РГР №6. В-1

$y'' + 4y' - 12y = 8\sin 2x$ ЛНДУ

1) $y'' + 4y' - 12y = 0$

$k^2 + 4k - 12 = 0$

$D = 4^2 - 4 \cdot 1 \cdot (-12) = 16 + 48 = 64$

$k_1 = \dfrac{-4 + 8}{2 \cdot 1} = 2$

$k_2 = -6$

$y_0 = C_1 \cdot e^{2x} + C_2 \cdot e^{-6x}$

2) $Z = 8\sin 2x$

$Z = A\cos 2x + B\sin 2x$

$Z' = 2B\cos 2x - 2A\sin 2x = -2(A\sin 2x -$

$- B\cos 2x)$

$Z'' = -4(A\cos 2x + B\sin 2x)$

$\cancel{-4(A\cos 2x + A\sin 2x) + 4 \cdot (-2)(A\sin 2x - B\cos 2x)}$

$\cancel{-12 \cdot (A\cos 2x + B\sin 2x) = 8\sin 2x}$

$-4(A\cos 2x + B\sin 2x) + 4\cdot(-2)\cdot(A\sin 2x -$
$-B\cos 2x) - 12\cdot(A\cos 2x + B\sin 2x) = 8\sin 2x$

$-4A\cos 2x - 4B\sin 2x - 8A\sin 2x + 8B\cos 2x$
$-12A\cos 2x - 12B\sin 2x = 8\sin 2x$

$-16A\cos 2x - 16B\sin 2x - 8A\sin 2x + 8B\cos 2x =$
$= 8\sin 2x$

$\begin{cases} -16A + 8B = 0 \\ -16B - 8A = 8 \end{cases}$ $\begin{cases} A = -\frac{1}{5} \\ B = -\frac{2}{5} \end{cases}$

$z = -\frac{1}{5}\cos 2x - \frac{2}{5}\sin 2x$

$y = y_0 + z = (C_1\cdot e^{2x} + C_2\cdot e^{-6x}) + (-\frac{1}{5}\cos 2x -$
$-\frac{2}{5}\sin 2x)$ общее решение ЛНДУ

3) $y(0) = 0;\ y'(0) = 0$

1. $(C_1 + C_2) + (-\frac{1}{5}) = 0$

2. $((C_1\cdot e^{2x} + C_2\cdot e^{-6x}) + (-\frac{1}{5}\cos 2x - \frac{2}{5}\sin 2x))'$

$= (C_1\cdot e$
$= 2C$
$2C_1 -$
$2C_1$
$C_1 =$
$y = ($
ча

$$= (C_1 \cdot e^{2x} + C_2 \cdot e^{-6x})' + \left(\frac{2}{5}\sin 2x - \frac{4}{5}\cos 2x\right) =$$

$$= 2C_1 \cdot e^{2x} - 6C_2 \cdot e^{-6x} + \frac{2}{5}\sin 2x - \frac{4}{5}\cos 2x$$

$$2C_1 - 6C_2 - \frac{4}{5} = 0$$

$$\underline{2C_1 - 6C_2 = \frac{4}{5}}$$

$$\underline{C_1 = \frac{1}{4}\ ;\ C_2 = -\frac{1}{20}}$$

$$y = \left(\frac{1}{4} \cdot e^{2x} - \frac{1}{20} \cdot e^{-6x}\right) + \left(-\frac{1}{5}\cos 2x - \frac{2}{5}\sin 2x\right)$$

частное решение задачи Коши.