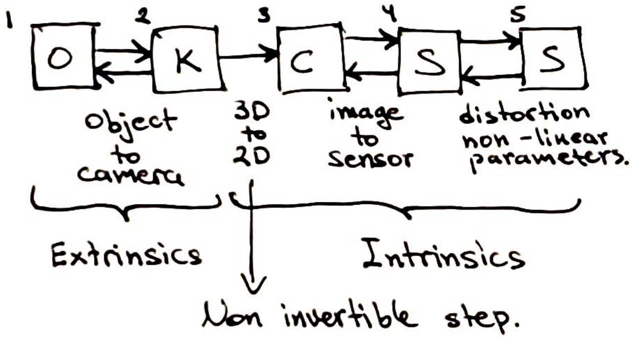
Camera Parameters (Intrinsics & Extrinsics)

- Transformations from 3D world point to 2D image point.

$$\underbrace{X_i}_{\text{image point}} = \underbrace{P}_{\text{Transformations}} \underbrace{X_o}_{\text{object/world point}} = P \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}^{S_s} = \underbrace{{}^{S}H_c}_{\text{imag to sensor}} \, \underbrace{{}^{c}P_k}_{\text{camera to image}} \, \underbrace{{}^{k}H_o}_{\text{object to camera}} \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}^{S_o}$$

- Systems of coordinates.
  1. World/Object system, S_o.
  2. Camera coordinate system, S_K.
  3. Image coordinate system, S_c.
  4. Sensor coordinate system, S_s.



1-4: Describes a linear model.

5: Deviation from the linear model. Even thou distortion usually happens at step 2 to 3, we add it as an extra step to simplify and account for non linear errors.

• Intrinsic parameters.

• This is a process of projecting from camera coord. sys. to sensor coord. sys..

• Describes what happens inside camera.

• Upper triangular matrix.

• 5D of freedom

• K is a calibration matrix.

$$K = \begin{vmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} f & s & c_x \\ 0 & af & c_y \\ 0 & 0 & 1 \end{vmatrix}$$

f : focal lengths.

s : skew between sensor axis.

c_x, c_y : principal point, or image center.

a : aspect ratio.

• How to obtain K?

One of the popular methods is Direct Linear Transformation (DLT) and extension of that the Zhang's method.

• Intrinsic parameters will stay constant as you move the camera.

• Extrinsic parameters.

• Describes what happens outside the camera.

• Describes the pose of the camera, translation (3D) and orientation/rotation of the camera (3D).

• Has 6D of freedom.

• $$[R \mid T] = \left|\begin{array}{ccc|c} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{array}\right|$$

R: Rotation.

T: Translation.

• To perform camera localization we would need to know our camera pose.

• How to obtain R, T?

• Can be done using DLT, or fundamental/essential matrix. They (F and E) are found using 8 and 5 point algoriths respectively.

• As you move the camera extrinsic parameters change from frame to frame.

- **Camera matrix / projection matrix.**

$$x_i = P X_o \text{ , where } P = K[R|T] \text{ or also written as} \ldots$$

$$P = KR[I_3 | -T]$$

- camera matrix contains both intrinsic and extrinsic matrix.

- **Distortion coefficients.** (Non linear errors).
  - This is non-linear errors.
  - Example: barrel distortion.
  - Reasons why they occur; imperfect lenses.
  - To resolve the distortion, we perform a shift on every pixel. This shift will need to be location dependent.

$${}^a x = {}^s x + \Delta x(x_i, q)$$

$${}^a y = {}^s y + \Delta y(x_i, q)$$

Shift for each pixel that depends on x_i, image point and q, non linear parameters.

Arbitrary point.

Original sensor point.

$$(1)\quad K(x_i, q) = \begin{vmatrix} f & s & x_p + \Delta x(x_i, q) \\ 0 & af & y_p + \Delta y(x_i, q) \\ 0 & 0 & 1 \end{vmatrix}$$

*p = principal point.

- We can rewrite K to take this shift into account. (1)
- We can model barrel distortion using the following equations...

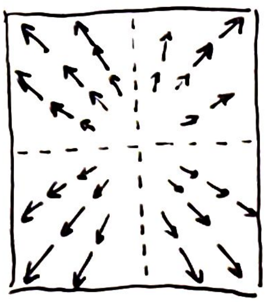
$${}^a x = x(1 + q_1 r^2 + q_2 r^4)$$

$${}^a y = y(1 + q_1 r^2 + q_2 r^4)$$

r: distance of the pixel to principal point.

q_1, q_2: these are distortion coefficients.

Radial distortion field example



As we go further from center, the distortion can increase.

- We can use other models to compensate for distortion and they can get very complicated.