Problem 1

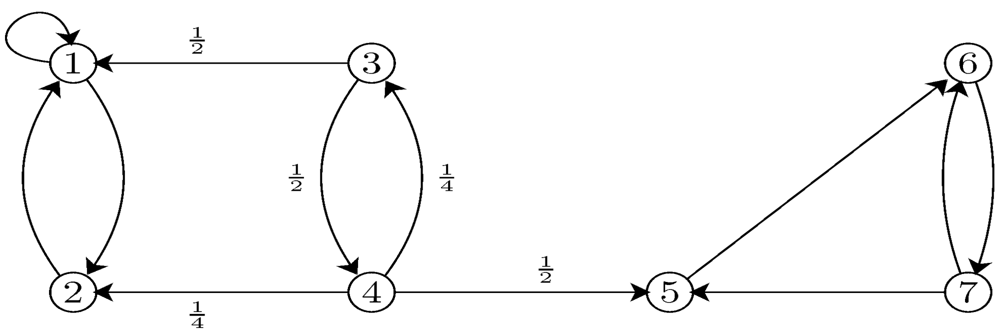
Consider the Markov chain with three states, $S = \{1, 2, 3\}$, that has the following transition matrix

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}.$$

a. Draw the state transition diagram for this chain.
b. If we know $P(X_1 = 1) = P(X_1 = 2) = \frac{1}{4}$, find $P(X_1 = 3, X_2 = 2, X_3 = 1)$.

Problem 2

Consider the Markov chain in Figure. There are two recurrent classes, $R_1 = \{1, 2\}$, and $R_2 = \{5, 6, 7\}$. Assuming $X_0 = 3$, find the probability that the chain gets absorbed in R_1.



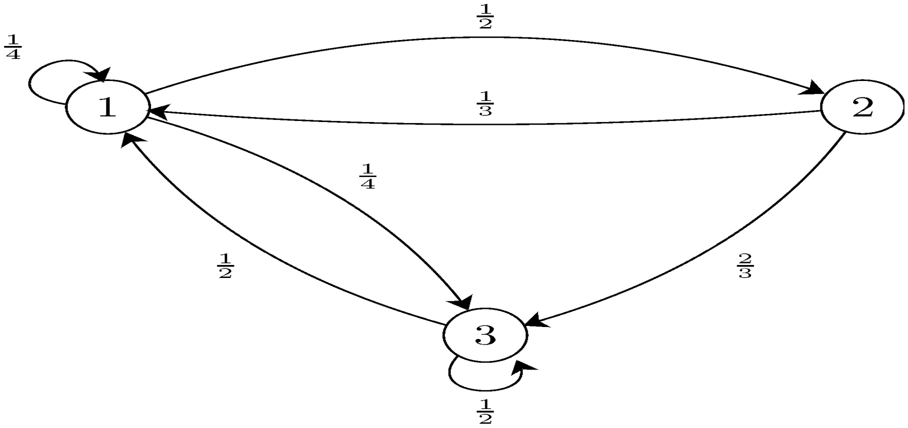
Problem 3

Consider the Markov chain of Problem 2. Again assume $X_0 = 3$. We would like to find the expected time (number of steps) until the chain gets absorbed in R_1 or R_2. More specifically, let T be the absorption time, i.e., the first time the chain visits a state in R_1 or R_2. We would like to find $E[T|X_0 = 3]$.

Problem 4

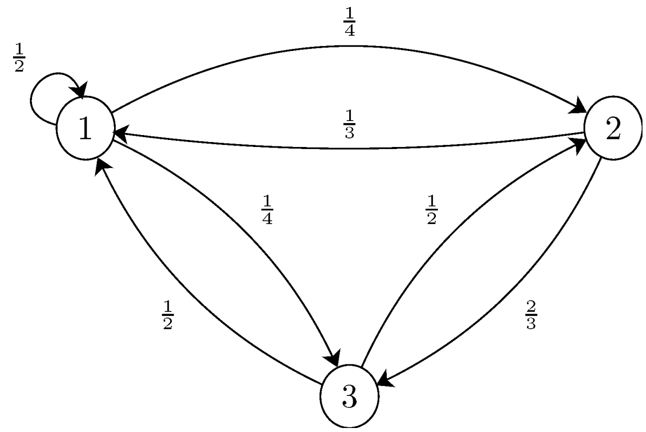
Consider the Markov chain shown in Figure. Assume $X_0 = 1$, and let R be the first time that the chain returns to state 1, i.e., $R = \min\{n \geq 1 : X_n = 1\}$.

Find $E[R|X_0 = 1]$.



Problem 5

Consider the Markov chain shown in Figure .



a. Is this chain irreducible?
b. Is this chain aperiodic?
c. Find the stationary distribution for this chain.
d. Is the stationary distribution a limiting distribution for the chain?

Problem 6

Consider the Markov chain shown in Figure. Assume that $\frac{1}{2} < p < 1$. Does this chain have a limiting distribution? For all $i, j \in \{0,1,2,\cdots\}$, find

$$\lim_{n\to\infty} P(X_n = j|X_0 = i).$$