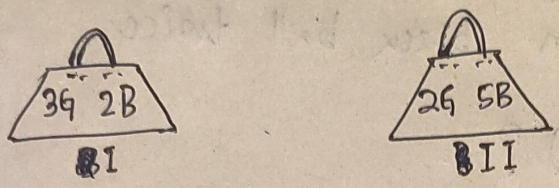
Example. Let two bags identical in appearance. First bag contains 3 green and 2 black balls. Second bag contains 2 green and 5 black balls. One bag is selected at random and a ball is drawn from it. Find probability that it is black.



Let A_1 be the event of selecting I bag.

$P(A_1) = \frac{1}{2}$

Let A_2 be the event of selecting II bag.

$P(A_2) = \frac{1}{2}$.

Let B be the event of getting black ball

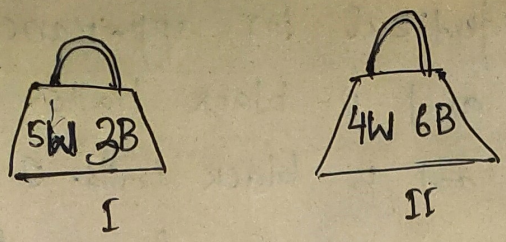
$P(\text{black ball}) = P(B)$ (∵ Total Prob. Thm)

$= P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2)$.

∵ $P(B|A_1) = \frac{2}{5}$ and $P(B|A_2) = \frac{5}{7}$

$= \frac{2}{5} \cdot \frac{1}{2} + \frac{5}{7} \cdot \frac{1}{2} = \frac{32}{35}$.

Example. Let there are two identical bags. Bag 1 contains 5 white and 3 black balls. Whether, Bag 2 contains 4 white and 6 black balls. A bag is selected at random and 2 balls are drawn, each found to be black. Find the probability that Bag 1 was selected.



A_1: Event of selecting bag I ; $P(A_1) = \frac{1}{2}$

A_2: Event of selecting bag II ; $P(A_2) = \frac{1}{2}$.

B: Event of drawn black ball twice.

To find $P(A_i | B)$?

$$= \frac{P(B|A_1) \cdot P(A_1)}{P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2)}.$$

$\therefore P(B|A_1) = \frac{3}{8} \cdot \frac{2}{7} = \frac{3}{28}$ $P(B|A_2) = \frac{5}{10} \cdot \frac{4}{9} = \frac{1}{3}$

$$= \frac{\frac{3}{28} \cdot \frac{1}{2}}{\frac{3}{28} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2}} = \frac{0.053}{0.053 + 0.166} = 0.2196.$$

Exercises Sec 2.4 Selected Problems.

Q. 45, 47, 50, 55, 56, 58, 59, 63.

Sec 2.5. Q. 71, 72, 78, 80, 83, 87.