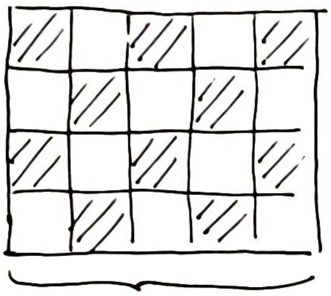
Zhangs Method

- Method used for camera calibration.



Chessboard

X_o = world object point

x_p = image point

$x_p = P X_o$ P = Transformations

K = intrinsic matrix

$$\begin{bmatrix} f & s & c_x \\ 0 & af & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = K \underbrace{\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix}}_{\text{Projection}}$$

$$X_o = \begin{vmatrix} x \\ y \\ Z \\ 1 \end{vmatrix}$$

This is zero

can be removed because X_o Z is zero.

$$H = \begin{vmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{vmatrix}, \quad x_p = \underbrace{K \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}}_{\text{This is a homography matrix H.}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- This is similar to DLT, but we do not care about extrinsics and no Z axis.
- We solve projection in DLT instead of H.

Lets say $AH = 0$.

$$\underbrace{\begin{bmatrix} -X_o, -Y_o, -1, 0, 0, 0, x_pX_o, x_pY_o, x_p \\ 0, 0, 0, -X_o, -Y_o, -1, y_pX_o, y_pY_o, y_p \\ \vdots \end{bmatrix}}_{A}$$

} row a_x } row a_y for point 1.

$\vdots$ for i points.

$$\begin{bmatrix} a_{x1} \\ a_{y1} \\ \vdots \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ \vdots \end{bmatrix} = 0$$

- Because its a homogeneous linear system we can use SVD to solve $AH=0$, as if $Ax=0$.

* Note that in reality it will not be zero, but close to zero.

Lets say $AH = err$.

- Now that we have $AH = err$, we try to find a solution for H that will minimize err to be close to 0. We do that, by doing SVD of A matrix. The solution of H will be the last column of the right singular vector.

$SVD = U \Sigma V^T$

Reminder.

- We now have $H = K[r_1, r_2, t]$.

• Now that we have found H we need to find K. In DLT it can be done using QR decomposition, now however, due to not having R we are not able to apply it.
So how do we go from H to K?
We exploit the constaints about K, r_1 and r_2.

① $r_1^T r_2 = 0$

② $r_1^T r_1 - r_2^T r_2 = 0$
or
$\|r_2\| = \|r_1\| = 1$
$r_1^T r_1 = r_2^T r_2 = 1$

• We come to such constraints because r_1 and r_2 are orthonormal. Simply, they are perpendicular.

• Another constrait is with $H = K[r_1, r_2, t]$. We multiply both sides by inverse K.

$[h_1, h_2, h_3] = K[r_1, r_2, t] \rightarrow \underset{③}{K^{-1}h_1 = r_1}$ and $\underset{④}{K^{-1}h_2 = r_2}$
*columns

Recall: $(M^{-1})^T = (M^T)^{-1}$

$(K^{-1}h_1)^T (K^{-1}h_2) = 0$ using ①③④
$h_1^T K^{-T} K^{-1} h_2 = 0$
$h_1^T K^{-T} K^{-1} h_1 = h_2^T K^{-T} K^{-1} h_2$ using ②③④ $\rightarrow h_1^T K^{-T} K^{-1} h_1 - h_2^T K^{-T} K^{-1} h_2 = 0$
↳ note if this was h_2 the equation would be equal to zero.

Let substitute $B = K^{-T} K^{-1}$ as we see it appear frequently.
$h_1^T B h_2 = 0$ and $h_1^T B h_1 - h_2^T B h_2 = 0$

Then we can use Cholesky decomp. to find K, if we know B.
$\text{Cholesky}(B) = AA^T$ where $A = K^{-T}$.

$$B = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix}$$

• We are only interested in the upper triangular values $b_{11}, b_{12}, b_{13}, b_{22}, b_{23}, b_{33}$.

Now we construct a system of linear equations $V.b = 0$, using previusly mentioned constraints. $V_{12}^T b = 0$ and $V_{11}^T b - V_{22}^T b = 0$.

$$V = \begin{vmatrix} V_{12}^T \\ V_{11}^T - V_{22}^T \\ \vdots \\ n \text{ images} \end{vmatrix}$$

image 1.

and $$V_{ij} = \begin{vmatrix} h_{1i}h_{1j} \\ h_{1i}h_{2j} + h_{2i}h_{1j} \\ h_{3i}h_{1j} + h_{1i}h_{3j} \\ h_{2i}h_{2j} \\ h_{3i}h_{2j} + h_{2i}h_{3j} \\ h_{3i}h_{3j} \end{vmatrix}$$

Lastly we use $Vb = err$ and use SVD to solve V to find b in same manner as last time.

Now that we have B we can easily find K using Cholesky method.