Point Estimation

Estimator and Estimate :-

Estimator is a function of random sample say $\underline{X} = (X_1, X_2, \cdots, X_n)$. Then $T(\underline{X})$ is said to be estimator. and the observed values of $\underline{X}$ say $(x_1, \cdots, x_n)$ is known as estimate. Estimator is used to estimate the unknown parameters present in the population.

Parameter Space.

This is the set of all possible values of the parameters and it is denoted by Θ.

Desired properties of Estimators.

i) Unbiased ii) Consistent iii) Sufficient (Not in syllabus)

iv) Efficient

Point Estimate

It is defined as a particular value of statistic which is used to estimate a given parameter. Point estimation is a single-valued estimation and is also called the estimation of the parameter.

Ex Suppose we want to estimate a true value of the parameter θ, say 1, i.e., $\theta = 1$ by point estimation.

Then if we provide a single point for example 1.1 or 0.99 or 0.98 or 1.12. so, these values are point estimate for $\theta = 1$.

Method of point estimation :

1) Maximum likelihood estimation

II) Method of Moments

III) Least square method.

<u>Likelihood Function</u> : Let $x_1, x_2, \ldots, x_n$ be a random sample of size 'n' from a population $f(x,\theta)$. Then the likelihood function of the sample values $x_1, x_2, \ldots, x_n$ usually denoted by $L = L(\theta)$ is their joint density or joint pmf and given as

$$L(\underline{x},\theta) = f(x_1,\theta)\cdot f(x_2,\theta)\cdots f(x_n,\theta)$$

$$= \prod_{i=1}^{n} f(x_i,\theta) \quad ; \theta \in \Theta .$$

The value of θ, say $\hat{\theta}(\underline{x})$ for which

$$L(\hat{\theta},\underline{x}) \geq L(\theta,\underline{x}) \quad \forall\ \theta \in \Theta$$ is

called MLE of θ.

<u>steps for MLE</u> :

Let $X_1, X_2, X_3, \ldots, X_n \overset{iid}{\sim} f(x,\theta)$.

① First write the likelihood function of θ.

$$L(\theta) = f(x_1,\theta)\cdot f(x_2,\theta)\cdots f(x_n,\theta)$$

$$L(\theta) = \prod_{i=1}^{n} f(x_i,\theta)$$

(2) Take logarithm of likelihood function (base 'e').
Since log is increasing function so resultant will be same either we maximize $L(\theta)$ or $\log L(\theta)$. But taking log will reduce the calculation difficulties.

$$l = \log L(\theta) = \ln f(x_1,\theta) + \ln f(x_2,\theta) + \cdots + \ln f(x_n,\theta)$$

$$= \sum_{i=1}^{n} \ln f(x_i,\theta) \qquad \left(\because \ln(x_1 x_2 \cdots x_n) = \ln x_1 + \ln x_2 + \cdots + \ln x_n \right)$$

③ Take partial derivative w.r.t. θ

$$\frac{\partial l}{\partial \theta} = \frac{\partial}{\partial \theta}\left[\sum_{i=1}^{n} \ln f(x_i;\theta)\right]$$

④ For maxima put $\frac{\partial l}{\partial \theta} = 0$ and find the value of θ in terms of x. and check that

$$\frac{\partial^2 l}{\partial \theta^2} < 0 \text{ then}$$

$$\hat{\theta} = \theta(x) \text{ is MLE for } \theta.$$

Ex

Let $X_1, X_2, \cdots, X_n \overset{iid}{\sim} P(\lambda)$. Find MLE for λ.

Sol^n

$$P(\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$L(\lambda) = \frac{e^{-\lambda}\lambda^{x_1}}{x_1!} \cdot \frac{e^{-\lambda}\lambda^{x_2}}{x_2!} \cdots \frac{e^{-\lambda}\lambda^{x_n}}{x_n!}$$

$$l = L(\lambda) = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!} = e^{-n\lambda} \cdot \lambda^{\sum_{i=1}^{n} x_i} \prod_{i=1}^{n} \frac{1}{x_i!}$$

Next, take log both side,

$$\ln l = -n\lambda + \left(\sum_{i=1}^{n} x_i\right)\ln\lambda + \ln\left(\prod_{i=1}^{n}\frac{1}{x_i!}\right)$$

Next take partial diff. wrt λ,

$$\frac{\partial \ln l}{\partial\lambda} = -n + \frac{\sum_{i=1}^{n} x_i}{\lambda} + 0$$

For maxima,

$$\frac{\partial \ln l}{\partial\lambda} = 0 \Rightarrow -n + \frac{\sum_{i=1}^{n} x_i}{\lambda} = 0$$

$$\Rightarrow \lambda = \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x}$$

Now, $$\frac{\partial^2 \ln l}{\partial\lambda^2} = \frac{-\sum_{i=1}^{n} x_i}{\lambda^2} < 0 \text{ for } \lambda = \bar{x},$$

Then, $\hat{\lambda} = \bar{x}$ is MLE for λ.