Problem 1.4 A wave traveling along a string is given by

$$y(x,t) = 2\sin(4\pi t + 10\pi x) \quad \text{(cm)},$$

where x is the distance along the string in meters and y is the vertical displacement. Determine: (a) the direction of wave travel, (b) the reference phase ϕ_0, (c) the frequency, (d) the wavelength, and (e) the phase velocity.

Solution:

(a) We start by converting the given expression into a cosine function of the form given by (1.17):

$$y(x,t) = 2\cos\left(4\pi t + 10\pi x - \frac{\pi}{2}\right) \quad \text{(cm)}.$$

Since the coefficients of t and x both have the same sign, the wave is traveling in the negative x-direction.

(b) From the cosine expression, $\phi_0 = -\pi/2$.

(c) $\omega = 2\pi f = 4\pi$,

$$f = 4\pi/2\pi = 2 \text{ Hz}.$$

(d) $2\pi/\lambda = 10\pi$,

$$\lambda = 2\pi/10\pi = 0.2 \text{ m}.$$

(e) $u_{\rm p} = f\lambda = 2 \times 0.2 = 0.4$ (m/s).

Problem 1.8 Two waves on a string are given by the following functions:

$$y_1(x,t) = 4\cos(20t - 30x) \quad \text{(cm)}$$
$$y_2(x,t) = -4\cos(20t + 30x) \quad \text{(cm)}$$

where x is in centimeters. The waves are said to interfere constructively when their superposition $|y_\mathrm{s}| = |y_1 + y_2|$ is a maximum, and they interfere destructively when $|y_\mathrm{s}|$ is a minimum.

(a) What are the directions of propagation of waves $y_1(x,t)$ and $y_2(x,t)$?

(b) At $t = (\pi/50)$ s, at what location x do the two waves interfere constructively, and what is the corresponding value of $|y_\mathrm{s}|$?

(c) At $t = (\pi/50)$ s, at what location x do the two waves interfere destructively, and what is the corresponding value of $|y_\mathrm{s}|$?

Solution:

(a) $y_1(x,t)$ is traveling in positive x-direction. $y_2(x,t)$ is traveling in negative x-direction.

(b) At $t = (\pi/50)$ s, $y_s = y_1 + y_2 = 4[\cos(0.4\pi - 30x) - \cos(0.4\pi + 3x)]$. Using the formulas from Appendix C,

$$2\sin x \sin y = \cos(x - y) - (\cos x + y),$$

we have

$$y_s = 8\sin(0.4\pi)\sin 30x = 7.61\sin 30x.$$

Hence,

$$|y_\mathrm{s}|_{\max} = 7.61$$

and it occurs when $\sin 30x = 1$, or $30x = \dfrac{\pi}{2} + 2n\pi$, or $x = \left(\dfrac{\pi}{60} + \dfrac{2n\pi}{30}\right)$ cm, where $n = 0, 1, 2, \ldots$.

(c) $|y_\mathrm{s}|_{\min} = 0$ and it occurs when $30x = n\pi$, or $x = \dfrac{n\pi}{30}$ cm.

Problem 1.13 The voltage of an electromagnetic wave traveling on a transmission line is given by $\upsilon(z,t) = 5e^{-\alpha z}\sin(4\pi \times 10^9 t - 20\pi z)$ (V), where z is the distance in meters from the generator.

(a) Find the frequency, wavelength, and phase velocity of the wave.

(b) At $z = 2$ m, the amplitude of the wave was measured to be 2 V. Find α.

Solution:

(a) This equation is similar to that of Eq. (1.28) with $\omega = 4\pi \times 10^9$ rad/s and $\beta = 20\pi$ rad/m. From Eq. (1.29a), $f = \omega/2\pi = 2 \times 10^9$ Hz $= 2$ GHz; from Eq. (1.29b), $\lambda = 2\pi/\beta = 0.1$ m. From Eq. (1.30),

$$u_{\mathrm{p}} = \omega/\beta = 2 \times 10^8 \text{ m/s}.$$

(b) Using just the amplitude of the wave,

$$2 = 5e^{-\alpha 2}, \qquad \alpha = \frac{-1}{2\text{ m}} \ln\left(\frac{2}{5}\right) = 0.46 \text{ Np/m}.$$

Problem 1.26 Find the phasors of the following time functions:

(a) $\upsilon(t) = 9\cos(\omega t - \pi/3)$ (V)

(b) $\upsilon(t) = 12\sin(\omega t + \pi/4)$ (V)

(c) $i(x,t) = 5e^{-3x}\sin(\omega t + \pi/6)$ (A)

(d) $i(t) = -2\cos(\omega t + 3\pi/4)$ (A)

(e) $i(t) = 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6)$ (A)

Solution:

(a) $\widetilde{V} = 9e^{-j\pi/3}$ V.

(b) $v(t) = 12\sin(\omega t + \pi/4) = 12\cos(\pi/2 - (\omega t + \pi/4)) = 12\cos(\omega t - \pi/4)$ V, $\widetilde{V} = 12e^{-j\pi/4}$ V.

(c)

$$\begin{aligned}
i(t) &= 5e^{-3x}\sin(\omega t + \pi/6) \text{ A} = 5e^{-3x}\cos[\pi/2 - (\omega t + \pi/6)] \text{ A} \\
&= 5e^{-3x}\cos(\omega t - \pi/3) \text{ A}, \\
\widetilde{I} &= 5e^{-3x}e^{-j\pi/3} \text{ A}.
\end{aligned}$$

(d)

$$\begin{aligned}
i(t) &= -2\cos(\omega t + 3\pi/4), \\
\widetilde{I} &= -2e^{j3\pi/4} = 2e^{-j\pi}e^{j3\pi/4} = 2e^{-j\pi/4} \text{ A}.
\end{aligned}$$

(e)

$$\begin{aligned}
i(t) &= 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6) \\
&= 4\cos[\pi/2 - (\omega t + \pi/3)] + 3\cos(\omega t - \pi/6) \\
&= 4\cos(-\omega t + \pi/6) + 3\cos(\omega t - \pi/6) \\
&= 4\cos(\omega t - \pi/6) + 3\cos(\omega t - \pi/6) = 7\cos(\omega t - \pi/6), \\
\widetilde{I} &= 7e^{-j\pi/6} \text{ A}.
\end{aligned}$$

Problem 1.27 Find the instantaneous time sinusoidal functions corresponding to the following phasors:

(a) $\widetilde{V} = -5e^{j\pi/3}$ (V)
(b) $\widetilde{V} = j6e^{-j\pi/4}$ (V)
(c) $\widetilde{I} = (6 + j8)$ (A)
(d) $\tilde{I} = -3 + j2$ (A)
(e) $\tilde{I} = j$ (A)
(f) $\tilde{I} = 2e^{j\pi/6}$ (A)

Solution:

(a)

$$\begin{aligned}\widetilde{V} &= -5e^{j\pi/3} \text{ V} = 5e^{j(\pi/3-\pi)} \text{ V} = 5e^{-j2\pi/3} \text{ V},\\ v(t) &= 5\cos(\omega t - 2\pi/3) \text{ V}.\end{aligned}$$

(b)

$$\begin{aligned}\widetilde{V} &= j6e^{-j\pi/4} \text{ V} = 6e^{j(-\pi/4+\pi/2)} \text{ V} = 6e^{j\pi/4} \text{ V},\\ v(t) &= 6\cos(\omega t + \pi/4) \text{ V}.\end{aligned}$$

(c)

$$\begin{aligned}\widetilde{I} &= (6 + j8) \text{ A} = 10e^{j53.1^\circ} \text{ A},\\ i(t) &= 10\cos(\omega t + 53.1^\circ) \text{ A}.\end{aligned}$$

(d)

$$\begin{aligned}\widetilde{I} &= -3 + j2 = 3.61\, e^{j146.31^\circ},\\ i(t) &= \mathfrak{Re}\{3.61\, e^{j146.31^\circ} e^{j\omega t}\} = 3.61 \cos(\omega t + 146.31^\circ) \text{ A}.\end{aligned}$$

(e)

$$\begin{aligned}\widetilde{I} &= j = e^{j\pi/2},\\ i(t) &= \mathfrak{Re}\{e^{j\pi/2} e^{j\omega t}\} = \cos(\omega t + \pi/2) = -\sin\omega t \text{ A}.\end{aligned}$$

(f)

$$\begin{aligned}\widetilde{I} &= 2e^{j\pi/6},\\ i(t) &= \mathfrak{Re}\{2e^{j\pi/6} e^{j\omega t}\} = 2\cos(\omega t + \pi/6) \text{ A}.\end{aligned}$$

5.

a. by inspection $\vec{E}(x,y,z,t) = |E_0| \cos(\omega t - kx + \phi)\,\hat{z}$

gives

$$\omega = 2\pi \times 10^8 \qquad = 2\pi f$$

$$k = \frac{8\pi}{3} \qquad = 2\pi/\lambda$$

$$\phi = -\pi/2$$

a. $f = 10^8 = 100\ MHz$

b. $k = 8\pi/3 \quad m^{-1}$

c. $\lambda = 3/4\ m = 75\ cm$

d. $v = \lambda f = \frac{3}{4} \times 10^8\ \frac{m}{s} = 7.5 \times 10^7\ m/s$

e. $v = \frac{c}{\sqrt{\epsilon_r \mu_r}} \qquad \epsilon_r = \frac{1}{\mu_r}\left(\frac{c}{v}\right)^2 = \left(\frac{3\times 10^8}{\frac{3}{4}\times 10^8}\right)^2$

$$\epsilon_r = 16$$

f. $\vec{E}(x,y,z,t) = Re\left\{ \vec{E}_\omega(x,y,z)\, e^{j\omega t} \right\}$

$$= Re\left\{ |E_0|\, e^{-j\pi/2}\, e^{j\omega t}\, e^{-j\,8\pi/3\,x} \right\} \hat{z}$$

$\Rightarrow$

$$\vec{E}_\omega(x,y,z) = |E_0|\, e^{-j\pi/2}\, e^{-j\frac{8\pi}{3}x}\ \hat{z}$$

g. $\vec{E}_\omega(x=0,y,z) = |E_0|\, e^{-j\pi/2}\,\hat{z} = -j\,|E_0|\,\hat{z}$

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6. Phasor $\vec{E}_\omega(x,y,z) = (e^{j\pi/4}\hat{x} - e^{-j\pi/4}\hat{z})e^{j\pi y}$

a. by inspection $k = \pi = \frac{2\pi}{\lambda} \Rightarrow \underline{\lambda = 2m}$

b. Propagation in air $\Rightarrow v = c = 3\times10^8$

Thus $v = f\lambda$ gives

$$f = v/\lambda = \frac{3\times10^8}{2} = 150\ MHz$$

c. $\vec{E}(x,y,z,t) = Re\{\vec{E}_\omega(x,y,z)e^{j\omega t}\}$

$$= Re\{e^{j\pi/4}e^{j(\omega t+ky)}\hat{x} - e^{-j\pi/4}e^{j(\omega t+ky)}\hat{z}\}$$

$$= \cos(\omega t+ky+\pi/4)\hat{x} - \cos(\omega t+ky-\pi/4)\hat{z}$$

$$= \cos(\omega t+ky+\pi/4)\hat{x} + \cos(\omega t+ky+\tfrac{3\pi}{4})\hat{z}$$

$\omega = 2\pi\times1.5\times10^8$ rad/sec $= 3\pi\times10^8$ rads/sec

$k = \pi\ m^{-1}$

d. From the expressions it is clear that the wave is moving in the $-\hat{y}$ direction.

Problem 1.9 Give expressions for $y(x,t)$ for a sinusoidal wave traveling along a string in the negative x-direction, given that $y_{\max} = 40$ cm, $\lambda = 30$ cm, $f = 10$ Hz, and

(a) $y(x,0) = 0$ at $x = 0$,

(b) $y(x,0) = 0$ at $x = 3.75$ cm.

Solution: For a wave traveling in the negative x-direction, we use Eq. (1.17) with $\omega = 2\pi f = 20\pi$ (rad/s), $\beta = 2\pi/\lambda = 2\pi/0.3 = 20\pi/3$ (rad/s), $A = 40$ cm, and x assigned a positive sign:

$$y(x,t) = 40\cos\left(20\pi t + \frac{20\pi}{3}x + \phi_0\right) \quad \text{(cm)},$$

with x in meters.

(a) $y(0,0) = 0 = 40\cos\phi_0$. Hence, $\phi_0 = \pm\pi/2$, and

$$\begin{aligned} y(x,t) &= 40\cos\left(20\pi t + \frac{20\pi}{3}x \pm \frac{\pi}{2}\right) \\ &= \begin{cases} -40\sin\left(20\pi t + \frac{20\pi}{3}x\right) \text{ (cm), if } \phi_0 = \pi/2, \\ 40\sin\left(20\pi t + \frac{20\pi}{3}x\right) \text{ (cm),} \quad \text{if } \phi_0 = -\pi/2. \end{cases} \end{aligned}$$

(b) At $x = 3.75$ cm $= 3.75 \times 10^{-2}$ m, $y = 0 = 40\cos(\pi/4 + \phi_0)$. Hence, $\phi_0 = \pi/4$ or $5\pi/4$, and

$$y(x,t) = \begin{cases} 40\cos\left(20\pi t + \frac{20\pi}{3}x + \frac{\pi}{4}\right) \text{ (cm),} \quad \text{if } \phi_0 = \pi/4, \\ 40\cos\left(20\pi t + \frac{20\pi}{3}x + \frac{5\pi}{4}\right) \text{ (cm), if } \phi_0 = 5\pi/4. \end{cases}$$

Problem 1.15 A laser beam traveling through fog was observed to have an intensity of 1 (μW/m^2) at a distance of 2 m from the laser gun and an intensity of 0.2 (μW/m^2) at a distance of 3 m. Given that the intensity of an electromagnetic wave is proportional to the square of its electric-field amplitude, find the attenuation constant α of fog.

Solution: If the electric field is of the form

$$E(x,t) = E_0 e^{-\alpha x} \cos(\omega t - \beta x),$$

then the intensity must have a form

$$\begin{aligned} I(x,t) &\approx [E_0 e^{-\alpha x} \cos(\omega t - \beta x)]^2 \\ &\approx E_0^2 e^{-2\alpha x} \cos^2(\omega t - \beta x) \end{aligned}$$

or

$$I(x,t) = I_0 e^{-2\alpha x} \cos^2(\omega t - \beta x)$$

where we define $I_0 \approx E_0^2$. We observe that the magnitude of the intensity varies as $I_0 e^{-2\alpha x}$. Hence,

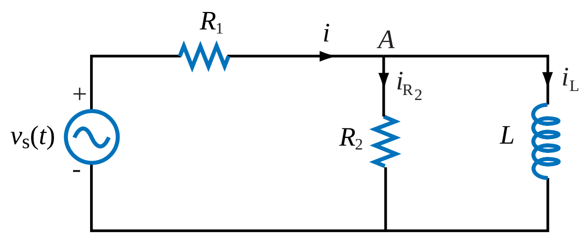
$$\begin{aligned} &\text{at } x = 2 \text{ m}, \qquad I_0 e^{-4\alpha} = 1 \times 10^{-6} \quad (\text{W/m}^2), \\ &\text{at } x = 3 \text{ m}, \qquad I_0 e^{-6\alpha} = 0.2 \times 10^{-6} \quad (\text{W/m}^2). \end{aligned}$$

$$\begin{aligned} \frac{I_0 e^{-4\alpha}}{I_0 e^{-6\alpha}} &= \frac{10^{-6}}{0.2 \times 10^{-6}} = 5 \\ e^{-4\alpha} \cdot e^{6\alpha} &= e^{2\alpha} = 5 \\ \alpha &= 0.8 \quad (\text{NP/m}). \end{aligned}$$

Problem 1.29 The voltage source of the circuit shown in Fig. P1.29 is given by

$$v_s(t) = 25\cos(4\times 10^4 t - 45^\circ) \quad \text{(V)}.$$

Obtain an expression for $i_L(t)$, the current flowing through the inductor.



$R_1 = 20\ \Omega$, $R_2 = 30\ \Omega$, $L = 0.4$ mH

Figure P1.29:

Solution: Based on the given voltage expression, the phasor source voltage is

$$\widetilde{V}_s = 25e^{-j45^\circ} \quad \text{(V)}. \tag{9}$$

The voltage equation for the left-hand side loop is

$$R_1 i + R_2 i_{R_2} = v_s \tag{10}$$

For the right-hand loop,

$$R_2 i_{R_2} = L\frac{di_L}{dt}, \tag{11}$$

and at node A,

$$i = i_{R_2} + i_L. \tag{12}$$

Next, we convert Eqs. (2)–(4) into phasor form:

$$R_1\widetilde{I} + R_2\widetilde{I}_{R_2} = \widetilde{V}_s \tag{13}$$

$$R_2\widetilde{I}_{R_2} = j\omega L\widetilde{I}_L \tag{14}$$

$$\widetilde{I} = \widetilde{I}_{R_2} + \widetilde{I}_L \tag{15}$$

Upon combining (6) and (7) to solve for $\widetilde{I}_{R_2}$ in terms of $\widetilde{I}$, we have:

$$\widetilde{I}_{R_2} = \frac{j\omega L}{R_2 + j\omega L} I\,. \tag{16}$$

Substituting (8) in (5) and then solving for $\widetilde{I}$ leads to:

$$
\begin{aligned}
R_1\widetilde{I} + \frac{jR_2\omega L}{R_2+j\omega L}\widetilde{I} &= \widetilde{V}_{\mathrm{s}} \\
\widetilde{I}\left(R_1 + \frac{jR_2\omega L}{R_2+j\omega L}\right) &= \widetilde{V}_{\mathrm{s}} \\
\widetilde{I}\left(\frac{R_1R_2 + jR_1\omega L + jR_2\omega L}{R_2+j\omega L}\right) &= \widetilde{V}_{\mathrm{s}} \\
\widetilde{I} &= \left(\frac{R_2+j\omega L}{R_1R_2+j\omega L(R_1+R_2)}\right)\widetilde{V}_{\mathrm{s}}. \qquad (17)
\end{aligned}
$$

Combining (6) and (7) to solve for $\widetilde{I}_{\mathrm{L}}$ in terms of $\widetilde{I}$ gives

$$
\widetilde{I}_{\mathrm{L}} = \frac{R_2}{R_2+j\omega L}\widetilde{I}. \qquad (18)
$$

Combining (9) and (10) leads to

$$
\begin{aligned}
\widetilde{I}_{\mathrm{L}} &= \left(\frac{R_2}{R_2+j\omega L}\right)\left(\frac{R_2+j\omega L}{R_1R_2+j\omega L(R_1+R_2)}\right)\widetilde{V}_{\mathrm{s}} \\
&= \frac{R_2}{R_1R_2++j\omega L(R_1+R_2)}\widetilde{V}_{\mathrm{s}}.
\end{aligned}
$$

Using (1) for $\widetilde{V}_{\mathrm{s}}$ and replacing R_1, R_2, L and ω with their numerical values, we have

$$
\begin{aligned}
\widetilde{I}_{\mathrm{L}} &= \frac{30}{20\times 30 + j4\times 10^4 \times 0.4 \times 10^{-3}(20+30)}\ 25e^{-j45^\circ} \\
&= \frac{30\times 25}{600+j800}\ e^{-j45^\circ} \\
&= \frac{7.5}{6+j8}\ e^{-j45^\circ} = \frac{7.5e^{-j45^\circ}}{10e^{j53.1^\circ}} = 0.75e^{-j98.1^\circ} \quad \text{(A)}.
\end{aligned}
$$

Finally,

$$
\begin{aligned}
i_{\mathrm{L}}(t) &= \mathfrak{Re}[\widetilde{I}_{\mathrm{L}}e^{j\omega t}] \\
&= 0.75\cos(4\times 10^4 t - 98.1^\circ) \quad \text{(A)}.
\end{aligned}
$$