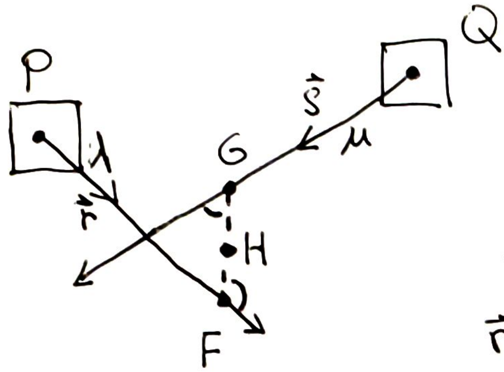
# Triangulation

We try to estimate 3D location of points using relative information of our camera(s).

- Triangulation Using Geometric Approach.




- P, Q are view points.
- $\lambda, \mu$ are scalar lines from respective cameras. Show how far to go, along vector.

*Note, as in 3D the lines may not intersect, so we need to find point H where the distance between the two lines is minimal.

$\vec{r}, \vec{s}$ are line vectors, show direction.

Point G : $g = q + \mu s$ Point F : $f = p + \lambda r$

$p = X_{0'}$

$q = X_{0''}$ } Projection centers.

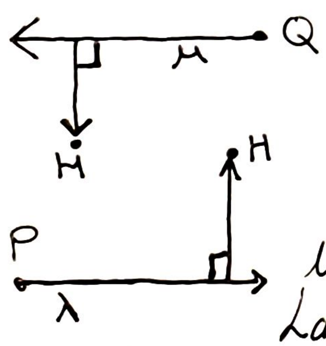
$S = R''^{T} \, {}^{k}X''$ ← Ray direction vectors → $r = R'^{T} \, {}^{k}X'$ *Assuming calibrated camera.

with ${}^{k}X' = (x', y', c)^T$, ${}^{k}X'' = (x'', y'', c)^T$

p, q are known projection centers.

${}^{k}X$ are points in camera coord..

The line between G, and F is orthogonal to both lines $\lambda$, and $\mu$.




This leads to multiple constraints.

$(g-f)\cdot r = 0$, and $(g-f)\cdot s = 0$ ① ← *These are correct.

*There is a mistake in a lecture with equations ①.

$(q + \lambda s - p - \mu r)\cdot s = 0$, and $(q + \lambda s - p - \mu r)\cdot r = 0$

Now we have 2 equations, with 2 unknown $(\lambda, \mu)$.

Lastly, we obtain $\lambda, \mu$ by solving the two equations, which leads to G and F. We use them to compute H as the middle of two lines.

Replcasing some of the variables in the two equation, leads to a more concrete representation...

$$(X_{o'} + \lambda r - X_{o''} - \mu s)^T r = 0, \text{ and } (X_{o'} + \lambda r - X_{o} - \mu s)^T s = 0.$$

and further to...

$$(X_{o'} - X_{o''})^T r + \lambda r^T r - \mu s^T r = 0$$
$$(X_{o'} - X_{o''})^T s + \lambda r^T s - \mu s^T s = 0$$

with the following matrix form...

$$\overbrace{\begin{bmatrix} r^T r & -s^T r \\ r^T s & -s^T s \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} \lambda \\ \mu \end{bmatrix}}^{x} = \overbrace{\begin{bmatrix} (X_{o''} - X_{o'})^T \\ (X_{o''} - X_{o'})^T \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}}^{b}$$

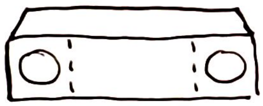
↳ unkown

which is of the form $Ax = b$.

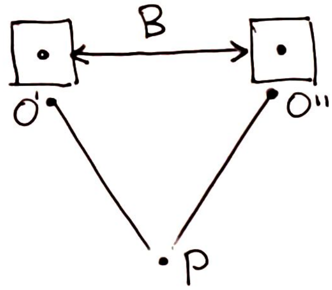
The 3D point $H$ is then found using $H = \frac{F+G}{2}$.

- Geometric solution is not statistically optimal, does not take uncertainties into the account.
- There is a special case, using a stereo normal set up. Thats when camera 1 and 2 only have translation difference, and orientation is the same.

Example,



Stereo Camera.




* I wont be going into this.
Please watch the lecture, see other notes.
My current set up uses a single moving camera.

- After obtaining our 3D points, we can use bundle adjustment to optimize our 3D point data, for an optimal solution.