



MA1001E - MATHEMATICS I
(Common to EC, EE, CH & BT branches)

Time: 180 Minutes

Max Marks: 50

Answer all questions. Calculators are not permitted.

1. Solve the initial value problem $\frac{dx}{dt} = -2x + y$, $\frac{dy}{dt} = x - 2y$, $x(0) = 1$ and $y(0) = 0$. (5)

2. Test the convergence of the series

$$1 + \frac{(\alpha + 1)}{(\beta + 1)} + \frac{(\alpha + 1)(2\alpha + 1)}{(\beta + 1)(2\beta + 1)} + \frac{(\alpha + 1)(2\alpha + 1)(3\alpha + 1)}{(\beta + 1)(2\beta + 1)(3\beta + 1)} + \dots, \alpha, \beta > 0. \quad (4)$$

3. Discuss the convergence of the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$, $x \in R$. (5)

4. Find the half range Fourier cosine series expansion of $f(x) = x^2 - 2x + 1$ in the interval $0 < x < 1$. Using this expansion, evaluate

i) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ ii) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. (5)

5. Find the Fourier integral representation of $f(x) = \begin{cases} \frac{\pi}{2} & \text{if } |x| < 1, \\ 0 & \text{if } |x| > 1 \end{cases}$. Hence evaluate

$$\int_0^{\infty} \frac{\sin \omega}{\omega} d\omega. \quad (5)$$

6. Find the Fourier transform of $f(x) = e^{-\frac{x^2}{2}}$. (4)

7. For the function

$$f(x, y) = \begin{cases} \frac{x^2 - y^2}{x - y}, & (x, y) \neq (1, -1) \\ 0, & (x, y) = (1, -1) \end{cases}$$

check the continuity of $f(x, y)$ and existence of its partial derivatives at $(1, -1)$. (5)

8. Find Taylor's series expansion for $f(x, y) = e^x \sin y$ about the point $(1, \frac{\pi}{2})$ upto third degree term. (4)

9. Locate all relative extrema and saddle points of the function

$$f(x, y) = (6 - x)(6 - y)(x + y - 6) \quad (5)$$

10. Let $f(x, y) = x^2y + y^3$. Find the directional derivative of f at the point $(1, 2)$ in the direction of the vector $\vec{v} = 3\vec{i} + 4\vec{j}$. (3)
11. For the curve $\vec{r}(t) = 3\cos(t)\vec{i} + 3\sin(t)\vec{j} + 4t\vec{k}$, find the unit tangent vector, unit normal vector, curvature and torsion at t . (5)

Question Nos.	1	2	3	4	5	6	7	8	9	10	11
Course Outcomes	CO2	CO4	CO4	CO5	CO5	CO5	CO3	CO3	CO3	CO3	CO3
Difficulty Level*	4	3	4	2	3	1	2	4	3	2	1
Marks	5	4	5	5	5	4	5	4	5	3	5

Course Outcomes:

- CO1: Formulate some engineering problems as ODEs and hence solve such problems.
- CO2: Solve linear ODEs with constant coefficients
- CO3: Find the limits, check for continuity and differentiability of real valued functions of two variables
- CO4: Test for the convergence of sequences and series.
- CO5: Find the Fourier series representing periodic functions.

1. Knowledge / Recall Level; 2. Understand / Comprehend Level; 3. Apply / Analyze Level; 4. Evaluate / Create Level