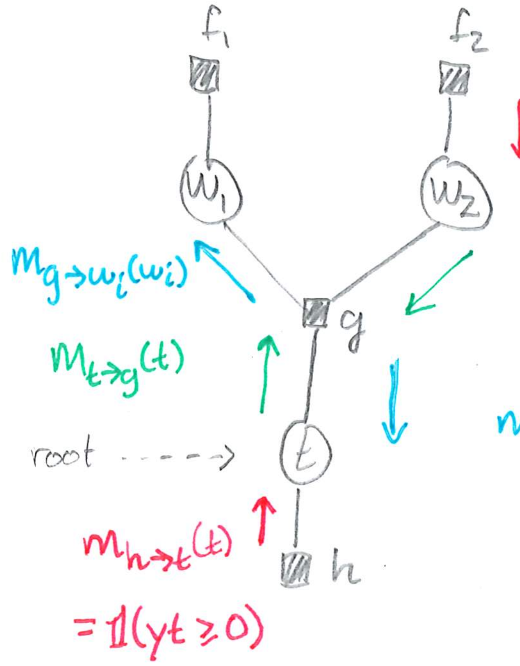
Message-passing in TrueSkill

Start with one game



$$\cancel{m}_{f_i \to w_i}(w_i) = f_i(w_i) = N(w_i | 0, \sigma_0^2)$$

$$m_{w_i \to g}(w_i) = m_{f_i \to w_i}(w_i) = N(w_i | 0, \sigma_0^2)$$

$$m_{g\to t}(t) = \int \underbrace{g(t, w_1, w_2)}_{= N(t | w_1 - w_2, 1)} N(w_1 | 0, \sigma_0^2) N(w_2 | 0, \sigma_0^2) dw_1 dw_2$$

$$= N(t | 0, 1 + 2\sigma_0^2)$$

After the downward pass we have computed the marginal distribution of t, $p(t) = m_{g\to t}(t)$!

Posterior marginal (belief) at node t,

$$\pi(t) \propto \mathbb{1}(yt \geq 0)\, m_{g\to t}(t)$$

This is a truncated Gaussian

[Pres → first problem]

[Draw messages in figure when continuing]

Using the Gaussian approximation, we can compute

$$m_{t\to g}(t) = \frac{q(t)}{m_{g\to t}(t)} = \frac{N(t \mid \tilde{\mu}, \tilde{\sigma}^2)}{N(t \mid 0, 1+2\sigma_0^2)}$$

Dividing by a PDF can be thought of as subtracting information. It is convenient to work with information form.

$$\Rightarrow \quad m_{t\to g}(t) \propto N_I(t \mid \nu_{t\to g}, \lambda_{t\to g})$$

$$\begin{cases} \lambda_{t\to g} = \dfrac{1}{\tilde{\sigma}^2} - \dfrac{1}{1+2\sigma_0^2} \\[2ex] \nu_{t\to g} = \dfrac{\tilde{\mu}}{\tilde{\sigma}^2} - 0 \end{cases}$$

derived from here

For game → skill messages we get

$$m_{g\to w_1}(w_1) = \int g(t, w_1, w_2)\, m_{t\to g}(t)\, m_{w_2\to g}(w_2)\, dt\, dw_2$$

Note that $g(t, w_1, w_2) = N(t \mid w_1 - w_2, 1)$

$$\propto N(w_1 \mid t + w_2, 1)$$

$$\Rightarrow \quad m_{g\to w_1}(w_1) \propto N(w_1 \mid \mu_{t\to g} + 0, 1 + \sigma_{t\to g}^2 + \sigma_0^2)$$

and similarly for w_2 (but note the sign difference)

Finally, we can compute the skill marginals

$$\pi(w_i) \approx q(w_i) \propto f_i(w_i)\, m_{g \to w_i}(w_i)$$

$$\propto \mathcal{N}_I\left(w_i \mid \nu_{g \to w_i}, \lambda_{g \to w_i} + \frac{1}{\sigma_0^2}\right)$$

$\boxed{\text{Pres} \to \text{rest}}$