

Rodrigues Formula

$$R(\hat{n}, \theta) = I + \sin \theta [\hat{n}]_x + (1 - \cos \theta) [\hat{n}]_x^2$$

$$[\hat{n}]_x = \begin{vmatrix} 0 & -\hat{n}_z & \hat{n}_y \\ \hat{n}_z & 0 & -\hat{n}_x \\ -\hat{n}_y & \hat{n}_x & 0 \end{vmatrix} : \text{Rotation by } 90^\circ \text{ using cross product.}$$

• Normalized unit vector
 $\hat{n} = (\hat{n}_x, \hat{n}_y, \hat{n}_z)$.

R is rotation matrix corresponding to rotation by θ around normal axis \hat{n} .

$\omega = \theta \hat{n} = (\omega_x, \omega_y, \omega_z)$ is a rotation vector, a minimal representation of 3D rotation.

• Rotation vector to rotation matrix

This can be done using above Rodrigues equation.

function (vector) \rightarrow rotation matrix

$v = \text{vector } (3 \times 1)$

$v_m = \sqrt{v[0]^2 \dots}$ # vector norm.

$v_r = v / v_m$ # only if $v_m > 0$.

$$v_n = \begin{vmatrix} 0 & -v_r[2] & v_r[1] \\ v_r[2] & 0 & -v_r[0] \\ -v_r[1] & v_r[0] & 0 \end{vmatrix}$$

return $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \sin(v_m) \cdot v_n + (1 - \cos(v_m)) \cdot (v_n \cdot v_n)$