

Theorem 1. $S K K = I$.

Proof.

$$(\lambda x y z.x z (y z)) (\lambda x y.x) (\lambda x y.x) = \lambda x.x$$

$$\downarrow \alpha, normal\ strategy$$

$$((\lambda x y z.x z (y z)) (\lambda x' y'.x')) (\lambda x'' y''.x'') = \lambda x.x$$

$$\downarrow \beta$$

$$(\lambda y z.(\lambda x' y'.x') z (y z)) (\lambda x'' y''.x'') = \lambda x.x$$

$$\downarrow \beta$$

$$\lambda z.(\lambda x' y'.x') z ((\lambda x'' y''.x'') z) = \lambda x.x$$

$$\downarrow normal\ strategy$$

$$\lambda z.(\lambda x' y'.x') z ((\lambda x'' y''.x'') z) = \lambda x.x$$

$$\downarrow \beta$$

$$\lambda z.(\lambda y'.z) ((\lambda x'' y''.x'') z) = \lambda x.x$$

$$\downarrow \beta$$

$$\lambda z.z = \lambda x.x$$

$$\downarrow \alpha$$

$$\lambda x.x = \lambda x.x$$

□