For first order,

$$\left.\frac{dM_x(t)}{dt}\right|_{t=0} = n(pe^t+q)^{n-1}\, pe^t \Big|_{t=0}$$

$$= pn(\underbrace{p+q}_{1})^{n-1}$$

$$= np.$$

Exercise 1) Find Moment generating fun. of Poisson's distribution.

11) Given an example where $E(X)$ doesn't exist.

Soln: 1) $M_x(t) = E(e^{tx})$

$$= \sum_{x=0}^{\infty} e^{tx} \cdot \frac{e^{-\mu} \cdot \mu^x}{x!}$$

$$= e^{-\mu} \sum_{x=0}^{\infty} \frac{e^{tx} \cdot \mu^x}{x!}$$

$$= e^{-\mu} \sum_{x=0}^{\infty} \frac{(\mu e^t)^x}{x!}$$

$$= e^{-\mu}\, e^{\mu e^t}$$

$$= e^{\mu e^t - \mu} = e^{\mu(e^t-1)}.$$

$$\left.\frac{dM_x(t)}{dt}\right|_{t=0} = e^{\mu(e^t-1)} \cdot \mu e^t \Big|_{t=0}$$

$$= \mu.$$

Q. Let X be a discrete rv with pmf $P_X(x) = \frac{K}{x^3}$, where $x = 1, 2, 3, \dots$ Find $E(X)$. Does it exist.

Soln $E(X) = \sum_{x=1}^{\infty} x \cdot \frac{K}{x^3}$ $\qquad \underbrace{1 + \frac{1}{4} + \frac{1}{9} + \cdots}_{} = \frac{\pi^2}{6}$

$= \sum_{x=1}^{\infty} \frac{K}{x^2} = K \sum_{x=1}^{\infty} \frac{1}{x^2}$

$= \frac{K\pi^2}{6}$ //

ii) Soln Let $p_X(x) = \frac{K}{x^2}$, $x = 1, 2, \dots$

$E(X) = \sum_{x=1}^{\infty} x \frac{K}{x^2}$

$= K \sum_{x=1}^{\infty} \frac{1}{x} \longrightarrow$ Divergent Series.

Note $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$

" is divergent if $0 \leq p \leq 1$.

Discrete Uniform Distribution

A random variable X has a discrete ~~random variable~~ uniform distribution if each of the 'n' values in its range, say $x_1, x_2, \dots, x_n$, has equal probability.

Then X has pmf

$$p(x) = \begin{cases} \frac{1}{n} & \text{for } x = 1, 2, \dots, n \\ 0 & \text{Otherwise.} \end{cases}$$

Compute $E(X)$ and $V(X)$.

x	1	2	$\cdots$	n
$p(x)$	$1/n$	$1/n$	$\cdots$	$1/n$

$$\therefore E(X) = \sum_{x=1}^{n} x\, p(x) = \sum_{x=1}^{n} x \cdot \frac{1}{n}$$

$$= \frac{1}{n} \sum_{x=1}^{n} x$$

$$= \frac{1}{n} \frac{n(n+1)}{2} = \frac{n+1}{2} \, //$$

$$E(X^2) = \sum_{x=1}^{n} x^2\, p(x) = \frac{1}{n} \sum_{x=1}^{n} x^2$$

$$= \frac{1}{n} \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{(n+1)(2n+1)}{6}.$$

$$\therefore V(X) = E(X^2) - (E(X))^2$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = \frac{n^2-1}{12} \, //$$

Ex. Roll a die and let X be the upward face showing. Find $E(X)$ and $V(X)$.

Sol^n It's a uniform distribution.

$$E(X) = \frac{6+1}{2} = \frac{7}{2}$$

$$V(X) = \frac{6^2-1}{12} = \frac{35}{12} \, //$$