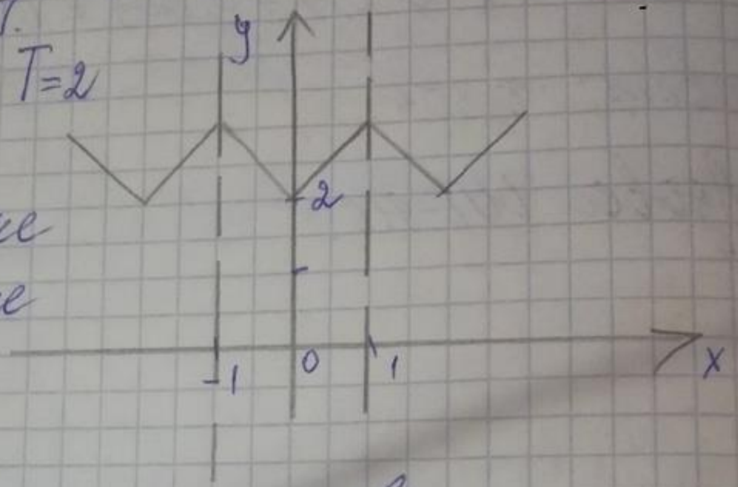
В-1 Разложить в ряд Фурье функцию

РГР№3. $f(x)=2+|x|$, заданную на отрезке $[-1;1]$.

$T=2$



Функция чётная

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ряд Фурье имеет вид:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\frac{\pi n x}{l}$$

Вычислим a_0 и a_n:

- $a_0=\frac{1}{l}\int_{-l}^{l} f(x)\cdot dx$

- $a_n=\frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{\pi n x}{l}\,dx$

- $l=\frac{T}{2}=1$

$* \; a_0 = \frac{1}{1}\int_{-1}^{1}(2+|x|)\,dx = \int_{-1}^{1} 2\,dx + \int_{-1}^{1}|x|\,dx =$

$= 4 + \begin{cases}\int_{-1}^{0} -x\,dx \\ \int_{0}^{1} x\,dx\end{cases} = 4 + \begin{cases} -\frac{x^2}{2}\Big|_{-1}^{0} \\ \frac{x^2}{2}\Big|_{0}^{1}\end{cases} =$

$= 4 + \begin{cases}\frac{1}{2} \\ \frac{1}{2}\end{cases} = 5$

$\bullet \; a_0 = 5$

$* \; a_n = \frac{1}{1}\int_{-1}^{1}(2+|x|)\cos\frac{\pi n x}{1}\,dx =$

$= \overset{1}{\int_{-1}^{1} 2\cos\pi n x\,dx} + \overset{2}{\int_{-1}^{1}|x|\cos\pi n x\,dx} =$

$\overset{1}{=} \left| \begin{matrix} z = \pi n x, \; dz = \pi n, \\ dx = \frac{1}{\pi n}\,dz \end{matrix} \right| = 2\int_{-1}^{1}\cos\pi n x\cdot\frac{1}{\pi n}\,dz$

$= 2\int_{-1}^{1}\cos(z)\cdot\frac{1}{\pi n}\,dz = \frac{2}{\pi n}(\sin\pi n - \sin(-\pi n))$

$= \frac{2}{\pi n}\cdot 2\sin\pi n \overset{1}{=} \frac{4\sin\pi n}{\pi n}$

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$$\int_{-1}^{1} |x| \cos \pi n x\, dx = \begin{cases} \int_{-1}^{0} -x \cos \pi n x\, dx \\ \int_{0}^{1} x \cos \pi n x\, dx \end{cases} =$$

$$= \left| \begin{matrix} u = x, \; du = dx, \\ dv = \cos \pi n x\, dx, \end{matrix} \; v = \frac{\sin \pi n x}{\pi n} \right| =$$

$$= \left| \begin{matrix} u = -x, \; du = -dx, \\ v = \frac{\sin \pi n x}{\pi n}, \; dv = \cos \pi n x\, dx \end{matrix} \right| =$$

$$= \begin{cases} \left. \frac{-x \sin \pi n x}{\pi n} \right|_{-1}^{0} \Rightarrow \int_{-1}^{0} \frac{\sin \pi n x}{\pi n} dx \\ \left. \frac{x \sin \pi n x}{\pi n} \right|_{0}^{1} \Rightarrow \int_{0}^{1} \frac{\sin \pi n x}{\pi n} dx \end{cases} =$$

$$\int \frac{\sin \pi n x}{\pi n} dx = \frac{1}{\pi n} \int \sin \pi n x\, dx =$$

$$= \left| \begin{matrix} t = \pi n x, \; t' = \pi n \\ dx = \frac{1}{\pi n} dt \end{matrix} \right| = \frac{1}{\pi n} \cdot \int \sin(\pi n x) \frac{1}{\pi n} dt =$$

$$= \frac{1}{\pi^2 n^2} \int \sin \pi n \frac{t}{\pi n} =$$

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$$= \frac{1}{\pi^2 n^2} \int (-\cos t) = -\frac{1}{\pi^2 n^2} \cos \pi n x$$

$$= \begin{cases} -\frac{x \sin \pi n x}{\pi n} \Big|_{-1}^{0} + \frac{1}{\pi^2 n^2} \cos \pi n x \Big|_{-1}^{0} \\ \frac{x \sin \pi n x}{\pi n} \Big|_{0}^{1} + \frac{1}{\pi^2 n^2} \cos \pi n x \Big|_{0}^{1} \end{cases} =$$

$$= \begin{cases} -\frac{0 \cdot \sin 0}{\pi n} + \frac{(-1)\sin(-\pi n)}{\pi n} + \frac{1}{\pi^2 n^2} \cos 0 + \\ + \left(-\frac{1}{\pi^2 n^2} \cos(-\pi n)\right) \\ \frac{\sin \pi n}{\pi n} - \frac{0 \sin 0}{\pi n} + \frac{1}{\pi^2 n^2} \cos \pi n - \\ -\frac{0 \cos 0}{\pi^2 n^2} \end{cases} =$$

$$= \begin{cases} -\frac{\sin \pi n}{\pi n} + \frac{1}{\pi^2 n^2} - \frac{\cos \pi n}{\pi^2 n^2} \\ \frac{\sin \pi n}{\pi n} + \frac{1}{\pi^2 n^2} \cos \pi n \end{cases}$$

$$= \frac{2 \sin \pi n}{\pi n} + \frac{1}{\pi^2 n^2}$$

$$a_n = \frac{4\sin \pi n}{\pi n} + \frac{2\sin \pi n}{\pi n} + \frac{1}{\pi^2 n^2} =$$

$$= \frac{6\sin \pi n}{\pi n} + \frac{1}{\pi^2 n^2}$$

$$f(x) = \frac{5}{2} + \sum_{n=1}^{\infty} \left(\frac{6\sin \pi n}{\pi n} + \frac{1}{\pi^2 n^2} \right) \cdot \cos nx$$

$\Longrightarrow$

Ответ:

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