(i) $P_X(x_i) \geq 0$

(ii) $\sum_{x_i \in R_X} P_X(x_i) = 1$.

This is the pmf for random variable X.

$X = x_i$	0	1	2	3
$P_X(x_i)$	1/8	3/8	3/8	1/8

Cumulative Distribution Function (cdf)

The commulative distribution function (cdf) $F(x)$ of a discrete random variable X with pmf $p(x)$ is defined for every number x by

$$F(x) = P(X \leq x) = \sum_{y : y \leq x} p(y)$$
$$= P(-\infty < X \leq x)$$

where x = observed value / values from range space.

Ex. Find cdf in the above example.

$$F_X(2) = F(2) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}.$$

$$F_X(1) = F(1) = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$$

$$F_X(0) = F(0) = \frac{1}{8}.$$

properties of cdf

(1) $F_X(\infty) = P(-\infty < x \leq \infty) = 1$.

(2) $F(-\infty) = 0$

(3) $0 \leq F_X \leq 1$.

(4) cdf is non-decreasing function, that is, either it increases or remain constant.

(5) cdf is right continious, that is,

$$\lim_{h \to 0} F_X(x+h) = F_X(x).$$

(6) $P(X > x) = 1 - P(X \leq x)$.

Interval properties of cdf:

For $a < b$:

(1) $P(a < X \leq b) = F(b) - F(a)$.

(2) $P(a < X < b)$

$= P(a < X < b) + P(X = b) - P(X = b)$

$= P(a < X \leq b) - P(X = b)$

$= F(b) - F(a) - P(X = b)$.

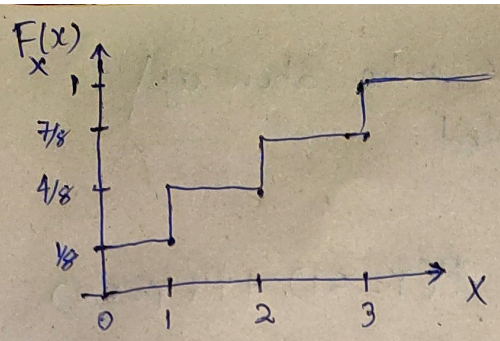
(3) $P(a \leq X < b)$

$= P(a < X < b) + P(X = a)$.

$= F(b) - F(a) + P(X = a) - P(X = b)$.

plot the cdf of last example:

$F(0) = \frac{1}{8}$, $F(1) = \frac{4}{8}$, $F(2) = \frac{7}{8}$, $F(3) = 1$.



$$F_x(x) = \begin{cases} 0 & x < 0 \\ 1/8 & 0 \le x < 1 \\ 4/8 & 1 \le x < 2 \\ 7/8 & 2 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

Ex. Find pmf by above cdf.

$$\boxed{P_x(x_i) = F_x(x_i) - F_x(\bar{x}_i)}$$

So, $P_x(3) = F_x(3) - F_x(2) = 1 - \frac{7}{8} = \frac{1}{8}$.

$P_x(2) = F_x(2) - F_x(1) = \frac{7}{8} - \frac{4}{8} = \frac{3}{8}$

$P_x(1) = F_x(1) - F_x(0) = \frac{3}{8}$.

$P_x(0) = F_x(0) - f_x(-\infty) = \frac{1}{8} - 0 = \frac{1}{8}$.

Ex. Q12 (Sec 3.2)

A plane with 50 seats, 55 passangers have ticket.

Y = no. of ticketed passangers who actually show up for the flight.

pmf of Y is given in the table.

(a) P(accommodate all ticketed passangers who show up)

$= P(Y \le 50)$

$= P(Y=45) + P(Y=46) + P(Y=47) + P(Y=48) + P(Y=49) + P(Y=50)$.

$= 0.05 + 0.1 + 0.12 + 0.14 + 0.25 + 0.17$

$= 0.83$.

(b) P(not all ticketed passangers who show up can be accommodated)

$= P(Y > 50)$

$= 1 - P(Y \leq 50)$

$= 1 - 0.83 = 0.17.$

(or) $P(Y=51) + P(Y=52) + P(Y=53) + P(Y=54) + P(Y=55) = 0.17.$

(c) $P(Y \leq 49) = P(45) + P(46) + P(47) + P(48) + P(49)$

$= 0.66.$

$P(Y \leq 47) = P(45) + P(46) + P(47) = 0.27.$