

Chapter 3Random Variable:

Random variable is a real valued function whose domain is a sample space.

i.e., If X is a random variable and S is sample space then $X: S \rightarrow \mathbb{R}$

$$(or) \quad X(x_i) = w_i \text{ where } x_i \in S \text{ and } w_i \in \mathbb{R}.$$

Example 1. Toss 3 coins, sample space.

$$S = \{ HHH, HHT, HTH, HTT, THH, THT, TTH, TTT \}$$

Define X = number of heads in any trial.

$$X(HHH) = 3, \quad X(HHT) = X(HTH) = X(THH) = 2$$

$$X(HTT) = X(THT) = X(TTH) = 1$$

$$X(TTT) = 0.$$

Example 2. 2 white balls, 1 red ball, X = number of red ball

$$S = \{ \omega_1\omega_2, \omega_1\omega_1, \omega_2\omega_1, \omega_2\omega_2, \omega_1\Omega, \omega_2\Omega, \Omega\omega_1, \Omega\omega_2, \Omega\Omega \}$$

$$X(\omega_1\omega_2) = X(\omega_1\omega_1) = X(\omega_2\omega_1) = X(\omega_2\omega_2) = 0$$

$$X(\omega_1\Omega) = X(\omega_2\Omega) = X(\Omega\omega_2) = 1$$

$$X(\Omega\Omega) = 2$$

probability Distribution $x_1 \quad x_2 \quad x_3 \quad x_4$

In Example 1, $X = 0 \quad 1 \quad 2 \quad 3$

$$P(X=0) = P(\text{no head}) = \frac{1}{8} \quad P \quad X: 0 \quad 1 \quad 2 \quad 3$$

$$P(X=1) = P(1 \text{ head occurs}) = \frac{3}{8} \quad P(X) = P(X=0) P(X=1) P(X=2) P(X=3)$$

$$P(X=2) = P(2 \text{ head occurs}) = \frac{3}{8}$$

$$P(X=3) = P(3 \text{ head occurs}) = \frac{1}{8}.$$

Classification of random variable (rv)

There are three types of random variables

- a) Discrete random variable
- b) Continuous random variable.
- c) Mixed type random variable.

A discrete random variable is an rv whose possible values either constitute a finite set or countably infinite.

→ It depends on the range or codomain of the rv.
probability Mass function (pmf).

Let X be discrete random variable, associated with sample space S . Let $R_X = \{x_1, x_2, \dots, x_n\}$ be the range space of X and each x_i is associated with a number $p_X(x_i) = P(X=x_i)$, then the number $p_X(x_i)$, $i=1, 2, \dots, n$ satisfies the following properties.

$$(i) p_X(x_i) \geq 0,$$

$$(ii) \sum_{x_i \in R_X} p_X(x_i) = 1,$$

is called probability mass function (pmf) of X .

Example. Tossing a coin thrice.; $X = \text{no. of heads.}$

$$R_X = \{0, 1, 2, 3\} \quad p_X(x_1) = p_X(0) = \frac{1}{8}$$

$$p_X(2) = \frac{3}{8}, \quad p_X(3) = \frac{1}{8}$$

$$p_X(1) = \frac{3}{8}$$

$$(c) p_x(x_i) \geq 0$$

$$(ii) \sum_{x_i \in R_X} p_x(x_i) = 1.$$

This is the pmf for random variable X .

$$X = x_i \quad 0 \quad 1 \quad 2 \quad 3$$

$$p_x(x_i) \quad 1/8 \quad 3/8 \quad 3/8 \quad 1/8$$