Estimating Fundamental and Essential Matrix

- First lets tackle F and 8 point algorithm.
  We will estimate F using coplanarity constraint.

$$x'Fx''=0, \quad [x'_n, y'_n, 1]\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}\begin{bmatrix} x''_n \\ y''_n \\ 1 \end{bmatrix} = 0$$

x' and x'' are known, we use them to estimate F.
To get F we will need at least 8 points, $n=8$ as you can guess based on the name of the algorithm.

This can be represented as a linear function...

$$x''_n F_{11} x'_n + x''_n F_{21} y'_n + \dots = 0$$

then like we do in DLT and Zhang's method we seperat x, y coefficients and fundamental matrix $Af=0$.

$$A = \begin{vmatrix} x''_0 F_{11} x'_0 + \dots \\ \vdots \\ n=1 \\ \vdots \\ n \end{vmatrix}, \quad f = \begin{vmatrix} F_{11} \\ F_{21} \\ \vdots \\ F_{33} \end{vmatrix}$$

Again, like in Zhang's method we solve SVD, for A and use last column from V, right singular vector. $U\Sigma V^T = SVD(A)$.
So $f=$ last column of V.

- Why 8 points? Matrix A has at most rank 8.

Now this is not a full solution because f might not be of rank 2. (Check F and E notes.) Hence, we take another SVD, now of f.
This way we can enforce f to be rank 2.
$U\Sigma V^T = SVD(f)$ then we manipulate to get $\Sigma' = \begin{vmatrix} \Sigma_{11} & 0 & 0 \\ 0 & \Sigma_{22} & 0 \\ 0 & 0 & 0 \end{vmatrix}$
Using the new Σ' we compute F.

$F = U\Sigma' V^T$ and F is our solution.