

# Unit 4: Inference for numerical data

## 2. Bootstrapping

Sta 101 - Fall 2015

Duke University, Department of Statistical Science

## 1. Housekeeping

## 2. Main ideas

1. Bootstrapping = sampling with replacement from the observed sample
2. Bootstrap percentile intervals: middle XX% of the bootstrap distribution
3. Bootstrap SE intervals: point estimate  $\pm$  ME
4. Bootstrap testing for a single numerical variable requires shifting the bootstrap distribution to be centered at the null value

## 3. Summary

- ▶ Feedback from midterm evaluation:  $n = 10$  :( or :)?
  - Majority think pace is about right
  - Majority like videos and application exercises
  - Majority like flipped / team-based aspect, though feelings on teams not unanimous
- ▶ Read the project instructions one more time before lab tomorrow, and go to lab with questions.

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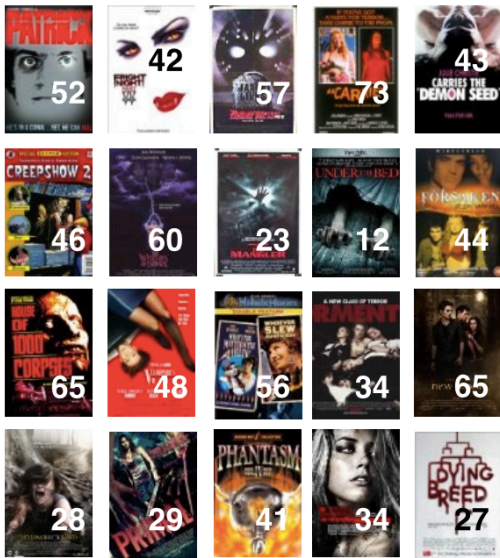
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## 3. Summary

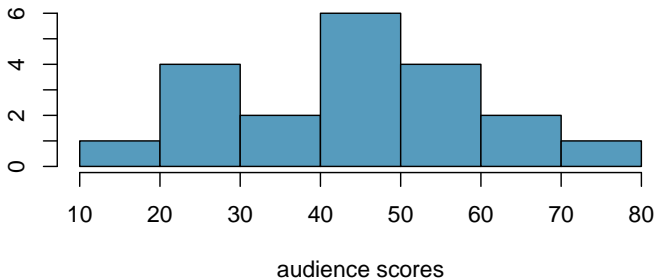


is a movie aggregator, where the audience is also able to review and score the movies. We want to estimate the average audience score of horror movies on RottenTomatoes.com. We start with a random sample of 20 horror movies.



	title	audience_score
1	Patrick	52
2	Demon Seed	43
3	Tormented	34
4	Under the Bed	12
5	Phantasm IV: Oblivion	41
6	Fright Night Part 2	42
7	House of 1000 Corpses	65
8	Creepshow 2	46
9	The Forsaken	44
10	All the Boys Love Mandy Lane	34
11	Jason Lives: Friday the 13th Part VI	57
12	Vampire's Kiss	48
13	The Witches of Eastwick	60
14	Yellowbrickroad	28
15	Dying Breed	27
16	Carrie	73
17	Whoever Slew Auntie Roo?	56
18	The Mangler	23
19	Primal	29
20	The Twilight Saga: New Moon	65

The histogram below shows the distribution of the audience scores of these movies (ranging from 0 to 100). The median score in the sample is 43.5. Can we apply CLT based methods we have learned so far to construct a confidence interval for the median RottenTomatoes score of horror movies. Why or why not?





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- ▶ This term comes from the phrase “pulling oneself up by one’s bootstraps”, which is a metaphor for accomplishing an impossible task without any outside help.
- ▶ In this case the impossible task is estimating a population parameter, and we’ll accomplish it using data from only the given sample.



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- ▶ The XX% bootstrap confidence interval can be estimated by
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  - OR
  - $point\ estimate \pm t^* SE_{boot}$

**(1) Take a bootstrap sample:**

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	title	audience_score
1	Vampire's Kiss	48
2	Phantasm IV: Oblivion	41
3	House of 1000 Corpses	65
4	Dying Breed	27
5	Whoever Slew Auntie Roo?	56
6	The Forsaken	44
7	The Twilight Saga: New Moon	65
8	The Twilight Saga: New Moon	65
9	Whoever Slew Auntie Roo?	56
10	The Twilight Saga: New Moon	65
11	The Mangler	23
12	Dying Breed	27
13	Creepshow 2	46
14	House of 1000 Corpses	65
15	Whoever Slew Auntie Roo?	56
16	Tormented	34
17	Jason Lives: Friday the 13th Part VI	57
18	Vampire's Kiss	48
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**(2) Calculate the median of the bootstrap sample:**

**(1) Take a bootstrap sample:**

	title	audience_score
1	Vampire's Kiss	48
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3	House of 1000 Corpses	65
4	Dying Breed	27
5	Whoever Slew Auntie Roo?	56
6	The Forsaken	44
7	The Twilight Saga: New Moon	65
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**(2) Calculate the median of the bootstrap sample:**

23, 27, 27, 29, 34, 41, 44, 46, 48, 48, 56, 56, 56, 57, 60, 65, 65, 65, 65, 65

median =  $(48 + 56) / 2 = 52$

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	title	audience_score
1	Vampire's Kiss	48
2	Phantasm IV: Oblivion	41
3	House of 1000 Corpses	65
4	Dying Breed	27
5	Whoever Slew Auntie Roo?	56
6	The Forsaken	44
7	The Twilight Saga: New Moon	65
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18	Vampire's Kiss	48
19	Primal	29
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**(2) Calculate the median of the bootstrap sample:**

23, 27, 27, 29, 34, 41, 44, 46, 48, 48, 56, 56, 56, 57, 60, 65, 65, 65, 65, 65

median =  $(48 + 56) / 2 = 52$

**(3) Record this value**



**(1) Take another bootstrap sample:**

**(1) Take another bootstrap sample:**

	title	audience_score
1	Fright Night Part 2	42
2	Carrie	73
3	The Forsaken	44
4	The Mangler	23
5	Primal	29
6	Patrick	52
7	Jason Lives: Friday the 13th Part VI	57
8	The Mangler	23
9	Vampire's Kiss	48
10	All the Boys Love Mandy Lane	34
11	The Twilight Saga: New Moon	65
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**(2) Calculate the median of the bootstrap sample:**

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19	House of 1000 Corpses	65
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**(2) Calculate the median of the bootstrap sample:**

23, 23, 23, 28, 29, 34, 34, 34, 41, 42, 44, 48, 48, 52, 52, 57, 65, 65, 65, 73

median =  $(42 + 44) / 2 = 43$

**(1) Take another bootstrap sample:**

	title	audience_score
1	Fright Night Part 2	42
2	Carrie	73
3	The Forsaken	44
4	The Mangler	23
5	Primal	29
6	Patrick	52
7	Jason Lives: Friday the 13th Part VI	57
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18	Patrick	52
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**(2) Calculate the median of the bootstrap sample:**

23, 23, 23, 28, 29, 34, 34, 34, 41, 42, 44, 48, 48, 52, 52, 57, 65, 65, 65, 73

median =  $(42 + 44) / 2 = 43$

**(3) Record this value**

**(1) Take another bootstrap sample:**

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	title	audience_score
1	Tormented	34
2	The Witches of Eastwick	60
3	The Witches of Eastwick	60
4	The Witches of Eastwick	60
5	The Mangler	23
6	The Witches of Eastwick	60
7	Patrick	52
8	Phantasm IV: Oblivion	41
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12	Jason Lives: Friday the 13th Part VI	57
13	Fright Night Part 2	42
14	Primal	29
15	Fright Night Part 2	42
16	Whoever Slew Auntie Roo?	56
17	Fright Night Part 2	42
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19	Under the Bed	12
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1	Tormented	34
2	The Witches of Eastwick	60
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16	Whoever Slew Auntie Roo?	56
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18	Fright Night Part 2	42
19	Under the Bed	12
20	Phantasm IV: Oblivion	41

**(2) Calculate the median of the bootstrap sample:**

12, 23, 28, 28, 29, 34, 41, 41, 42, 42, 42, 42, 52, 56, 57, 57, 60, 60, 60, 60

median =  $(42 + 42) / 2 = 42$

**(1) Take another bootstrap sample:**

	title	audience_score
1	Tormented	34
2	The Witches of Eastwick	60
3	The Witches of Eastwick	60
4	The Witches of Eastwick	60
5	The Mangler	23
6	The Witches of Eastwick	60
7	Patrick	52
8	Phantasm IV: Oblivion	41
9	Yellowbrickroad	28
10	Jason Lives: Friday the 13th Part VI	57
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12	Jason Lives: Friday the 13th Part VI	57
13	Fright Night Part 2	42
14	Primal	29
15	Fright Night Part 2	42
16	Whoever Slew Auntie Roo?	56
17	Fright Night Part 2	42
18	Fright Night Part 2	42
19	Under the Bed	12
20	Phantasm IV: Oblivion	41

**(2) Calculate the median of the bootstrap sample:**

12, 23, 28, 28, 29, 34, 41, 41, 42, 42, 42, 42, 52, 56, 57, 57, 60, 60, 60, 60

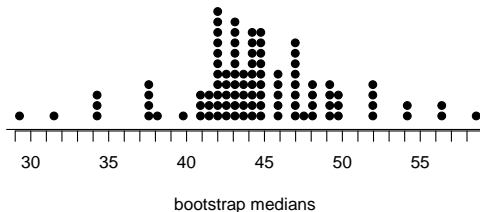
median =  $(42 + 42) / 2 = 42$

**(3) Record this value**

... repeat

### Clicker question

The dot plot below is the bootstrap distribution of medians constructed using 100 simulations. What does each dot on the dot plot represent?



- (a) Score of a horror movie in the original sample
- (b) Score of a horror movie in the population
- (c) Median from one bootstrap sample from the original sample
- (d) Median from one sample from the population

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2. Bootstrap percentile intervals: middle XX% of the bootstrap distribution

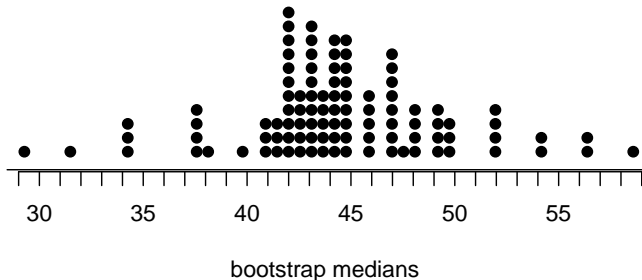
3. Bootstrap SE intervals: point estimate  $\pm$  ME

4. Bootstrap testing for a single numerical variable requires shifting the bootstrap distribution to be centered at the null value

## 3. Summary

### Clicker question

The dot plot below shows the distribution of 100 bootstrap medians. Estimate the 90% bootstrap confidence interval for the median RT score of horror movies using the percentile method.



(a) (29, 58.5)

(c) (37.5, 52)

(b) (34, 57)

(d) (40, 49.5)

## 1. Housekeeping

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1. Bootstrapping = sampling with replacement from the observed sample

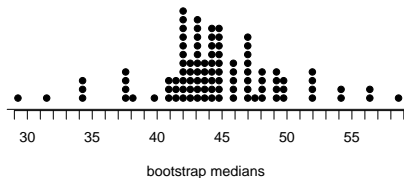
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4. Bootstrap testing for a single numerical variable requires shifting the bootstrap distribution to be centered at the null value

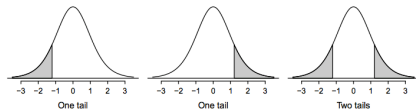
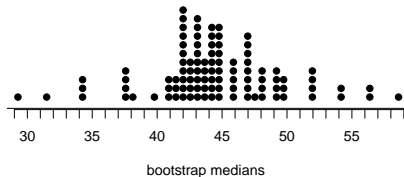
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The dot plot below shows the distribution of 100 bootstrap medians. The median of the original sample is 43.5 and the bootstrap standard error is 4.88. Estimate the 90% bootstrap confidence interval for the median RT score of horror movies using the standard error method.





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one tail	0.100	0.050	0.025	0.010	0.005
two tails	0.200	0.100	0.050	0.020	0.010
df 60	1.30	1.67	2.00	2.39	2.66
70	1.29	1.67	1.99	2.38	2.65
80	1.29	1.66	1.99	2.37	2.64
90	1.29	1.66	1.99	2.37	2.63

### Application exercise: 4.2 Bootstrap intervals

See the course webpage for details.

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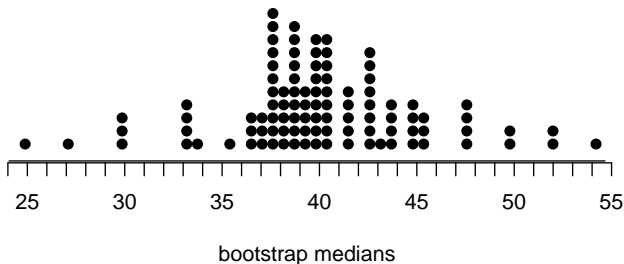
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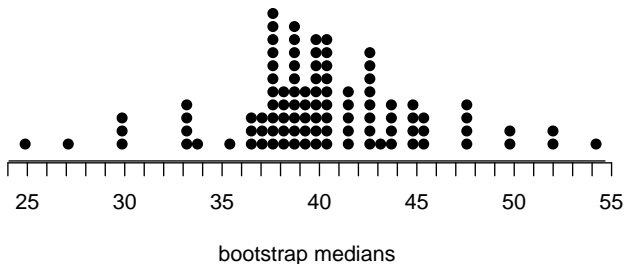
- ▶ This is very similar to bootstrapping, i.e. we randomly sample with replacement from the sample, but this time we shift the bootstrap distribution to be centered at the null value.

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- ▶ The p-value is then defined as the proportion of simulations that yield a sample statistic at least as favorable to the alternative hypothesis as the observed sample statistic.

Do these data provide convincing evidence that the median audience score of horror movies is greater than 40?  
Remember that the median of the original sample was 43.5.



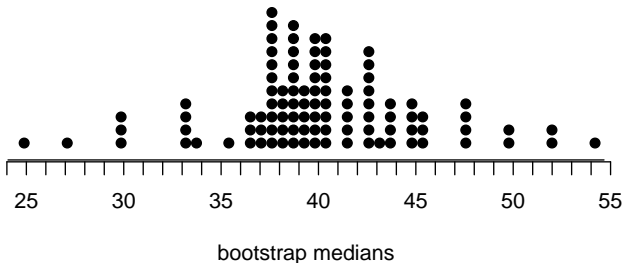
Do these data provide convincing evidence that the median audience score of horror movies is greater than 40?  
Remember that the median of the original sample was 43.5.



$$H_0 : \text{median} = 40$$

$$H_A : \text{median} > 40$$

Do these data provide convincing evidence that the median audience score of horror movies is greater than 40?  
Remember that the median of the original sample was 43.5.



$H_0$  : *median* = 40

$H_A$  : *median* > 40

p-value: proportion of simulations where the simulated bootstrap sample median is at least as extreme as the one observed (43.5).  $\rightarrow 20 / 100 = 0.20$



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