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Sorting Algorithms

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CSA121 : DSA

Sorting

The process of **arranging the elements** in a specific order. Imagine sorting a list of names, numbers, or search results.

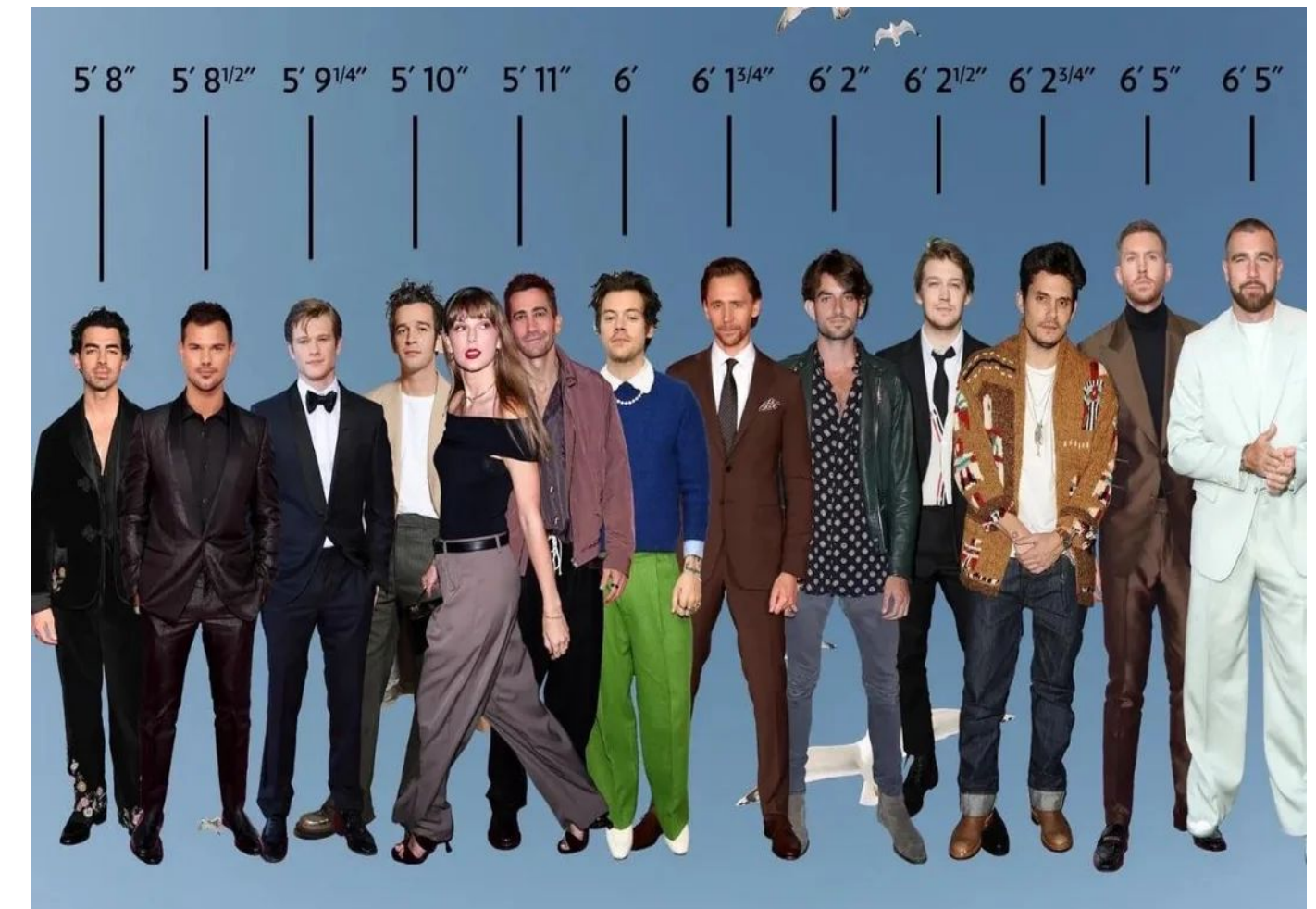


Example :

Example : We arrange students according to their height for a prayer assembly in the school.

What if we have just have the data of students in a list ?

A **sorting algorithm** can be used to find the sorted order of the student's height.



Bubble Sort

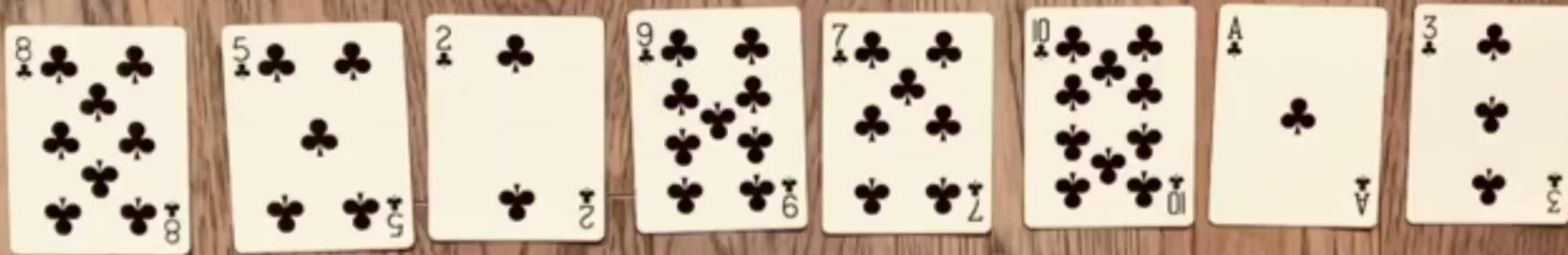
Why is it called Bubble Sort ?

- Works similarly to bubbles rising to the surface in water.
- During each pass through the list, larger elements "bubble up" to their correct positions
- This happens by repeatedly comparing and swapping adjacent elements, just like a bubble rising through liquid.



Working principle of Bubble Sort

Sorting Deck of Cards



Working principle

It works by **repeatedly stepping through the list, comparing adjacent** elements and swapping them if they are in the wrong order.

➔ **Step 1: Comparison**

The algorithm compares the **first two adjacent elements** in the list.

➔ **Step 2: Swapping**

If the elements are in the **wrong order**, they are swapped.

➔ **Step 3: Iteration**

The process is **repeated for the next pair of elements**, and so on.

Bubble Sort – Example

Example – Bubble Sort

Index | **Data Items**

0 5

1 1

5	1	4	2	8	9
---	---	---	---	---	---

2 4

3 2

4 8

5 9

Iteration-1:

1	4	2	5	8	9
---	---	---	---	---	---

Index | Data Items

0	5	1	1	1	1	1
1	1	5	4	4	4	4
2	4	4	5	2	2	2
3	2	2	2	5	5	5
4	8	8	8	8	8	8
5	9	9	9	9	9	9

Swap

Swap

Swap

No Swap

No Swap

Comparisons = 5

Swaps = 3

9 is fixed at (n-1)th -
5th index

Iteration-2:

1	2	4	5	8	9
---	---	---	---	---	---

Index | Data Items

0	1	1	1	1	1
1	4	4	2	2	2
2	2	2	4	4	4
3	5	5	5	5	5
4	8	8	8	8	8

No Swap

Swap

No Swap

No Swap

Comparisons = 4

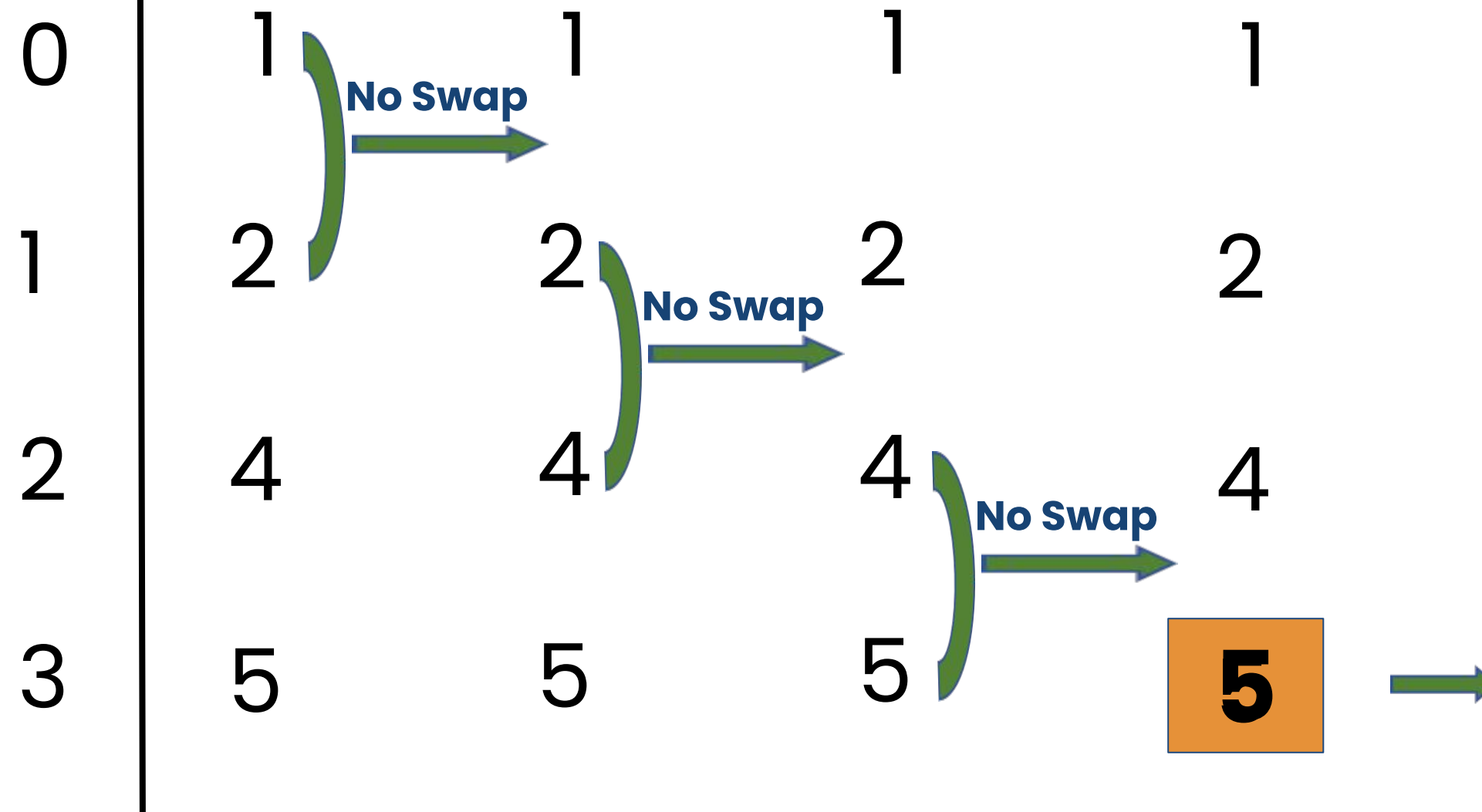
Swaps = 1

**8 is fixed at (n-2)th -
4th index**

Iteration-3 :

1	2	4	5	8	9
---	---	---	---	---	---

Index | Data Items



Comparisons = 3

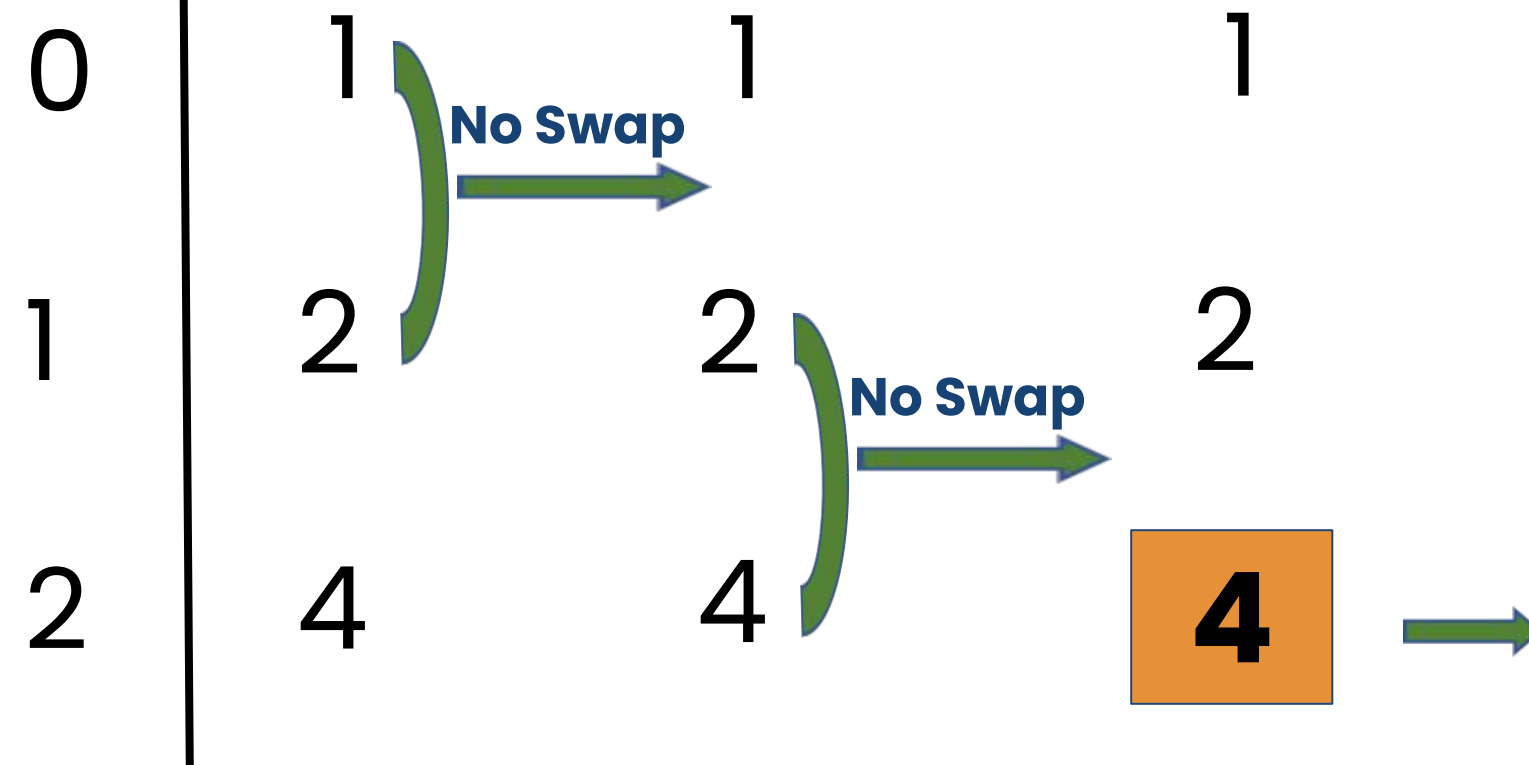
Swaps = 0

**5 is fixed at (n-3)th -
3rd index**

Iteration-4 :

1	2	4	5	8	9
---	---	---	---	---	---

Index | Data Items



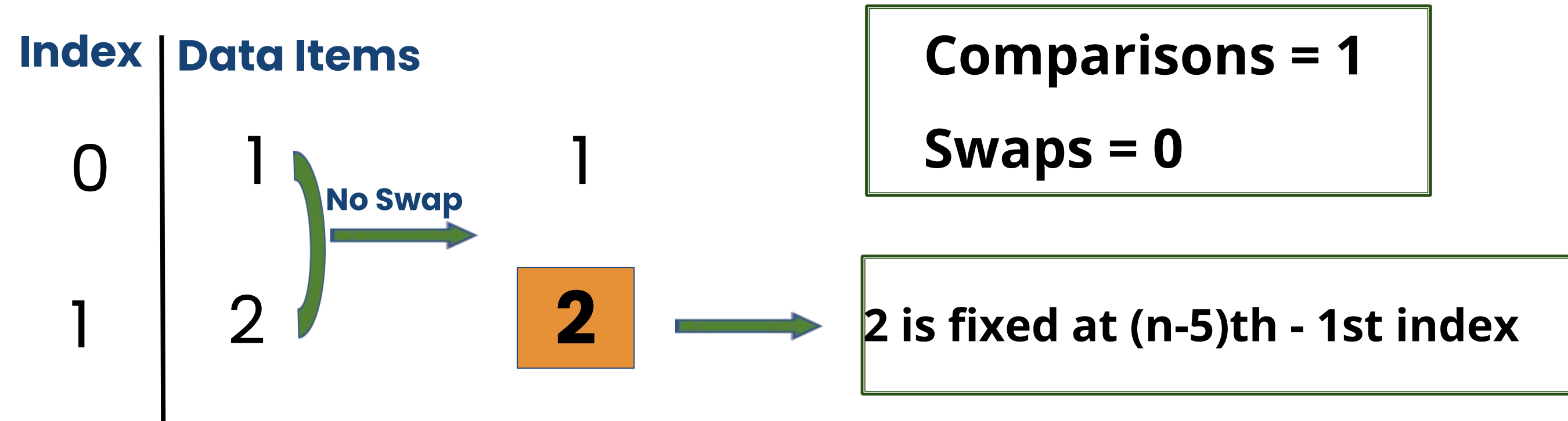
Comparisons = 2

Swaps = 0

**4 is fixed at (n-4)th -
2nd index**

Iteration-5 :

1	2	4	5	8	9
---	---	---	---	---	---



Post Iteration – 5 :

After Iteration 5, only one element remains unsorted in the array.

A **single element is inherently sorted**, as there are no other elements to compare or swap with.

Index	Data Items
0	1

Bubble Sort – Worst Case

Example – Bubble Sort

Index | **Data Items**

0 15

1 14

15	14	13	12	11
----	----	----	----	----

2 13

3 12

4 11

Iteration-1:

14	13	12	11	15
----	----	----	----	----

Index | Data Items

0	15	14	14	14	14
1	14	15	13	13	13
2	13	13	15	12	12
3	12	12	12	15	11
4	11	11	11	11	15

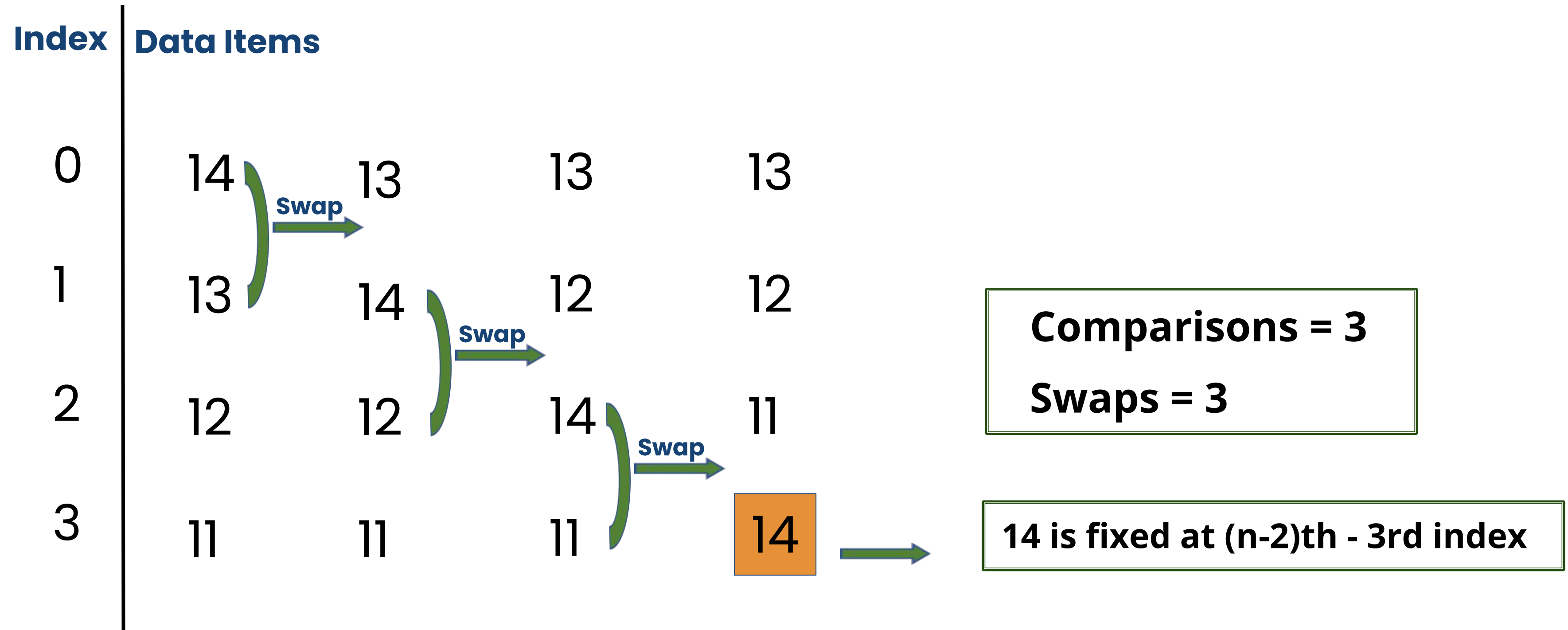
Comparisons = 4

Swaps = 4

15 is fixed at (n-1)th - 4th index

Iteration-2:

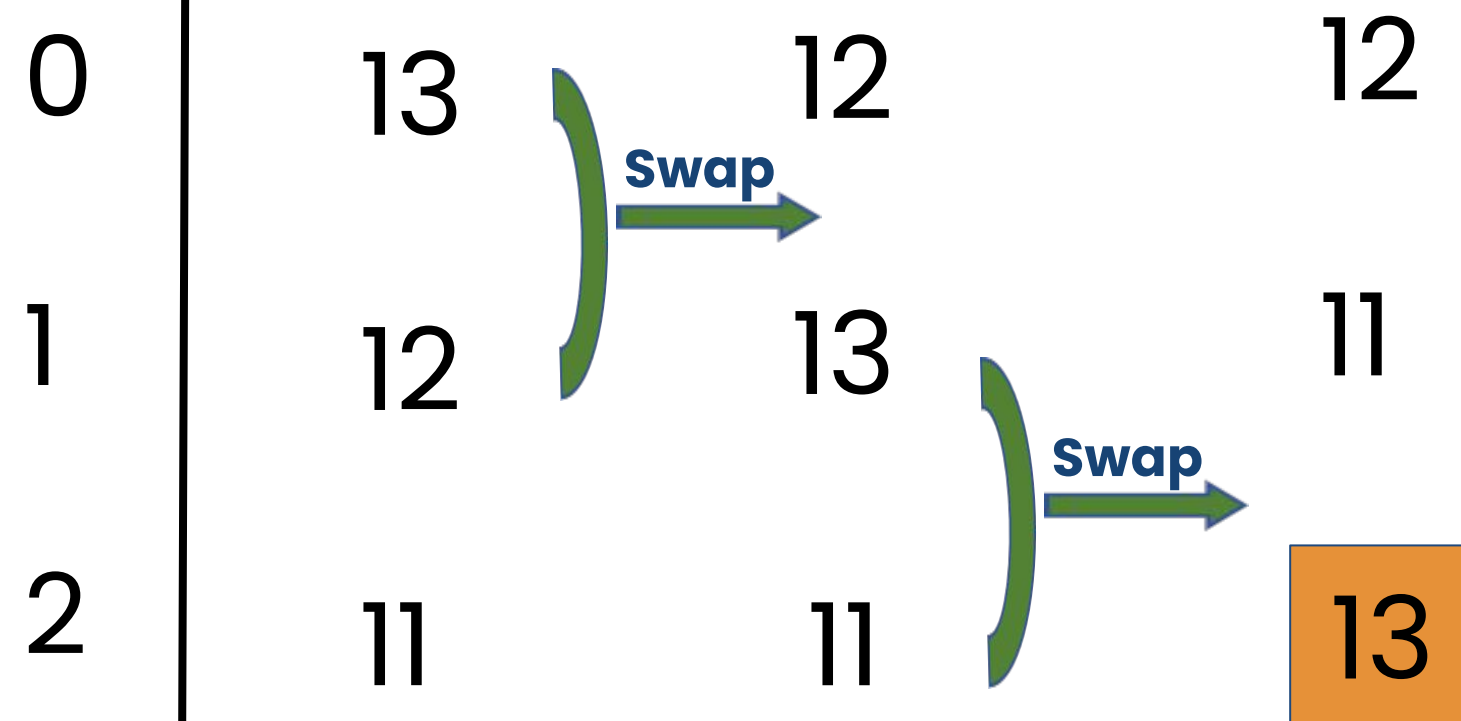
13	12	11	14	15
----	----	----	----	----



Iteration-3 :

12	11	13	14	15
----	----	----	----	----

Index | Data Items



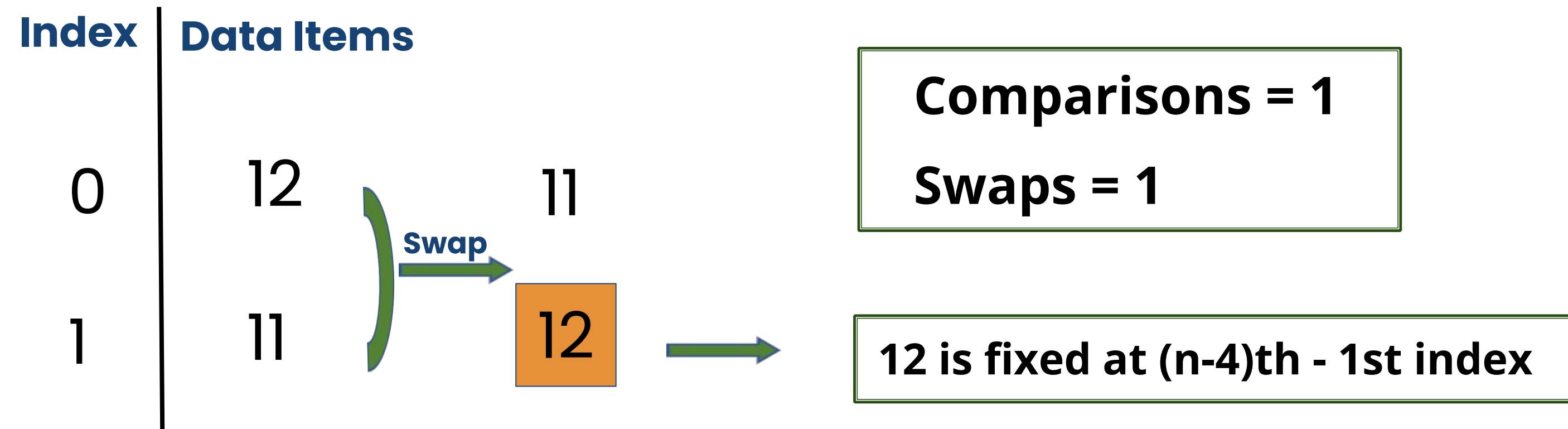
Comparisons = 2

Swaps = 2

13 is fixed at (n-3)th - 2nd index

Iteration-4 :

11	12	13	14	15
----	----	----	----	----



Post Iteration-4 :

After Iteration 4, only one element remains unsorted in the array.

A **single element is inherently sorted**, as there are no other elements to compare or swap with.

Index	Data Items
0	11

Code Implementation :

```
def bubble_sort(arr):  
    n = len(arr)  
    for i in range(n):  
        for j in range(n - i - 1):  
            if arr[j] > arr[j + 1]:  
                arr[j], arr[j + 1] = arr[j + 1], arr[j]  
    return arr
```

Performance Analysis – Worst Case

	Number of Comparisons	Number of Swaps
1st Iteration	n-1	n-1
2nd Iteration	n-2	n-2
3rd Iteration	n-3	n-3
• •	• •	• •
(N-1)th Iteration	1	1

Key Observations :

- **After the first pass**, the largest element reaches its rightful spot at the end.
- **After each pass, the "comparison window"** shrinks since the last elements are already sorted.
- **After each subsequent pass**, the next largest is placed before it, and so on.

Performance Analysis – Worst Case

The worst-case scenario for Bubble Sort occurs when the input list is sorted in reverse order :

5	4	3	2	1
---	---	---	---	---

- For each element, the algorithm needs to compare it with every other element in the list to ensure proper placement.
- The total number of **comparisons and swaps required is maximum.**

Performance Analysis – Worst Case

Total number of comparisons = $n (n-1) / 2$

Total number of swaps = $n (n-1) / 2$

$$\text{Worst Case TC} = \frac{n(n-1)}{2} + \frac{n(n-1)}{2} = n(n-1) = O(n^2)$$

How many passes would be required , if input array is already sorted ?

```
def bubble_sort(arr):  
    n = len(arr)  
    for i in range(n):  
        for j in range(n - i - 1):  
            if arr[j] > arr[j + 1]:  
                arr[j], arr[j + 1] = arr[j + 1], arr[j]  
    return arr
```

Bubble Sort – Best Case

Iteration - 1

Index | **Data Items**

0

11

1

12

11

12

13

14

15

2

13

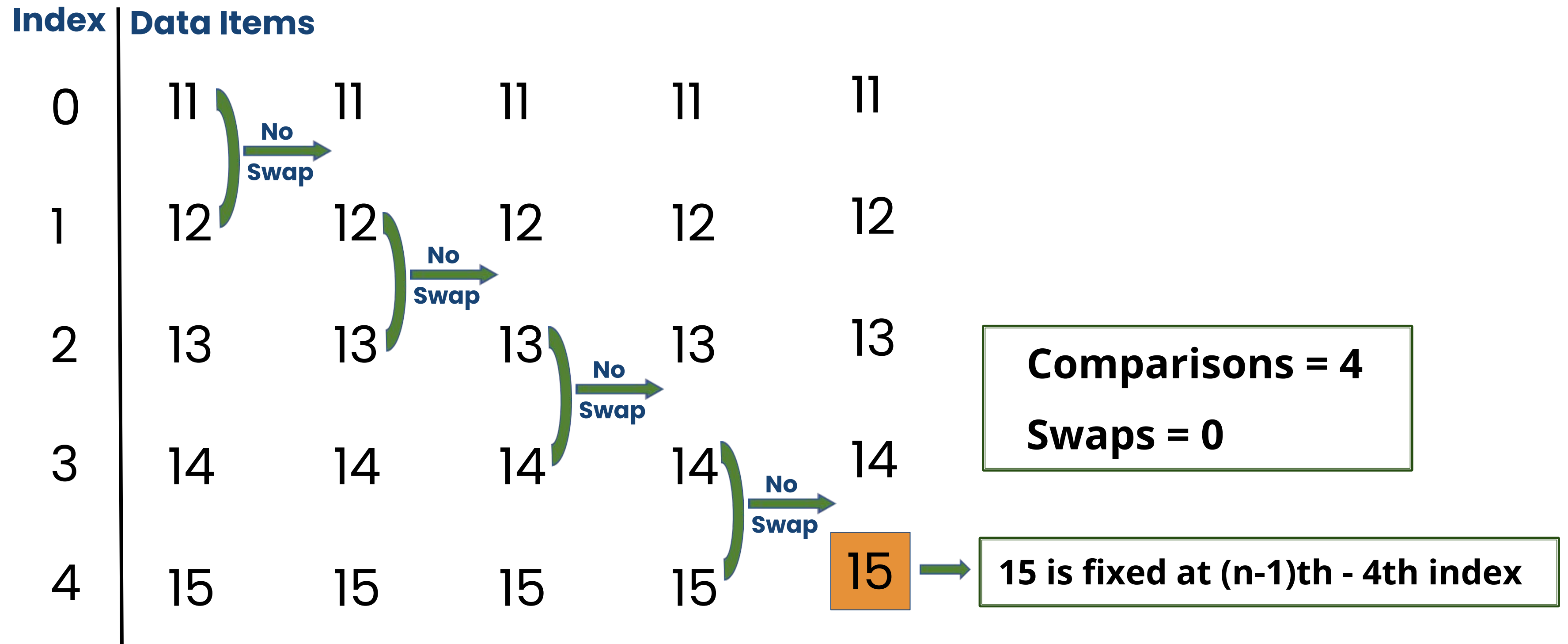
3

14

4

15

Iteration - 1



Performance Analysis – Best Case :

Input list is already sorted in ascending order

	Number of Comparisons	Number of Swaps
1st Iteration	$n-1$	0

Total number of comparisons = $n-1$

Total number of swaps = 0

Best Case Time Complexity = $O(n)$

Early exit – Optimisation :

```
def bubble_sort_optimized(arr):  
    n = len(arr)  
    for i in range(n):  
        swapped = False  
        for j in range(n - i - 1):  
            if arr[j] > arr[j + 1]:  
                arr[j], arr[j + 1] = arr[j + 1], arr[j]  
                swapped = True  
        if not swapped: # Exit early if no swaps occurred in this pass  
            break  
    return arr
```

Time Complexity of Bubble Sort :

Worst Case

$O(n^2)$

Average Case

$O(n^2)$

Best Case

$O(n)$

Space Complexity of Bubble Sort

Bubble Sort has a space complexity of $O(1)$

$O(1)$ because it is an **in-place sorting algorithm**.

It **does not require any additional space** proportional to the input size.

Sorting is achieved by swapping adjacent elements directly within the input list.

About Bubble Sort

- **Stable** Sorting Algorithm.
- **Inplace** Sorting Algorithm.
- **Rarely used** due to poor performance.

Advantages and Disadvantages :

Advantages	Disadvantages
Simple and easy to implement.	Inefficient for large datasets due to $O(n^2)$ time complexity.
Requires only $O(1)$ additional space (in-place sorting).	High number of comparisons and swaps in the worst case.

END