CV5100 – MUDE
Modeling, Uncertainty, and Data for Engineers
Ch3 – Numerical Modelling

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Numerical Modelling, Linear Algebra, Optimization

Week	Lecture/ Practical	Topic
1	1	**Course overview and Introduction** to modelling - classification
	2	**Modelling concepts –** Choosing a model, validation, performance
	3	**Numerical Modelling –** DE, FDM, Taylor series
	4	**Practical**
2	5	**Numerical Modelling** – Numerical integration
	6	**Numerical Modelling** – IVP/BVP for ODE
	7	**Numerical Modelling** – BVP for ODE numerical methods (accuracy, stability)
	8	**Practical**
3	9	**Numerical Modelling** – PDE basics and PDE types, Nabla and Laplacian operations
	10	**Numerical Modelling** – FEM
	11	**Linear Algebra –** Vector spaces, span, linear dependence
	12	**Practical**
4	13	**Linear Algebra** – Basis, dimension, examples, tensor vs. matrix
	14	**Linear Algebra** – System of linear equations, matrix form, solution approach - direct
	15	**Linear Algebra** – Matrix equations, solution approach - iterative
	16	**Practical**
5	17	**Linear Algebra** – Eigenvalue problem, solution approaches
	18	**Linear Algebra** – Complexity and scaling
	19	**Optimization** – Classification and types of problems
	20	**Practical**
6	21	**Optimization** – Mathematical formulations and key concepts
	22	**Optimization** – Gradient and non-gradient approaches
	23	**Optimization** – Gradient and non-gradient approaches
	24	**Practical**

Outline

- **DEs in structural engineering**
- **ODE types**
- **Analytical vs. Numerical solutions**

https://mude.citg.tudelft.nl/book/2024/modelling/overview.html

Differential Eqs in Struct Engg

- DEs are mathematical models of a physical phenomenon
- **Examples:**

1. Beam Bending (Euler-Bernoulli Beam Theory)

- **Equation:**
  $EI\frac{d^4y}{dx^4} = q(x)$
- **Description:** Models the deflection $y(x)$ of a beam under a distributed load $q(x)$, where E is the modulus of elasticity and I is the moment of inertia.

2. Vibration of Structures

- **Equation:**
  $m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$
- **Description:** Represents the dynamic response of a single-degree-of-freedom system (mass-spring-damper), where m is mass, c is damping, k is stiffness, and $F(t)$ is external force.

Differential Eqs in Struct Engg

- DEs are mathematical models of a physical phenomenon
- **Examples:**

3. Stability of Columns (Buckling)

- **Equation:**
  $EI\frac{d^2y}{dx^2} + Py = 0$
- **Description:** Describes the buckling behavior of a column under axial load P.

4. Divergence Equation of Structural Mechanics (Equilibrium Equation)

- **Equation:**
  $\nabla \cdot \boldsymbol{\sigma} + \mathbf{b} = \rho\ddot{\mathbf{u}}$
- **Purpose:** Represents the conservation of linear momentum in a deformable solid.
- **Parameters:**
  - $\boldsymbol{\sigma}$: Stress tensor
  - $\mathbf{b}$: Body force per unit volume
  - ρ: Density
  - $\ddot{\mathbf{u}}$: Acceleration of the displacement field
- **Application:** Fundamental in continuum mechanics and finite element analysis (FEA) for stress-strain modeling.

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + b_x = \rho\frac{\partial^2 u}{\partial t^2}$$
$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + b_y = \rho\frac{\partial^2 v}{\partial t^2}$$
$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = \rho\frac{\partial^2 w}{\partial t^2}$$

ODE vs. PDE

- Identify and characterize the following DEs

$$\frac{dx(t)}{dt} = \cos t$$

$$\frac{\partial c(x,t)}{\partial t} + u\frac{\partial c(x,t)}{\partial x} = 0$$

$$\frac{d^3y}{dx^3} - x\frac{d^2y}{dx^2} + y = 0$$ third-order linear ODE

$$\frac{dy}{dx} = y^2 + x$$ first-order non-linear ODE

$$\frac{d^2y}{dx^2} + y\left(\frac{dy}{dx}\right)^2 = \sin(y)$$ second-order non-linear ODE

Analytical vs. Numerical Solutions

- Let us use the teachbook & live code

https://mude.citg.tudelft.nl/book/2024/numerical_methods/1-revision-of-concepts.html