The standard Normal Distribution

(*) If X is a normal rv with parameters μ and σ, then $$P(a \leq X \leq b) = \int_a^b \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$$

(*) The normal distribution with parameter values $\mu = 0$ and $\sigma = 1$ is called the standard normal distribution.

(*) A rv having a standard normal distribution is called a standard normal rv and will be denoted by Z. The pdf of Z is

$$f(z;0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad \text{for } -\infty < z < \infty.$$

(*) The graph of $f(z;0,1)$ is called the standard normal or (z) curve.

(*) The cdf of Z is $$P(Z \leq z) = \int_{-\infty}^{z} f(y;0,1)\,dy = \Phi(z).$$

Proposition

If X has a normal distribution with mean μ and standard deviation σ, then $Z = \frac{X-\mu}{\sigma}$, has a standard normal distribution. Thus,

$$P(a \leq X \leq b) = P\left(\frac{a-\mu}{\sigma} \leq Z \leq \frac{b-\mu}{\sigma}\right)$$
$$= \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

$\therefore P(X \leq a) = \Phi\left(\frac{a-\mu}{\sigma}\right)$; $P(X \geq b) = 1 - \Phi\left(\frac{b-\mu}{\sigma}\right)$.

Q// Let Z be a standard normal rv and calculate the following probabilies.

a) $P(0 \leq Z \leq 2.17) = \Phi(2.17) - \Phi(0)$

Check Appendix Table A.3.

$= 0.985 - 0.5 = 0.4850.$

e) $P(Z \leq 1.37) = \Phi(1.37) = 0.9147$

f) $P(-1.75 < Z) = 1 - \Phi(-1.75)$

$= 1 - 0.0401 = 0.9599.$

Q Determine the value of the constant 'c',

a) $\Phi(c) = 0.9838.$

From Table A3, $\Phi(2.14) = 0.9838$

$\Rightarrow c = 2.14$

b) $P(0 \leq Z \leq c) = 0.291$

$\Phi(c) - \Phi(0) = 0.291$

$\Rightarrow \Phi(c) = 0.291 + \Phi(0) = 0.291 + 0.5$

$\Rightarrow c = 0.81$

Q// Suppose the force acting on a coloumn that helps to support a building is a normally distributed rv X with $\mu = 15.0$ kips and $\sigma = 1.25$ kips. Compute the following probabilities by standardizing.

Selected Questions: 28 (a, c, e, f, i, j)

29, 32, 35, 43, 47, 53, 57.

a) $P(X \leqslant 15)$ standardizing gives.

$X \leqslant 15$ iff $\quad \frac{X-\mu}{\sigma} \leqslant \frac{15-15}{1.25}$

$$\frac{X-15}{1.25} \leqslant 0$$

$$Z \leqslant 0$$

$$\therefore P(X \leqslant 15) = P(Z \leqslant 0) = \Phi(0)$$

$$= 0.5.$$

b) $P(X \geqslant 10)$

$X \geqslant 10$ iff $\quad \frac{X-15}{1.25} \geqslant \frac{10-15}{1.25} = \frac{-5}{1.25}$

$$Z \geqslant -4$$

$$P(X \geqslant 10) = P(Z \geqslant -4)$$

$$= 1 - \Phi(-4)$$

$$= 1 - 0.0001 = 0.9999.$$

c) $P(14 \leqslant X \leqslant 18)$

$14 \leqslant X \leqslant 18$ iff $\quad \frac{14-15}{1.25} \leqslant \frac{X-15}{1.25} \leqslant \frac{18-15}{1.25}$

$$-0.8 \leqslant \frac{X-15}{1.25} \leqslant 2.4.$$

$$-0.8 \leqslant Z \leqslant 2.4$$

$$P(14 \leqslant X \leqslant 18) = P(-0.8 \leqslant Z \leqslant 2.4)$$

$$= \Phi(2.4) - \Phi(-0.8)$$

$$= 0.9918 - 0.2119 = 0.7799.$$