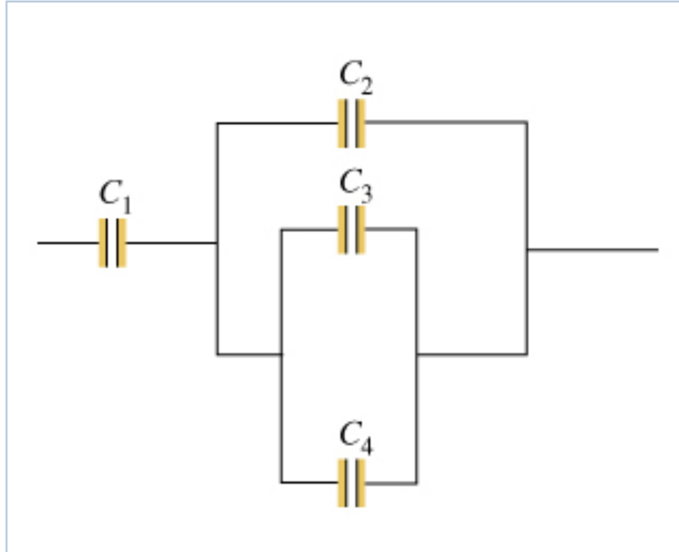
HW due 6/15

Due: 7:00am on Wednesday, June 15, 2016

To understand how points are awarded, read the Grading Policy for this assignment.

Equivalent Capacitance

Consider the combination of capacitors shown in the diagram, where C_1 = 3.00 $\mu\mathrm{F}$, C_2 = 11.0 $\mu\mathrm{F}$, C_3 = 3.00 $\mu\mathrm{F}$, and C_4 = 5.00 $\mu\mathrm{F}$.



Part A

Find the equivalent capacitance C_A of the network of capacitors.

Express your answer in microfarads.

Hint 1. How to reduce the network of capacitors

To find the equivalent capacitance of the given network of capacitors, it is most convenient to reduce the network in successive stages. First, replace the capacitors C_2, C_3, and C_4, which are in parallel, with a single capacitor with an equivalent capacitance. By doing so, you will reduce the network to a series connection of two capacitors. At this point, you only need to find their equivalent capacitance.

Hint 2. Find the capacitance equivalent to C_2, C_3, and C_4

Find the capacitance C_{234} equivalent to the parallel connection of the capacitors C_2, C_3, and C_4.

Express your answer in microfarads.

Hint 1. Find the capacitance equivalent to C_3 and C_4

Find the capacitance C_{34} equivalent to the parallel connection of the capacitors C_3 and C_4.

Express your answer in microfarads.

Hint 1. Two capacitors in parallel

Consider two capacitors of capacitance $C_{\rm a}$ and $C_{\rm b}$ connected in parallel. They are equivalent to a capacitor with capacitance $C_{\rm eq}$ given by

$$C_{\rm eq} = C_{\rm a} + C_{\rm b}\,.$$

ANSWER:

C_{34} = 8.00 $\mu\rm F$

ANSWER:

C_{234} = 19.0 $\mu\rm F$

Correct

If you replace the capacitors C_2, C_3, and C_4 with a capacitor of capacitance C_{234}, the resulting network would be a series connection between C_1 and C_{234}. Its equivalent capacitance is also the equivalent capacitance of the original network.

Hint 3. Two capacitors in series

Consider two capacitors of capacitance $C_{\rm a}$ and $C_{\rm b}$ connected in series. They are equivalent to a capacitor of capacitance $C_{\rm eq}$ that satisfies the following relation:

$$\frac{1}{C_{\rm eq}} = \frac{1}{C_{\rm a}} + \frac{1}{C_{\rm b}}.$$

ANSWER:

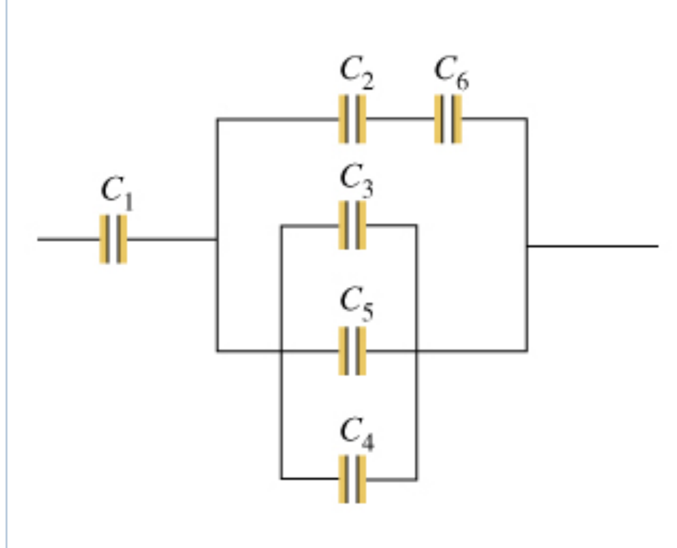
$C_{\rm A}$ = 2.59 $\mu\rm F$

Correct

Part B

Two capacitors of capacitance C_5 = 6.00 $\mu\rm F$ and C_6 = 3.00 $\mu\rm F$ are added to the network, as shown in the diagram. Find the equivalent capacitance $C_{\rm B}$ of the new network of capacitors.

Express your answer in microfarads.



Hint 1. How to reduce the extended network of capacitors

To determine the equivalent capacitance of the extended network of capacitors, it is again convenient to reduce the network in successive stages. First, determine the equivalent capacitance of the series connection of the capacitors C_2 and C_6. Then, combine it with the equivalent capacitance of the parallel connection of C_3, C_4, and C_5, and replace the five capacitors with their equivalent capacitor. The resulting network will consist of two capacitors in series. At this point, you only need to find their equivalent capacitance.

Hint 2. Find the equivalent capacitance of C_2, C_3, C_4, C_5, and C_6

Find the equivalent capacitance C_{2-6} of the combination of capacitors C_2, C_3, C_4, C_5, and C_6.

Express your answer in microfarads.

Hint 1. Find the equivalent capacitance of C_2 and C_6

Find the equivalent capacitance C_{26} of the series connection of C_2 and C_6.

Express your answer in microfarads.

Hint 1. Two capacitors in series

Consider two capacitors of capacitance C_{a} and C_{b} connected in series. They are equivalent to a capacitor of capacitance C_{eq} that satisfies the following relation:

$$\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_{\mathrm{a}}} + \frac{1}{C_{\mathrm{b}}}.$$

ANSWER:

C_{26} = 2.36 $\mu\mathrm{F}$

Hint 2. Find the equivalent capacitance of C_3, C_4 , and C_5

Find the equivalent capacitance C_{345} of the parallel connection of C_3, C_4 , and C_5

Express your answer in microfarads.

Hint 1. Three capacitors in parallel

Consider three capacitors of capacitance $C_{\rm a}$, $C_{\rm b}$, and $C_{\rm c}$ connected in parallel. They are equivalent to a capacitor with capacitance $C_{\rm eq}$ given by

$$C_{\rm eq} = C_{\rm a} + C_{\rm b} + C_{\rm c}.$$

ANSWER:

C_{345} = 14.0 $\mu\mathrm{F}$

ANSWER:

C_{2-6} = 16.4 $\mu\mathrm{F}$

Correct

If you replace the capacitors C_2, C_3, C_4, C_5, and C_6 with a capacitor of capacitance C_{2-6}, the resulting network would be a series connection between C_1 and C_{2-6}. Its equivalent capacitance is also the equivalent capacitance of the original network.

Hint 3. Two capacitors in series

Consider two capacitors of capacitance $C_{\rm a}$ and $C_{\rm b}$ connected in series. They are equivalent to a capacitor of capacitance $C_{\rm eq}$ that satisfies the following relation:

$$\frac{1}{C_{\rm eq}} = \frac{1}{C_{\rm a}} + \frac{1}{C_{\rm b}}.$$

ANSWER:

$C_{\rm B}$ = 2.54 $\mu\mathrm{F}$

Correct

± Potential Difference and Electric-Field Energy of a Spherical Capacitor

A spherical capacitor is formed from two concentric spherical conducting shells separated by vacuum. The inner sphere has radius 10.0 centimeters, and the separation between the spheres is 1.50 centimeters. The magnitude of the charge on each sphere is 3.30 nanocoulombs.

Part A

What is the magnitude of the potential difference ΔV between the two spheres?

Hint 1. How to approach the problem

Find an appropriate Gaussian surface for the system; then use Gauss's law to calculate the electric field between the concentric spheres. Use the calculated electric field to find the potential difference between the spheres.

Hint 2. Choosing the Gaussian surface

Because of the spherical symmetry of the spherical capacitor, the best Gaussian surface to use for this system is a sphere with radius r that satisfies the criterion $r_{\mathrm{inner}} < r < r_{\mathrm{outer}}$, where r_{inner} is the radius of the inner sphere and r_{outer} is the radius of the outer sphere.

Hint 3. Find the electric field

Find an expression for the magnitude E of the electric field on the Gaussian surface. Note that the spherical symmetry of the situation ensures that the electric field will have constant magnitude over a particular spherical surface.

Express your answer in terms of some or all of the following variables: the radius r of the Gaussian surface, the permittivity of free space ϵ_0, and the magnitude of the charge Q on the inner sphere.

Hint 1. Calculate the integral in Gauss's law

Gauss's law states that $\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\mathrm{encl}}}{\epsilon_0}$. Using a sphere of radius r as a Gaussian surface, evaluate the integral on the left-hand side of the equation.

Express your answer in terms of the magnitude E of the field and the radius r of the Gaussian surface.

Hint 1. Direction of the electric field

By symmetry, the electric field $\vec{E}$ points radially at all points between the spheres, so for any surface element $d\vec{A}$ on the Gaussian surface, the normal of the surface element will be parallel to the electric field at all points, so $\vec{E} \cdot d\vec{A} = E\,dA$.

ANSWER:

$\oint_S \vec{E} \cdot d\vec{A} =$ $E{\cdot}4\pi r^2$

All attempts used; correct answer displayed

ANSWER:

$E = \frac{Q}{4\pi\epsilon_0 r^2}$

Incorrect; Try Again; 3 attempts remaining

Hint 4. How to use the electric field to calculate the potential difference

To calculate the potential difference, use the equation $V_{\mathrm{inner}} - V_{\mathrm{outer}} = \Delta V = \int_{r_{\mathrm{inner}}}^{r_{\mathrm{outer}}} \vec{E} \cdot d\vec{l}$. If a radial path is chosen to go from the inner sphere to the outer sphere, then the electric field and the path will be parallel, and therefore $\vec{E} \cdot d\vec{l} = E\,dr$.

ANSWER:

ΔV = 38.7 V

Correct

Part B

What is the electric-field energy stored in the capacitor?

Hint 1. How to calculate the electric-field energy

The electric-field energy is calculated by the equation $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$. Use the simplest of these equations, given the variables already known and/or calculated.

ANSWER:

6.38×10^{-8} J

Correct

Score Summary:

Your score on this assignment is 100%.

You received 10 out of a possible total of 10 points.