Project:

Quarter Car model using active suspension

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MEE 5630

12/2/2020

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1. Introduction

Automotive suspension designs must compromise between three criteria:

1. road holding
2. load carrying,
3. Passenger comfort.

Usually, the suspension must support the vehicle, improve handling while steering, and assure passenger isolation and loads from road disturbances. Therefore, a Good ride needs a soft suspension, whereas an applied load requires stiff suspension. According to ISO 2631-1 standard, exposing the human body to vibrations with frequencies between 0.5-80 Hz could cause a significant risk of injury to the vertebrae in the lumbar region and the nerves connected to these segments.
Due to these conflicting demands, suspension design had to be determined by the type of use for which the vehicle was designed. However, Active suspensions are considered a way to auto adapt suspension stiffness according to the road needs. Therefore, Active suspension is a very fertile topic with innovations each year.

2. Quarter Car model

The quarter car model is frequently used for suspension system analysis and design due to its simplicity and ability to present many important parameters. Figure 1 presents a Quarter car suspension system. The car body is denoted as a sprung mass, and the tire is denoted un-sprung mass. A single wheel axle is connected to the car body's quarter portion through a passive spring and damper. The tire is assumed to have a spring constant and a damper having contact with the road. The road serves as an external disturbance to the system. The diagram used is as follows:

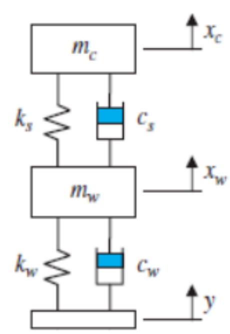

Figure 1 Quarter car model.

The equations of motion and free body diagrams used can be given as follows:

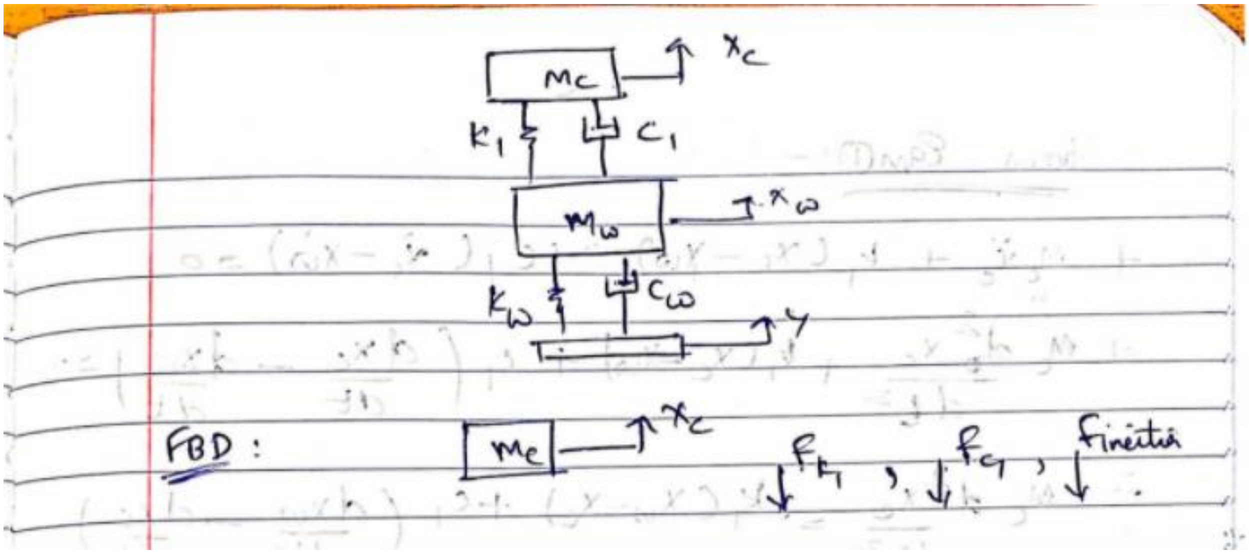

Figure 2 Free body diagram of Sprung Mass.

The equation of motion can be given as follows:

$$M_c\ddot{x}_c + K_1(x_c - x_w) + C_1(\dot{x}_c - \dot{x}_w) = 0$$

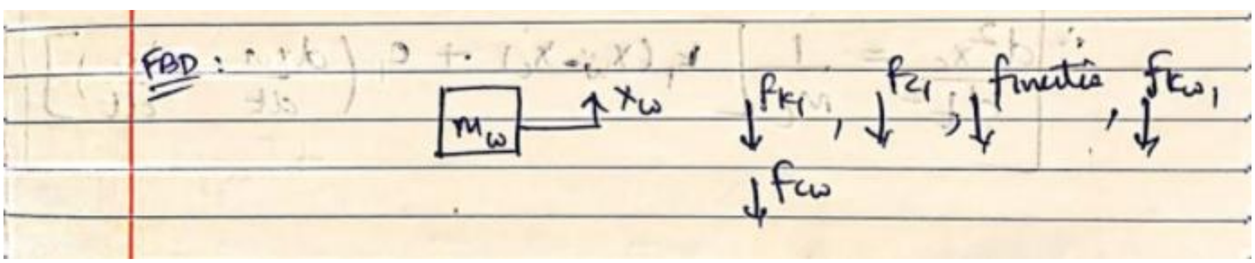

Figure 3 Free body diagram of Unsprung Mass.

The equation of motion can be given as follows:

$$M_w\ddot{x}_w + K_1(x_w - x_c) + C_1(\dot{x}_w - \dot{x}_c) + K_w(x_w - y) + C_w(\dot{x}_w - y) = 0$$

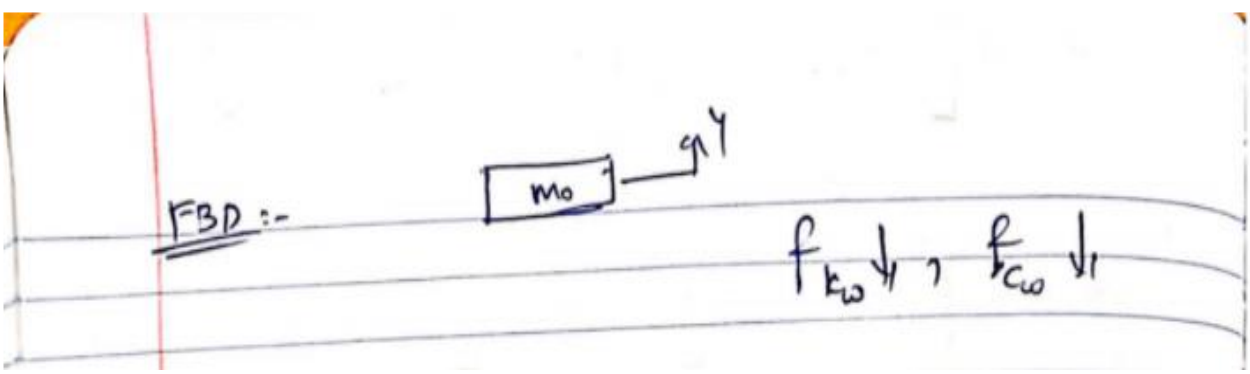

Figure 4 Free body diagram of the surface making contact with the road. (Assuming mass = 0)

The equation of motion can be given as follows:

$$K_w(y - x_w) + C_w(\dot{y} - \dot{x}_w) = 0$$

Where;

- $m_c = Sprung\ Mass$
- $m_w = Unsprung\ Mass$
- $c_s = Damping\ Coefficient\ of\ sprung\ mass$
- $c_w = Damping\ Coefficient\ of\ unsprung\ mass$
- $K_1 = Stiffness\ Coefficient\ of\ sprung\ mass$
- $K_w = Stiffness\ Coefficient\ of\ unsprung\ mass$
- Y = $Input\ displacement$
- $x_c = Absolute\ displacement\ of\ sprung\ Mass$
- $x_w = Absolute\ displacement\ of\ \ unsprung\ Mass$

3. Model Parameters

The model parameters taken can be shown in the following table:

Name of the parameter	Parameter Symbol	Value
$Sprung\ Mass$	m_c	504.5
$Unsprung\ Mass$	m_w	62
$Damping\ Coefficient\ of\ sprung\ mass$	c_s	13100
$Stiffness\ Coefficient\ of\ sprung\ mass$	K_1	252000
$Stiffness\ Coefficient\ of\ unsprung\ mass$	K_w	400
$Damping\ Coefficient\ of\ unsprung\ mass$	c_w	2500

Table 1 Model Parameters with their respective values.

4. Simulink model of Quarter Car

The Simulink model of the quarter car can be seen in the following figure:

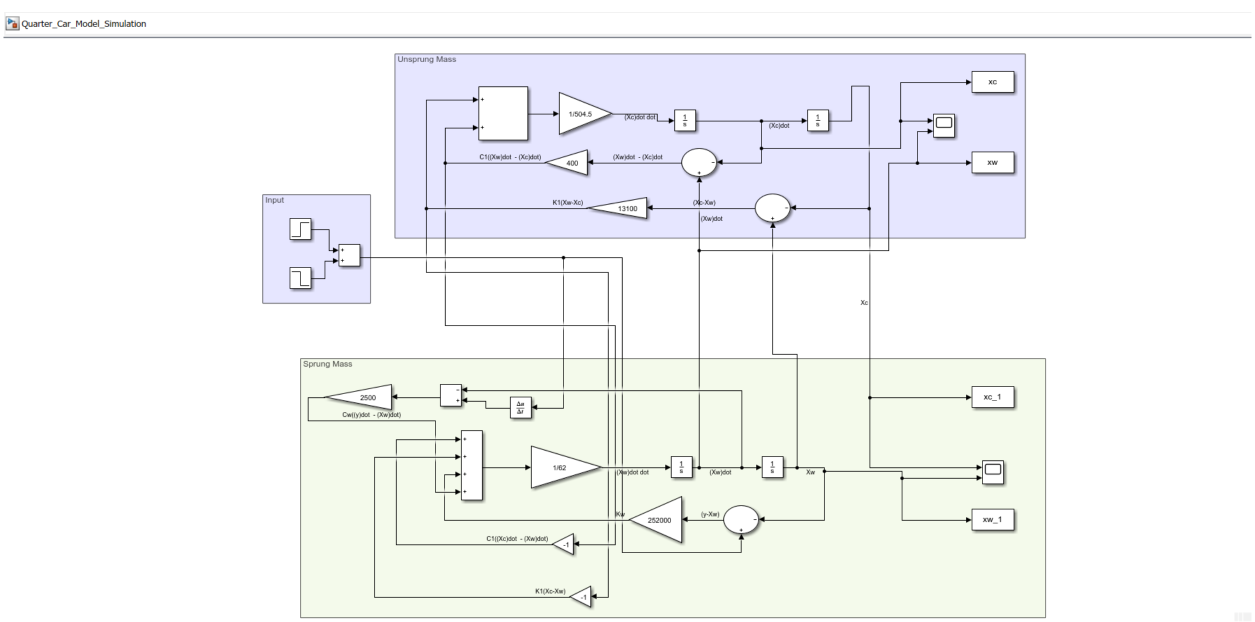

Figure 5 Simulink Model Of the Quarter Car.

The values were selected for different parameters and based on the equation of motion achieved; they were carefully entered into the different Simulink gain blocks. The wiring was done based on the equations of motions acquired from the quarter car.

5. Quarter car model with a PID controller:

To further test the quarter car, a PID controller was added to record the readings of an actuator force (f_a) applied to the quarter car model. The quarter car, along with an actuator force, can be seen in the following figure.

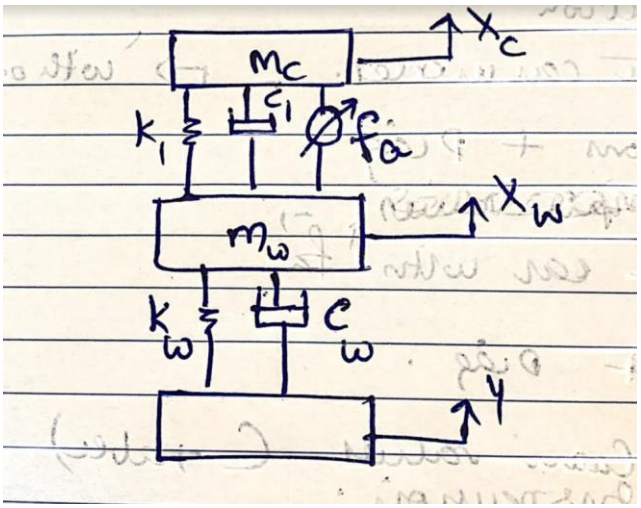

Figure 6 Quarter Car model containing an Actuator Force.

The equations of motion are shown below:

$$M_c\ddot{x}_c + K_1(x_c - x_w) + C_1(\dot{x}_c - \dot{x}_w) - f_a = 0$$

$$M_w\ddot{x}_w + K_1(x_w - x_c) + C_1(\dot{x}_w - \dot{x}_c) + K_w(x_w - y) + C_w(\dot{x}_w - y) + f_a = 0$$

Where;

- $m_c = Sprung\ Mass$
- $m_w = Unsprung\ Mass$
- $c_s = Damping\ Coefficient\ of\ sprung\ mass$
- $c_w = Damping\ Coefficient\ of\ unsprung\ mass$
- $K_1 = Stiffness\ Coefficient\ of\ sprung\ mass$
- $K_w = Stiffness\ Coefficient\ of\ unsprung\ mass$
- Y = *Input displacement*
- $x_c = Absolute\ displacement\ of\ sprung\ Mass$
- $x_w = Absolute\ displacement\ of\ unsprung\ Mass$
- $f_a = Actuator\ Force$

5.1. PID Controller:

PID controller has been widely used in industry because of its simplicity and effectiveness. Despite many PID controller uses, its standard structure has constant gain parameters and is not good to decrease velocity control error. Therefore, an exponential function is added to the derivate component of the conventional PID controller.

The used PID controller was adapted from (Eski and Yildirim, 2009) and can be described as follows:

$$f_a = G.(K_p e(t) + \ K_i \int e(t)dt + K_d \frac{de(t)}{dt}(K_N e^{-K_N t}))$$

Where;

- $K_p = \ Proportional$
- $K_i = Integral$
- $K_d = Derivative$
- $K_N = Controller\ Gain$
- G = Gain

The gain has been used to scale the values.

5.1.1. Tuning Parameters

To tune the parameters, the following values were taken:

Parameters	Values
K_p	4.971
K_i	4.948
K_d	0.3614
K_N	414.1968
G	15000

Table 2 Parameters with their values.

5.1.2. Quarter car model with active suspension

Active suspension system requires actuator force to provide external control; this actuator's goal is to provide a better ride and handling. The force f_a is given by a PID controller that aims to bring the body position of (m_c) to a constant value (zero). The system has two-step input added to a variable (y) and three outputs (x_c, x_w and f_a). The Simulink model of my quarter car with active suspension can be seen in the following figure.

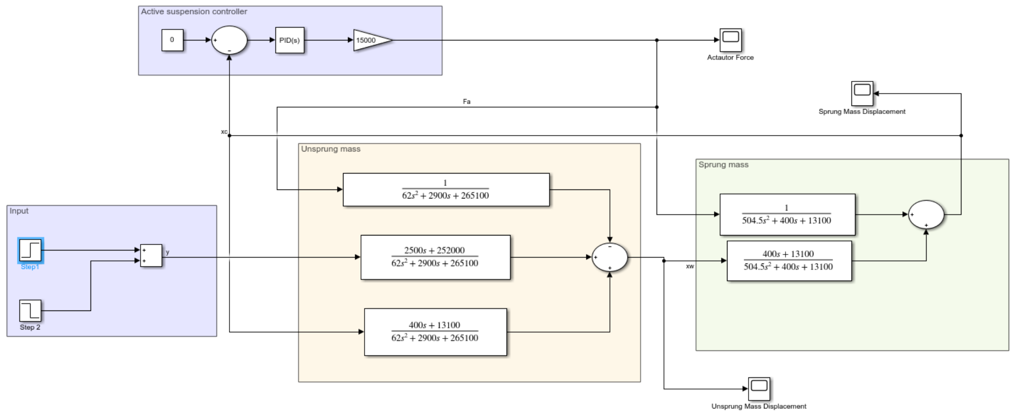

Figure 7 Quarter Car Model using active suspension.

In the above Simulink, two inputs were added and sent to the PID controller. Active suspension using constant reference (having value = 0) was added to the input and sent to the PID controller. The graph was plotted in the scope of Simulink. The step input values taken can be shown in the following table.

Initial Value	Final Value
3	10
5	-10

Table 3 Step input values taken.

6. Results

The results obtained can be seen in the following figure.

6.1. Quarter Car Model without active suspension

The input of my model can be seen in the following figure.

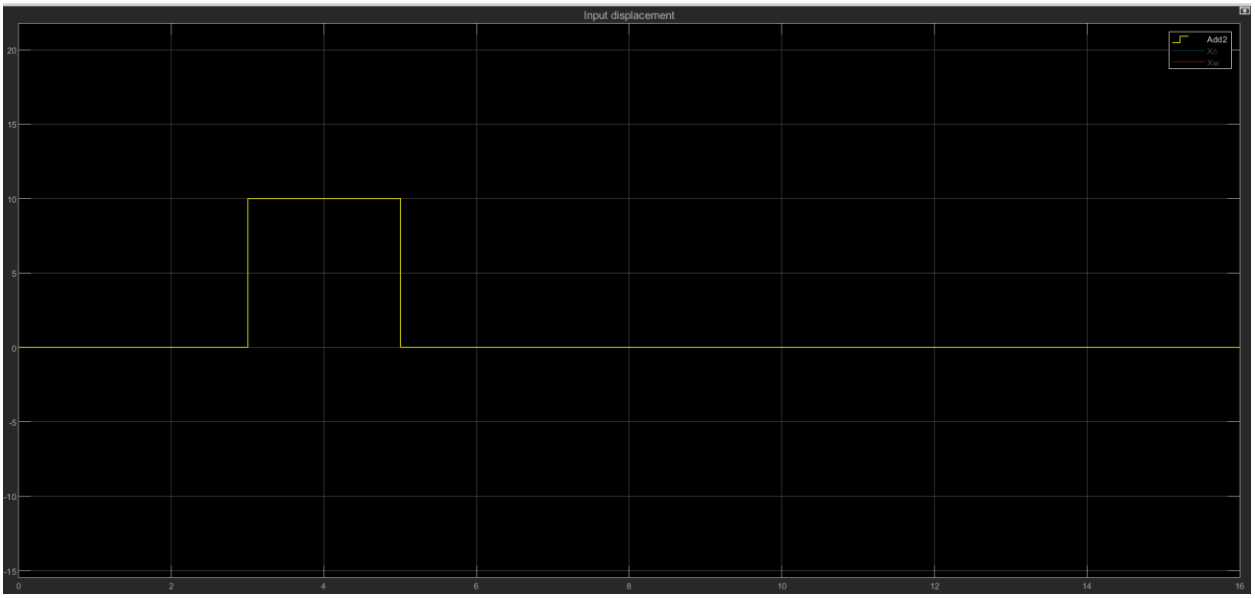

Figure 8 Input displacement of my quarter car model.

The input graph is a square graph, whose value starts from 0, reaches a point 10 and then back to 0 and stays at 0.

The outputs can be shown in the following figures.

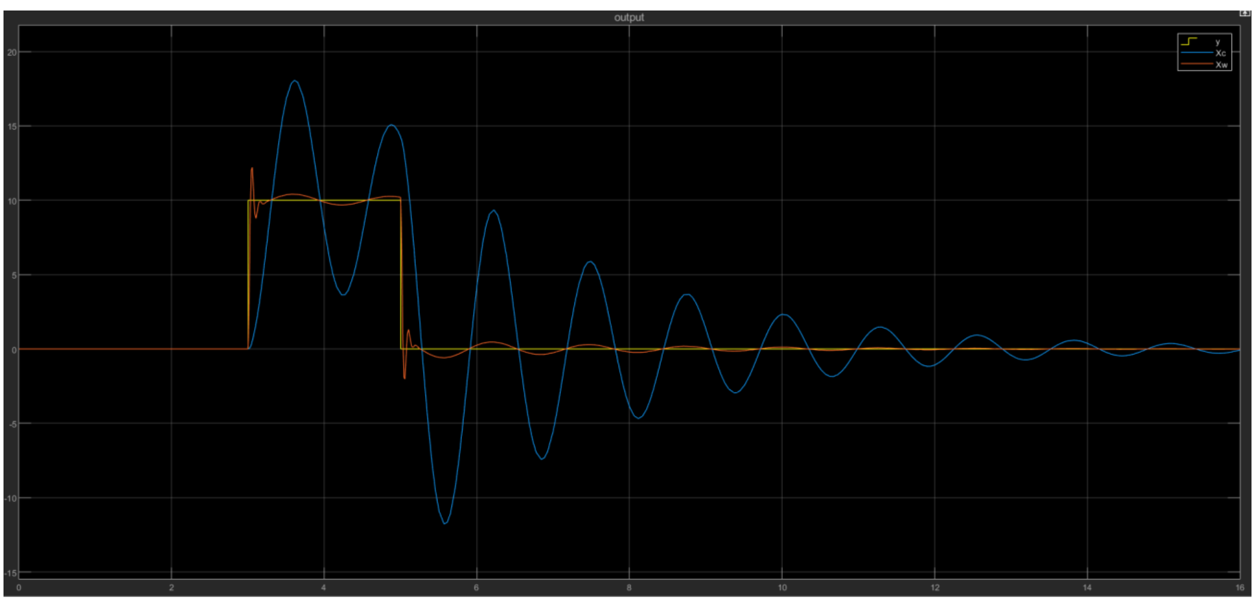

Figure 9 Output obtained.

From the above graph, we can see that according to the given input(y), the wheel displacement(Xw) tries to trace with the input, and the displacement of the car Xc has a lot of disturbance initially and then stabilizes at a fixed road disturbance 0.

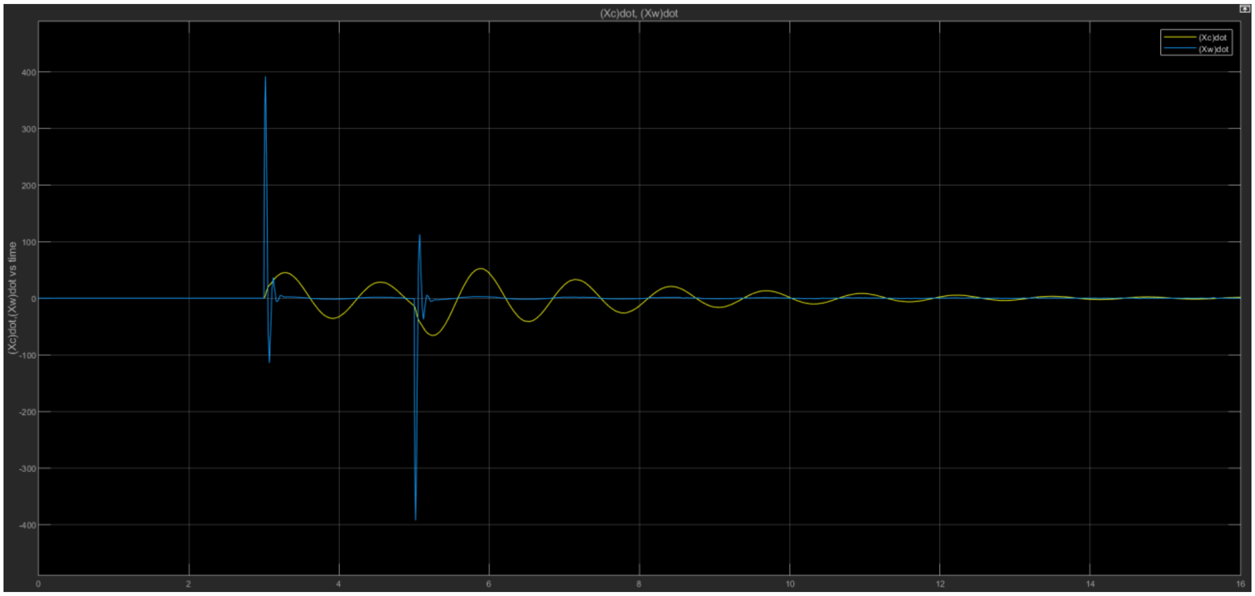

Figure 10 Velocity (Xc dot and Xw dot) vs time.

From the above figure, we can see that there is a sudden changes in the road surface, there is a steep increase and decrease in the graph for $\dot{x}_w$ and then it slowly stabilizes after a particular point. The $\dot{x}_c$ graph portrays a sinusoidal wave graph and, in turn, stabilizes at point 0.

6.2. Quarter Car with active suspension.

The input graph can be shown in the following figure.

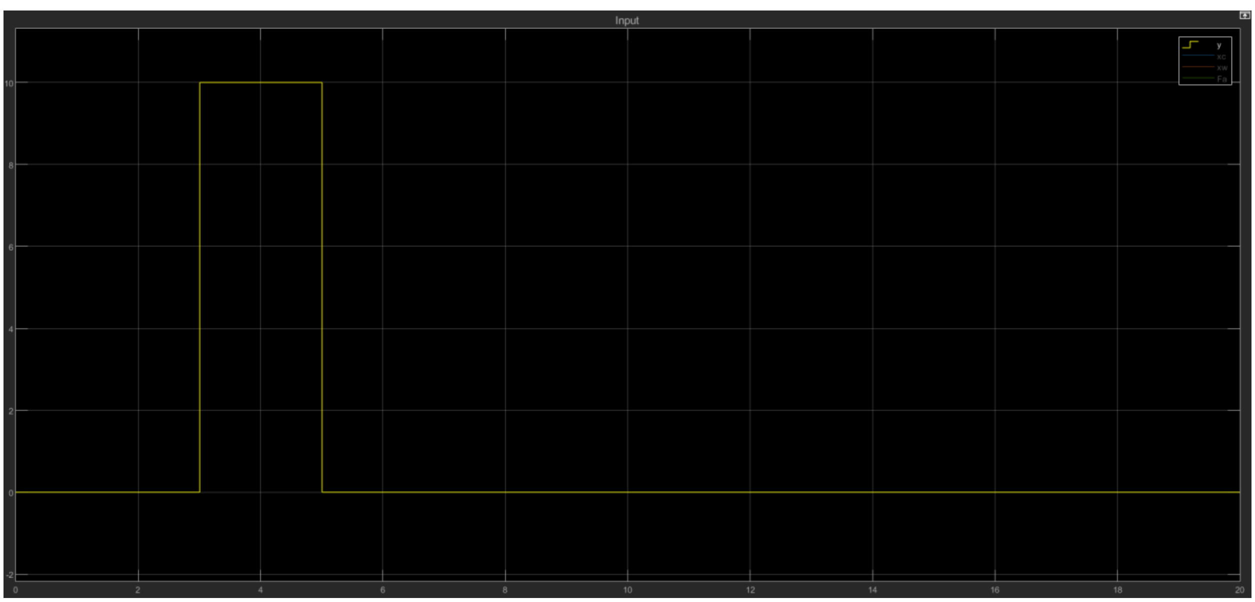

Figure 11 Input displacement of my quarter car model with active suspension.

The input graph is a square graph, whose value starts from 0, reaches a point 10 and then back to 0 and stays at that 0.

The outputs can be shown in the following figure.

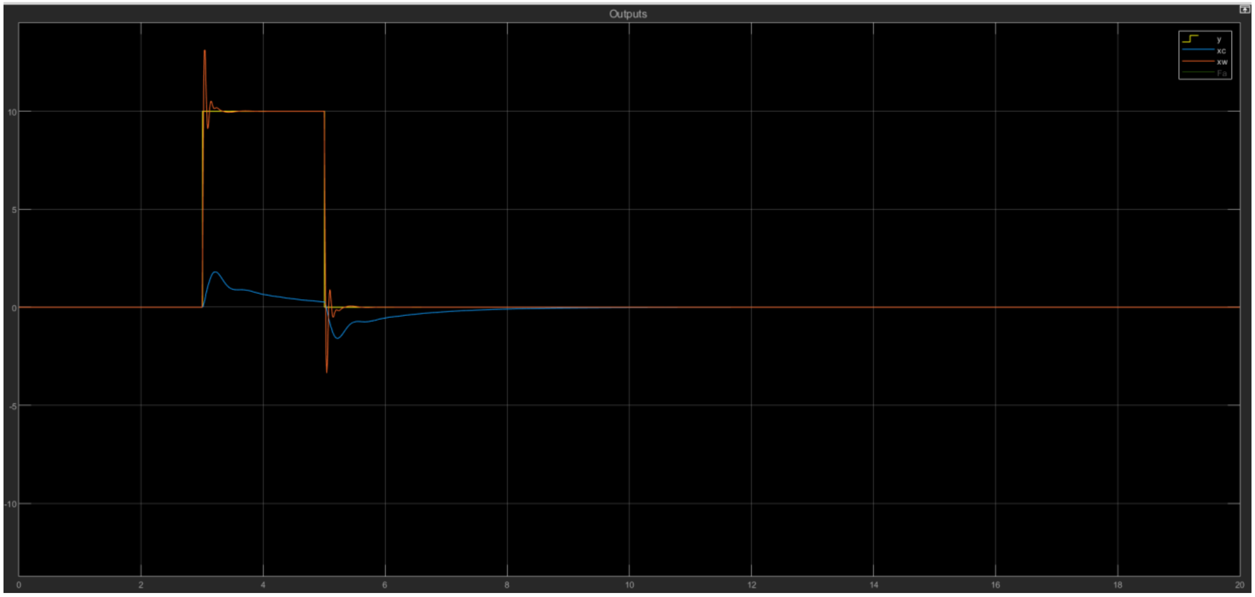

Figure 12 Output with active suspension.

For Xw vs time, we can see that the displacement caused by the wheel almost traces with the input(y). The displacement (Xc) of the car is so negligible that the passenger sitting in the car will not feel most of the disturbance.

The graph obtained for the quarter car model with active suspension with a constant reference 0 can be shown in the following figure.

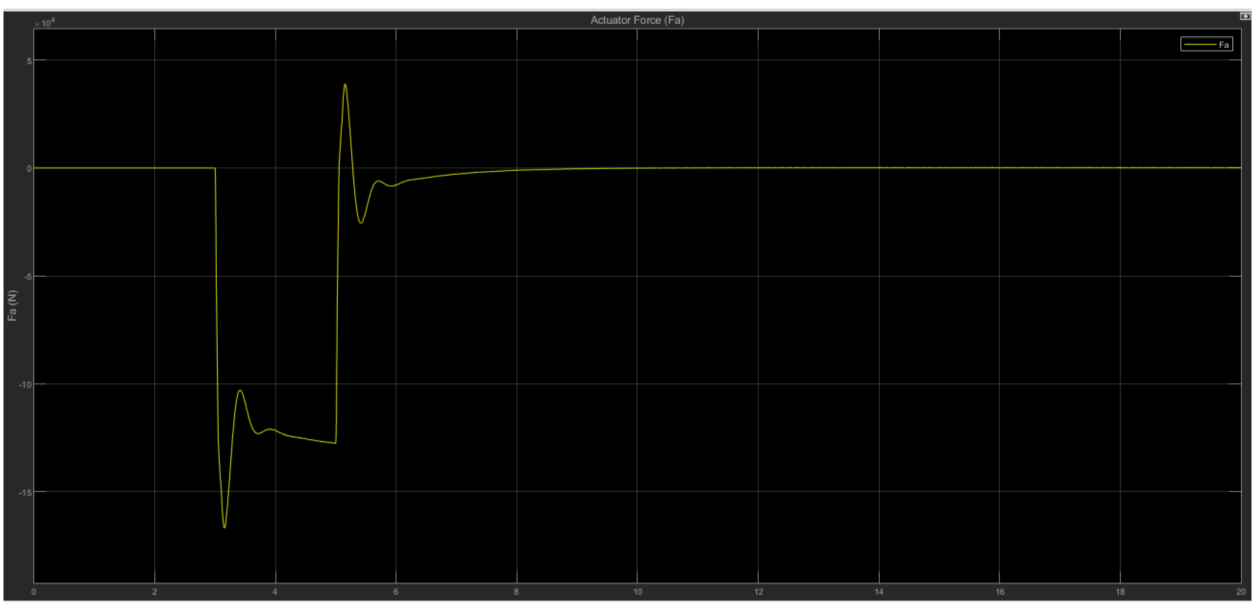

Figure 13 Actuator force vs time.

From the above graph, we can see a sudden change in the actuator with the road disturbance.