M110C Quiz#4 Spring 2022.

Rubric.

Show up to the meeting: 4pts
problems1,2,3 : 1 pt, 2pts, 3pts respectively.
Total possible: 10 pts

1. Compute the limit, if possible. If it's not possible, explain why the limit doesn't exist.

a) $\lim_{(x,y)\to(5,1)} \dfrac{x-y-1}{\sqrt{x-y}-1}$

b) $\lim_{(x,y)\to(0,0)} \dfrac{4xy}{x^2+y^2}$

2.

$f(x,y)=\cos(x)\cos(2y).$

a) Using the tabulated data to the right, approximate the partial derivatives $f_x(1,3)$ and $f_y(1,3)$.

b) Compute those partial derivatives exactly.

f(x,y)=cos(x)cos(2y)	y							
x		0.7	0.8	0.9	1	1.1	1.2	1.3
	2.7	0.49	0.44	0.39	0.34	0.29	0.23	0.17
	2.8	0.59	0.54	0.48	0.42	0.35	0.28	0.21
	2.9	0.68	0.62	0.55	0.48	0.40	0.32	0.24
	3	0.73	0.67	0.60	0.52	0.44	0.35	0.26
	3.1	0.76	0.69	0.62	0.54	0.45	0.36	0.27
	3.2	0.76	0.69	0.62	0.54	0.45	0.36	0.27
	3.3	0.73	0.66	0.59	0.51	0.43	0.34	0.25

3. $f(x,y)=x^2y-xy^2, \quad (a,b)=(1,3)$

a) Compute the partial derivatives $f_x(x,y)$ and $f_y(x,y)$.
b) Compute the gradient $\nabla f(a,b)$.
c) Find the directional derivative of f at the point (a,b) in the direction of the vector $\mathbf{u}=<3,4>$.
d) In which direction does the function increase the fastest at the point (a,b)?
e) What is the magnitude of the largest rate of increase at the point (a,b)?
f) What is the equation of the tangent plane to f at the point (a,b)?
g) What is the equation of the 2nd order Taylor Polynomial approximation to f at (a,b)?

4. This problem is optional extra credit.

The blue graph on the right is a level surface of
$w(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z}$.

There is a tangent plane to this surface at the point A, which intersects the x,y and z axes at the points E,F and G, respectively.

Claim: The sum of the distances from the points E,F and G to the origin (i.e. the sum of their non-zero coordinates) is constant over all points A.

Prove this claim, and find that constant.