NISQ ALGORITHM TO SOLVE NON-LINEAR PDE'S

QUANTUM SCIENCE AND TECHNOLOGY

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5th Year Integrated M.Sc.

COLLABORATORS

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Along with other group members of PDE subgroup

- "The rule of simulation that I would like to have is that the number of computer elements required to simulate a large physical system is only to be proportional to the space-time volume of the physical system. I don't want to have an explosion"-

Feynman, 1982

VARIATIONAL QUANTUM ALGORITHM TO SOLVE NON-LINEAR PDE'S

STEPS TO FOLLOW:

1. EXACT LOADING OF THE INITIAL CONDITION
2. TIME EVOLUTION OF THE GIVEN PDE
3. EXACT READ-OUT
4. NOISY-SIMULATION OF THE ABOVE STEPS

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OVERVIEW OF THE TALK

1. NISQ DEVICES
2. VARIATIONAL QUANTUM TECHNOLOGY
3. NON-LINEAR PDE'S
4. CLASSICAL & QUANTUM FINITE DIFFERENCE METHODS
5. ANSATZ DESIGN & READ-IN

NOISY INTERMEDIATE-SCALE QUANTUM (NISQ) TECHNOLOGY

Quantum Computing in the NISQ era and beyond

John Preskill

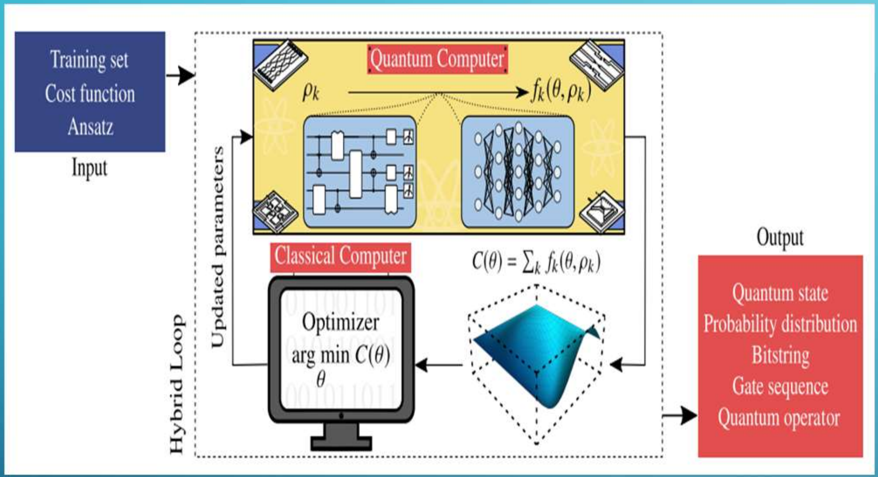
Institute for Quantum Information and Matter and Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena CA 91125, USA

30 July 2018

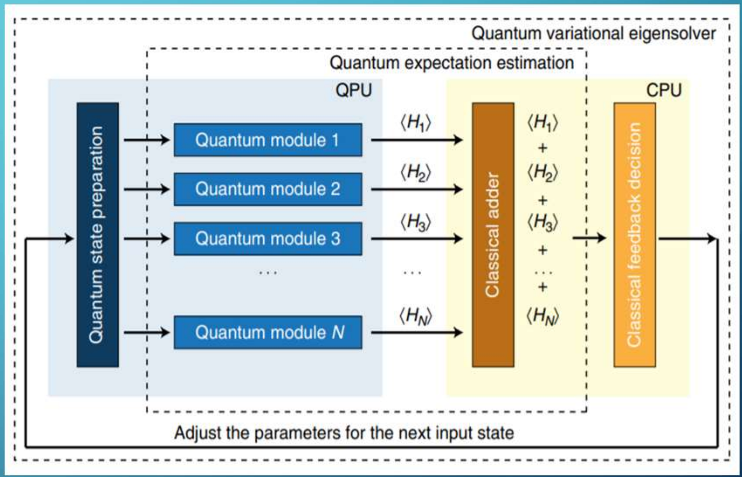
Noisy Intermediate-Scale Quantum (NISQ) technology will be available in the near future. Quantum computers with 50-100 qubits may be able to perform tasks which surpass the capabilities of today's classical digital computers, but noise in quantum gates will limit the size of quantum circuits that can be executed reliably. NISQ devices will be useful tools for exploring many-body quantum physics, and may have other useful applications, but the 100-qubit quantum computer will not change the world right away — we should regard it as a significant step toward the more powerful quantum technologies of the future. Quantum technologists should continue to strive for more accurate quantum gates and, eventually, fully fault-tolerant quantum computing.

VARIATIONAL QUANTUM TECHNOLOGY

OVERVIEW



OVERVIEW



APPLICATIONS SUCH AS SIMULATING LARGE QUANTUM SYSTEMS OR SOLVING LARGE-SCALE LINEAR ALGEBRA PROBLEMS ARE IMMENSELY CHALLENGING FOR CLASSICAL COMPUTERS DUE TO THEIR EXTREMELY HIGH COMPUTATIONAL COST.

https://www.nature.com/articles/ncomms5213.pdf

VARIATIONAL QUANTUM ALGORITHM TO SOLVE PDE'S

Variational quantum algorithms for nonlinear problems

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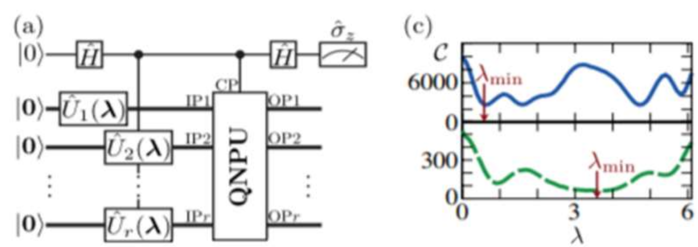
We show that nonlinear problems including nonlinear partial differential equations can be efficiently solved by variational quantum computing. We achieve this by utilizing multiple copies of variational quantum states to treat nonlinearities efficiently and by introducing tensor networks as a programming paradigm. The key concepts of the algorithm are demonstrated for the nonlinear Schrödinger equation as a canonical example. We numerically show that the variational quantum ansatz can be exponentially more efficient than matrix product states and present experimental proof-of-principle results obtained on an IBM Q device.

Nonlinear problems are ubiquitous in all fields of science and engineering and often appear in the form of nonlinear partial differential equations (PDEs). Standard numerical approaches seek solutions to PDEs on discrete grids. However, many problems of interest require extremely large grid sizes for achieving accurate results, in particular in the presence of unstable or chaotic behaviour that is typical for nonlinear problems [1–



NON-LINEAR PDE'S

CONSIDER THE PROBLEM OF SOLVING NON-LINEAR WAVE EQ.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}.$$

NON-LINEARITY

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0,$$

$$\partial_t u(x,t) = \hat{O}u(x,t)$$

Initial conditions

$$(x,t) \in [0,1] \times [0,T]$$

$$u(0,t) = u(1,t) \ , \ \forall t$$

$$u(x,0) = u_0(x)$$

CLASSICAL FINITE DIFFERENCE METHOD

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0,$$

$$U(t+\tau) = (\mathbf{1} + \tau\hat{O})U(t) + \mathcal{O}(\tau^2)$$

Forward Euler's method

$$\frac{\partial u}{\partial x} = \frac{u(x_{k+1},t) - u(x_{k-1},t)}{2\Delta x} + \mathcal{O}(\Delta x^2)$$

$$\mathcal{C}(U(t+\tau)) = \|U(t+\tau) - (\mathbf{1}+\tau\hat{O})U(t)\|^2$$

Forward Euler cost function

QUANTUM FINITE DIFFERENCE METHOD

$$\frac{\partial}{\partial t}\left|u(t)\right\rangle = \hat{O}\left|u(t)\right\rangle$$

$$\left|u(t)\right\rangle = \sum_{k=0}^{N-1} u(x_k,t)\left|k\right\rangle$$

$$\left|u(t)\right\rangle = \lambda_0 \hat{U}(\boldsymbol{\lambda})\left|0\right\rangle^{\otimes n} = \lambda_0\left|\psi(\boldsymbol{\lambda})\right\rangle$$

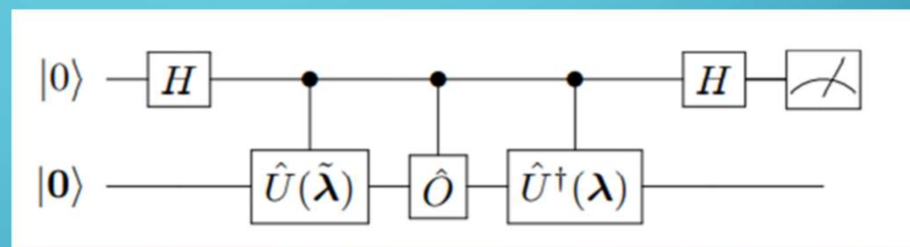
$$U(t+\tau) = (\mathbf{1} + \tau\hat{O})U(t) + \mathcal{O}(\tau^2)$$

$$\hat{O} = -\tilde{\lambda}_0 \hat{D}_{\tilde{\psi}} \hat{\nabla}.$$

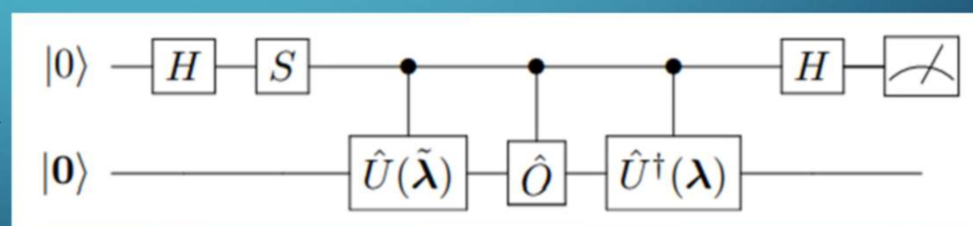
$$\begin{aligned}\mathcal{C}(\lambda_0,\boldsymbol{\lambda}) &= \|u(t+\tau) - (\mathbf{1}+\tau\hat{O})u(t)\|^2 \\ &= \langle u(t+\tau) - (\mathbf{1}+\tau\hat{O})u(t) | u(t+\tau) - (\mathbf{1}+\tau\hat{O})u(t)\rangle \\ &= \langle \lambda_0\psi(\boldsymbol{\lambda}) - (\mathbf{1}+\tau\hat{O})\tilde{\lambda}_0\tilde{\psi} | \lambda_0\psi(\boldsymbol{\lambda}) - (\mathbf{1}+\tau\hat{O})\tilde{\lambda}_0\tilde{\psi}\rangle \\ &= \lambda_0^2 - 2\lambda_0\tilde{\lambda}_0 \mathfrak{Re}\{\langle\psi(\boldsymbol{\lambda})|\mathbf{1}+\tau\hat{O}|\tilde{\psi}\rangle\} + \text{const.}\end{aligned}$$

QUANTUM CIRCUIT FOR QUANTIZED COST FUNCTION

To calculate *REAL* part of the expectation value



To calculate *IMAGINARY* part of the expectation value

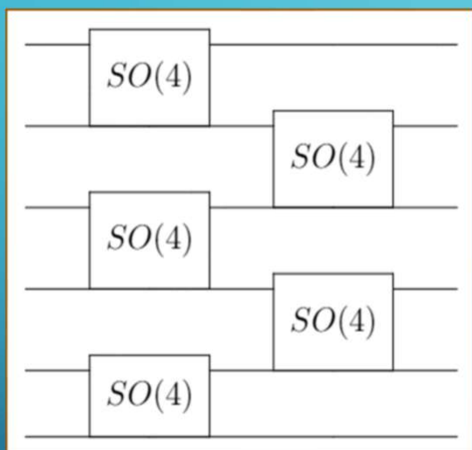


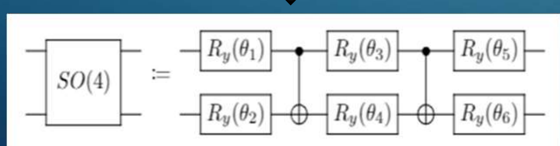
DESIGNING THE ANSATZ TO OBTAIN EXACT RAD-IN

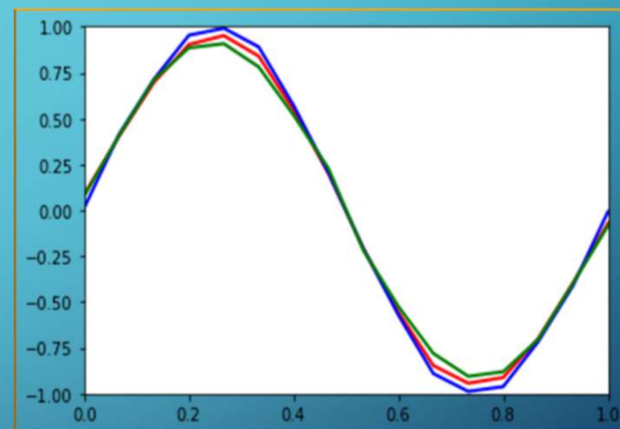
USING GOOGLE CIRQ SIMULATOR

ANSATZ DESIGN & READ-IN

Loading information about initial conditions





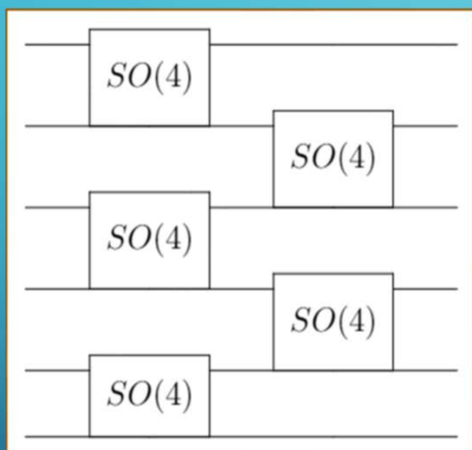


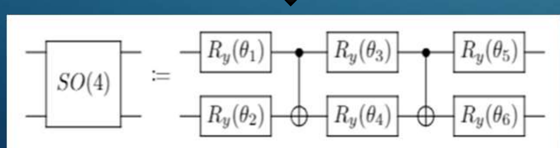
: Directly From circuit

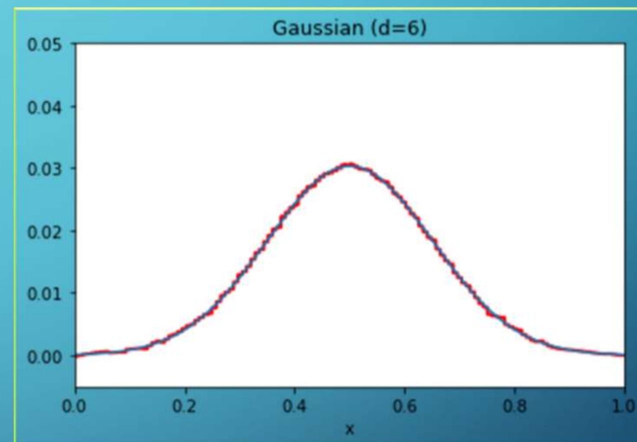
/ : By solving the PDE using cost function with t=0 and the initial condition is set to be sin(x)

ANSATZ DESIGN & READ-IN

Loading information about initial conditions





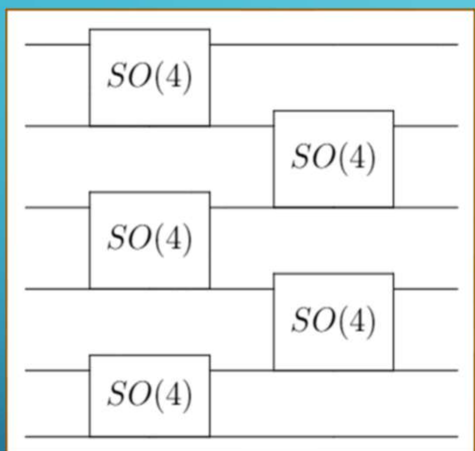


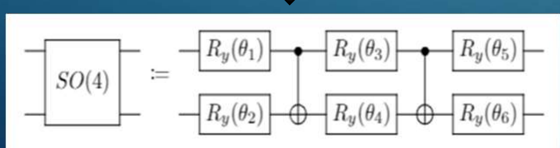
: From Quantum circuit

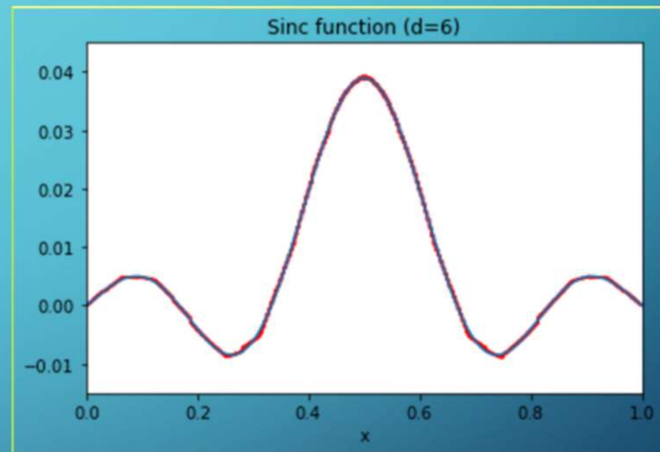
: Actual Gaussian curve

ANSATZ DESIGN & READ-IN

Loading information about initial conditions







: From Quantum circuit

: Actual Gaussian curve

SCALING THE PROBLEM

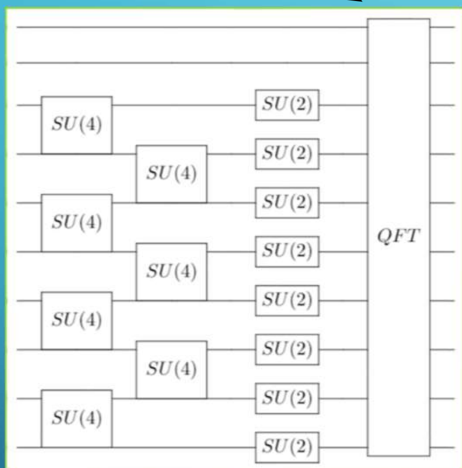
Restrictions:

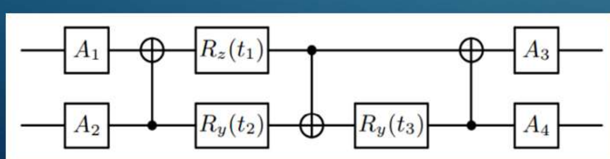
1. No of gates
2. Total gate complexity
3. Hardware efficiency

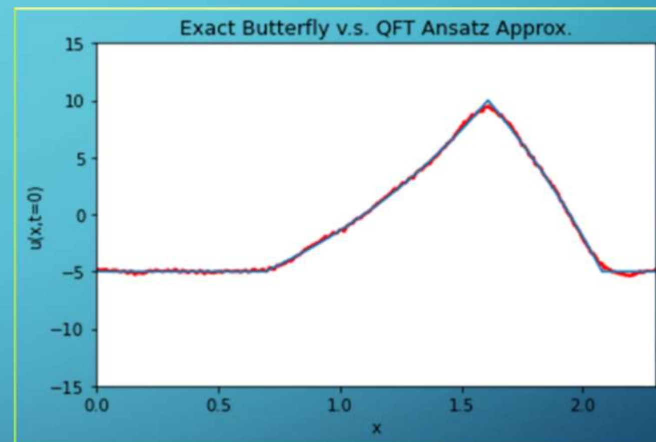
ANSATZ DESIGN & READ-IN

Scaling the problem with 10 Qubits

Loading information about initial conditions







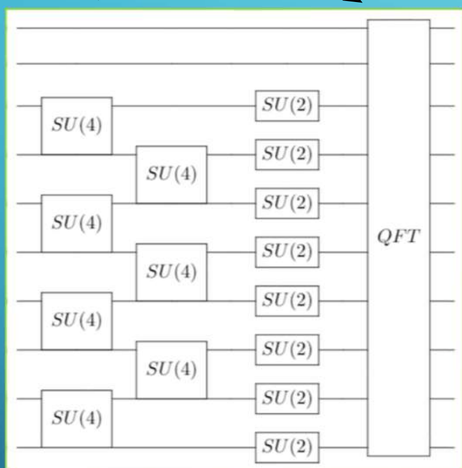
: From Quantum circuit

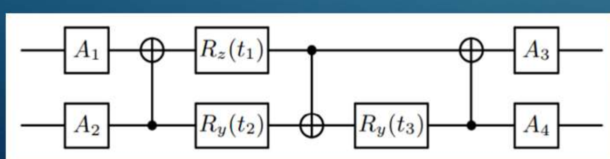
: Actual Put-Butterfly curve

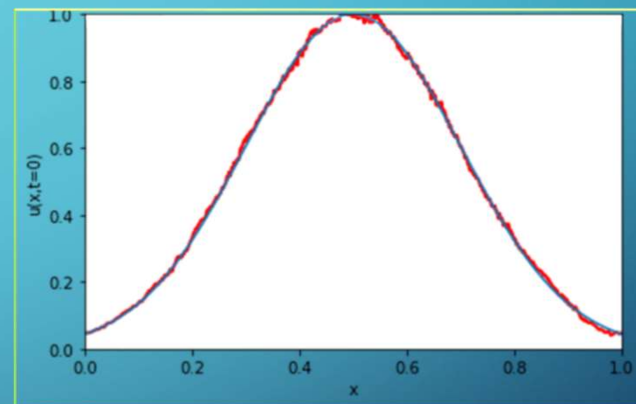
ANSATZ DESIGN & READ-IN

Scaling the problem with 10 Qubits

Loading information about initial conditions







: From Quantum circuit

: Actual Gaussian curve

ANSATZ DESIGN & READ-IN

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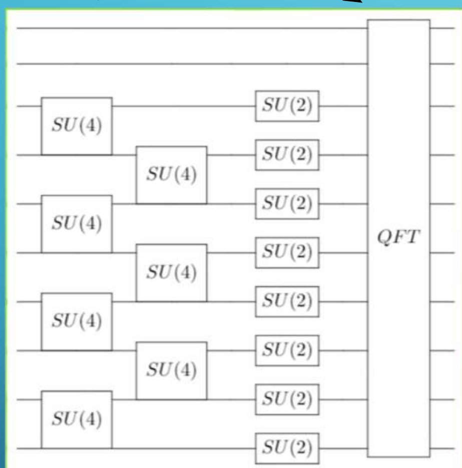
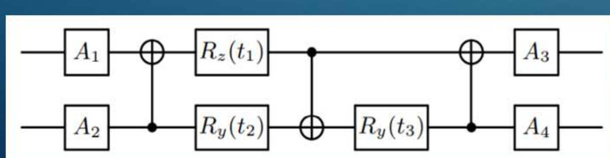
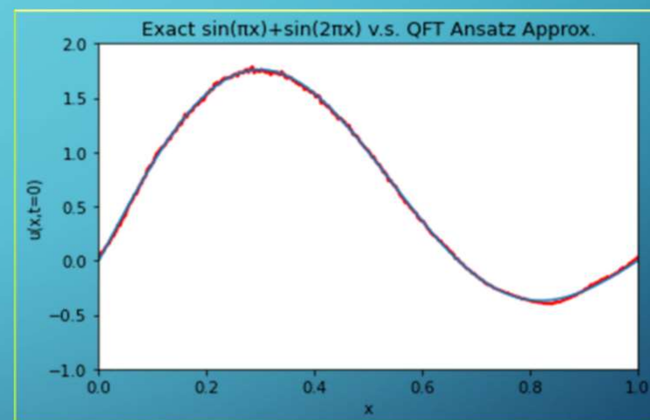
Loading information about initial conditions







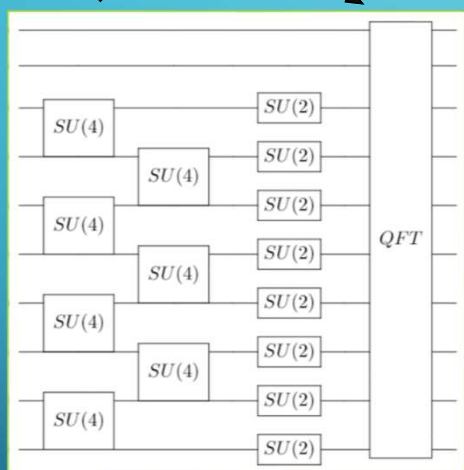
: From Quantum circuit

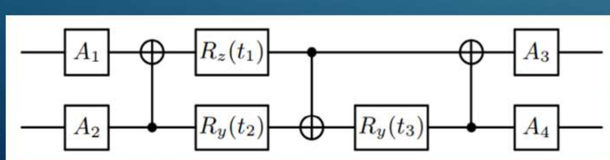
: Actual sine curve

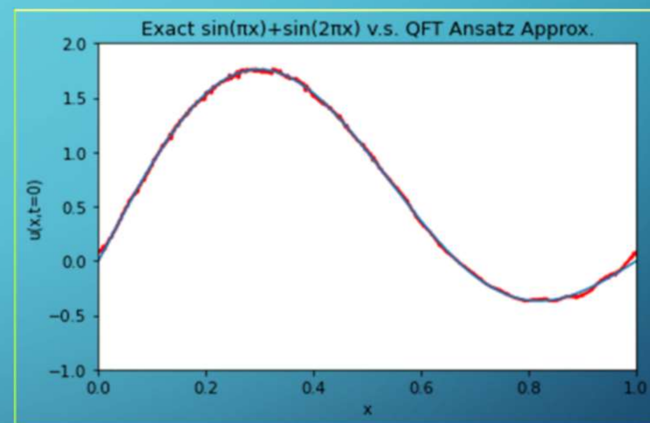
ANSATZ DESIGN & READ-IN

Scaling the problem with 12 Qubits

Loading information about initial conditions







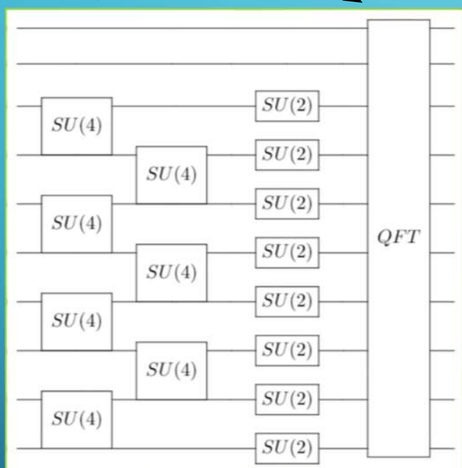
: From Quantum circuit

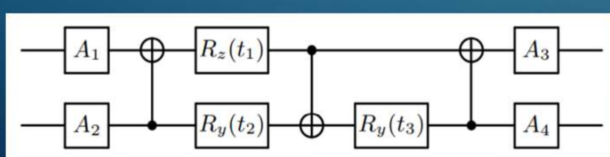
: Actual sine curve

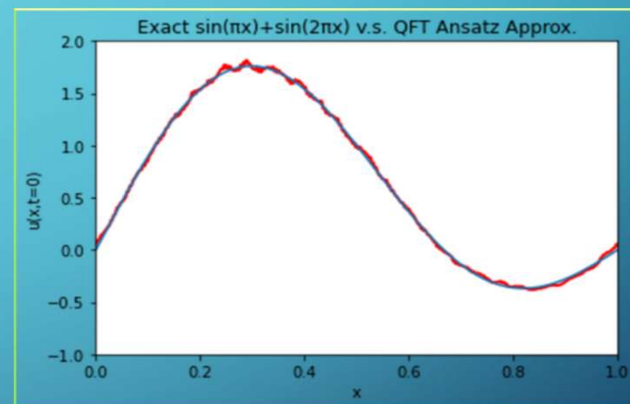
ANSATZ DESIGN & READ-IN

Scaling the problem with 15 Qubits

Loading information about initial conditions





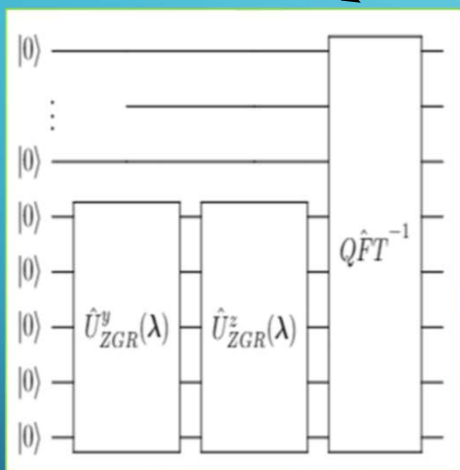


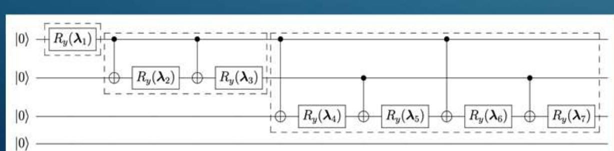
: From Quantum circuit

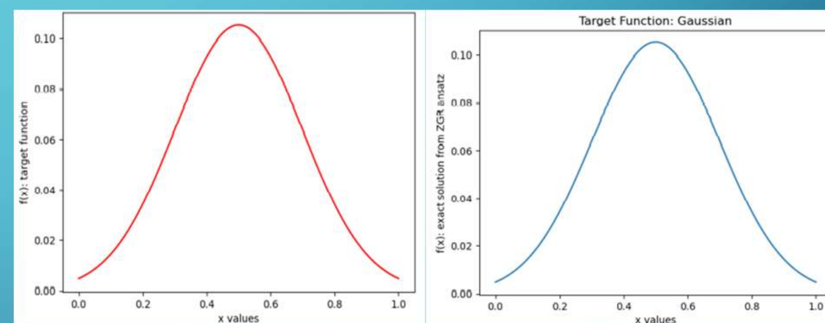
: Actual sine curve

ANSATZ DESIGN & READ-IN

Loading information about initial conditions







: From Quantum circuit

: Actual Gaussian curve

THANK YOU VERY MUCH!