Homework 3
ENE4014 Programming Languages, Spring 2021
due: 5/10(Mon), 24:00

Exercise 1 Consider the following programming language, called `miniML`, that features (recursive) procedures and explicit references.

Syntax The syntax is defined as follows:

$$
\begin{array}{rcl}
P & \to & E \\
E & \to & n \\
 & | & x \\
 & | & E + E \mid E - E \mid E * E \mid E/E \\
 & | & E - E \\
 & | & \texttt{iszero}\ E \\
 & | & \texttt{if}\ E\ \texttt{then}\ E\ \texttt{else}\ E \\
 & | & \texttt{let}\ x = E\ \texttt{in}\ E \\
 & | & \texttt{letrec}\ f(x) = E\ \texttt{in}\ E \\
 & | & \texttt{proc}\ x\ E \\
 & | & E\ E \\
 & | & \texttt{ref}\ E \\
 & | & !\ E \\
 & | & E := E \\
 & | & E; E \\
 & | & \texttt{begin}\ E\ \texttt{end}
\end{array}
$$

Semantics The semantics is defined with the following domain:

$$
\begin{array}{rcl}
\mathit{Val} & = & \mathbb{Z} + \mathit{Bool} + \mathit{Procedure} + \mathit{RecProcedure} + \mathit{Loc} \\
\mathit{Procedure} & = & \mathit{Var} \times E \times \mathit{Env} \\
\mathit{RecProcedure} & = & \mathit{Var} \times \mathit{Var} \times E \times \mathit{Env} \\
\rho \in \mathit{Env} & = & \mathit{Var} \to \mathit{Val} \\
\sigma \in \mathit{Mem} & = & \mathit{Loc} \to \mathit{Val}
\end{array}
$$

and evaluation rules:

$$\frac{}{\rho, \sigma \vdash n \Rightarrow n, \sigma} \qquad \frac{}{\rho, \sigma \vdash x \Rightarrow \rho(x), \sigma}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow n_1, \sigma_1 \qquad \rho, \sigma_1 \vdash E_2 \Rightarrow n_2, \sigma_2}{\rho, \sigma_0 \vdash E_1 \oplus E_2 \Rightarrow n_1 \oplus n_2, \sigma_2} \quad \oplus \in \{+, -, *, /\}$$

$$\frac{\rho, \sigma_0 \vdash E \Rightarrow 0, \sigma_1}{\rho, \sigma_0 \vdash \texttt{iszero}\ E \Rightarrow \mathit{true}, \sigma_1} \qquad \frac{\rho, \sigma_0 \vdash E \Rightarrow n, \sigma_1}{\rho, \sigma_0 \vdash \texttt{iszero}\ E \Rightarrow \mathit{false}, \sigma_1}\ n \neq 0$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow \mathit{true}, \sigma_1 \qquad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash \texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3 \Rightarrow v, \sigma_2}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow \mathit{false}, \sigma_1 \qquad \rho, \sigma_1 \vdash E_3 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash \texttt{if}\ E_1\ \texttt{then}\ E_2\ \texttt{else}\ E_3 \Rightarrow v, \sigma_2}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow v_1, \sigma_1 \qquad [x \mapsto v_1]\rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash \texttt{let}\ x = E_1\ \texttt{in}\ E_2 \Rightarrow v, \sigma_2}$$

$$\frac{[f \mapsto (f, x, E_1, \rho)]\rho, \sigma_0 \vdash E_2 \Rightarrow v, \sigma_1}{\rho, \sigma_0 \vdash \texttt{letrec}\ f(x) = E_1\ \texttt{in}\ E_2 \Rightarrow v, \sigma_1}$$

$$\frac{}{\rho, \sigma \vdash \texttt{proc}\ x\ E \Rightarrow (x, E, \rho), \sigma}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow (x, E, \rho'), \sigma_1 \qquad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2 \qquad [x \mapsto v]\rho', \sigma_2 \vdash E \Rightarrow v', \sigma_3}{\rho, \sigma_0 \vdash E_1\ E_2 \Rightarrow v', \sigma_3}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow (f, x, E, \rho'), \sigma_1 \quad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2 \quad [x \mapsto v, f \mapsto (f, x, E, \rho')]\rho', \sigma_2 \vdash E \Rightarrow v', \sigma_3}{\rho, \sigma_0 \vdash E_1\ E_2 \Rightarrow v', \sigma_3}$$

$$\frac{\rho, \sigma_0 \vdash E \Rightarrow v, \sigma_1}{\rho, \sigma_0 \vdash \texttt{ref}\ E \Rightarrow l, [l \mapsto v]\sigma_1}\ l \notin \mathrm{Dom}(\sigma_1)$$

$$\frac{\rho, \sigma_0 \vdash E \Rightarrow l, \sigma_1}{\rho, \sigma_0 \vdash !\ E \Rightarrow \sigma_1(l), \sigma_1}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow l, \sigma_1 \qquad \rho, \sigma_1 \vdash E_2 \Rightarrow v, \sigma_2}{\rho, \sigma_0 \vdash E_1 := E_2 \Rightarrow v, [l \mapsto v]\sigma_2}$$

$$\frac{\rho, \sigma_0 \vdash E_1 \Rightarrow v_1, \sigma_1 \qquad \rho, \sigma_1 \vdash E_2 \Rightarrow v_2, \sigma_2}{\rho, \sigma_0 \vdash E_1; E_2 \Rightarrow v_2, \sigma_2}$$

$$\frac{\rho, \sigma_0 \vdash E \Rightarrow v, \sigma_1}{\rho, \sigma_0 \vdash \texttt{begin}\ E\ \texttt{end} \Rightarrow v, \sigma_1}$$

Implement an interpreter of `miniML`. Raise an exception `UndefinedSemantics` whenever the semantics is undefined. Skeleton code will be provided (before you start, see `README.md`).