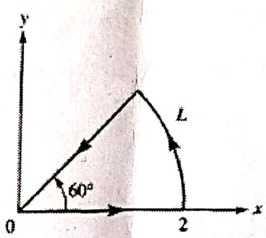
NATIONAL INSTITUTE OF TECHNOLOGY CALICUT
Department of Physics
PH1003E Electricity and Magnetism
Midsem Examination - Winter 2024-25

Marks: 30
Time: 2 hours

5 February 2025

Answer All Questions

1. Calculate the closed line integral of $\vec{A} = s\ \cos\phi\ \hat{s} + z\ \sin\phi\ \hat{z}$ around the circular wedge defined by $0 \le s \le 2,\quad 0 \le \phi \le 60°, z = 0$ as shown in the figure below. (3 marks)



2. The charge density of a sphere of radius R is $\rho(x, y, z) = \rho_0 e^{(x^2+y^2+z^2)^{3/2}}$, where ρ_0 is a constant. Calculate the charge enclosed within a unit radius. (3 marks)

3. Given that $\vec{E} = (y + az)\hat{x} + (z + bx)\hat{y} + (x + cy)\hat{z}$ represents a static electric field.
   (a) Find the values of a, b and c.
   (b) Find the potential corresponding to $\vec{E}$ and the charge density that generates $\vec{E}$. Reference point of the potential is taken to be $\vec{r} = 0$ i.e, $V(0, 0, 0) = 0$.

(3 marks)

4. A ring of radius R has a total charge $+Q$ uniformly distributed on it. Calculate the electric field at any point on the axis of symmetry of the ring. What will be the nature of this field as $z \to \infty$, where z is the distance from the centre of the ring, along the axis? (3 marks)

5. An inverted hemispherical bowl of radius R carries a uniform surface charge density σ. Find the potential difference between the "north pole" and the centre. (3 marks)

6. Two spheres, each of radius R and carrying uniform charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap. Considering the vector from the positive charge center to the negative charge center as $\vec{d}$, show that the field in the region of overlap is constant, and find its value. (3 marks)

7. Consider a coaxial cable which consists of a inner solid conducting cylinder of radius a and a coaxial cylindrical shell of negligible thickness with radius $b > a$. Find it's capacitance per unit length. Assume that the spacing between the two cylinders is empty. (3 marks)

8. A metal sphere of radius R, carrying charge q, is surrounded by a thick concentric metal shell (inner radius a, outer radius b) carrying no charge.

   (a) Find the surface charge density at R, a and b.

   (b) Find the potential at the center. (3 marks)

9. The polarization in a dielectric cube of side length L centered at the origin is given by $\vec{P} = k(x\hat{x} + y\hat{y} + z\hat{z})$, where k is a constant.

   (a) Determine the surface and volume bound charge densities.

   (b) Show that the total bound charges is zero. (3 marks)

10. Consider a spherical shell of radius R and uniform charge density σ.

   (a) Write down the electric field inside and outside the shell.

   (b) Using the expression for the electric field, find the total energy of the configuration.

   (c) For the same constant value of σ, sketch the graph of energy vs the radius of the shell. (3 marks)

Some useful vector derivatives

Spherical coordinates:

$$d\mathbf{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$$

$$\vec{\nabla}t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\mathbf{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\hat{\phi}$$

Cylindrical coordinates:

$$d\mathbf{l} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z}$$

$$\vec{\nabla}t = \frac{\partial t}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\phi} + \frac{\partial t}{\partial z}\hat{z}$$

$$\vec{\nabla}\cdot\mathbf{A} = \frac{1}{s}\frac{\partial}{\partial s}(sA_s) + \frac{1}{s}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\mathbf{A} = \left[\frac{1}{s}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sA_\phi) - \frac{\partial A_s}{\partial\phi}\right]\hat{z}$$