Note: The poission's distribution is also applicable, when the random variable is defined below:

(i) Number of car accident in a year on a road.

ii) Number of earthquake in a year.

iii) Number of breakdown of an electric computer.

iv) The number of printing mistakes at each page of a book.

Ex If a publisher of nontechnical books takes great pains to ensure that its books are free of typographical errors, so that the probability of any given page containing at least one such error is 0.005 and errors are independent from page to page, what is the probability one of its 400 pages novels will contain exactly one page with errors? At most 3 pages with errors?

Ans

X = Number of pages with at least one errors

A = page containing at least one error.

B = error-free page.

$$\therefore P(X=1) = b(x; n, p) = b(1; 400, 0.005)$$

$$= {}^{400}C_1 \cdot (0.005)^1 (1-0.005)^{400-1}$$

$$= 0.2707$$

$$P(X=1) = p(x; \underset{np}{\mu}) = p(1; 2) = \frac{e^{-2} \cdot 2^1}{1!} = 0.2707$$

$$\therefore b(1; 400, 0.005) \approx p(1; 2) \approx 0.2707$$

$$p(x \le 3) = p_x(0) + p_x(1) + p_x(2) + p_x(3) \quad \text{for}$$

$$= e^{-2}\left(\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!}\right)$$

$$= e^{-2}\left(1 + 2 + 2 + \frac{8}{6}\right) = 0.8571$$

Ex There are 50 telephone lines in an exchange. The probability of them being busy is 0.1. What is the probability that all lines are busy.

Soln $n = 50,\ p = 0.1$

$\mu = np = 50 \times 0.1 = 5$ $\qquad$ X = number of lines being busy.

$$p(X = 50) = e^{-5}\frac{5^{50}}{50!} = 1.97 \times 10^{-32}$$

Ex Births in a hospital occur randomly at an average rate of 1.8 births per hour. What is the prob of observing 4 births in a given hour at the hospital? What about the probability of observing more than or equal to 2 births in a given hour at the hospital?

Soln Let X = number of births in a given hour.

Events occur randomly.
Mean rate $\mu = 1.8$ $\Big\}$ $X \sim Po(1.8)$

$$\therefore\ P(X = 4) = e^{-1.8}\frac{(1.8)^4}{4!} = 0.0723.$$

$$P(X \geq 2) = P(X = 2) + P(X = 3) + \cdots$$

$$= 1 - P(X < 2)$$

$$= 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - \left[e^{-1.8}\frac{1.8^0}{0!} + e^{-1.8}\frac{1.8^1}{1!}\right]$$

$$= 1 - (0.16529 + 0.29753)$$

$$= 0.537.$$