Interval Estimation

Let $x_1, x_2, ..., x_n$ be a random sample from a population with pdf $f(x_1, \theta)$, $\theta \in \Theta$.

- (a) A random Interval is an interval whose end points are trandom variables.
- (b) A confidence interval for θ with confidence—coefficient $(1-\alpha)$; $0<\alpha<1$ is a random interval whose end points are statistics, say $L(x_1,x_2,...,x_n)$ and $U(x_1,x_2,...,x_n)$

Such that $L(X) \in U(X)_g$ where $X = (X_1, X_2, \dots, X_n)$ and

Then [L(X), U(X)] is called 100(1-x)% confidence interval for θ .

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1 Confidence Interval for Mor the Normal distribution
  with oit known motors to the company in more
  step I. Choose a confidence level (957, 997. for)
   Step II. Defermine the corrresponding c'
   33134 383+019018+001951 00019 00199 00199
          c 1.645 1.960 2.576 3.291
  step III compute the mean is of the sample x1, x2, --, x3
 step 14 Compute k = \frac{26}{\sqrt{n}}. The confidence Byterival
 17008-84 100 CONESTER 3707 5104 ( 113-8607 )
            CONF, { x-k & M & x+k }.
 Ex Defermine and 15% confidence Interval for the
     mean of a normal dist. with variance 62=9, using
   a sample of n=100 values with mean \(\tilde{\pi} = 5.
  5017 Step 5 / 1 = 0.95
   stip II Determine the solution of the political
        step 111 7 = 5 (given) = 00) 7
   (1-10) Stepilly (10) K=1-C16 t= 1:960 x13 = 0.588. mail
  Hence 5-1- 5-0.588 = 4.412 - 300 1 10 2000 196
  monthus, 95 %, confédence interval forz ne is
              (CONF 95 } A. A12 Sq M & 5. 5.88 } . 70 3
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Find al 95% confidence Intervals for most à normal destribation with standard deviation 1.4. (from the sample 30, 42, 40, 34, 48, 50. some step I 9 = 0.95 step II octemine the commodalities it gots Step IV $K = \frac{1.96 \times 4}{\sqrt{n}} = 3.2006$. Now (x-k, x+k) will be 95% confidence interval for u, that is, CONF 237,459 & M & 43-8607}. CONF, { Set & M & STAN ? I A II Confidence Interval for M of the Normal Distribution with unknown of ence Level & 1957, 99%, or EPO = 1 (the like July step II. Determine the solution, ich of their equation F(c) = = (14) == == 111 gold from the table of t-distribution with (n-1)degrees of freedomine 38820-2 = N-R (Table A9 in Appendix; where n=sample size) Step III, Compute the mean it and the sample vaniance

step 111, Compute the mean \bar{x} and the sample variance s^2 of the sample (x_1, x_2, \dots, x_n) .

Hepflying Computer : Komacs no fransque is the The confidence intereval Ps. 19 CONF, \$ x-K & M & x+K]. Is seen some a gg i Cuffiginic Anterval Chi-Square Distribution Of $X \sim N(0,1)$, then $X^2 \sim X^2$. () (chi-square) with 1-degree of freedom) Degree of Freedom of distribution is sum of squares of standard norzmal distribution. Eg. to If (X ~ N(0,1) Then (3) x2 ~ X2 (1).4.6 ② If $x_1, x_2 \stackrel{\text{cig}}{\sim} N(0,1)$ then $x_1^2 + x_2^2 \sim x_2^2$: Hitshit dalf fall fall = \overline{x} 3 If X1, X2, *.., Xm is N(0,1) then $\sum_{i=1}^{N} x_{i}^{2} \sim \chi_{(n)}^{2}$. The pdf of chi-squarce distribution is given as $\frac{f(x) = \frac{x^{\frac{1}{2}-1} e^{-\frac{x}{2}}}{(e^{-\frac{x}{2}+1} - 1)^{\frac{1}{2}} + (e^{-\frac{x}{2}})^{\frac{1}{2}}} = \frac{x^{\frac{1}{2}-1} e^{-\frac{x}{2}}}{(e^{-\frac{x}{2}+1} - 1)^{\frac{1}{2}} + (e^{-\frac{x}{2}})^{\frac{1}{2}}} = \frac{x^{\frac{1}{2}-1} e^{-\frac{x}{2}}}{(e^{-\frac{x}{2}+1} - 1)^{\frac{1}{2}} + (e^{-\frac{x}{2}})^{\frac{1}{2}}} = \frac{x^{\frac{1}{2}-1} e^{-\frac{x}{2}}}{(e^{-\frac{x}{2}+1} - 1)^{\frac{1}{2}}} = \frac{x^{\frac{1}{2}-1} e^{-\frac{x}{2}}}}{(e^{-\frac{x}{2}+1} - 1)^{\frac{1}{2}}} = \frac{x^{\frac{1}{2}-1} e^{-\frac{x}{2}}}}{(e$ Students t- distribution. If $X \sim N(0,1)_X$ and $Y \sim \chi^2(n)_Y$ and X and Y are independent, then CONFIGENCE INTERVED IS SUMMERS THOUS SUMMERS TO MIXING THE SUMBLE SUMBLE

Eg. Find independent measurements of the point of inflammation of Diesel oil gove the values 14A 147 146 142 144. Assuming normality, determine a 99% confidence înterval for the mean. Ohi - Square Distribution Solo step 1. To 0:99 with cools x (repried step 11. (F(0) = 1 = (1+7) E with divisit to me loss of the colors of the sone of Here n=5, 250 the value of 62 for which F(c) = 0.995 with degree of Freedom (5-1)=4 is 4.60.

Step 111 $\overline{\chi} = \frac{144+147+146+142+144}{5}$ 3 15 81, X2, x ... Xm . (1914) then 5 x2 ... x ... x ... 21 ... - (1) (149-144.6) + (147-14A.6) + (146-144.6) 2 + (42-144.6)2+(144-144.6)2. = 3.8 Step 1V" $K = \frac{14.6 \times \sqrt{3.8}}{\sqrt{5}} = 4.01$

The confidence interval is confidence interval is

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Il Determination rofo a confidence Interval for the
  variance of a moremal distribution, whose mean
 need not be known: and pass was not per to de
 step s. choose à confidence level j' (95, 99% or live this)
           Determine solms. C, and c2 of the equations
          F(c_1) = \frac{(1-y)}{2} \cdot F(c_2) = \frac{(1+y)}{2}
      from the table of the chi-square distribution
     with n-1 degrees of freedom (Table 10 in App. 5)
      11. Compute k_1 = \frac{(n-1)s^2}{c_1} and k_2 = \frac{(n-1)s^2}{c_2}.

The confidence interval is \frac{(x)}{(x)}
 Step III.
                     CONFY & Ky Soft Sono sould send
Eg. Determine (av 95 % confidence (Interval for
   the variance with sample
           89, 84, 87, 81, 89, 86, 91, 90, 78, 89, 87, 99, 89, 89.
       step 1. 21.4 = (0.95 - cons) $ = 0000 0
2012
        step 11. For n-1=13, we have
         F(G) = \frac{1}{2} (1-0.95) = 0.025 : \Rightarrow C_1 = F^{-1}(0.025) = 5.01
         F(C_2) = \frac{1}{2} (1 + 0.95) = 0.975 \Rightarrow C_2 = F^{-1} (0.975) = 24.74
                             (From X table with 13 degree of freedom)
                     K_1 = \frac{13 \text{ s}^2}{C_1} = \text{ and } K_2 = \frac{13 \text{ s}^2}{C_2}
      step 111.
 13/03: Where 300 05? = 13 So (x, 1) 2 and x = 89 +84+ ... +89
            I the Load to exceed section, 2000 kg.
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If the weight X of bags of cement is normally distributed, within a mean of a 40 kg and according 3D of 2kg, how many bags can ra delivery touce (carry so that the probability of the total load exceeding 2000 kg will be 5%. Soli Given E(x) = 40 and $6 = 2(3) = 6^2 = 4$ nother interior answers 7,000) = 10.05 last out most (2.99A m GIP (1X) \$ 2000) = 150.05 = 0.95 = 0.95 = 0.95 Using the standardization process, we get $p\left(\frac{x^{2}-E(x)}{\sqrt{V(x)}} \le \frac{2000-40n}{2\sqrt{n}}\right) = 0.95$ Since, there are I'm bags, 17 7400 THE (x) = 40 m and V(x) = 4 morring costs of - 18 3 Pr(Z8 83 82000 - 4000) PE 98958, AR. P8 $\Rightarrow \Phi(2) \Rightarrow \Phi(\frac{2000-40\eta}{2\sqrt{\eta}}) = 0.95$ => 2000-40n > 1445-10 stor . 11 doh 16.0 = (2000) = = p = 200.0 = (200-1) = (p) 7 Fig = (29) = (2000 - 40m - 2.9 \bar{m} \bar{m} \bar{m}) = (p) 7 F(6)= 2 (1+0.95) = -7.11 7.7.63. > Jn = 7.03 - 19 5 m 2 49 ban 3 5 5 = 18 i We need 495 bags loaded on a tower, in order for the load to exceed 200 kg.