The negative binomial distribution (Inverse)

1- The experiment consists of a sequence of independent trials.

2- Each trial can result in either a "S" or "F"

3- The probability of success is constant from trial to trial that is, $P(S \text{ on trial } i) = p$.

4. The experiment continues until a total of "r" success have been observed.

X = Number of ~~trials~~ failures to get "r" number of successes.
= negative binomial random variable.

r: fixed number of success

p: probability of S

q: probability of F.

The pmf of the negative binomial rv X with parameters 'r' and 'p' is

$$nb(x; r, p) = \binom{x+r-1}{r-1} p^r (1-p)^x \quad \text{for } x = 0, 1, 2, \ldots, \infty$$

a) $E(X) = \frac{r(1-p)}{p}$ = Average number of trials to get 'r' number of success.

b) $V(X) = \frac{r(1-p)}{p^2}$.

Note: The above pmf is called negative binomial of 2nd kind.

Q.15 E.g. Suppose that probability of mail birth = p = 0.5. A couple wishes to have exactly two female children in their family. They will have children until this condition is fulfilled.

a. What is the probability that the family has 'x' male children?

b. What is the probability that the family has four children?

c. What is the prob. that the family has at most four children?

d. How many male children would you expect this family to have? How many children would you expect this family to have?

Soln Given $p = P(\text{mail birth}) = 0.5$.

r = number of successes = 2.

$P(\text{female birth}) = 1 - 0.5 = 0.5$.

a) $P(x \text{ mail children}) = nb(x; 2, 0.5)$

$$= \binom{x+2-1}{2-1} (0.5)^2 (1-0.5)^x$$

$$= \binom{x+1}{1} 0.25 (1-0.5)^x$$

$$= 0.25 (x+1) (0.5)^x.$$

b) $P(X=2) = \binom{2+2-1}{2-1} (0.5)^2 (0.5)^2$

family has 4 children

$= {}^3C_1 (0.5)^4 = 0.1875.$

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2 males + 2 females

c) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = 0.25 + 0.25 + 0.1875.$

d) $E(X) = \frac{r(1-p)}{p} = \frac{2(1-0.5)}{0.5} = 2$ $\therefore$ We expect 2 male children in the family.

$\therefore$ Total children = 2 expect male + 2 female. = 4.

Q74. A second-stage smog alert has been called in a certain area of Los Angeles Country in which there are 50 industrial firms. An inspector will visit 10 randomly selected firms to check for violations of regulations.

a. If 15 of the firms are actually violating at least one regulation, what is the pmf of the number of firms visited by the inspector that are in violation of at least one regulation?

b. If there are 500 firms in the area, of which 150 are in violation, appr. the pmf of part (a) by a simpler pmf.

c. For X = the number among the 10 visited that are in violation, compute $E(X)$ and $V(X)$ both for the exact pmf and the appr. pmf in part (b).

Soln, Given $n = 10$

N = population size = 50

M = Number of observed successes = 15

a) $\therefore$ X = no. of firms visited by the inspector that are in violation of at least one regulation.

$$\therefore p(x) = \frac{\binom{15}{x}\binom{50-15}{10-x}}{\binom{50}{10}} = \frac{\binom{15}{x}\binom{35}{10-x}}{\binom{50}{10}}.$$

($\because$ X = Number of successes in sample size 'n'.)