First Semester B.Tech Mid Semester Test, October 2024

MA1001E - MATHEMATICS I

(Common to EC, EE, CH & BT branches)

Time: 90 Minutes | Max Marks: 30

Answer all questions

1. Find a second order homogeneous linear ordinary differential equation for which the functions $\cosh\frac{3x}{2}$, $\sinh\frac{3x}{2}$ are solutions. (2)

2. Does the initial value problem $y' = xy^{1/3}, y(0) = 0$ have a unique solution in a region around the initial point? Justify your answer. Also find all solutions, if it exists. (3)

3. Check whether the differential equation

$$(y \log y)dx + (x - \log y)dy = 0$$

is exact. If not, find an integrating factor and solve it. (4)

4. Find the orthogonal trajectories of the family of curves $y = x + ce^{-x}$, c is a parameter. (4)

5. Solve $y'' + 2y' + 4y = 2x^2 + 3e^{-x}$ by the method of undetermined coefficients. (4)

6. A spring with a mass of 2 Kg has damping constant 14. A force of $6N$ is required to keep the spring stretched $0.5m$ beyond its natural length. The spring is pushed to $1m$ above the equilibrium position and then released with zero velocity. Find the position of the mass at any time t. (4)

7. Solve the differential equation $x^2y'' - 3xy' + 5y = \sin(\ln x)$. (5)

8. Check for convergence or divergence of the following series

(a) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$. (3)

(b) $\sum_{n=1}^{\infty} \frac{n}{3n+1}$. (1)