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Binary Search : Problem solving

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CSA 101 : Problem Solving with Programming

Newton School of Technology

Koko Eating Bananas Problem

Koko Eating Bananas :

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➢ Koko is a monkey **who loves to eat** bananas.

➢ You're given **a list of banana pile sizes and hours H** in which Koko has to eat all the piles of bananas.

Koko Eating Bananas :

Piles

3	6	7	11

- ➢ **The Monkey has to eat all the piles of Bananas at a certain rate.**
- ➢ **Given hours H = 8 hours**

Input Constraints

Number of Banana Piles (n):

$1 <= n <= 10^4$

Number of Bananas in Each Pile (piles[i]):

$1 <= piles[i] <= 10^9$

Total Hours Available (H):

$n <= H <= 10^9$

Data Types:

piles is a **list of integers**.

H is an **integer**.

Brute Force Approach :

Piles				Time
3	6	7	11	H = 8 Hours

- ➢ Koko eats bananas **at certain rate** **(k bananas / hour)**

- ➢ **Minimum eating Speed** = **the minimum integer k** such that Koko can eat all the bananas within H hours.

Brute Force Approach :

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Given				
Piles				Time
3	6	7	11	H = 8 Hours

Brute Force Approach :

Given				
Piles				Time
3	6	7	11	H = 8 Hours

Find
Speed (k) = bananas/hour ? Such that hours_taken <= H

Brute Force Approach :

Let Speed (k) = 1 bananas/hour ?

Piles	3	6	7	11
hours_taken	3	6	7	11

= 27

hours_taken <= H ?

27 <= 8 ?

NO

Brute Force Approach : Speed (k) = bananas / hour

Piles	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11
3	3	2	1	1	1	1	1	1	1	1	1
6	6	3	2	2	2	1	1	1	1	1	1
7	7	4	3	2	2	2	1	1	1	1	1
11	11	6	4	3	3	2	2	2	2	2	1
hours_taken	**27**	**15**	**10**	**8**	**8**	**6**	**5**	**5**	**5**	**5**	**4**
hours_taken <= H	**NO**	**NO**	**NO**	**YES**	**YES**	**YES**	**YES**	**YES**	**YES**	**YES**	**YES**

Time Complexity - Brute Force :

O(max(Piles) * n)

Can we optimise this ?

Observation :

Speed (k)	1	2	3	4	5	6	7	8	9	10	11

k values are ascending in nature

k value can vary from 1 to max(piles)

Can we apply binary search on this range ?

Binary Search Approach :

Speed (k)	1	2	3	4	5	6	7	8	9	10	11

L = 1

M = 6

R = 11

k = 6

Piles	3	6	7	11
hours_taken at k = 6	1	1	2	2

$\boxed{= 6}$

hours_taken <= H ?

6 <= 8 ?

YES

Binary Search Approach :

Is K = 6 bananas / hour is the minimum speed of eating bananas with the condition hours_taken <= H ?

Answer :   Try for slower speed for k .

Binary Search Approach :

Speed (k)	1	2	3	4	5	6	7	8	9	10	11

proceed your search in left half

M = 6

Right half

Binary Search Approach :

Speed (k)	1	2	3	4	5

L = 1

M = 3

R = 5

K = 3

Piles	3	6	7	11
hours_taken at k = 3	1	2	3	4

$\boxed{= 10}$

hours_taken <= H ?

10 <= 8 ?

NO

Binary Search Approach :

Since hours_taken > 8 , So mid (k) = 3 banana/hour can not be the eating rate.

Answer : Try for faster speed for k .

Binary Search Approach :

Speed (k)	1	2	3	4	5

Left half

M = 3

proceed your search

in Right half

Binary Search Approach :

Speed (k)	4	5

L = 4

R = 5

M = 4

K = 4

Piles	3	6	7	11
hours_taken at k = 4	1	2	2	3

= 8

hours_taken <= H ?

8 <= 8 ?

YES

Binary Search Approach :

Is K = 4 bananas / hour is the minimum speed of eating bananas with the condition hours_taken <= H ?

Answer :   Try for slower speed for k .

Binary Search Approach :

Speed (k)	3	4	5

proceed your search in left half

L = 4

M = 4

Right half

Speed (k)	3	4

R = 3

L = 4

Stop the search here as Left > Right

Binary Search Approach :

So K = 4 bananas/hour is the minimum eating rate of koko

such that hours_taken <= H

8 <= 8

TIME COMPLEXITY : BINARY SEARCH APPROACH

$$O(n) * \log_2(\max(piles))$$

$$\Downarrow$$

$$O(n) * \log_2(11)$$

Solving Quadratic Equation Problem : Using Binary Search

Understanding Monotonic Function

$f(x) = x+5$

Given $f(x) = 13$, find $x = ?$

Understanding Monotonic Function

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Definition :

Functions are known as monotonic if **they are increasing or decreasing** in their entire domain.

Understanding Monotonic Function

Monotonically Increasing Function: The function never decreases as the input increases.

Mathematically: If x1 < x2, then f(x1) ≤ f(x2)

Example of Monotonically Increasing Function:

$f(x) = 2x + 3$, $f(x) = \log(x)$, $f(x) = e^x$

→ As x increases, f(x) also increases.

Understanding Monotonic Function

Monotonically Decreasing Function: The function never increases as the input increases.

Mathematically: If x1 < x2 , then f(x1) ≥ f(x2)

Example of Monotonically Decreasing Function:

$f(x) = -x^5$ and $f(x) = e^{-x}$

→ As x increases, f(x) decreases.

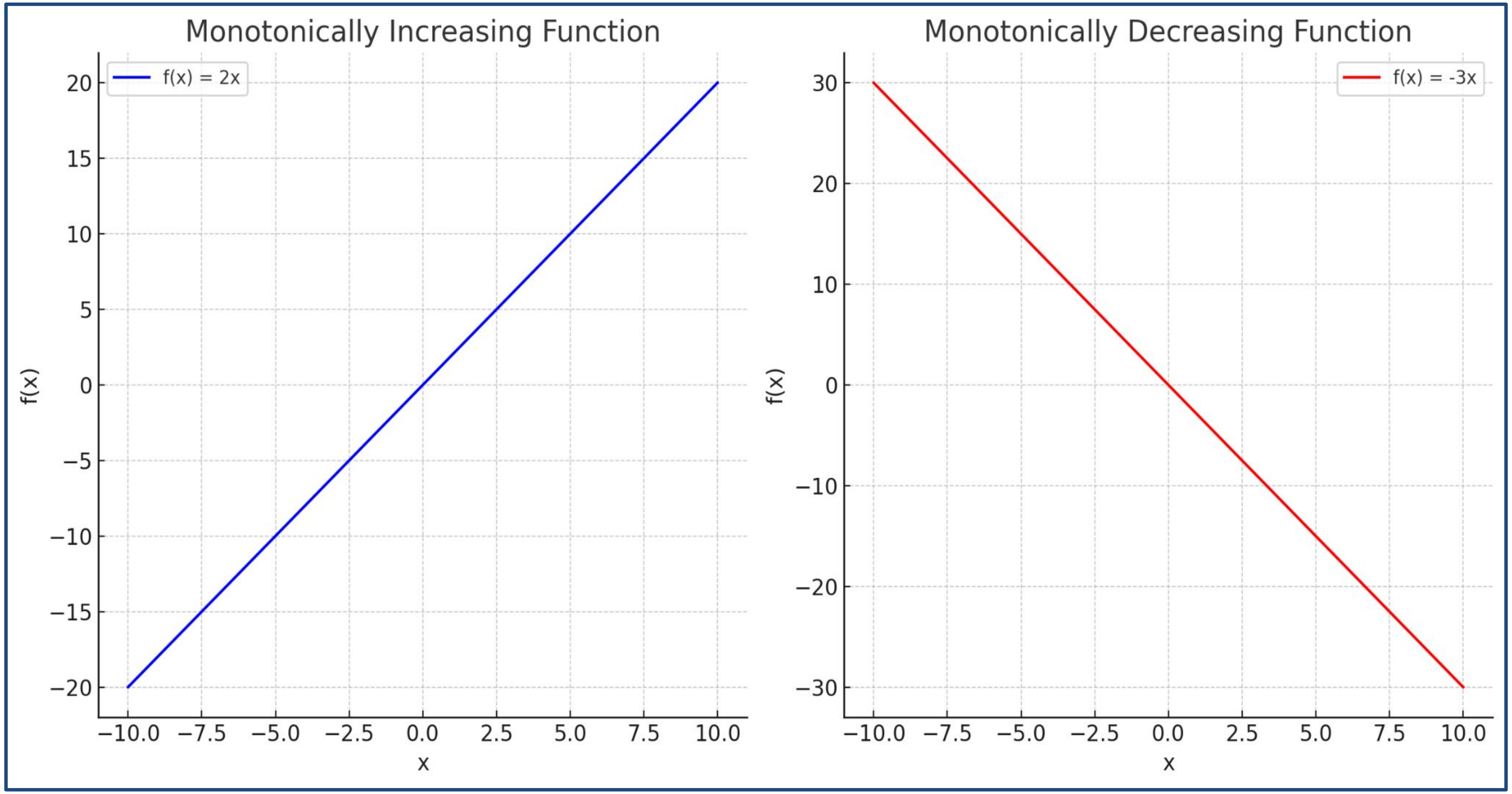
Understanding Monotonic Function

Graph Representation:

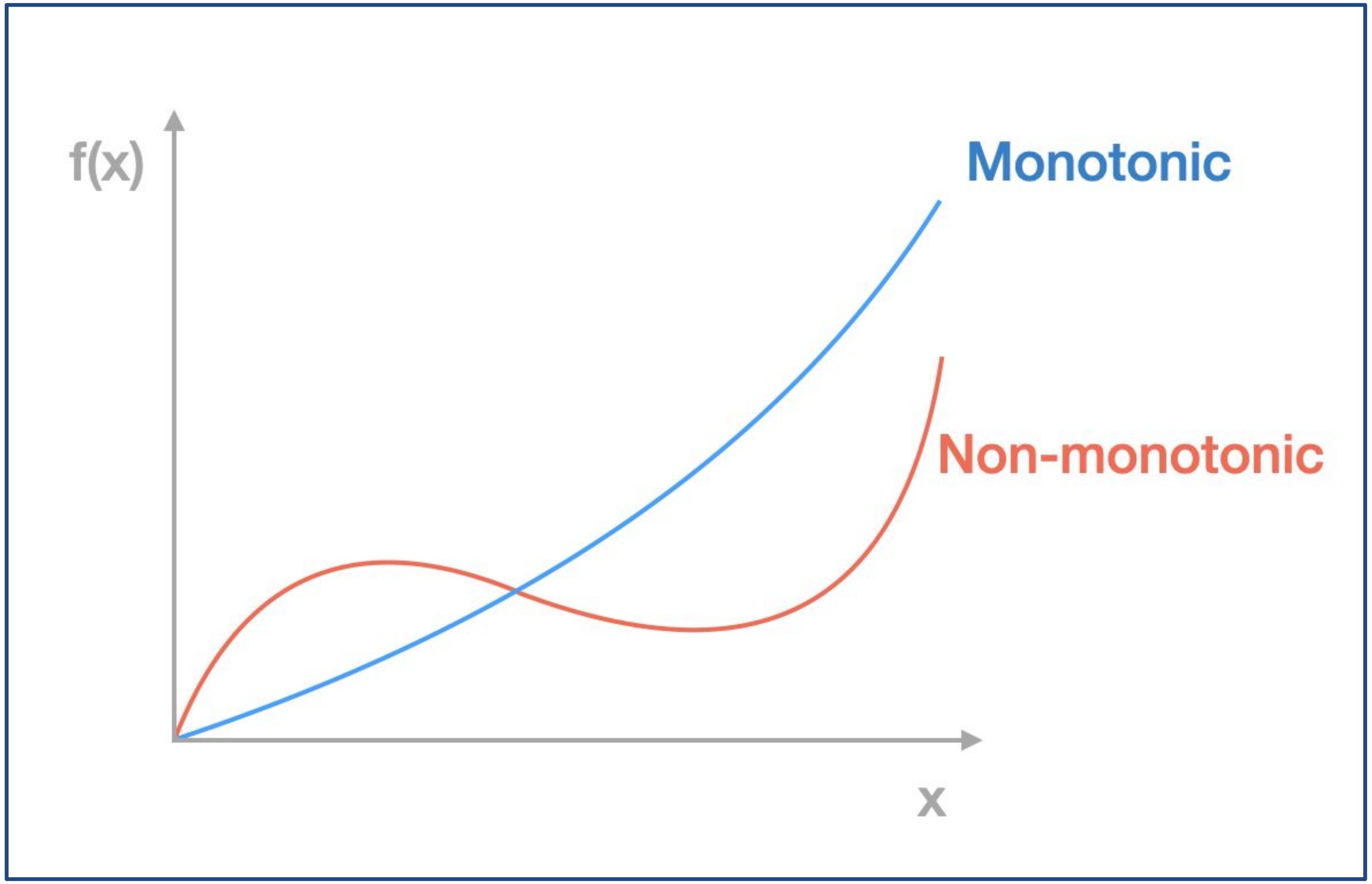
- **Increasing Function:** Rises steadily from left to right.
- **Decreasing Function:** Falls steadily from left to right.

Understanding Monotonic Function

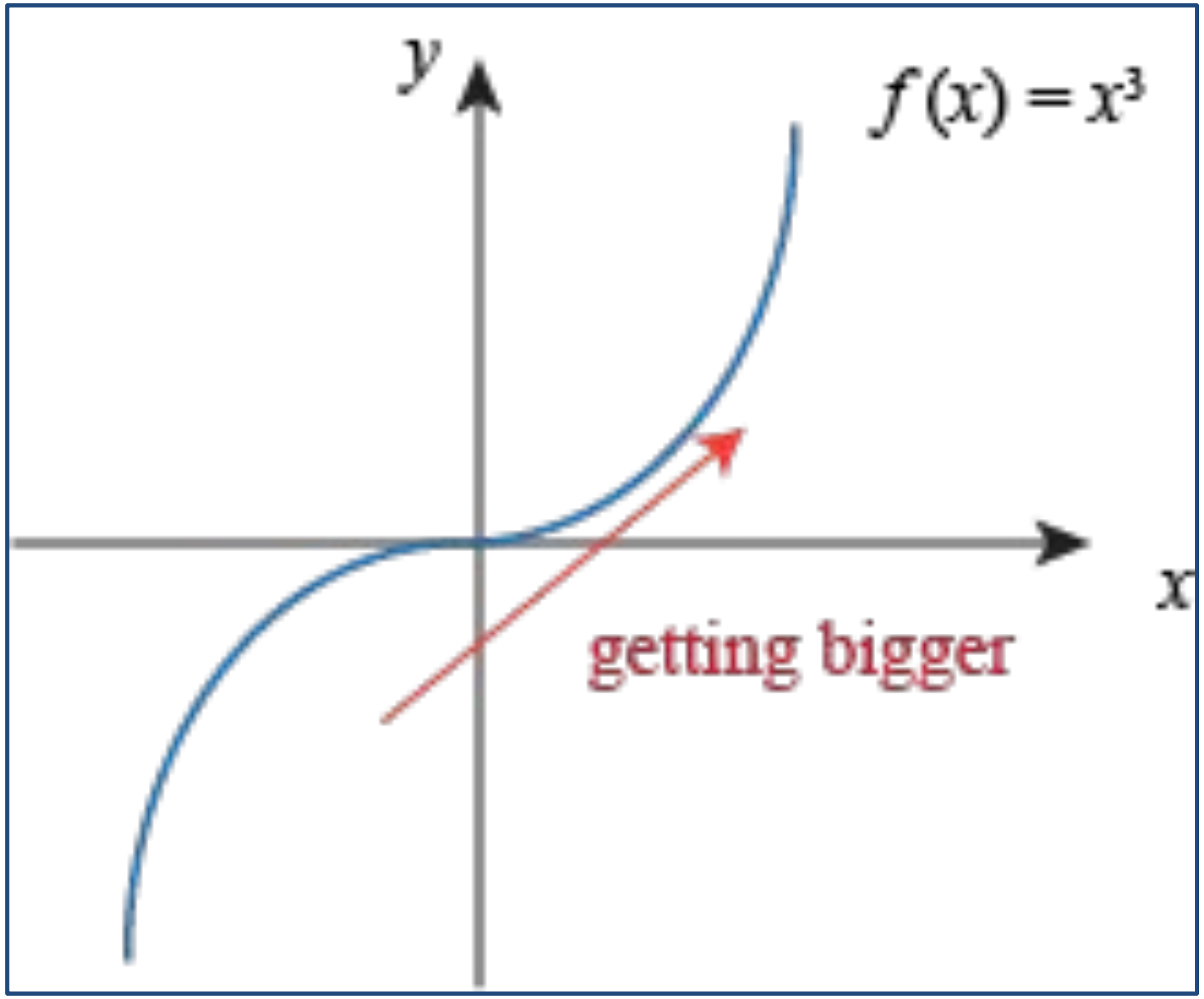
Graph Representation



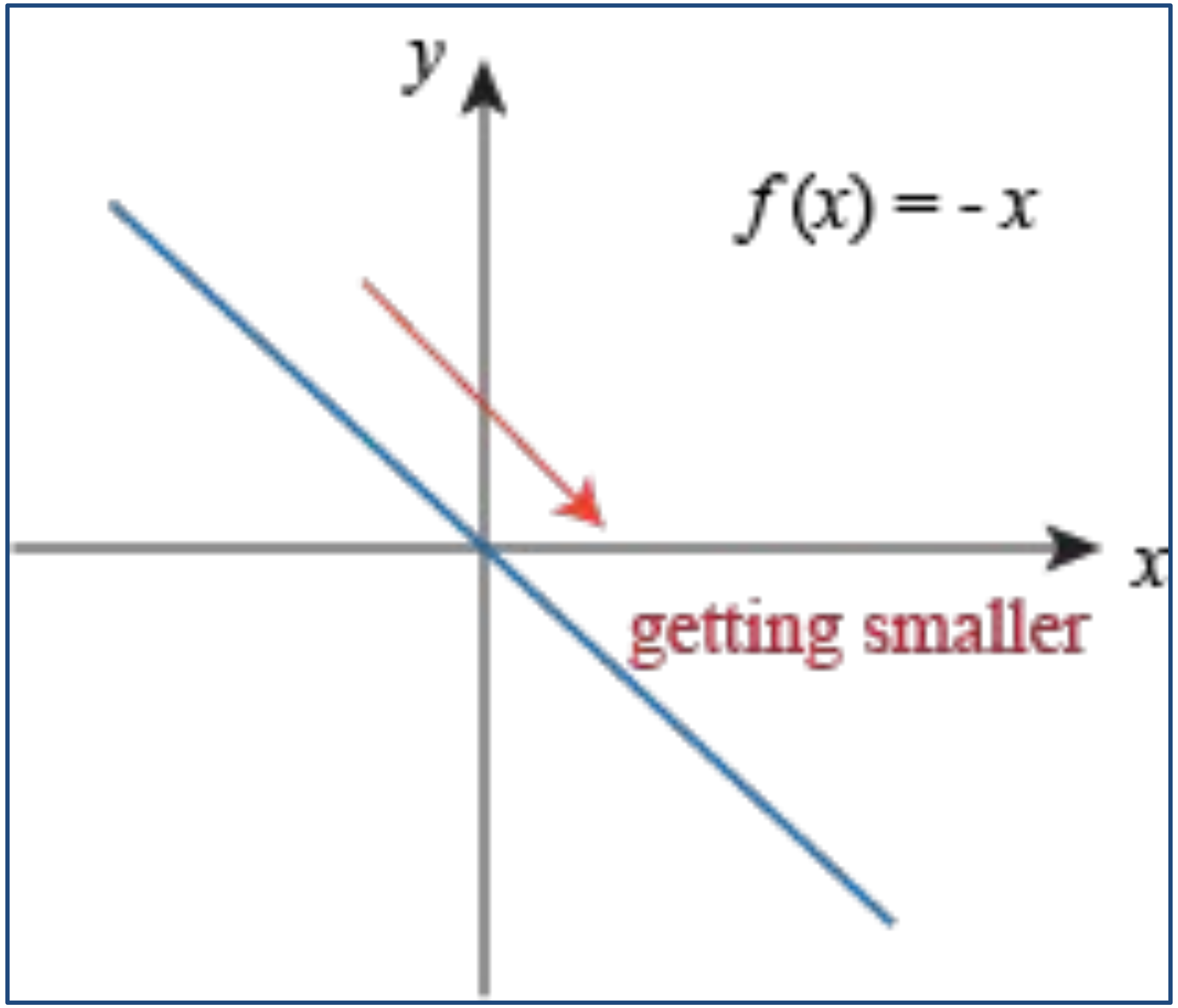
Understanding Monotonic Function



Understanding Monotonic Function



Understanding Monotonic Function



Solving Quadratic Equation Problem : Using Binary Search

Question :

Given: An integer K, **find a positive integer x** such that

$K = 2x^2 + 5x$.

→ **If no such positive integer x exists, return -1.**

Input Constraints :

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K is a positive integer

→ $1 \leq K \leq 10^2$

Output Constraints :

- **If a valid integer x exists that satisfies the equation:**

  → Return the integer value of x .

- **if no valid integer x exists:**

  → Return -1

- The output must be **produced within 1 second** for large values of K

- **Precision is not required** as the solution demands an exact integer match.

Solving Quadratic Equation Problem : Using Binary Search

Given: An integer K, **find a positive integer x** such that

$K = 2x^2 + 5x$.

- **Minimum x : 1** (since x is a positive integer).

- **Maximum x :** A value where $2x^2 + 5x$ exceeds K. This can be initially **set to a value like K** (since $2x^2 + 5x$ grows faster than linear).

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Solving Quadratic Equation Problem : Using Binary Search

Monotonic Relationship:

- ➢ The function $f(x) = 2x^2 + 5x$ is strictly increasing for $x > 0$.
- ➢ Thus, as x increases, f(x) increases.

Solving Quadratic Equation Problem : Using Binary Search

Given: An integer K = 18 , find a positive integer x such that

$K = 2x^2 + 5x.$

	Minimum	Maximum (x = k)
x	1	18

Observation :

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

x values are ascending in nature

x value can vary from 1 to 18

Can we apply binary search on this range ?

Solving Quadratic Equation Problem : Using Binary Search

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

L = 1

M = 9

R = 18

X = 9

Find value of k for x = 9
calculated_k = $2*9^2 + 5*9 = 207$

calculated_k == given_k ?

207 == 18 ?

NO

Solving Quadratic Equation Problem : Using Binary Search

Since calculated_k > given_k

↓

Try for smaller value of x .

Solving Quadratic Equation Problem : Using Binary Search

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

M = 9

proceed your search

in left half

Right half

Solving Quadratic Equation Problem : Using Binary Search

x	1	2	3	4	5	6	7	8

L = 1

M = 4

R = 8

X = 4

calculated_k == given_k ?

52 == 18 ?

NO

Find value of k for x = 9
calculated_k = $2*4^2 + 5*4 = 52$

Solving Quadratic Equation Problem : Using Binary Search

Since calculated_k > given_k

Try for smaller value of x .

Solving Quadratic Equation Problem : Using Binary Search

x	1	2	3	4	5	6	7	8

proceed your search in left half

M = 4

Right half

Solving Quadratic Equation Problem : Using Binary Search

x	1	2	3

L = 1

M = 2

R = 3

X = 2

Find value of k for x = 9
calculated_k = $2*2^2 + 5*2 = 18$

calculated_k == given_k ?

18 == 18 ?

NO

Solving Quadratic Equation Problem : Using Binary Search

Since calculated_k == given_k at x=2

↓

Stop the search process here .

↓

Return x = 2

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Complexity Analysis :

$\boxed{O(\log k)}$

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Thank You!