

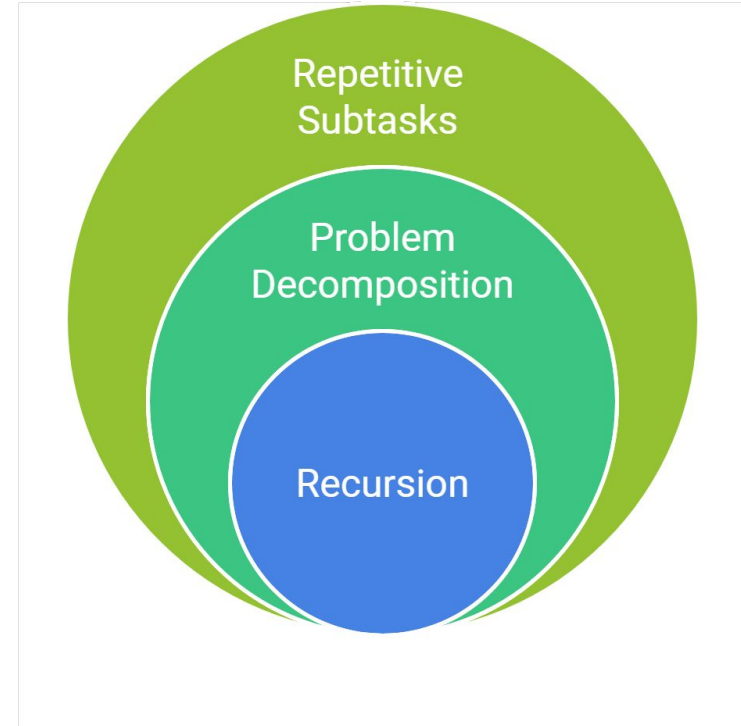
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# Recursion – Part IV

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# Lecture Agenda :

1. Count of subsets with sum  $k$
2. Find all subsets with sum  $k$



# Count of Subsets with sum K

# Description

You are given **an array** `nums` of size `n` containing positive integers, and a **target sum** `k`.

Your task is to find the number of ways to select a subset of elements from the array such that the sum of the chosen elements is equal to the target sum `k`

# Example

Input :  $K = 5$  ,

1	4	4	5
---	---	---	---

Output : 3

Explanation :

1	4	1	4	5
---	---	---	---	---

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# How many different paths/possibilities we have?

To find the subset we have 2 options :

- Include current element
- Other option ?



# How many different paths/possibilities we have?

To find the subset we have 2 options :

- Include current element
- Exclude current element



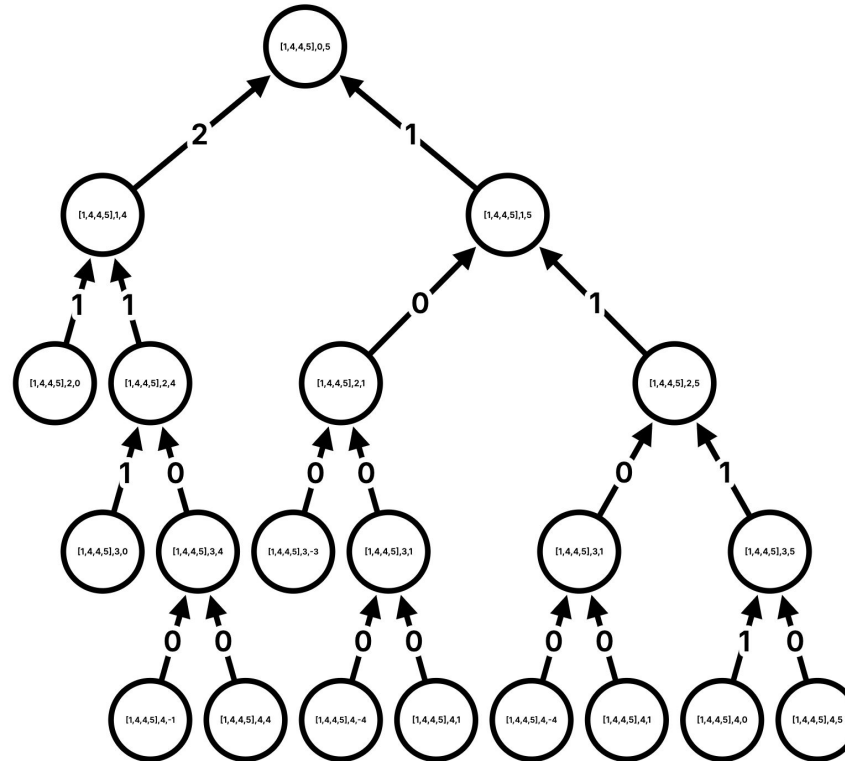
## When should i stop or what will be my base case?

1. When subset sum value is equals to target value
2. When we reached the ending index





# Recursive Tree:



# Approach

## Base Case

1. If the target sum  $k$  becomes 0, it means we have successfully found a valid subset. Return 1 to count this subset.
2. If the current index goes out of bounds (i.e.,  $\text{index} == \text{len}(\text{nums})$ ) or the target sum becomes negative, return 0 as no valid subset is possible.

## Recursive Call

At each index, consider two possibilities:

1. **Include the current element** in the subset:
  - Subtract the value of the current element from  $k$ , and move to the next index.
2. **Exclude the current element** from the subset:
  - Keep  $k$  unchanged and move to the next index.

## Return Type

The result for the current function call is the sum of these two recursive calls. The function returns an **integer**, representing the total number of valid subsets that achieve the target sum  $k$ .

# Code

```
1  def countSubsets(nums, index, k):
2      # Base Case: If the target sum is achieved
3      if k == 0:
4          return 1
5
6      # Base Case: If we've exhausted the array or target is invalid
7      if index == len(nums) or k < 0:
8          return 0
9
10     # Recursive Call: Include or exclude the current element
11     include = countSubsets(nums, index + 1, k - nums[index])
12     exclude = countSubsets(nums, index + 1, k)
13
14     # Return total count
15     return include + exclude
16
17 def findWays(nums, k):
18     return countSubsets(nums, 0, k)
19
20 # Example Usage
21 n, k = map(int, input().split()) # Input for size of array and target
22 nums = list(map(int, input().split())) # Input for the array
23 print(findWays(nums, k)) # Output the number of ways
24
```

# Code Analysis

**Time Complexity :  $O(2^N)$**

**Space Complexity :  $O(N)$**



**Find all subsets  
with target sum equals K**

# Description

You are given an integer array `arr` of size `n` and a target integer `target`.

Your task is to find and Return all the subsets of `arr` such that the sum of the elements in each subset is equal to `target`.

# Example

Input :  $K = 6$ ,

1	2	3	4
---	---	---	---

Output :

1	2	3	2	4
---	---	---	---	---

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## How can we store the subset for different options?

For including can I add current element into list ?

For excluding can I remove the current element that we just added ?





## When should i stop or what will be my base case?

1. When subset sum value is equals to target value
2. When we reached the ending index



# Approach

## Base Case

1. **Target Sum Reached:** If the current sum of the elements in the `current_subset` equals the target, it means we've found a valid subset. We should add this subset to the result list.
  - Condition: if `current_sum == target`:
  - Action: Add `current_subset` to result.
2. **End of Array:** If we've reached the end of the array (i.e., the index exceeds the length of the array), the recursion should terminate without adding any further subsets.
  - Condition: if `index == len(arr)`:
  - Action: Return (end of recursion).

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## Recursive Call

At each index, we have two recursive choices:

1. **Include the current element** in the subset:
  - Add the current element (`arr[index]`) to the `current_subset`, and call the recursive function for the next index, increasing the sum by `arr[index]`.
  - Recursive Call: `recursive(index + 1, current_subset, current_sum + arr[index])`

# Approach

## 2) **Exclude the current element** from the subset:

- Remove the last element from the `current_subset` (backtrack), and call the recursive function for the next index, keeping the sum unchanged.
- We are not including the current element into `current_sum`. So, needs to remove it from the `current_subset` also to find the new subset.
- Recursive Call: `recursive(index + 1, current_subset, current_sum)`

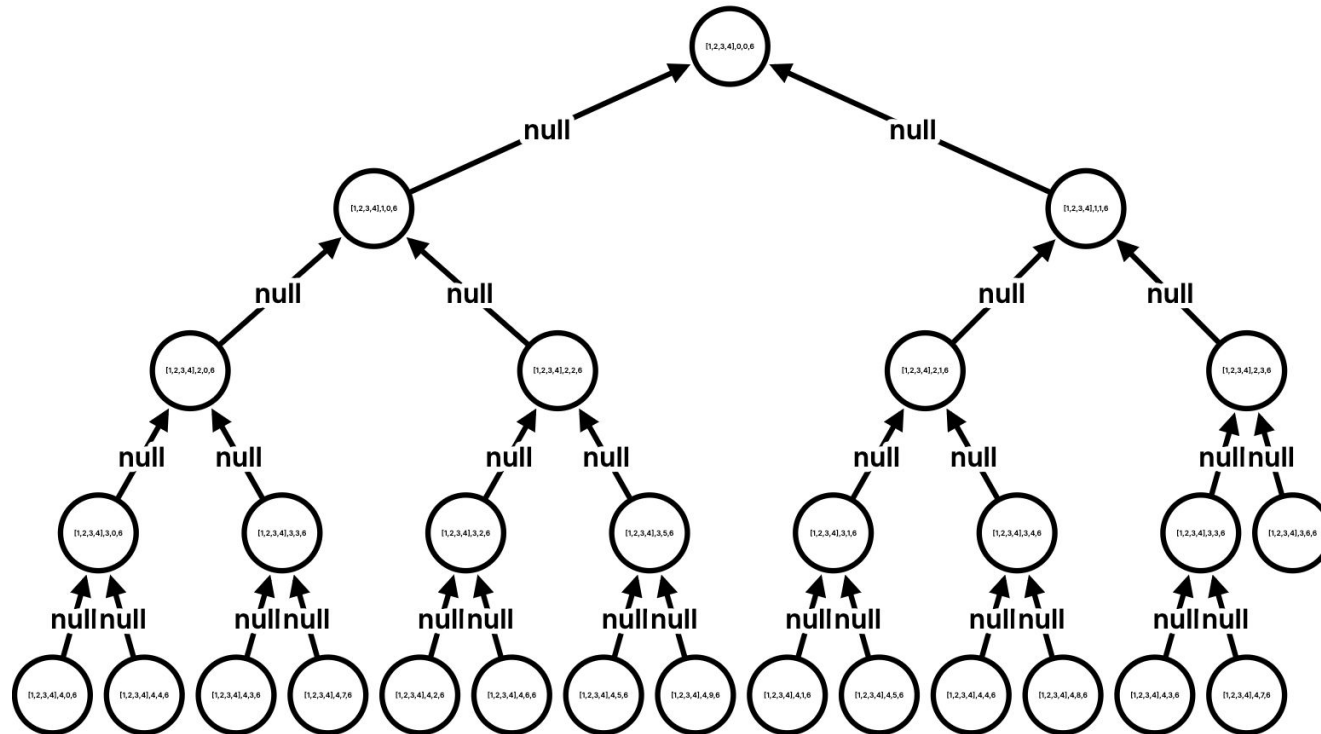
## 3) **Return Type**

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The function returns a list of lists (`result`), where each inner list represents a subset whose sum equals the target.

- **Type:** `List[List[int]]`
  - Each element of the list (`result`) is a list that represents a subset of elements whose sum equals the target.

# Recursive Tree:



# Code

```
1 def find_subsets(arr, target):
2     result = [] # List to store valid subsets
3
4     def helper(index, cur_sum, cur_subset):
5         # Base Case: If target sum is achieved, store the subset
6         if cur_sum == target:
7             result.append(cur_subset[:]) # Append a copy of the subset
8             return
9
10        # Base Case: If we've exhausted the array
11        if index == len(arr):
12            return
13
14        # Include the current element
15        helper(index + 1, cur_sum + arr[index], cur_subset + [arr[index]])
16
17        # Exclude the current element
18        helper(index + 1, cur_sum, cur_subset)
19
20    helper(0, 0, [])
21    return result
```

# Code Analysis

**Time Complexity :**  $O(2^N * N)$

**Space Complexity :**  $O(2^N * N)$



# Summary

In this lecture we studied about problems in which we have made two recursive calls and two parameters are changing

In first problem , **count subsets with target sum K** , index and K are changing parameters

In second problem , **find all subsets with target sum K** , index and K are changing parameters



**Thanks for  
Attending!**