More than two random variables.

If $X_1, X_2, \ldots, X_n$ are all discrete rv's, the joint pmf of the variables is the function

$$p(x_1, x_2, \ldots, x_n) = P(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n).$$

If the variables are continuous, the joint pdf of $X_1, X_2, \ldots, X_n$ is the function $f(x_1, x_2, \ldots, x_n)$, s.t. for any 'n' intervals $[a_1, b_1], \ldots, [a_n, b_n]$,

$$P(a_1 \leq X_1 \leq b_1, \ldots, a_n \leq X_n \leq b_n) = \int_{a_1}^{b_1} \cdots \int_{a_n}^{b_n} f(x_1, \ldots, x_n)\, dx_n \cdots dx_1$$

The random variables $X_1, X_2, \ldots, X_n$ are said to be independent if for every subset $X_{i_1}, X_{i_2}, \ldots, X_{i_k}$ of the variables, the joint pmf or pdf of the subset is equal to the product of the marginal pmf's or pdf's.

Conditional distributions.

Let X and Y be two continious rv's with joint pdf $f(x,y)$ and marginal X ~~pdf~~ pmf $f_X(x)$. Then for any X value x for which $f_X(x) > 0$, the conditional probability density function of Y given that $X = x$ is

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)}, \quad -\infty < y < \infty.$$

(f) Determine the conditional pdf of Y given that $X=x$ and the conditional pdf of X given that $Y=y$.

$\therefore$ The conditional pdf of Y given that $X=x$ is

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{K(x^2+y^2)}{10Kx^2+0.05} \quad \text{for } 20 \le y \le 30.$$

The conditional pdf of x given that $Y=y$ is

$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{K(x^2+y^2)}{10Ky^2+0.05} \quad \text{for } 20 \le x \le 30.$$

(g) If the pressure in the right tire is found to be 22 psi, what is the probability that the left tire has a pressure of at least 25 psi? compare this to $P(Y \ge 25)$.

$$P(Y \ge 25 \mid X=22) = \int_{25}^{30} f_{Y|X}(y|22)\,dy$$

$$= \int_{25}^{30} \frac{K(22^2+y^2)}{10K \times 22^2 + 0.05}\,dy$$

$$= \int_{25}^{30} \frac{K \times 22^2}{10 \times 22^2 \times K + 0.05}\,dy + \int_{25}^{30} \frac{Ky^2}{10 \times 22^2 \times K + 0.05}\,dy$$

$$= 0.56.$$

$$P(Y \ge 25) = \int_{25}^{30} f_Y(y)\,dy = \int_{25}^{30} (10Ky^2 + 0.05)\,dy$$

$$= 10K \left.\frac{y^3}{3}\right|_{25}^{30} + 0.05(30-25)$$

$$= 0.75.$$

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(h) If the pressure in the right tire is found to be 22 psi, what is the expected pressure in the left tire, and what is the standard deviation of pressure in this tire?

$$E(Y|X=22) = \int_{-\infty}^{\infty} y\, f_{Y|X}(y|22)\, dy$$

$$= \int_{20}^{30} y \, \frac{K(22^2+y^2)}{10K \cdot 22^2 + 0.05}\, dy = 25.37$$

$$V(Y|X) = E(Y^2|X) - [E(Y|X)]^2$$

$$E(Y^2|X) = \int_{-\infty}^{\infty} y^2 f_{Y|X}(y|22)\, dy$$

$$= \int_{20}^{30} y^2 \cdot \frac{K(22^2+y^2)}{10K \cdot 22^2 + 0.05}\, dy = 652.03$$

$$\therefore V(Y|X) = \sigma_{Y|X} = 652.03 - (25.37)^2$$

$$= 8.3931$$

Note.

Conditional probability mass function of Y given that $X = x$ is

$$p_{Y|X}(y|x) = \frac{p(x,y)}{p_X(x)} \quad \text{for } -\infty < y < \infty$$

when X and Y are discrete rv's.