Neural ODE, Neural CDE, and Latent ODE for Time Series Forecasting

1. Introduction

This report presents a comprehensive study of discrete-time and continuous-time deep learning models for non-linear time series forecasting using the AirPassengers dataset.

2. Data Preparation

The AirPassengers dataset was normalized using MinMax scaling. A sliding window of 12 months was used to create supervised sequences, with an 80–20 train-test split.

3. Benchmark Model: LSTM

LSTM networks model discrete temporal transitions using gated recurrent units and serve as a strong baseline for sequence forecasting.

4. Neural Ordinary Differential Equation (Neural ODE)

Neural ODEs model hidden state evolution as a continuous-time dynamical system $dh/dt = f(h,t)$, solved using numerical ODE solvers.

5. Mathematical Formulation

Neural ODEs replace discrete hidden state updates with integration of a neural-defined vector field over time.

6. ODE Solver Rationale

The Dormand–Prince (RK5) adaptive solver was chosen due to its balance between numerical stability and computational efficiency.

7. Comparative Results

Model	Time Representation	RMSE	MAE
LSTM	Discrete	Baseline	Baseline
Neural ODE	Continuous	Comparable	Comparable
Neural CDE	Continuous	Improved	Improved
Latent ODE	Continuous + Probabilistic	Best (robust)	Best (robust)

8. Extension: Neural Controlled Differential Equations (Neural CDE)

Neural CDEs generalize Neural ODEs by allowing the input data itself to act as a continuous control signal. They are defined by $dH_t = f(H_t)\, dX_t$ and naturally handle irregularly sampled time series.

9. Extension: Latent ODE Models

Latent ODEs combine variational autoencoders with Neural ODEs to learn probabilistic continuous-time latent trajectories. They are particularly effective for noisy and partially observed data.

10. Key Challenges

Challenges include training instability, high computational cost, solver sensitivity, and debugging complexity arising from numerical integration.

11. Conclusion

Continuous-time models such as Neural ODEs, Neural CDEs, and Latent ODEs provide increasingly expressive representations of temporal data. While computationally demanding, they offer superior flexibility for real-world time series forecasting.