Unit 2: Probability and distributions

1. Probability and conditional probability

Sta 101 - Fall 2015

Duke University, Department of Statistical Science

Dr. Çetinkaya-Rundel

Slides posted at *http://bit.ly/sta101_f15*

- Piazza – make sure you're enrolled for our class: Sta 101.002
- Team names – if we're missing yours you must email David asap (after class), or you won't be eligible for today's RA scores

1. Housekeeping

2. Readiness assessment

3. Main ideas
    1. Disjoint and independent do not mean the same thing
    2. Application of the addition rule depends on disjointness of events
    3. Bayes' theorem works for all types of events

4. Summary

- 15 minutes individual – turn your clicker over when you're done
- 10 minutes team – put your team name on the front of the scratch off sheet + Lab Time + put **only** the names of the members who are present today on the back

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4. Summary

- *Disjoint (mutually exclusive) events* cannot happen at the same time
  - A voter cannot register as a Democrat and a Republican at the same time
  - But s/he might be a Republican and a Moderate at the same time – *non-disjoint events*
  - For disjoint A and B: $P(A \ and \ B) = 0$

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  - A voter cannot register as a Democrat and a Republican at the same time
  - But s/he might be a Republican and a Moderate at the same time – *non-disjoint events*
  - For disjoint A and B: $P(A\ and\ B) = 0$
- If A and B are *independent events*, having information on A does not tell us anything about B (and vice versa)
  - If A and B are independent:
    - $P(A \mid B) = P(A)$
    - $P(A\ and\ B) = P(A) \times P(B)$

2. Application of the addition rule depends on disjointness of events

- *General addition rule:* P(A or B) = P(A) + P(B) - P(A and B)
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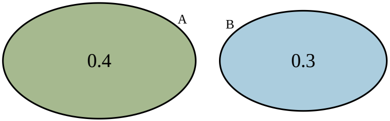
disjoint events:

P(A or B)
= P(A) + P(B) - P(A and B)
= 0.4 + 0.3 - 0 = 0.7



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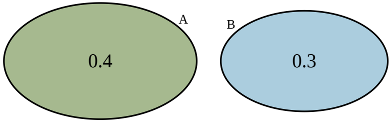
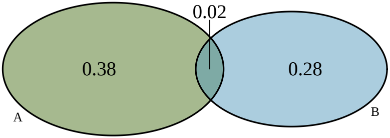


non-disjoint events:

P(A or B)
= P(A) + P(B) - P(A and B)
= 0.4 + 0.3 - 0.02 = 0.68



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- P(A and B)
  = P(A | B) × P(B)
  = *P(A)* × P(B)

Application exercise: 2.1 Probability and conditional probability

See the course website for instructions.

1. Disjoint and independent do not mean the same thing
2. Application of the addition rule depends on disjointness of events
3. Bayes' theorem works for all types of events