

NISQ ALGORITHM TO SOLVE NON-LINEAR PDE'S

QUANTUM SCIENCE AND TECHNOLOGY

ARUNAVA MAJUMDER,
5th Year Integrated M.Sc.



COLLABORATORS

PETER L. MCMAHON



THOMAS WATTS



Along with other group members of
PDE subgroup



- “The rule of simulation that I would like to have is that the number of computer elements required to simulate a large physical system is only to be proportional to the space-time volume of the physical system. I don’t want to have an explosion”-

Feynman, 1982

VARIATIONAL QUANTUM ALGORITHM TO SOLVE NON-LINEAR PDE'S

STEPS TO FOLLOW:

1. EXACT LOADING OF THE INITIAL CONDITION
2. TIME EVOLUTION OF THE GIVEN PDE
3. EXACT READ-OUT
4. NOISY-SIMULATION OF THE ABOVE STEPS

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OVERVIEW OF THE TALK

1. NISQ DEVICES
2. VARIATIONAL QUANTUM TECHNOLOGY
3. NON-LINEAR PDE'S
4. CLASSICAL & QUANTUM FINITE DIFFERENCE METHODS
5. ANSATZ DESIGN & READ-IN



NOISY INTERMEDIATE-SCALE QUANTUM (NISQ) TECHNOLOGY

Quantum Computing in the NISQ era and beyond

John Preskill

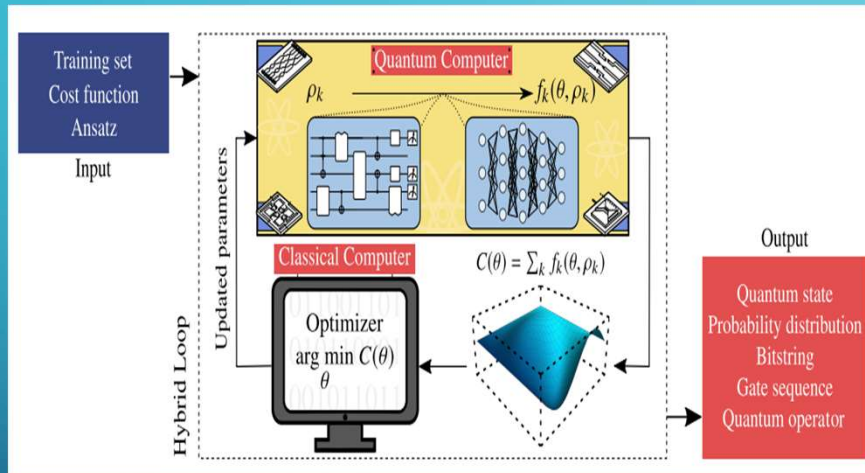
Institute for Quantum Information and Matter and Walter Burke Institute for Theoretical Physics,
California Institute of Technology, Pasadena CA 91125, USA
30 July 2018

Noisy Intermediate-Scale Quantum (NISQ) technology will be available in the near future. Quantum computers with 50-100 qubits may be able to perform tasks which surpass the capabilities of today's classical digital computers, but noise in quantum gates will limit the size of quantum circuits that can be executed reliably. NISQ devices will be useful tools for exploring many-body quantum physics, and may have other useful applications, but the 100-qubit quantum computer will not change the world right away — we should regard it as a significant step toward the more powerful quantum technologies of the future. Quantum technologists should continue to strive for more accurate quantum gates and, eventually, fully fault-tolerant quantum computing.

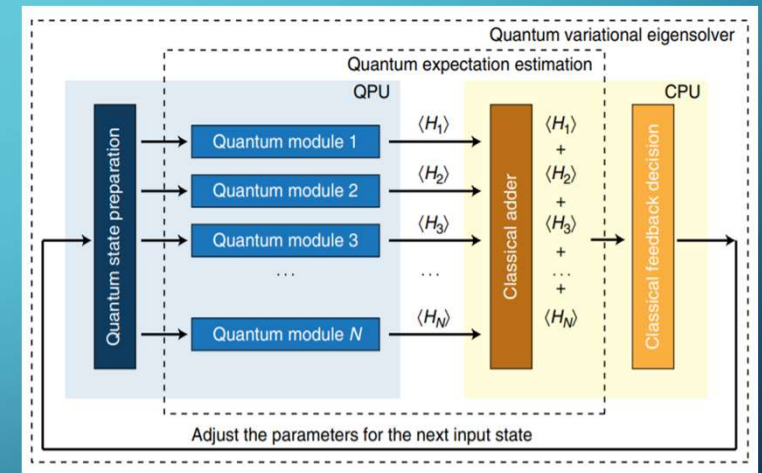


VARIATIONAL QUANTUM TECHNOLOGY

OVERVIEW



OVERVIEW



APPLICATIONS SUCH AS SIMULATING LARGE QUANTUM SYSTEMS OR SOLVING LARGE-SCALE LINEAR ALGEBRA PROBLEMS ARE IMMENSELY CHALLENGING FOR CLASSICAL COMPUTERS DUE TO THEIR EXTREMELY HIGH COMPUTATIONAL COST.

VARIATIONAL QUANTUM ALGORITHM TO SOLVE PDE'S

Variational quantum algorithms for nonlinear problems

Michael Lubasch¹, Jaewoo Joo¹, Pierre Moinier², Martin Kiffner^{3,1}, and Dieter Jaksch^{1,3}
Clarendon Laboratory, University of Oxford, Parks Road, Oxford OX1 3PU, United Kingdom¹

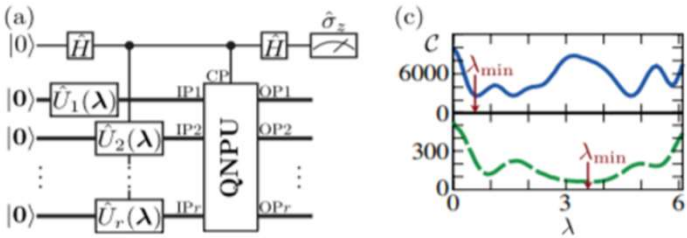
*BAE Systems, Computational Engineering, Buckingham House,
 FPC 267 PO Box 5, Filton, Bristol BS34 7QW, United Kingdom² and*

Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543³

We show that nonlinear problems including nonlinear partial differential equations can be efficiently solved by variational quantum computing. We achieve this by utilizing multiple copies of variational quantum states to treat nonlinearities efficiently and by introducing tensor networks as a programming paradigm. The key concepts of the algorithm are demonstrated for the nonlinear Schrödinger equation as a canonical example. We numerically show that the variational quantum ansatz can be exponentially more efficient than matrix product states and present experimental proof-of-principle results obtained on an IBM Q device.

Dec 2019

Nonlinear problems are ubiquitous in all fields of science and engineering and often appear in the form of nonlinear partial differential equations (PDEs). Standard numerical approaches seek solutions to PDEs on discrete grids. However, many problems of interest require extremely large grid sizes for achieving accurate results, in particular in the presence of unstable or chaotic behaviour that is typical for nonlinear problems [1–





NON-LINEAR PDE'S

CONSIDER THE PROBLEM OF
SOLVING NON-LINEAR WAVE EQ.

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}.$$

NON-LINEARITY

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0,$$

$$\partial_t u(x, t) = \hat{O}u(x, t)$$

Initial conditions

$$(x, t) \in [0, 1] \times [0, T]$$

$$u(0, t) = u(1, t), \forall t$$

$$u(x, 0) = u_0(x)$$



CLASSICAL FINITE DIFFERENCE METHOD

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0,$$



$$U(t + \tau) = (\mathbf{1} + \tau \hat{O})U(t) + \mathcal{O}(\tau^2)$$

Forward Euler's
method



$$\frac{\partial u}{\partial x} = \frac{u(x_{k+1}, t) - u(x_{k-1}, t)}{2\Delta x} + \mathcal{O}(\Delta x^2)$$



$$\mathcal{C}(U(t + \tau)) = \|U(t + \tau) - (\mathbf{1} + \tau \hat{O})U(t)\|^2$$

Forward Euler
cost function

QUANTUM FINITE DIFFERENCE METHOD

$$\frac{\partial}{\partial t} |u(t)\rangle = \hat{O} |u(t)\rangle$$



$$|u(t)\rangle = \sum_{k=0}^{N-1} u(x_k, t) |k\rangle$$

$$|u(t)\rangle = \lambda_0 \hat{U}(\lambda) |0\rangle^{\otimes n} = \lambda_0 |\psi(\lambda)\rangle$$

$$U(t + \tau) = (\mathbf{1} + \tau \hat{O})U(t) + \mathcal{O}(\tau^2)$$



$$\hat{O} = -\tilde{\lambda}_0 \hat{D}_{\tilde{\psi}} \hat{\nabla}.$$

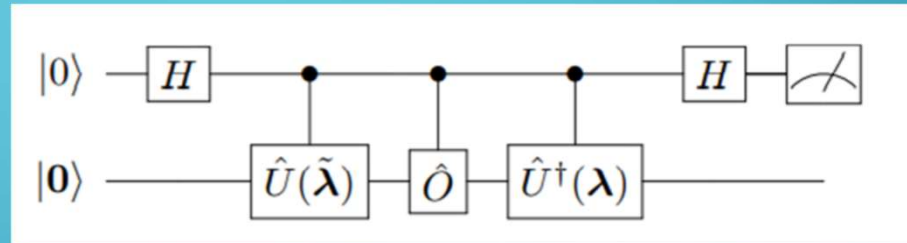


$$\begin{aligned} \mathcal{C}(\lambda_0, \lambda) &= \|u(t + \tau) - (\mathbf{1} + \tau \hat{O})u(t)\|^2 \\ &= \langle u(t + \tau) - (\mathbf{1} + \tau \hat{O})u(t) | u(t + \tau) - (\mathbf{1} + \tau \hat{O})u(t) \rangle \\ &= \langle \lambda_0 \psi(\lambda) - (\mathbf{1} + \tau \hat{O})\tilde{\lambda}_0 \tilde{\psi} | \lambda_0 \psi(\lambda) - (\mathbf{1} + \tau \hat{O})\tilde{\lambda}_0 \tilde{\psi} \rangle \\ &= \lambda_0^2 - 2\lambda_0 \tilde{\lambda}_0 \Re\{\langle \psi(\lambda) | \mathbf{1} + \tau \hat{O} | \tilde{\psi} \rangle\} + \text{const.} \end{aligned}$$

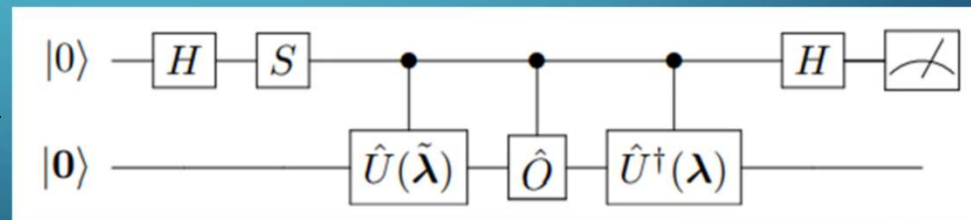


QUANTUM CIRCUIT FOR QUANTIZED COST FUNCTION

To calculate *REAL* part of the expectation value



To calculate *IMAGINARY* part of the expectation value



11/23



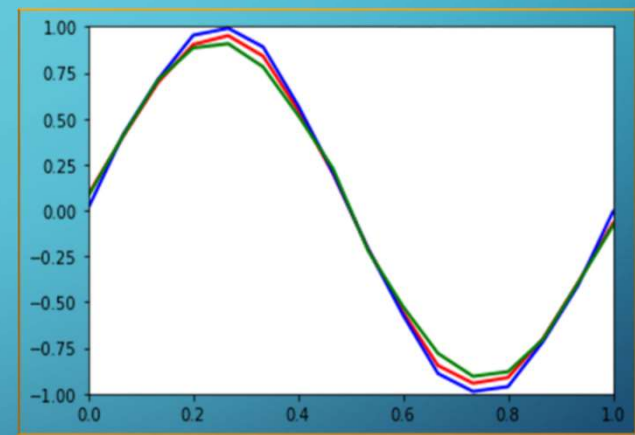
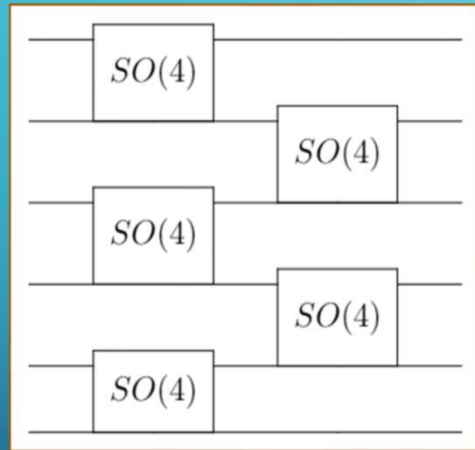
DESIGNING THE ANSATZ TO OBTAIN EXACT RAD-IN

USING GOOGLE CIRQ SIMULATOR

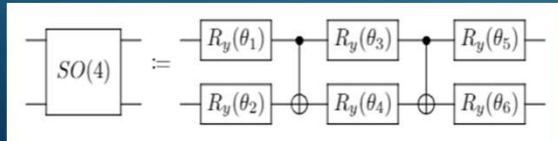


ANSATZ DESIGN & READ-IN

Loading information about initial conditions



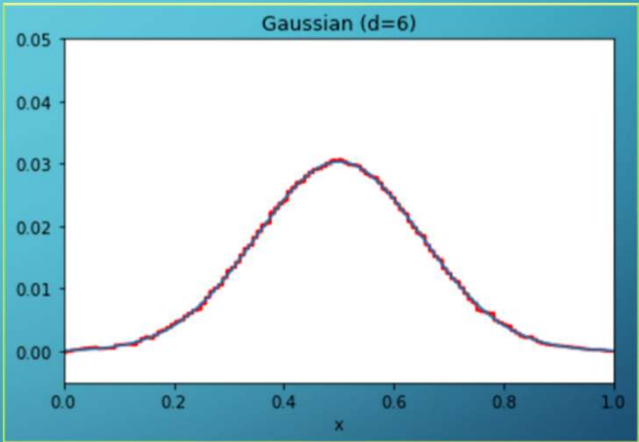
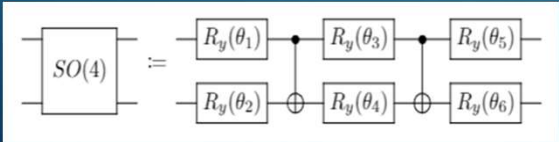
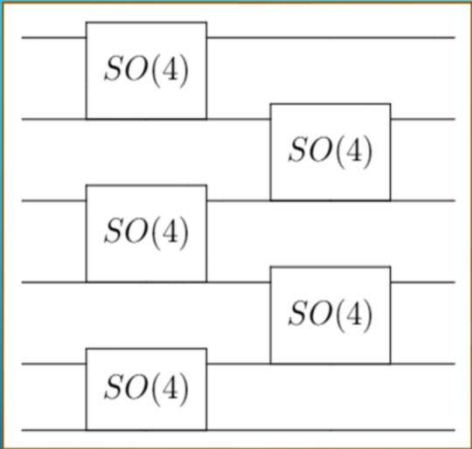
█ : Directly From circuit
█ / █ : By solving the PDE using cost function with $t=0$ and the initial condition is set to be $\sin(x)$





ANSATZ DESIGN & READ-IN

Loading information about initial conditions

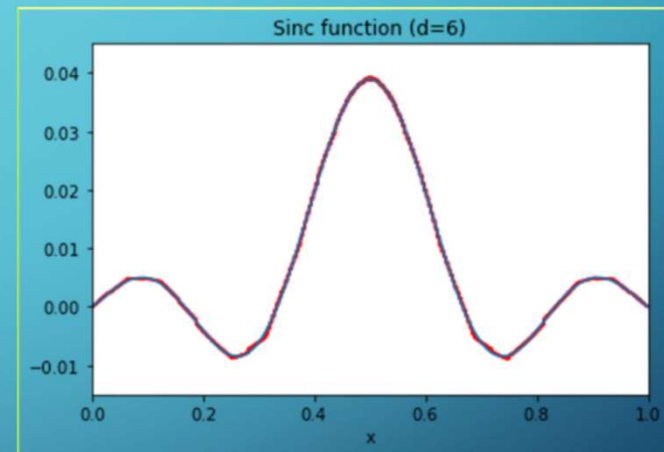
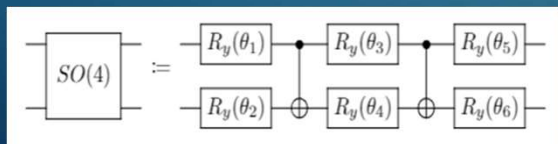
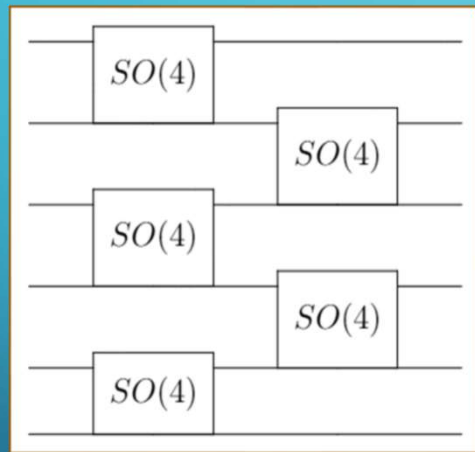


■ : From Quantum circuit
■ : Actual Gaussian curve



ANSATZ DESIGN & READ-IN

Loading information about initial conditions



— : From Quantum circuit
— : Actual Gaussian curve

SCALING THE PROBLEM

Restrictions:

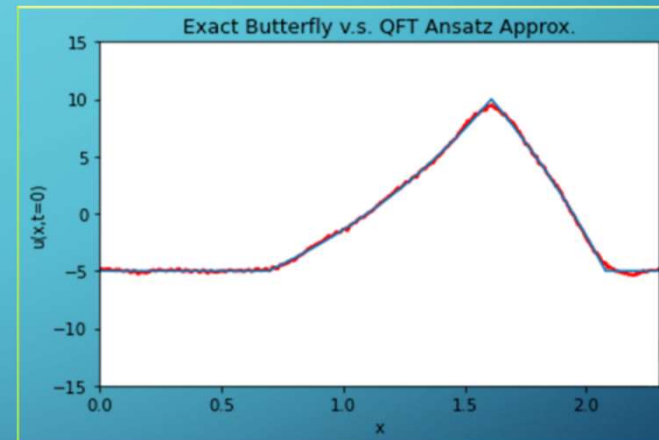
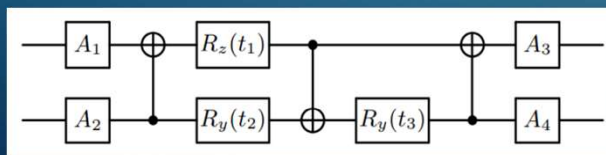
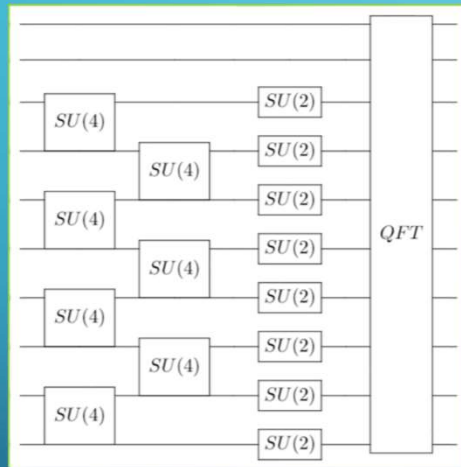
1. No of gates
2. Total gate complexity
3. Hardware efficiency



ANSATZ DESIGN & READ-IN

Scaling the problem with 10 Qubits

Loading information about initial conditions



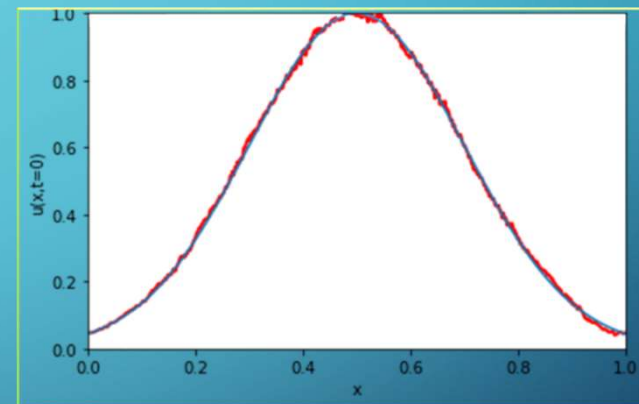
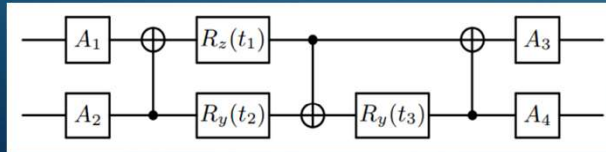
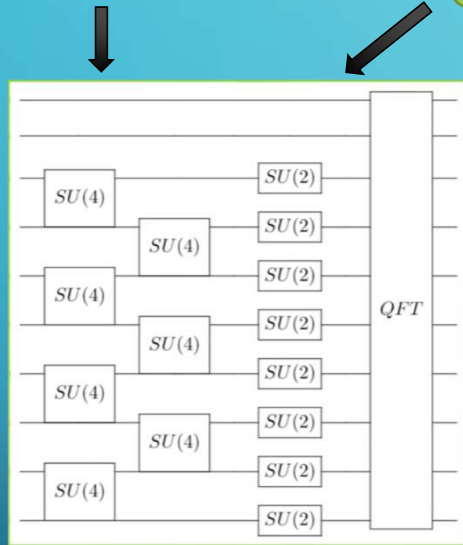
■ : From Quantum circuit
■ : Actual Gaussian curve



ANSATZ DESIGN & READ-IN

Scaling the problem with 10 Qubits

Loading information about initial conditions



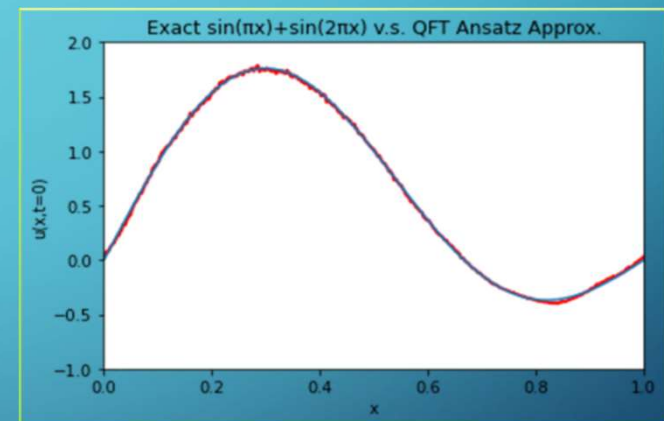
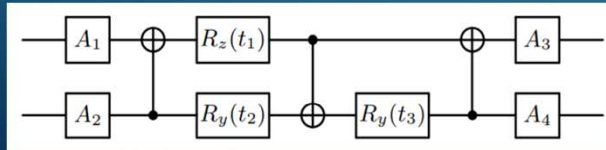
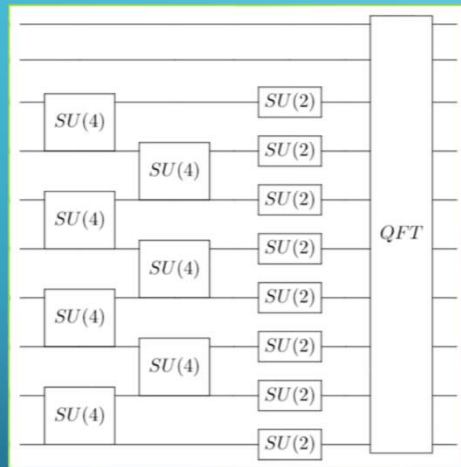
■ : From Quantum circuit
■ : Actual Gaussian curve



ANSATZ DESIGN & READ-IN

Scaling the problem with 10 Qubits

Loading information about initial conditions



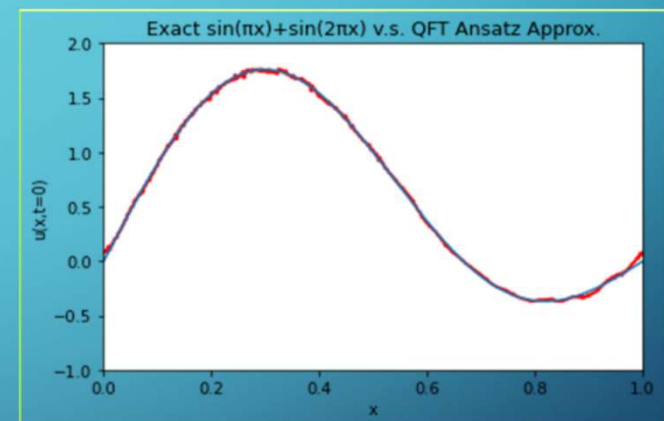
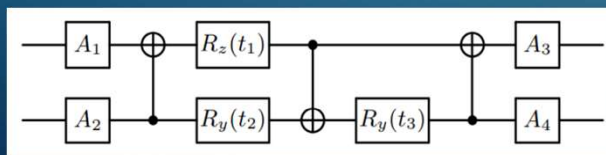
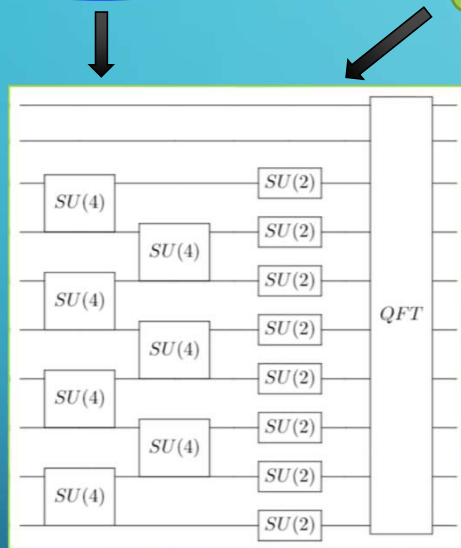
— : From Quantum circuit
— : Actual Gaussian curve



ANSATZ DESIGN & READ-IN

Scaling the problem with 12 Qubits

Loading information about initial conditions



■ : From Quantum circuit

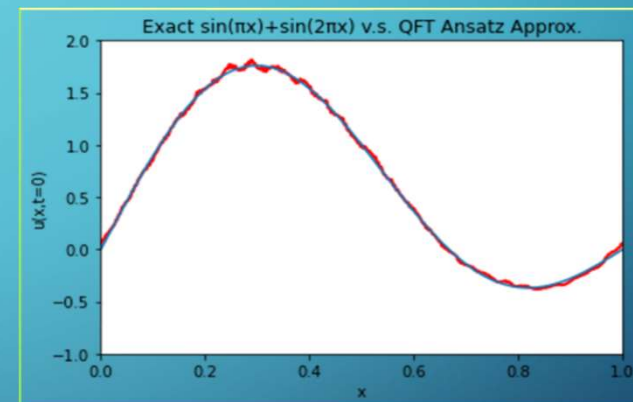
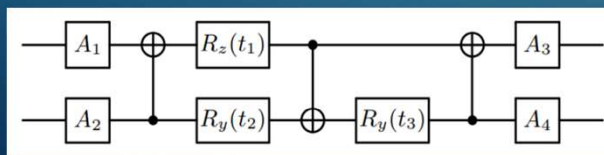
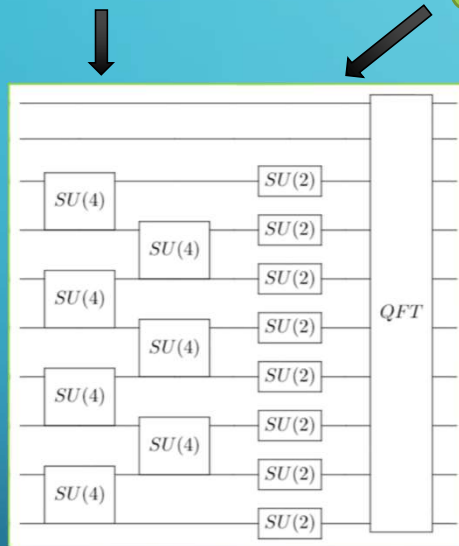
■ : Actual Gaussian curve



ANSATZ DESIGN & READ-IN

Scaling the problem with 15 Qubits

Loading information about initial conditions



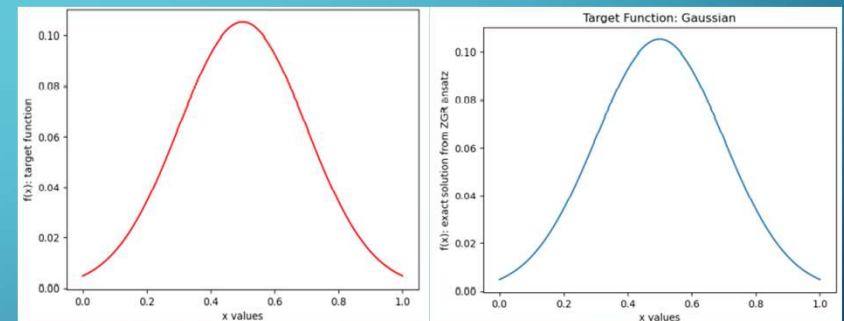
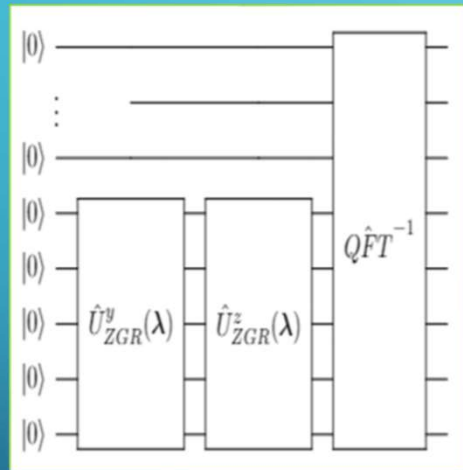
From Quantum circuit

Actual Gaussian curve

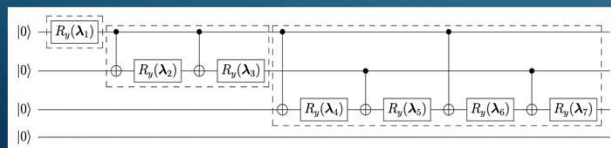


ANSATZ DESIGN & READ-IN

Loading information about initial conditions



■ : From Quantum circuit
■ : Actual Gaussian curve



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THANK YOU VERY MUCH!

