$P(A|B) = 0.42$

$P(A|B') = 0.9$

Events B and B' form partitions of the sample space S.

By total probability theorem; we have

$$P(A) = P(B)\, P(A|B) + P(B')\, P(A|B')$$

$$= 0.45 \times 0.42 + 0.55 \times 0.9 = 0.684.$$

$\therefore$ P(the job will be completed on time) = 0.684.

Independence of More than two events.

Events $A_1, \ldots, A_n$ are mutually independent if for every k $(k = 2, 3, \ldots, n)$ and every subset of indices $i_1, i_2, \ldots, i_k$;

$$P(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_k}) = P(A_{i_1}) \cdot P(A_{i_2}) \cdots P(A_{i_k}).$$

(*) The evnts are mutually independent (or collectively-independent) intuitively means that each event is independent of any combination of other events in the collection.

Bayes' Theorem. (R. T. Bayes, 1702)

Let $A_1, A_2, \ldots, A_n$ ~~are~~ be mutually exclusive and exhaustive events- ~~Then for other event B,~~ with non-zero probability of a random experiment, form a partition of S. If B is any event from the sample space S, then

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(B|A_i)\cdot P(A_i)}{\sum_{j=1}^{n} P(B|A_j)\cdot P(A_j)}$$

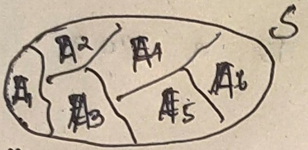
Multipl. formula.

Total prob.

for $i = 1, 2, \ldots, n$.

More Details

Partition of S.



Mutually Exclusive

i) $A_i \cap A_j = \phi \;\forall\; i \neq j$

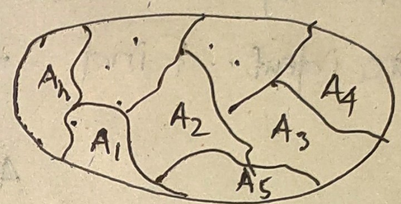
Exhaustive.

ii) $\bigcup_{j=1}^{n} A_j = S$ or

iii) $0 \leq P(A_i) \leq 1 \;\forall\; i$

$A_1, A_2, \ldots, A_n$ form a partition of S.

Theorem of probability



$A_1, A_2, \ldots, A_n$ form a partition of S.

B : event.

$P(B) = ?$

Baye's Theorem

let $A_1, A_2, \ldots, A_n$ form a partition of S.

If B is any event which occurs with A_1 or A_2 or ... or A_n, then $P(A_i|B) = ?$

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)}$$

$$= \frac{P(B|A_i)\cdot P(A_i)}{\sum_{j=1}^{n} P(B|A_j)\, P(A_j)}$$

We know,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

or

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

Example. An insurance co. insured 3000 scooters
4000 cars
5000 trucks

prob. of accident due to a scooter = 0.02

" " " car = 0.03

" " " truck = 0.04

One of the insured vehicles meets with an accident. Find the prob. that it is a car.

Accident Occured

↙ ↓ ↘

Scooter Car Truck.

?

A_1 : event of driving a scooter

A_2 : " " " car

A_3 : " " " truck.

} Mutually exclusive

exhaustive.

~~P(A2|B)=~~

B : event of occurence of accident

$$P(A_2|B) = \frac{P(B|A_2)\cdot P(A_2)}{P(B|A_1)\cdot P(A_1) + P(B|A_2)\cdot P(A_2) + P(B|A_3)\cdot P(A_3)}$$

$P(A_1) = \frac{1}{4}$, $P(A_2) = \frac{1}{3}$, $P(A_3) = \frac{5}{12}$.

$P(B|A_1) = 0.02$, $P(B|A_2) = 0.03$, $P(B|A_3) = 0.04$

$$= \frac{0.03 \times \frac{1}{3}}{0.02 \times \frac{1}{4} + 0.03 \times \frac{1}{3} + 0.04 \times \frac{5}{12}} = \frac{6}{19}.$$

Example. Let two bags identical in apperance. First bag contains 3 green and 2 black balls. Second bag contains 2 green and 5 black balls. One bag is selected at random and a ball is drawn from it. Find probability that it's black.