Boston Housing

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Overview

We will investigate data from the Boston Housing Market. We have data from 14 variables for individual housing units. Our goal is to investigate these variables & realationships among them, to ultimately build a Machine Learning system that could predict house prices(our response variable) with high accuracy given instances of new data containing these 13 predictor variables.

Data Investigation

```
The 14 variables are as listed below

crim- per capita crime rate by town.

zn- proportion of residential land zoned for lots over 25,000 sq.ft.

indus- proportion of non-retail business acres per town.

chas- Charles River dummy variable (= 1 if tract bounds river; 0 otherwise).

nox- nitrogen oxides concentration (parts per 10 million).

rm- average number of rooms per dwelling.

age- proportion of owner-occupied units built prior to 1940.

dis- weighted mean of distances to five Boston employment centres.

rad- index of accessibility to radial highways.

tax- full-value property-tax rate per $10,000.

ptratio- pupil-teacher ratio by town.

black- 1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town.

lstat- lower status of the population (percent).

medv(target variable)- median value of owner-occupied homes in $1000s.
```

Import Data

Data Visualization

We will look at distributions of our given variables & their bivariate plots with respect to the the target variable (medv) to identify key variables that may act as significant features in predicting our response variable so we may build a Machine Learning system with minimal noise as input

Models that will be considering are:

- 1. Multiple Linear Regression
- we will look for linear relationships between variables & target variable
- 2. Decision Trees
- we will look for splits within the distribution of the variables that may help us identify between the 3 categories of housing prices

I have divided, the target variable (medv-housing prices), into 3 categories: low price(less than equal to \$15K), medium price(\$15K-30K), & high price (\$35K).

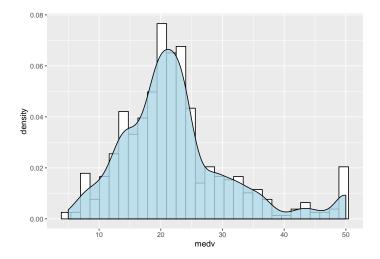
```
classifier =function(x)
{
    if(x<=15)
    {
        return ("low")
    }
    else if(x>15 & x<35)
    {
        return ("med")
    }
    else
    {
        return ("high")
    }
}
housing<-housing%>%mutate(class=sapply(medv,classifier))
```

Medv (Target Variable)

Distribution for our Target Variable, & it's Distribution divided by Categorie

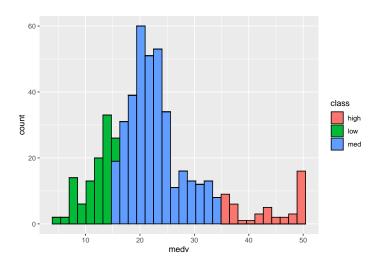
```
ggplot(housing, aes(x=medv)) +
   geom_histogram(aes(y=..density..), colour="black", fill="white")+
   geom_density(alpha=.8, fill="lightblue")
```

[`]stat_bin()` using `bins = 30`. Pick better value with `binwidth`.



ggplot(housing, aes(x=medv,fill=class)) + geom_histogram(colour="black")

`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.



count(filter(housing,class=="low"))/length(class)

n 1 0.1920792

count(filter(housing,class=="med"))/length(class)

n 1 0.7128713

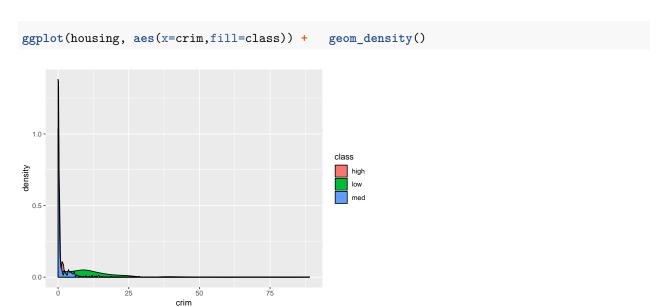
count(filter(housing,class=="high"))/length(class)

n 1 0.0950495

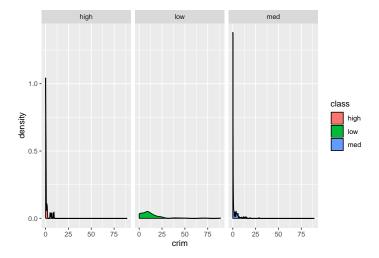
Observation:

The distribution has a peak around Housing Prices of medium level (\$15K-\$35), accounting for 71% of Prices, low level (below \$15K) accounting for 19%, & high level (above \$35K) accounting for 9%

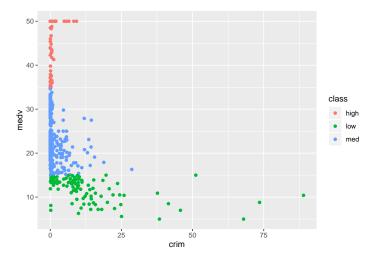
Variable # 1: Crim





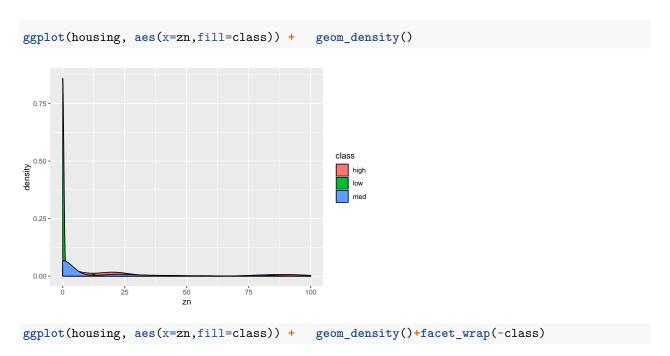


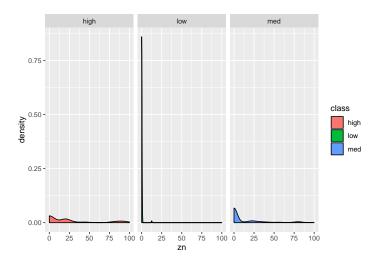
ggplot(housing, aes(x=crim,y=medv,color=class)) + geom_point()



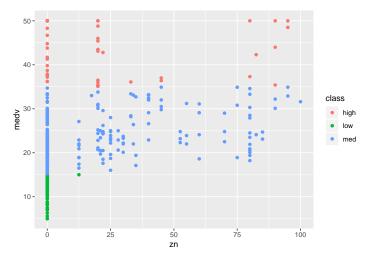
The distribution for crimes appears to have a higher spread for lower house price, and less for medium and high. Crime rates past 25% all appear to be of lower income, while those below 25% have a high concentration of all 3.

Variable # 2: Zn





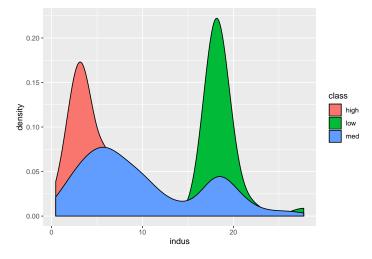
ggplot(housing, aes(x=zn,y=medv,color=class)) + geom_point()

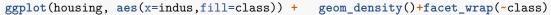


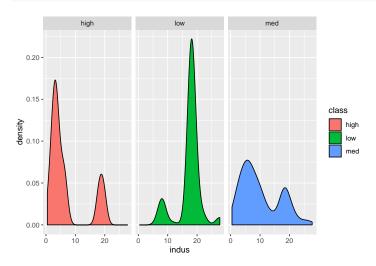
The distribution for proportion of residential land zoned for lots appears to have equal distribution for values of 0, while greater values have a blend of med & high, no apparent relationship seems to be apparent as there is a high variance in response variable (medv)

Variable # 3: Indus

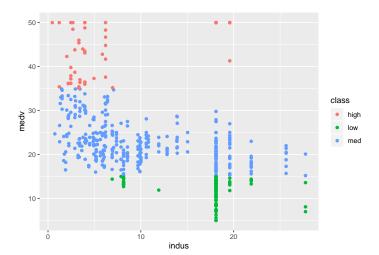
```
ggplot(housing, aes(x=indus,fill=class)) + geom_density()
```







ggplot(housing, aes(x=indus,y=medv,color=class)) + geom_point()



The distribution for proportion of non-retail business acres per town appears to have apparent distinctions for each categorie of house prices. There seems to be an apparent negative linear relationship, as proportion increases house prices get lower

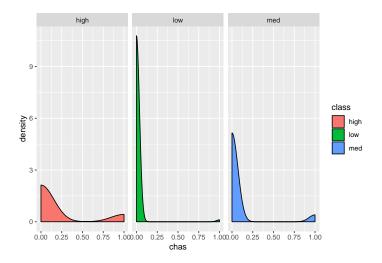
Variable # 4: Chas

0.25

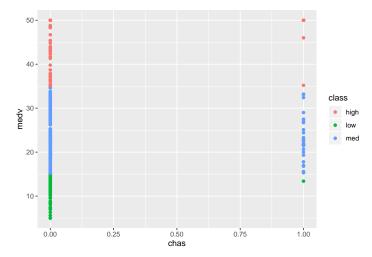
0.00

ggplot(housing, aes(x=chas,fill=class)) + geom_density()+facet_wrap(~class)

0.75

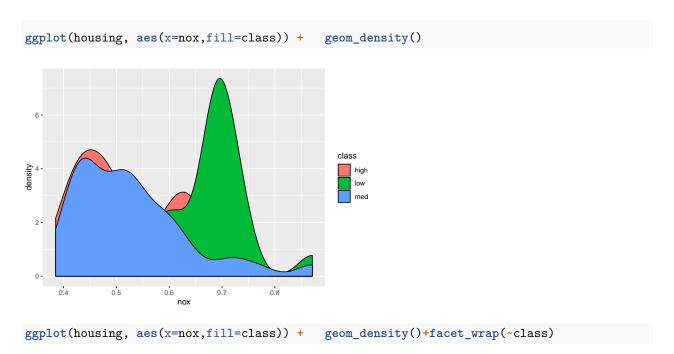


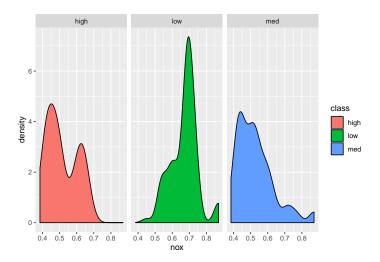
ggplot(housing, aes(x=chas,y=medv,color=class)) + geom_point()



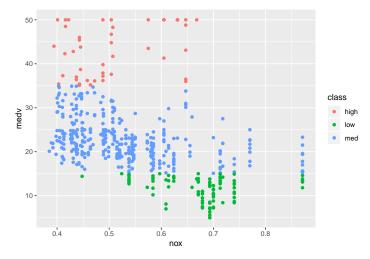
We can see that house prices are of medium range when closer to the river while those that arent are of equal distribution

Variable # 5: Nox





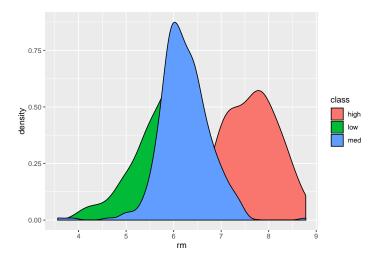
ggplot(housing, aes(x=nox,y=medv,color=class)) + geom_point()



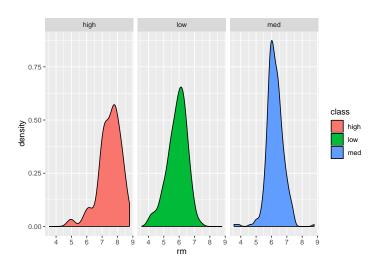
The distribution for nitrogen oxide concentration appears to have apparent distinctions for low vs med/high. There seems to be an apparent negative linear relationship, as concentration increases house prices get lower. However there seems to be high response variability for values of med and high

Variable # 6: Rm

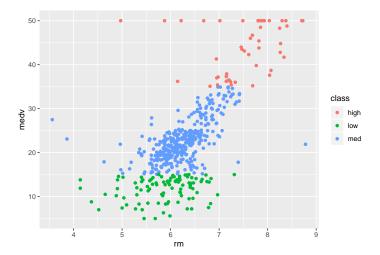
```
ggplot(housing, aes(x=rm,fill=class)) + geom_density()
```



ggplot(housing, aes(x=rm,fill=class)) + geom_density()+facet_wrap(~class)



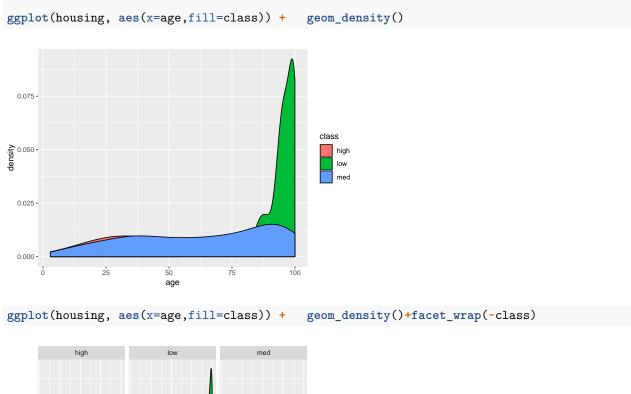
ggplot(housing, aes(x=rm,y=medv,color=class)) + geom_point()

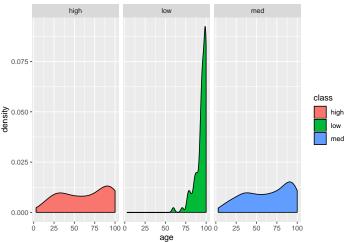


There is a high correlation between average number of rooms per dwelling & house prices, very apparent with a positive linear relationship, although there exists high variance for lower prices houses, but still follows

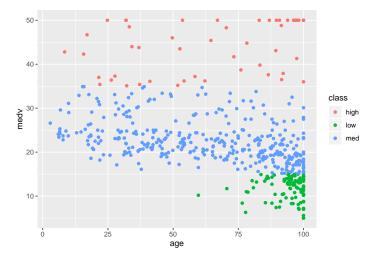
linear relationship regardless with a few exceptions that we may be able to capture with the help of other variables

Variable # 7: Age



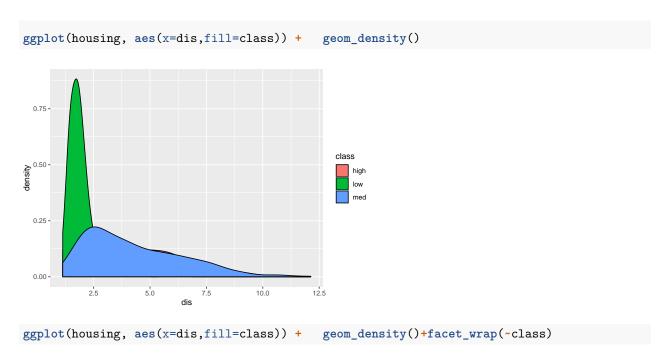


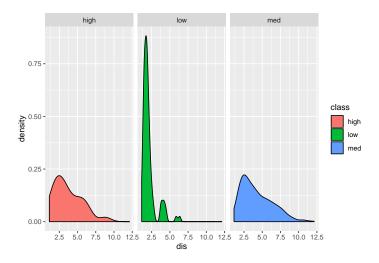
ggplot(housing, aes(x=age,y=medv,color=class)) + geom_point()



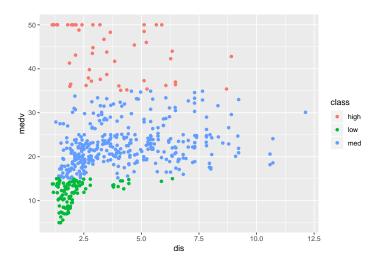
The distributions shows all low priced houses being of old age, but distributions for med & low are the same distribution with high variance in response and predictor

Variable # 8: Dis





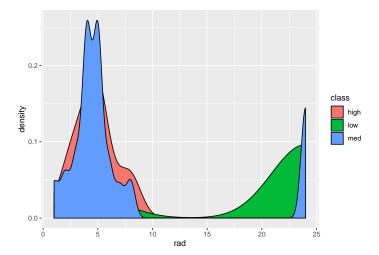
ggplot(housing, aes(x=dis,y=medv,color=class)) + geom_point()



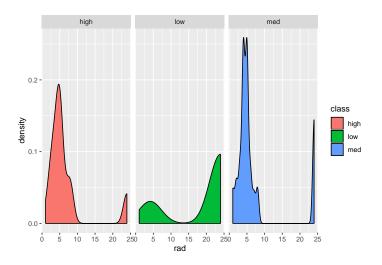
The distribution shows as weighted mean of distances to five Boston employment centres increases, the house prices are more towards the median and high side, with lower values having a mix of all 3, however the variance is quite high

Variable # 9: Rad

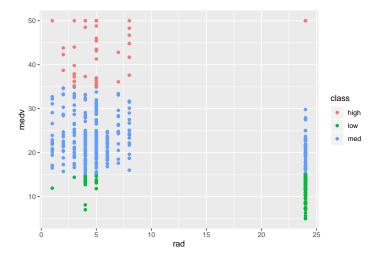
```
ggplot(housing, aes(x=rad,fill=class)) + geom_density()
```



ggplot(housing, aes(x=rad,fill=class)) + geom_density()+facet_wrap(~class)



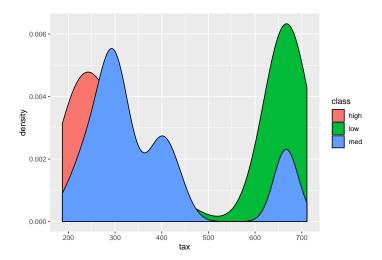
ggplot(housing, aes(x=rad,y=medv,color=class)) + geom_point()



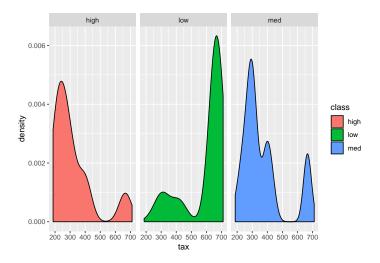
We can see that index of accessibility to radial highways, help indicate the prices of higher valued houses but fail to distinguish between low and med

Variable # 10: Tax

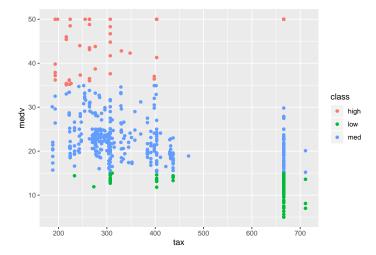
```
ggplot(housing, aes(x=tax,fill=class)) + geom_density()
```



ggplot(housing, aes(x=tax,fill=class)) + geom_density()+facet_wrap(~class)

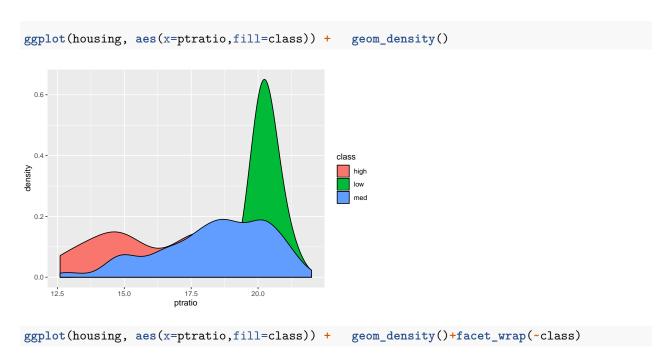


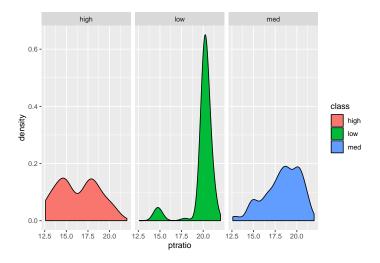
ggplot(housing, aes(x=tax,y=medv,color=class)) + geom_point()



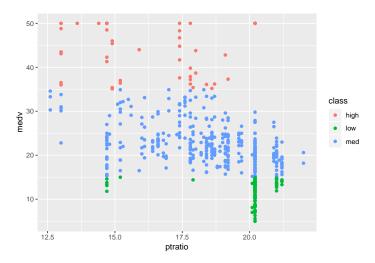
We see from the distribution that it is easy to view the distributional difference between low houses and med/high houses, and there exists large variability in med/low houses

Variable # 11: Ptratio





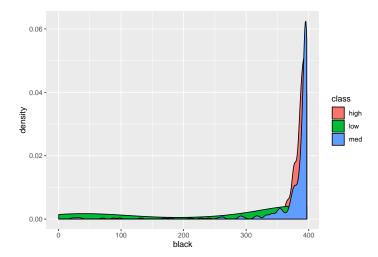
ggplot(housing, aes(x=ptratio,y=medv,color=class)) + geom_point()



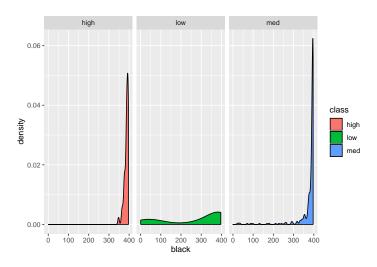
The distribution seems to have a negative liner relationship, however a very high level of variance

Variable # 12: Black

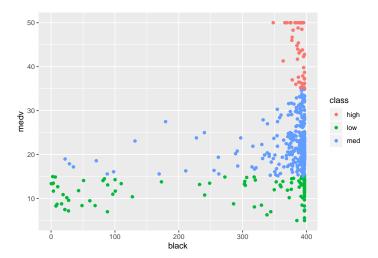
```
ggplot(housing, aes(x=black,fill=class)) + geom_density()
```



ggplot(housing, aes(x=black,fill=class)) + geom_density()+facet_wrap(~class)



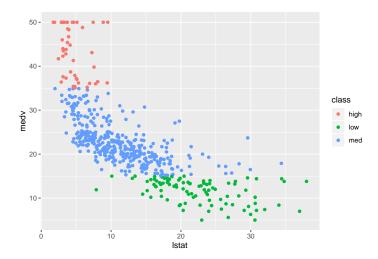
ggplot(housing, aes(x=black,y=medv,color=class)) + geom_point()



Aside from the fact that lower values tend to reflect a lower prices in houses, it seems to show no other pattern for values over 400

Variable # 13: Lstat

```
ggplot(housing, aes(x=lstat,fill=class)) + geom_density()
 0.2 -
density
 0.0 -
             10
                                  30
ggplot(housing, aes(x=1stat,fill=class)) + geom_density()+facet_wrap(~class)
                                  10
         20
                    10
ggplot(housing, aes(x=lstat,y=medv,color=class)) + geom_point()
```



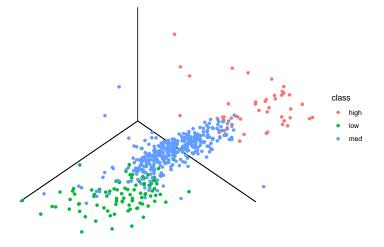
A clearly negatively linear correlation exists, with low variance comparetively as well, there seems to exist a little skew we may hope to capture

3D Plots

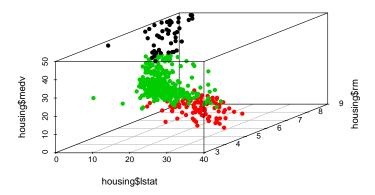
I shall plot 3 Dimensional Scatterplots of variables I believe give a strong indicator of house prices, and hypothesis on the multiple regression model to be fit

Lstat & Rm

```
ggplot(housing, aes(x=lstat, y=rm, z=medv, color=class)) +
    theme_void() +
    axes_3D() +
    stat_3D()
```

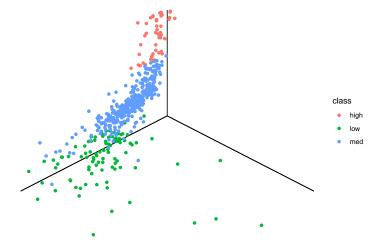


```
housing$class <- as.factor(housing$class)
scatterplot3d(x=housing$lstat, y=housing$rm, z=housing$medv,pch=16, color=as.numeric(housing$class))
```

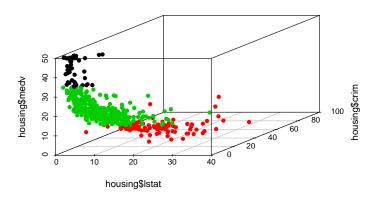


Lstat & Crim

```
ggplot(housing, aes(x=lstat, y=crim, z=medv, color=class)) +
    theme_void() +
    axes_3D() +
    stat_3D()
```

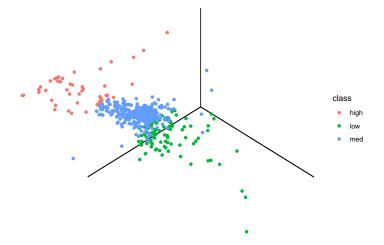


scatterplot3d(x=housing\$lstat, y=housing\$crim, z=housing\$medv,pch=16, color=as.numeric(housing\$class))

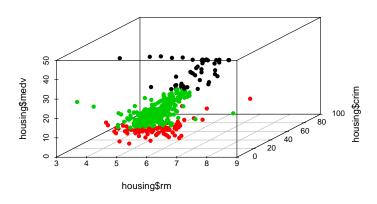


Rm & Crim

```
ggplot(housing, aes(x=rm, y=crim, z=medv, color=class)) +
    theme_void() +
    axes_3D() +
    stat_3D()
```

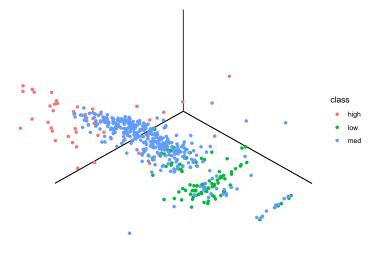


scatterplot3d(x=housing\$rm, y=housing\$crim, z=housing\$medv,pch=16, color=as.numeric(housing\$class))

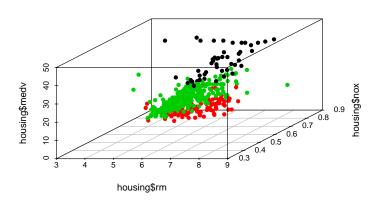


Rm & Nox

```
ggplot(housing, aes(x=rm, y=nox, z=medv, color=class)) +
    theme_void() +
    axes_3D() +
    stat_3D()
```

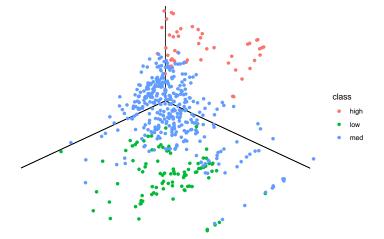


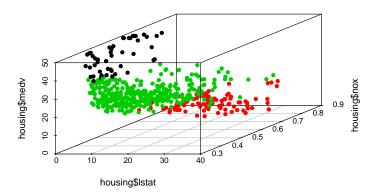
scatterplot3d(x=housing\$rm, y=housing\$nox, z=housing\$medv,pch=16, color=as.numeric(housing\$class))



Lstat & Nox

```
ggplot(housing, aes(x=lstat, y=nox, z=medv, color=class)) +
    theme_void() +
    axes_3D() +
    stat_3D()
```





Observation:

These variables studied above show clear linear realationships between the predictors and the response, the variability in Medv is explained by these inputs + Noise. We shall explore Building MLR models with them, and further investigate statistically significant variable detected beyond via R commands, this allows to to view insights beyond the eye test

Multiple Linear Regression

Feature Selection

We will try out a Multiple Linear Regression Model, but first we must ensure proper features are selected for our model to reduce noise, and we shall train the model & test performance on a test set

```
housing<-housing[1:14]
model <- lm(medv ~ crim+zn+indus+chas+nox+rm+age+dis+rad+tax+ptratio+black+lstat, data = housing)
summary(model)</pre>
```

Call:

```
lm(formula = medv ~ crim + zn + indus + chas + nox + rm + age +
dis + rad + tax + ptratio + black + lstat, data = housing)
```

Residuals:

```
Min 1Q Median 3Q Max
-15.5642 -2.7248 -0.5312 1.7687 26.1511
```

Coefficients:

		a		5 (: 1 : 1)	
	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	36.634941	5.102043	7.180	2.59e-12	***
crim	-0.107417	0.032847	-3.270	0.001150	**
zn	0.046121	0.013721	3.361	0.000836	***
indus	0.014269	0.061653	0.231	0.817071	
chas	2 671108	0.861115	3 102	0.002033	**

```
-17.633641
                        3.818719 -4.618 4.96e-06 ***
nox
             3.794307
                        0.417835 9.081 < 2e-16 ***
rm
                        0.013205
age
             0.001076
                                   0.081 0.935079
            -1.479179
                        0.199347 -7.420 5.19e-13 ***
dis
rad
             0.301534
                        0.066398
                                  4.541 7.04e-06 ***
                        0.003765 -3.202 0.001454 **
            -0.012053
tax
            -0.958874
                        0.130831 -7.329 9.60e-13 ***
ptratio
                                   3.467 0.000573 ***
black
             0.009305
                        0.002684
lstat
            -0.527600
                        0.050732 -10.400 < 2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 4.742 on 491 degrees of freedom Multiple R-squared: 0.7415, Adjusted R-squared: 0.7346 F-statistic: 108.3 on 13 and 491 DF, p-value: < 2.2e-16

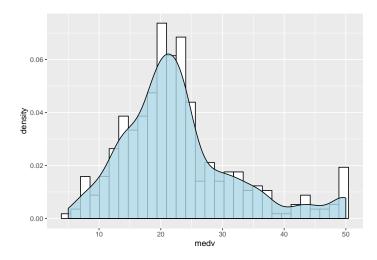
```
sample <- sample.split(medv, SplitRatio = .70)

train <- subset(housing, sample == TRUE)

test <- subset(housing, sample == FALSE)

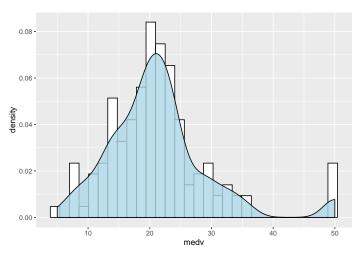
ggplot(train, aes(x=medv)) +
    geom_histogram(aes(y=..density..), colour="black", fill="white")+
    geom_density(alpha=.8, fill="lightblue")</pre>
```

`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.



```
ggplot(test, aes(x=medv)) +
   geom_histogram(aes(y=..density..), colour="black", fill="white")+
   geom_density(alpha=.8, fill="lightblue")
```

`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.



```
regfit.fwd=regsubsets(medv~.,data=train, nvmax=13,method="forward")
summary(regfit.fwd)
```

```
Subset selection object
```

Call: regsubsets.formula(medv ~ ., data = train, nvmax = 13, method = "forward")
13 Variables (and intercept)

LIADICS	(and	1 IIICEIC	ebei
Forced	in	Forced	out

crim	FALSE	FALSE
zn	FALSE	FALSE
indus	FALSE	FALSE
chas	FALSE	FALSE
nox	FALSE	FALSE
rm	FALSE	FALSE
age	FALSE	FALSE
dis	FALSE	FALSE
rad	FALSE	FALSE
tax	FALSE	FALSE
ptratio	FALSE	FALSE
black	FALSE	FALSE
lstat	FALSE	FALSE
41	- C 1-	

 $1\ \mbox{subsets}$ of each size up to 13

 ${\tt Selection\ Algorithm:\ forward}$

indus chas nox rm age dis rad tax ptratio black lstat 11 11 "*" 1 (1) "*" (1) "*" (1) "*" "*" (1) "*" 6 "*" (1 "*" 7 (1 "*" "*" "*" 9 (1) "*" "*" "*" "*" "*" 10 (1) "*" 11 "*" "*" (1) "*" "*" "*" "*" 13 (1) "*"

```
test.mat=model.matrix(medv~.,data=test)
validation.errors=rep(NA,13)
for(i in 1:13){
  coefi=coef(regfit.fwd,id=i)
  pred=test.mat[,names(coefi)]%*%coefi
  validation.errors[i]=mean((test$medv-pred)^2)
}
validation.errors
 [1] 36.21700 29.11124 25.94498 25.39358 23.32370 24.37883 23.77584 22.52542
 [9] 22.36304 21.77664 20.65098 20.85605 20.89573
which.min(validation.errors)
[1] 11
coef(regfit.fwd,id=which.min(validation.errors))
  (Intercept)
                       crim
                                       zn
                                                   chas
                                                                  nox
 40.708506655 -0.101416843
                              0.038176087
                                            1.915972649 -22.052695249
                                                              ptratio
                        dis
                                      rad
                                                    tax
  3.819294696 -1.581392947
                              0.254704983 -0.008969070 -1.027981738
       black
                      lstat
```

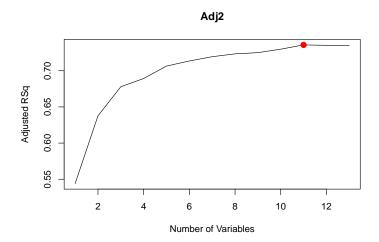
Observation:

0.008315321 -0.549929286

We can see that 11 of these variables have statistical significance in the model because of there contribution in explaining the variance ifor the variable. The variables not included are indus and age. We shall test which number of variables are most optimal with more performance measures, & the test set

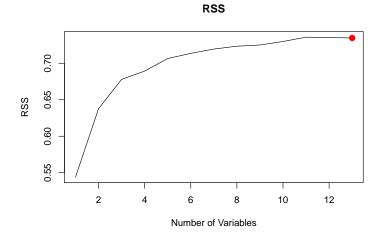
```
regfit.full=regsubsets(medv~.,data=housing,nvmax=13,method="forward")
reg.summary=summary(regfit.full)
which.max(reg.summary$adjr2)
```

```
[1] 11
```



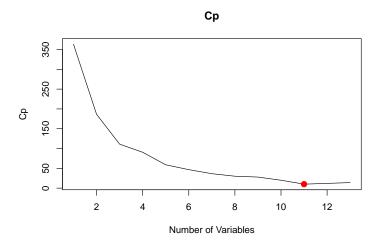
which.min(reg.summary\$rss)

[1] 13



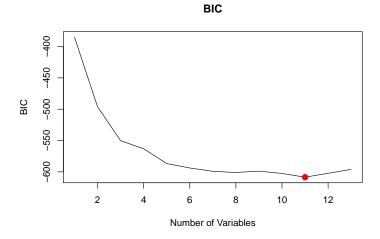
which.min(reg.summary\$cp)

[1] 11



```
which.min(reg.summary$bic)
```

[1] 11



Observation:

We further confirm that 11 variables is the most optimal model to select for out Muliple Linear Regression, we shall now average error and compare predictions with the actual target. we will plot the distribution of our prediction

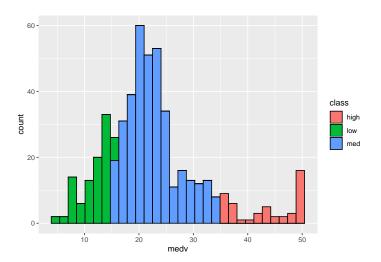
```
coefficients<-as.matrix(coef(regfit.fwd,id=which.min(validation.errors)))
predictions<-function()
{
    predictions<- c()
    for(i in 1:length(medv))
    {</pre>
```

```
input<-housing[i,][c("crim","zn","chas","nox","rm","dis","rad","tax","ptratio","black","lstat")</pre>
        input<-as.data.frame(c(1,input))</pre>
        input<-as.matrix(input)</pre>
        prediction<-t(coefficients) %*% t(input)</pre>
        predictions<-c(predictions, prediction)</pre>
    }
    return (predictions)
}
predictions<-as.data.frame(predictions())</pre>
targetAndPrediction<-cbind(medv,predictions)</pre>
targetAndPrediction<-as.data.frame(targetAndPrediction)</pre>
names(targetAndPrediction)<-c("target", "predictions")</pre>
filter(targetAndPrediction,predictions<0)</pre>
 target predictions
    13.8 -0.09870132
     7.0 -4.80441298
for(i in 1:length(targetAndPrediction$predictions)){
    if(targetAndPrediction$predictions[i]<0)</pre>
        targetAndPrediction$predictions[i]=0
    }
}
targetAndPrediction<-targetAndPrediction%>%mutate(error=abs(target-predictions))
head(targetAndPrediction)
  target predictions
                          error
            25.34659 3.7465946
1
   21.6
  34.7
            31.04083 3.6591669
3
   33.4 28.96020 4.4398007
4 36.2 28.23010 7.9699010
  28.7
            25.53852 3.1614848
5
6 22.9
            23.10215 0.2021473
mean(targetAndPrediction$error)
[1] 3.308083
filter(targetAndPrediction,error>=10)
```

```
target predictions
                          error
1
     14.4
             3.358644 11.04136
2
     50.0
            37.413933 12.58607
3
     50.0
            37.794343 12.20566
            36.343483 13.65652
4
     50.0
5
     23.7
            10.957131 12.74287
6
     46.7
            35.962094 10.73791
7
     48.3
            37.592931 10.70707
8
     42.8
            30.007106 12.79289
            36.372981 14.47298
9
     21.9
10
     27.5
            13.870271 13.62973
     23.1
            10.923490 12.17651
11
12
     50.0
            23.875270 26.12473
            31.957109 18.04289
     50.0
13
            33.947312 16.05269
14
     50.0
            24.926991 25.07301
15
     50.0
16
     50.0
            25.090505 24.90949
     13.8
            0.000000 13.80000
17
            25.054959 10.05496
18
     15.0
     7.2
            17.218790 10.01879
19
20
     17.9
             1.566281 16.33372
21
     11.9
            22.321902 10.42190
```

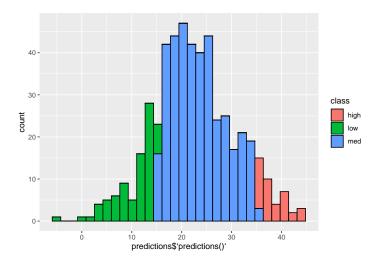
```
ggplot(housing, aes(x=medv,fill=class)) + geom_histogram(colour="black")
```





predictions<-predictions%>%mutate(class=sapply(predictions\$predictions()`,classifier))
ggplot(predictions, aes(x=predictions\$predictions()`,fill=class)) + geom_histogram(colour="black")

[`]stat_bin()` using `bins = 30`. Pick better value with `binwidth`.

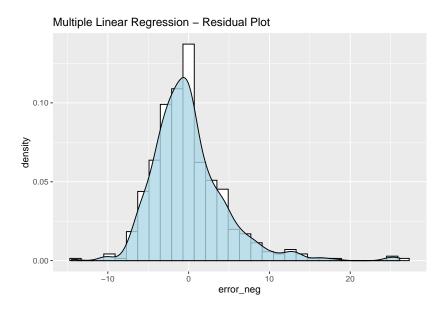


targetAndPrediction<-targetAndPrediction%>%mutate(error_neg=target-predictions)

Residual Plot-MLR

```
ggplot(targetAndPrediction, aes(x=error_neg)) +
    geom_histogram(aes(y=..density..), colour="black", fill="white")+
    geom_density(alpha=.8, fill="lightblue")+ggtitle("Multiple Linear Regression - Residual Plot")
```

`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.



Decision Trees

```
tree.boston=tree(medv~.,train)
plot(tree.boston)
text(tree.boston,pretty=0)
```

```
| stat < 14.915 | rm < 7.437 | stat < 11.455 | ptratio < 17.6 | stat < 1.6156 | stat < 1.6156
```

```
predictions_tree=predict(tree.boston,newdata=housing)

targetAndPrediction_tree<-cbind(medv,predictions_tree)

targetAndPrediction_tree<-as.data.frame(targetAndPrediction_tree)
names(targetAndPrediction_tree)<-c("target","predictions")

filter(targetAndPrediction_tree,predictions<0)</pre>
```

```
[1] target predictions
<0 rows> (or 0-length row.names)
```

```
for(i in 1:length(targetAndPrediction_tree$predictions)){
    if(targetAndPrediction_tree$predictions[i] < 0)
    {
        targetAndPrediction_tree$predictions[i] = 0
    }
}

targetAndPrediction_tree<-targetAndPrediction_tree%>%mutate(error=abs(target-predictions))
head(targetAndPrediction_tree)
```

```
target predictions
                        error
   21.6
           21.44043 0.1595745
1
2
   34.7
           33.73939 0.9606061
3
   33.4
           33.73939 0.3393939
4
  36.2
           33.73939 2.4606061
   28.7
           21.44043 7.2595745
   22.9
           21.44043 1.4595745
6
```

```
mean(targetAndPrediction_tree$error)
```

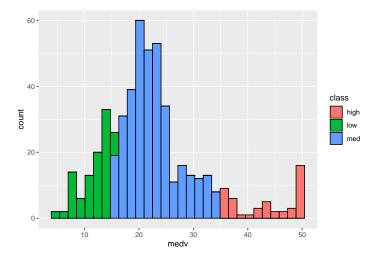
[1] 2.954402

filter(targetAndPrediction_tree,error>=10)

```
target predictions
                         error
     23.6
             33.73939 10.13939
1
2
     19.6
             31.56000 11.96000
3
     15.3
             31.56000 16.26000
            21.44043 14.75957
4
     36.2
5
    50.0
            37.03333 12.96667
6
     21.9
             37.03333 15.13333
7
     50.0
             31.56000 18.44000
8
     50.0
             28.11136 21.88864
     50.0
9
             33.73939 16.26061
10
     50.0
             31.56000 18.44000
             31.56000 18.44000
11
     50.0
12
     10.4
            20.74000 10.34000
13
     27.5
            11.93036 15.56964
             31.56000 16.56000
14
     15.0
15
     10.9
             21.44043 10.54043
             17.12632 10.12632
16
     7.0
```

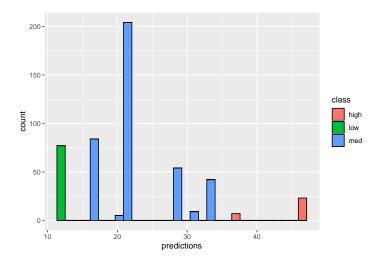
```
targetAndPrediction_tree<-targetAndPrediction_tree%>%mutate(error_neg=target-predictions)
targetAndPrediction_tree<-targetAndPrediction_tree%>%mutate(class=sapply(predictions, classifier))
ggplot(housing, aes(x=medv,fill=class)) + geom_histogram(colour="black")
```

`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.



```
ggplot(targetAndPrediction_tree, aes(x=predictions,fill=class)) + geom_histogram(colour="black")
```

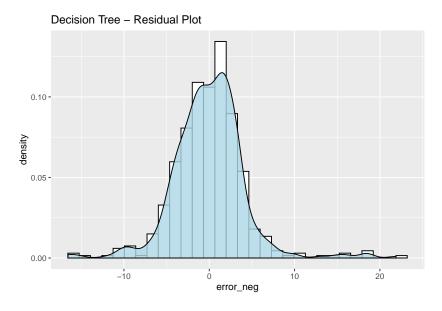
`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.



Residual Plot - Decision Tree

```
ggplot(targetAndPrediction_tree, aes(x=error_neg)) +
   geom_histogram(aes(y=..density..), colour="black", fill="white")+
   geom_density(alpha=.8, fill="lightblue")+ggtitle("Decision Tree - Residual Plot")
```

`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.



Conclusions:

We have investigated our data & made visualizations for understanding key variables and their relationships, we build models for predictions that have relatively high accuracy. Multiple Linear Regression Models &

Decision Tree Models were both used, Decision trees outperformed Multiple Linear Regression slightly, by an average of 0.4 over our testing data. Errors were plotted to show a normal distribution indicating our model captures variability in our response variable