Unit 3: Foundations for inference

3. Hypothesis tests

Sta 101 - Spring 2016

Duke University, Department of Statistical Science

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Slides posted at *http://bit.ly/sta101_s16*

1. Housekeeping

2. Main ideas
    1. Use hypothesis tests to make decisions about population parameters
    2. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree
    3. Results that are statistically significant are not necessarily practically significant
    4. Hypothesis tests are prone to decision errors

3. Summary

Midterm 1: Feb 24, Wed

- Preparation
  - Come to class with questions on Monday
  - Sample MT posted on course website
- Rules
  - Bring a calculator + cheat sheet (one sheet, both sides, typed or handwritten, must be prepared by you) + writing utensil
  - We'll provide tables

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Hypothesis testing framework:

1. Set the hypotheses.
2. Check assumptions and conditions.
3. Calculate a *test statistic* and a p-value.
4. Make a decision, and interpret it in context of the research question.

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   - $H_0 : \mu = null\ value$
   - $H_A : \mu <$ or $>$ or $\neq null\ value$

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$$Z = \frac{\bar{x} - \mu}{SE},\ where\ SE = \frac{s}{\sqrt{n}}$$

4. Make a decision, and interpret it in context of the research question
   - If p-value $< \alpha$, reject H_0, data provide evidence for H_A
   - If p-value $> \alpha$, do not reject H_0, data do not provide evidence for H_A

Application exercise: 3.2 Hypothesis testing for a single mean

See course website for details.

Clicker question

Which of the following is the correct interpretation of the p-value from App Ex 3.2?

(a) The probability that average GPA of Duke students has changed since 2001.

(b) The probability that average GPA of Duke students has not changed since 2001.

(c) The probability that average GPA of Duke students has not changed since 2001, if in fact a random sample of 63 Duke students this year have an average GPA of 3.58 or higher.

(d) The probability that a random sample of 63 Duke students have an average GPA of 3.58 or higher, if in fact the average GPA has not changed since 2001.

(e) The probability that a random sample of 63 Duke students have an average GPA of 3.58 or higher or 3.16 or lower, if in fact the average GPA has not changed since 2001.

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(e) *The probability that a random sample of 63 Duke students have an average GPA of 3.58 or higher or 3.16 or lower, if in fact the average GPA has not changed since 2001.*

1. P-value is the probability that the null hypothesis is true
   A p-value is the probability of getting a sample that results in a test statistic as or more extreme than what you actually observed (and in favor of the null hypothesis) if in fact the null hypothesis is correct. It is a conditional probability, conditioned on the null hypothesis being correct.

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3. A low p-value confirms the alternative hypothesis.
   A low p-value means the data provide convincing evidence for the alternative hypothesis, but not necessarily that it is confirmed.

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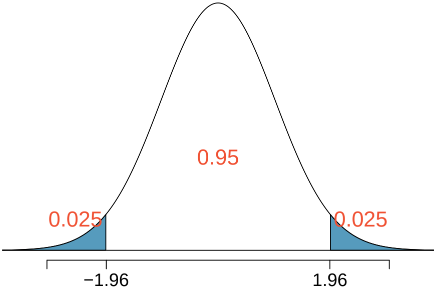
Two sided



95% confidence level
is equivalent to
two sided HT with $\alpha = 0.05$

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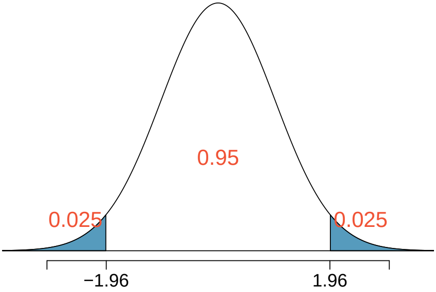

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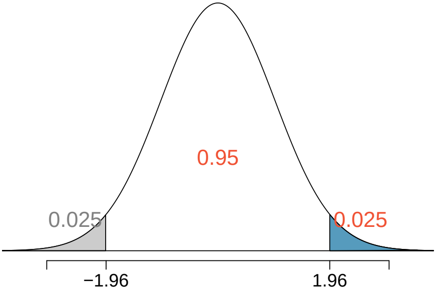
One sided



95% confidence level
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Clicker question

What is the confidence level for a confidence interval that is equivalent to a two-sided hypothesis test at the 1% significance level? *Hint: Draw a picture and mark the confidence level in the center.*

(a) 0.80
(b) 0.90
(c) 0.95
(d) 0.98
(e) 0.99

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Clicker question

A 95% confidence interval for the average normal body temperature of humans is found to be (98.1 F, 98.4 F). Which of the following is <u>true</u>?

(a) The hypothesis $H_0 : \mu = 98.2$ would be rejected at $\alpha = 0.05$ in favor of $H_A : \mu \neq 98.2$.

(b) The hypothesis $H_0 : \mu = 98.2$ would be rejected at $\alpha = 0.025$ in favor of $H_A : \mu > 98.2$.

(c) The hypothesis $H_0 : \mu = 98$ would be rejected using a 90% confidence interval.

(d) The hypothesis $H_0 : \mu = 98.2$ would be rejected using a 99% confidence interval.

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3. Results that are statistically significant are not necessarily practically significant

Clicker question

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

(a) $n = 100$
(b) $n = 10,000$

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Suppose $\bar{x} = 5$, $s = 2$, $H_0 : \mu = 4.5$, *and* $H_A : \mu > 4.5$.

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Suppose $\bar{x} = 5$, $s = 2$, $H_0 : \mu = 4.5$, *and* $H_A : \mu > 4.5$.

$$Z_{n=100} = \frac{5 - 4.5}{\frac{2}{\sqrt{100}}}$$

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$$Z_{n=100} = \frac{5 - 4.5}{\frac{2}{\sqrt{100}}} = \frac{5 - 4.5}{\frac{2}{10}} = \frac{0.5}{0.2} = 2.5, \quad \textit{p-value} = 0.0062$$

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$$
\begin{aligned}
Z_{n=100} &= \frac{5 - 4.5}{\frac{2}{\sqrt{100}}} = \frac{5 - 4.5}{\frac{2}{10}} = \frac{0.5}{0.2} = 2.5, \quad \textit{p-value} = 0.0062 \\
Z_{n=10000} &= \frac{5 - 4.5}{\frac{2}{\sqrt{10000}}}
\end{aligned}
$$

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As n increases - $SE \downarrow$, $Z \uparrow$, p-value $\downarrow$

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true		
	H_A true		

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	H_A true		

- A *Type 1 Error* is rejecting the null hypothesis when H_0 is true: α
  - For those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times
  - Increasing α increases the Type 1 error rate, hence we prefer to small values of α

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- A *Type 2 Error* is failing to reject the null hypothesis when H_A is true: β
- *Power* is the probability of correctly rejecting H_0, and hence the complement of the probability of a Type 2 Error: $1 - \beta$

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