Fundamental and Essential Matrix

- Fundamental matrix is used for uncalibrated camera.
- Essential matrix is used for calibrated camera.
- Both matrix F and E are 3x3 homogeneous matrices with rank dificiency rank(F) = rank(E) = 2. This rank is used in coplanarity constraint.
- To compute F we can use 8 point algorithm.
- To compute E we can use 5 point algorithm.
- E has 5 DoF, F has 7 DoF (includes calibration parameter information)

- So what information does E and F provide to us?

  It tells us where camera 2 is located with respect to camera 1 using a rotation matrix and translation matrix. Usually, we know in which direction camera 2 is from camera 1, but not exactly how far.

- Given two images we can reconstruct the object only up to a similarity transform.

- The photogrammetric model obtained using relative orientation is not up to scale. If we want to put the model into a real world, then we need absolute orientation. This means computing the scale of the object.

Calibrated camera

- Relative Orientation
  - 5 parameters
    3 Rotation, 2 Direction vector
    (we cannot estimate the lengths)
  - Relative orientation of camera 2 with respect to camera 1.
  - This can be computed by moving a camera and knowing nothing about a scene.

- Absolute Orientation
  - 7 parameters
    Additional R and T of camera 1 and Scale.
    3R, 3T and 1 Scale
  - Requires min 3, 3D points in real world to be known or camera parameters orientation.

- Relationship between E and F?
  - F corresponds to pixel coordinates.
  - E corresponds to normalized image coordinates.
  - As can be seen, F is a generalization of E.
  - $E = (K')^T F K$, $F = (K')^{-T} E K^{-1}$.

  K is the intrinsic matrix for camera 1, and K' for camera 2.

- How do we find R,T from E?

  We do SVD on E.

  $U \Sigma V^T = SVD(E)$. $\quad W = \begin{vmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ and $W^{-1} = W^T = \begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

  $R = U W^{-1} V^T$

  $[t]_x = \underbrace{U W \Sigma U^T}_{1}$ or $\underbrace{U Z U^T}_{2}$ depending on contraints. $\quad Z = \begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$

  - There are multiple solutions, above is one them.

    1 and 2 give two different results. 1 might not be suffient if Σ does not fulfill the constraints.