The LNM Institute of Information Technology

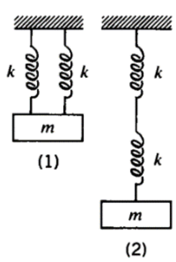
Department of Physics

Summer Term: Classical Physics: *Oscillations*

Assignment-1

Instructor: Dr. Ashok Garai

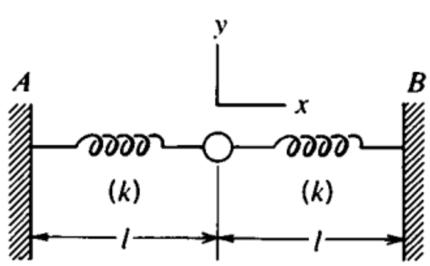
1. Verify that the differential equation $\frac{d^2y}{dx^2} = -ky$ has as its solution: $y = A\cos(kx) + B\sin(kx)$ where *A* and *B* are arbitrary constants. Show also that this solution can be written in the form $y = C\cos(kx + \phi)$ and express *C* and ϕ as functions of *A* and *B*.
2. (a) Justify the formulas $\cos\theta = (e^{\iota\theta} + e^{-\iota\theta})/2$ and $\sin\theta = (e^{\iota\theta} - e^{-\iota\theta})/2\iota$, using the appropriate series. (b) Display the above relationships geometrically by means of vector diagrams in the *xy* plane.
3. An object of mass *1* g is hung from a spring and set in oscillatory motion. At *t=0* the displacement is *43.785* cm and the acceleration is – *1.7514* cm/sec^2. What is the spring constant?



4. A mass *m* hangs from a uniform spring of spring constant *k*. (a) What is the period of oscillations in the system? (b) What would it be if the mass were hung so that (1) It was attached to two identical springs hanging side by side? (2) It was attached to the lower of two identical springs connected end to end?
5. A uniform rod of length *L* is nailed to a post so that two thirds of its length is below the nail. What is the period of small oscillations of the rod?
6. A cylinder of diameter *d* floats with l of its length submerged. The total height is L. Assume no damping. At time *t=0* the cylinder is pushed down a distance *B* and released. (a) what is the frequency of oscillation? (b)

Draw a graph of velocity versus time from *t=0* to *t= one period*. The correct amplitude and phase should be included.

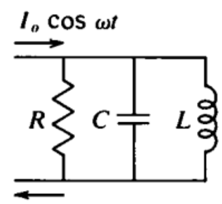
7. Verify that $x = A\, e^{-\alpha t} \cos \omega t$ is a possible solution of the equation $\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0$ and find α and ω in terms of γ and ω_0.
8. An object of mass *m* is hung from a spring whose spring constant is *k*. the object is subject to a resistive force given by $-rv$, where v is the velocity and r is the resistive force coefficient. Set up the differential equation of motion for free oscillations of the system. Solve the equation to obtain the position, and velocity. What is the expression of Q-value for the system?
9. (a) When the note "middle C" on the piano is struck, its energy of oscillation decreases to one half its initial value in about 1 sec. The frequency of middle C is 256 Hz, what is the Q of the system? (b) If the note an octave higher (512 Hz) takes about the same time for its energy to decay, what is its Q?
10. According to classical electromagnetic theory an accelerated electron radiates energy at the rate $\frac{Ke^2a^2}{c^3}$, where $K = 6 \times 10^9\ N - \frac{m^2}{C^2}$, *e* is the electronic charge (*C*), *a* is the instantaneous acceleration (*m/sec*^{2}), and c is the speed of light (*m/sec*). (a) If an electron were oscillating along a straight line with frequency ν (Hz) and amplitude *A*, how much energy would it radiate away during one cycle? (Assuming that the motion is described adequately by $x = A \sin 2\pi\nu t$ during any one cycle.) (b) What is the *Q* of this oscillator?



11. A mass m rests on a frictionless horizontal table and is connected to rigid supports via two identical supports via two identical springs each of relaxed length l_0 and spring constant *k* (see figure). Each spring is stretched to a length *l* considerably greater than l_0. Horizontal displacements on *m* from its equilibrium position are labeled *x* (along AB) and *y* (perpendicular to AB). (a) Write down the differential equation of

motion governing small oscillations in the *x*-direction. (b) Write down the differential equation of motion governing small oscillations in the *y*-direction (assume $y<<l$). (c) In terms of l and l_0, calculate the ratio of the periods of oscillation along *x* and *y*.

12. A simple pendulum has a length l of *1* m. In free vibration the amplitude of its swings falls off by a factor e in *50* swings. The pendulum is set into forced vibration by moving its point of suspension horizontally in SHM with an amplitude of *1* mm. (a) Show that if the horizontal displacement of the pendulum bob is *x*, and the horizontal displacement of the support is ξ, the equation of motion of the bob for small oscillations is $\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \frac{g}{l}x = \frac{g}{l}\xi$, solve this equation for steady-state motion, if $\xi = \xi_0 cos\omega t$. (put $\omega_0^2 = g/l$). (b) At exact resonance, what is the amplitude of the motion of the pendulum bob? (First, use the given information to find *Q*).



13. For the electrical system in the figure, find (a) the resonant frequency, ω_0. (b) The resonance width, γ. (c) The power absorbed at resonance.

14. Consider a large number (*N*) of coupled oscillators connected by uniform springs along a straight line and limited to motions along that line. (a) Write down the equation of motion of the *i-th* oscillator and solve it, (b) Determine the normal modes of the oscillations and calculate the highest possible mode when *N* is very large.