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Department of Mathematics

Second Semester B.Tech. Mid Semester Examination

Winter Semester 2024-25

MA1011E: Mathematics II

Time: 90 minutes **Maximum Marks:** 30

Instructions:

- Answer all the questions.
- Answers of question 1 must be written on the first page of the main sheet.
- $\mathbf{F}, \mathbf{f}, \mathbf{r}$ are vector fields and f_1, f_2, f_3, g, ϕ are scalar valued functions.
- Calculators/other assisting gadjets/materials are not allowed for this examination.
- Sketch the surfaces/curves whenever necessary.

1. (a) Write a parametric form for the sphere $x^2 + y^2 + z^2 = \alpha^2$, $\alpha > 0$. (1)

   (b) Find the line integral $\int_C \frac{27}{8}\, ds$, where C is the path from $(0,0)$ to $(4,8)$ along the curve $\mathbf{r}(t) = t\,\mathbf{i} + t^{\frac{3}{2}}\mathbf{j}$. (1)

   (c) If σ is the surface of the sphere $x^2 + y^2 + z^2 = 9$, then find $\iint_\sigma 7\, d\sigma$. (1)

   (d) Without evaluating, express $\int_0^2 \int_{\frac{x-2}{2}}^{\frac{2-x}{2}} x^2\, dy\, dx$, after changing the order of integration. (1)

   (e) Find $\int_0^3 \int_0^{3-x} \int_0^{3-y-x} y\, dV$. (1)

2. (a) Show that $\mathrm{div}(\phi\,\mathbf{f}) = \phi\,\mathrm{div}(\mathbf{f}) + \nabla\phi\cdot\mathbf{f}$, where $\phi = \phi(x,y,z)$ and $\mathbf{f} = f_1(x,y,z)\,\mathbf{i} + f_2(x,y,z)\,\mathbf{j} + f_3(x,y,z)\,\mathbf{k}$. (2)

   (b) Let $\mathbf{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$ and $r = \|\mathbf{r}\|$. If $g(r)$ is a differentiable function of the variable r and $\mathbf{F}(\mathbf{r}) = g(r)\,\mathbf{r}$, show that $\mathrm{div}(\mathbf{F}) = 3g(r) + rg'(r)$. (3)

3. Is the vector field $\mathbf{F}(x,y) = (2x + ye^{xy})\,\mathbf{i} + (2y + xe^{xy})\,\mathbf{j}$ conservative in the XY - plane? If conservative, find its scalar potential and also calculate the work done by $\mathbf{F}$ on a particle that moves it from $(-1,2)$ to $(2,3)$. (5)

4. Verify Green's theorem in the plane for

$$\oint_C (xy + y^2)\, dx + x^2\, dy,$$

where C is the closed boundary of the region bounded by $y = x$ and $y = x^2$. (5)

5. A cylindrical volume of $x^2 + y^2 = 4$ is removed from inside the cone $z = 4 - \sqrt{x^2 + y^2}$, $0 \leq z \leq 4$. Use double integral to find the volume of the portion between the cone and the cylinder. (5)

6. (a) Define the flux of a velocity vector field. (1)

   (b) Find the flux of $\mathbf{F}(x, y, z) = yz\,\mathbf{j} + z^2\,\mathbf{k}$ outward through the surface σ cut from the cylinder $y^2 + z^2 = 1$, $z \geq 0$ by the planes $x = 0$ and $x = 1$. (4)

Question Nos.	1	2	3	4	5	6
Course Outcomes	CO1	CO1	CO1	CO1	CO1	CO1
Difficulty Level*	2	1	3	2	4	2
Marks	5	5	5	5	5	5

Course Outcomes:

- CO1: Find the parametric representation of curves and surfaces in space and evaluate integrals over curves and surfaces
- CO2: Use Laplace transform and its properties to solve differential equations and integral equations.
- CO3: Test the consistency of the system of linear equations and solve it.
- CO4: Diagonalise symmetric matrices and use it to find the nature of quadratic forms.

*1. Knowledge / Recall Level; 2. Understand / Comprehend Level; 3. Apply / Analyze Level; 4. Evaluate / Create Level