Expected values of a random variable (Mean)

Let X be a discrete random variable, with pmf $p_X(x)$. The expected value or mean value of X, denoted by $E(X)$ or μ_X or just μ, is

$$E(x) = \sum_{x_i \in R_X} x_i \, p_X(x_i) .$$

Basically, it gives the average value of a random variable. Also, this gives the central-tendancy of observed data.

Example: In previous example find expected values

$x_i =$	0	1	2	3
$p_X(x_i) =$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$E(x) = x_1 p_X(x_1) + x_2 p_X(x_2) + x_3 p_X(x_3) + x_4 p_X(x_4)$$

$$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} .$$

$$= 1.5 .$$

$\therefore$ On average we get 1.5 times Head.

* properties of Expectation.

1) $E(a) = a$ for any constant a.

proof $\sum_{x_i \in R_X} a \cdot p_X(x_i) = a \sum_{x_i \in R_X} p_X(x_i)$

$$= a \times 1 = a .$$

2) For any constant b,

$$E(X + b) = E(X) + b .$$

3) $E(ax+b) = aE(x) + b.$

proof $\quad E(ax+b) = \sum_{x_i \in R_x} (ax+b)\, p(x)$

$$= a \sum_{x_i \in R_x} x\, p(x) + b \underbrace{\sum_{x_i \in R_x} p(x)}_{=1}$$

$$= a E(x) + b$$

The variance of X.

Let X have pmf $p(x)$ and expected value μ. Then the varience of X, denoted by $V(X)$ or σ_X^2 or just σ^2, is

$$V(X) = \sum_{x_i \in R_X} (x-\mu)^2\, p(x) = E[(X-\mu)^2].$$

The standard deviation (SD) of X is

$$\sigma_X = \sqrt{\sigma_X^2}$$

(*) Let X be a random variable, then variance of X is denoted by

$$V(X) = E[(X - E(X))^2]$$
$$= E[X^2 - 2XE(X) + (E(X))^2]$$
$$= E(X^2) - E(2XE(X)) + E((E(X))^2)$$
$$= E(X^2) - 2E(X)E(X) + (E(X))^2$$
$$= E(X^2) - 2(E(X))^2 + (E(X))^2$$
$$= E(X^2) - (E(X))^2$$

∴ Variance measures the variablity of the data from it's average value/expected value.

* For any function $g(x)$

$$E(g(x)) = \sum_{x \in R_X} g(x) \cdot P_X(x).$$

properties of Variance :-

1) $V(a) = E(a^2) - (E(a))^2$

$= a^2 - (a)^2 = 0.$

2) $V(ax+b) = E((ax+b)^2) - (E(ax+b))^2$

$= E(a^2x^2) + E(2axb) + E(b^2) - (aE(x)+b)^2$

$= a^2E(x^2) + \cancel{2ab E(x)} + \cancel{E(b^2)} - a^2(E(x))^2 - \cancel{2ab E(x)} - \cancel{b^2}$

$= a^2 [E(x^2) - (E(x))^2]$

$= a^2 V(x).$

3) Let X and Y be independent variables, then

$$Var(X+Y) = Var(X) + Var(Y).$$

4) If X and Y are not independent, then

$$Var(X+Y) = Var(X) + Var(Y) + 2\,cov(X,Y).$$