Estimating Fundamental and Essential Matrix

- First lets tackle F and 8 point algorithm.
  We will estimate F using coplanarity constraint.
  $x'Fx''=0$, $\begin{bmatrix} x'_n, y'_n, 1 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} x''_n \\ y''_n \\ 1 \end{bmatrix} = 0$

x' and x'' are known, we use them to estimate F.
To get F we will need at least 8 points, $n=8$ as you can guess based on the name of the algorithm.

This can be represented as a linear function...

$x''_n F_{11} x'_n + x''_n F_{21} y'_n + \dots = 0$ then like we do in DLT and Zhang's method we seperat x, y coefficients and funda mental matrix $Af=0$.

$$A = \begin{vmatrix} x''_0 F_{11} x'_0 + \dots \\ \vdots \\ n=1 \\ \vdots \\ n \end{vmatrix}, \quad f = \begin{vmatrix} F_{11} \\ F_{21} \\ \vdots \\ F_{33} \end{vmatrix}$$

Again, like in Zhang's method we solve SVD, for A and use last column from V, right singular vector. $U\Sigma V^T = SVD(A)$.
So $f =$ last column of V.

- Why 8 points? Matrix A has at most rank 8.

Now this is not a full solution because f might not be of rank 2. (Check F and E notes.) Hence, we take another SVD, now of f.
This way we can enforce f to be rank 2.
$U\Sigma V^T = SVD(f)$ then we manipulate to get $\Sigma' = \begin{vmatrix} \Sigma_{11} & 0 & 0 \\ 0 & \Sigma_{22} & 0 \\ 0 & 0 & 0 \end{vmatrix}$
Using the new Σ' we compute F.

$F = U\Sigma' V^T$ and F is our solution.

- Pseudocode of 8 point algorith...

$x' = n \times 3$ point coordinates.
$x'' = \nearrow$

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for i in range (0 ... len(x')) # len(x') == len(x'') should be
   A(n,:) = kron(x''(n,:), x'(n,:)) # true.
end
U, D, V = SVD(A)
f = reshape(V(:,9), 3, 3)
U, D, V = SVD(f)
F = U * diag(D(1,1), D(2,2), 0) * V.T
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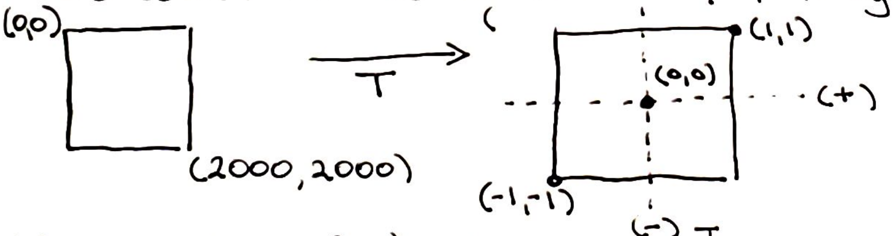
- Cons of 8 point algorithm.

1. Issue with large pixel coord.. Example $\begin{bmatrix} 2000 \\ 1500 \\ 1 \end{bmatrix}$, instead we want $\begin{bmatrix} 0.9 \\ 0.5 \\ 1 \end{bmatrix}$.
Performing normalization on the coord. sys. improves the stability.
So what we do is transform, T, n by m image to -1 by 1.

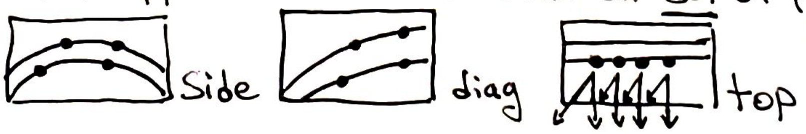


Then we can transform points using T and revert the scaled F_s to F using T as well.

We can think of it this way $x_s'^T F_s x_s'' = 0$ and then $F = T^T F_s T$, where $x_s = Tx$.

2. Singularity, $\det(F) = 0$.
   - That is rank of matrix A is less than 8.
   - There is no translation, its zero. This means projection centers are the same.

→ This can happen if we have collinear <u>set</u> of points.



As we can see the point lie on one plane.

- How to estimate E using 5 point algorith?

We start of in the same manner...

$E = R[t]_x \quad x_n'^T E x_n'' = 0$ for points $0 \ldots n$.

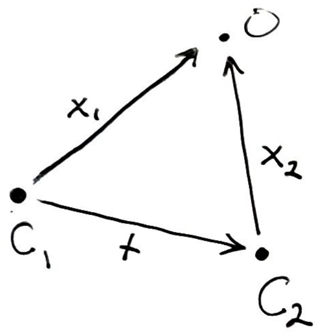
*Note that these x_n points are calibrated, $x_n = K^{-1} x_{un}$

x_{un} = uncalibrated point.

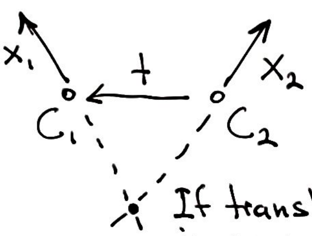
$$[x_n', y_n', c'] \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} \begin{bmatrix} x_n'' \\ y_n'' \\ c'' \end{bmatrix} = 0$$

, where C is a camera constant.

- Singularity of F also holds true for E, $\det(E) = 0$.
- Instead of 8, we only requlre 5 points.
- Its common to use 5 point algorith with RANSAC to remove outliers.



The above is what we are trying to find.



If translation is in wrong direction we find that points x_1 and x_2 meet at the back.

- As you can see we must make sure to find the right solotion. We need to make sure that both C_1 and C_2 are looking in forward direction and that point O is intersected by poth points x_1 and x_2.

We assume $\Sigma = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$ meaning a noise free essental matrix.

• So how do we solve it? (Hartley & Zisserman Solution)

Lets do, $E = U \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\Sigma} V^T$ and break Σ into Z and W.

$$\underbrace{Z}_{\text{Skew}} = \begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \qquad \underbrace{W}_{\text{Rotation}} = \begin{vmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$ZW = \Sigma$$

Another point/contraint we can mention is that U is a rotation matrix, meaning $U^T U = I$, is identity matrix. We can substitute this all into E.

$$E = \underbrace{UZU^T}_{\text{Translation}} \underbrace{UWV^T}_{\text{Rotation } (R^T)}$$

We can use variations of Z and W to get multiple solutions.

$$\left.\begin{aligned} \Sigma &= ZW \\ &= -Z^T W \\ &= Z(-W^T) \\ &= Z^T W^T \end{aligned}\right\}$$

This gives us 2 solutions for translation and 2 for rotation.

$$E_1 = UZU^T UWV^T$$
$$E_2 = UZ^T U^T UWV^T$$
$$E_3 = UZU^T UW^T V^T$$
$$E_4 = UZ^T U^T UW^T V^T$$

*Note, in book computer vision: alogoriths and applications (I read the second ed. draft) chapter 11.3, he talks about the 5point algorith. There he breaks down E into $[t]_x$ and R.

$[t]_x = U \Sigma W_{90°} U^T$ and $R = \pm U W_{\pm 90°} \cdot V^T$.

It puts two solutions very concisely.

We generate 4 points and keep 2 that have $\det(R) = 1$.