→ The term probability refers to the study of randomness and uncertainty.

→ In any situation in which one of a number of possible outcomes may occur, the discipline of probability provides methods for quantifying the chances, or likelihoods, associated with the various - outcomes.

Random Experiment:

An experiment is said to random experiment, if it satisfies the following conditions.

(i) All the outcomes are known in advance.

(ii) In a particular trial, we don't know what is the outcome.

(iii) We can repeat the experinment as many times as we wish.

Example: ① Tossing a coin.

Outcomes: $\{H, T\}$, $S = \{H, T\}$

② Rolling a dice.

Outcomes: $\{1, 2, 3, 4, 5, 6\}$, $\Omega = \{1, 2, 3, 4, 5, 6\}$.

Sample Space:

It is a collection of all possible outcomes of the random experiment, usually it is denoted as S or Ω.

Event: An event is any collection (subset) of outcomes contained in the sample space.

Example: Tossing Coin

$S = \{H, T\}$.

Events: $\underset{(\times)\parallel_{S}}{\{H,T\}}, \{H\}, \{T\}, \underset{(\times)}{\phi}$

Note: S and ϕ can not be events.

→ The **complement** of an event A, denoted by A', is the set of all outcomes in S that are not contained in A.

classical Definition of probability.

Let S be the sample space associated with random experinment. Let the total number of all possible outcomes in S be n (each and every-event are equally likely). Let A be the event and m be the outcome, that is favourable to A, then, probability of event A is

$$P(A) = \frac{m}{n} = \frac{\text{favourable to A}}{\text{total outcomes}}$$

Example:

Determine the probability of getting an even number when rolling a die.

→ 3 favorable outcomes because there are 3 even numbers in a die.

→ Number of possible outcomes would be 6.

$$\therefore P\left(\begin{array}{c}\text{getting an even number}\\ \text{when rolling a die}\end{array}\right) = \frac{3}{6} = \frac{1}{2}$$

Mutually Exclusive Event :

Let A and B are two events. from sample-space associated with some random experinment, then, event A and B are said to be mutually-exclusive if they can not happen together.

Example: 1. The two possible outcomes of a coin flip are mutually exclusive.

2. Rain and sunshine are not mutually exclusive.

<u>Exercise</u>: 1. Mutually exclusive events with a die.

a. Rolling a number div by 2 or rolling a no. div. by 3

b. Rolling a no. div. by 2 or rolling a no. that is multiple of 5

c. Rolling a prime no. or rolling an even no.

d. Rolling a non-prime no. or rolling an odd no.

Mathematically: $A \cap B = \phi$.

Mutually Exhaustive Event.

A set of event is said to be mutually-exhaustive if at least one of the event must occur.

Mathematically: $A \cup B = S$.

When more than two sets, it is called collectively exhaustive.

Example: When rolling a die, the events 1, 2, 3, 4, 5, and 6 are collectively exhaustive.

Axiomatic Definition of probability (Kolmogorov's Three Axioms.)

Let S be the sample space and B be the collection of all subsets of S then the probability of an event $A \in B$ is denoted as $P(A)$, and it must satisfy the following properties.

(i) $P(A) \geq 0 \quad \forall \ A \in B$.

(ii) $P(S) = 1$

(iii) If $A_1, A_2, \ldots, A_n$ are pairwise disjoint or mutually exclusive, then,

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) \quad , \text{ i.e.,}$$

$$P\left(A_1 \cup A_2 \cup \cdots \cup A_n\right) = P(A_1) + P(A_2) + \cdots + P(A_n).$$

Some important results:

① $P(\phi) = 0$, where ϕ is the null event.

proof: We know that $P(S) = 1$ and S, ϕ are disjoint, that is,

$$S \cup \phi \cup \phi \cup \cdots = S.$$

$$\Rightarrow P\left(S \cup \phi \cup \phi \cup \cdots\right) = P(S) = 1.$$

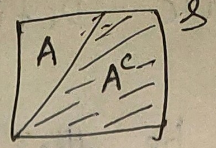
Since, S and ϕ are disjoint (mutually exclusive)

$$\Rightarrow P(S) + P(\phi) + P(\phi) + \cdots = 1$$

$$\Rightarrow 1 + P(\phi) + P(\phi) + \cdots = 1$$

$$\Rightarrow P(\phi) = 0.$$

② $P(A^c) = 1 - P(A)$



proof: We know that

$$A \cup A^c = S$$

$$\Rightarrow P(A \cup A^c) = P(S)$$

Since A and A^c are disjoint, we have.

$$\Rightarrow P(A) + P(A^c) = 1$$

$$\Rightarrow P(A^c) = 1 - P(A).$$

③ $0 \le P(A) \le 1$

proof: ~~P(A)≥0~~ $P(A) \geq 0$ (By Kolmogorov's axioms)

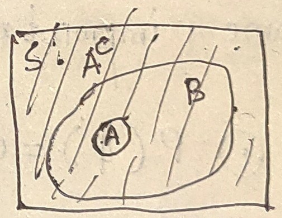
$$P(A^c) = 1 - P(A) \le 1 \quad (\text{since } P(A^c) \geq 0)$$

$$\Rightarrow P(A) \le 1$$

$$\Rightarrow 0 \le P(A) \le 1.$$

④ If A is a subset of B, then,

$$P(A) \le P(B).$$



proof: Let $A \subseteq B$.

$$\Rightarrow B = A \cup (A^c \cap B).$$

$$P(B) = P(A \cup (A^c \cap B))$$

$$= P(A) + P(A^c \cap B)$$ ($\because$ A and $A^c \cap B$ are mutually exclusive.)

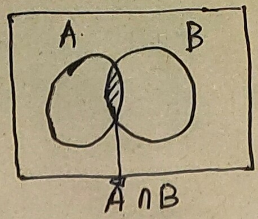
$$\Rightarrow P(B) - P(A) = P(A^c \cap B) \geq 0.$$

$$\Rightarrow P(B) - P(A) \geq 0$$

$$\Rightarrow P(B) \geq P(A).$$ //

Note: $P(A) \le P(B) \not\Rightarrow A \subseteq B$.

⑤ $P(A\cap B^c) = P(A) - P(A\cap B)$



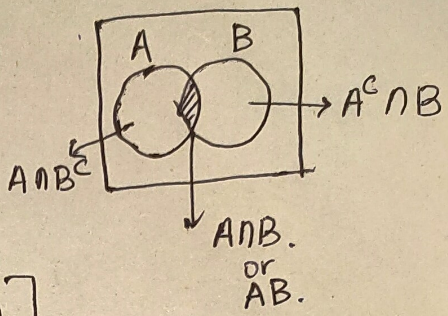
proof $A = (A\cap B) \cup (A\cap B^c)$

$P(A) = P((A\cap B) \cup (A\cap B^c))$

$\Rightarrow P(A) = P(A\cap B) + P(A\cap B^c).$

$\Rightarrow P(A\cap B^c) = P(A) - P(A\cap B).$

⑥ $P(A\cup B) = P(A) + P(B) - P(A\cap B).$



proof:

$A\cup B = (A\cap B^c) \cup (A\cap B) \cup (A^c\cap B).$

$P(A\cup B) = P[(A\cap B^c) \cup (A\cap B) \cup (A^c\cap B)]$

$= P(A\cap B^c) + P(A\cap B) + P(A^c\cap B)$

$= P(A) - P(A\cap B) + P(A\cap B) + P(B) - P(A\cap B)$

$= P(A) + P(B) - P(A\cap B).$ //

⑦ For any three events A, B and C,

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC).$

proof: $P(A\cup B\cup C)$

$= P(A) + P(B\cup C) - P(A\cap(B\cup C))$

$= P(A) + P(B) + P(C) - P(B\cap C) - P((A\cap B)\cup(A\cap C))$

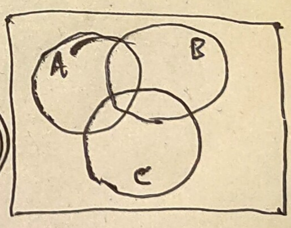


Figure $A\cup B\cup C$.

$= P(A) + P(B) + P(C) - P(B\cap C) - \big(P(A\cap B) + P(A\cap C) - P(A\cap B\cap C)\big)$

$= P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) - P(A\cap B\cap C).$ //