• So how do we solve it? (Hartley & Zisserman Solution)

Lets do, $E = U \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\Sigma} V^T$ and break Σ into Z and W.

$$Z = \begin{vmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \qquad W = \begin{vmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Skew

Rotation

$$ZW = \Sigma$$

Another point/contraint we can mention is that U is a rotation matrix, meaning $U^T U = I$, is identity matrix. We can substitute this all into E.

$$E = \underbrace{UZU^T}_{\text{Translation}} \; \underbrace{UWV^T}_{\text{Rotation } (R^T)}$$

We can use variations of Z and W to get multiple solutions.

$$\begin{aligned} \Sigma &= ZW \\ &= -Z^T W \\ &= Z(-W^T) \\ &= Z^T W^T \end{aligned}$$

This gives us 2 solutions for translation and 2 for rotation.

$$E_1 = UZU^T UWV^T$$
$$E_2 = UZ^T U^T UWV^T$$
$$E_3 = UZU^T UW^T V^T$$
$$E_4 = UZ^T U^T UW^T V^T$$

* Note, in book computer vision: alogoriths and applications (I read the second ed. draft) chapter 11.3, he talks about the 5 point algorith. There he breaks down E into $[t]_x$ and R.
$[t]_x = U \Sigma W_{90°} U^T$ and $R = \pm U W_{\pm 90°} \cdot V^T$.

It puts two solutions very concisely.
We generate 4 points and keep 2 that have $\det(R) = 1$.