

Modern Quantum Chemistry, Szabo & Ostlund

HW

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4 Configuration Interaction

4.1 Multiconfigurational Wave Functions and the Structure of Full CI Matrix

4.1.1 Intermediate Normalization and an Expression for the Correlation Energy

Ex 4.1 If $a \notin \{c, d, e\}$ and $r \notin \{t, u, v\}$,

$$\langle \Psi_a^r | \mathcal{H} | \Psi_{cde}^{tuv} \rangle = 0 \quad (4.1.1)$$

Let's suppose $a = e$, thus

$$\langle \Psi_a^r | \mathcal{H} | \Psi_{cde}^{tuv} \rangle = \langle \Psi_a^r | \mathcal{H} | \Psi_{acd}^{vtu} \rangle \quad (4.1.2)$$

if $r \neq v$, this term will still be zero, thus

$$\sum_{c < d < e, t < u < v} c_{cde}^{tuv} \langle \Psi_a^r | \mathcal{H} | \Psi_{cde}^{tuv} \rangle = \sum_{c < d, t < u} c_{acd}^{rtu} \langle \Psi_a^r | \mathcal{H} | \Psi_{acd}^{rtu} \rangle \quad (4.1.3)$$

Ex 4.2

$$\begin{vmatrix} -E_{\text{corr}} & K_{12} \\ K_{12} & 2\Delta - E_{\text{corr}} \end{vmatrix} = 0 \quad (4.1.4)$$

$$-E_{\text{corr}}(2\Delta - E_{\text{corr}}) - K_{12}^2 = 0 \quad (4.1.5)$$

$$E_{\text{corr}} = \frac{2\Delta \pm \sqrt{4\Delta^2 + 4K_{12}^2}}{2} = \Delta \pm \sqrt{\Delta^2 + K_{12}^2} \quad (4.1.6)$$

choosing the lowest eigenvalue,

$$E_{\text{corr}} = \Delta - \sqrt{\Delta^2 + K_{12}^2} \quad (4.1.7)$$

Ex 4.3 At $R = 1.4$,

$$\begin{aligned} \Delta &= \varepsilon_2 - \varepsilon_1 + \frac{1}{2}(J_{11} + J_{22}) - 2J_{12} + K_{12} \\ &= 0.6703 + 0.5782 + \frac{1}{2}(0.6746 + 0.6975) - 2 \times 0.6636 + 0.1813 \\ &= 0.78865 \end{aligned} \quad (4.1.8)$$

$$E_{\text{corr}} = \Delta - \sqrt{\Delta^2 + K_{12}^2} = 0.78865 - \sqrt{0.78865^2 + 0.1813^2} = -0.020571 \quad (4.1.9)$$

$$c = \frac{E_{\text{corr}}}{K_{12}} = \frac{-0.020571}{0.1813} = -0.1135 \quad (4.1.10)$$

As $R \rightarrow \infty$, $\varepsilon_2 - \varepsilon_1 \rightarrow 0$, all 2e integrals $\rightarrow \frac{1}{2}(\phi_1\phi_1|\phi_1\phi_1)$, thus

$$\lim_{R \rightarrow \infty} \Delta = 0 + \lim_{R \rightarrow \infty} \left[\frac{1}{2}(J_{11} + J_{22}) - 2J_{12} + K_{12} \right] = 0 \quad (4.1.11)$$

$$\lim_{R \rightarrow \infty} E_{\text{corr}} = - \lim_{R \rightarrow \infty} K_{12} \quad (4.1.12)$$

$$\lim_{R \rightarrow \infty} c = \lim_{R \rightarrow \infty} \frac{E_{\text{corr}}}{K_{12}} = -1 \quad (4.1.13)$$

As $R \rightarrow \infty$, the full CI wave function will be

$$|\Phi_0\rangle = |\Psi_0\rangle - |\Psi_{11}^{2\bar{2}}\rangle = |\psi_1\bar{\psi}_1\rangle - |\psi_2\bar{\psi}_2\rangle \quad (4.1.14)$$

Since

$$\psi_1 = \frac{1}{\sqrt{2(1 + S_{12})}}(\phi_1 + \phi_2) \quad (4.1.15)$$

$$\psi_2 = \frac{1}{\sqrt{2(1 - S_{12})}}(\phi_1 - \phi_2) \quad (4.1.16)$$

we get

$$|\psi_1\bar{\psi}_1\rangle = \frac{1}{2(1+S_{12})}(|\phi_1\bar{\phi}_1\rangle + |\phi_1\bar{\phi}_2\rangle + |\phi_2\bar{\phi}_1\rangle + |\phi_2\bar{\phi}_2\rangle) \quad (4.1.17)$$

$$|\psi_2\bar{\psi}_2\rangle = \frac{1}{2(1-S_{12})}(|\phi_1\bar{\phi}_1\rangle - |\phi_1\bar{\phi}_2\rangle - |\phi_2\bar{\phi}_1\rangle + |\phi_2\bar{\phi}_2\rangle) \quad (4.1.18)$$

As $R \rightarrow \infty$, $S_{12} \rightarrow 0$, thus

$$|\Phi_0\rangle = |\psi_1\bar{\psi}_1\rangle - |\psi_2\bar{\psi}_2\rangle = \frac{1}{2}(|\phi_1\bar{\phi}_2\rangle + |\phi_2\bar{\phi}_1\rangle) \quad (4.1.19)$$

Renormalize it, we get

$$|\Phi_0\rangle = \frac{1}{\sqrt{2}}(|\phi_1\bar{\phi}_2\rangle + |\phi_2\bar{\phi}_1\rangle) \quad (4.1.20)$$

4.2 Doubly Exited CI

4.3 Some Illustrative Calculations

4.4 Natural Orbitals and the 1-Particle Reduced DM

Ex 4.4

$$\gamma_{ij} = \int d\mathbf{x}_1 d\mathbf{x}'_1 \chi_i^*(\mathbf{x}_1) \gamma(\mathbf{x}_1, \mathbf{x}'_1) \chi_j(\mathbf{x}'_1) \quad (4.4.1)$$

$$\begin{aligned} \gamma_{ji}^* &= \int d\mathbf{x}_1 d\mathbf{x}'_1 \chi_j(\mathbf{x}_1) \gamma^*(\mathbf{x}_1, \mathbf{x}'_1) \chi_i^*(\mathbf{x}'_1) \\ &= \int d\mathbf{x}'_1 d\mathbf{x}_1 \chi_j(\mathbf{x}'_1) \gamma^*(\mathbf{x}'_1, \mathbf{x}_1) \chi_i^*(\mathbf{x}_1) \\ &= \int d\mathbf{x}'_1 d\mathbf{x}_1 \chi_j(\mathbf{x}'_1) \gamma(\mathbf{x}'_1, \mathbf{x}_1) \chi_i^*(\mathbf{x}_1) \\ &= \gamma_{ij} \end{aligned} \quad (4.4.2)$$

$\therefore \gamma$ is Hermitian.

Ex 4.5

$$\begin{aligned} \langle \Phi | \Phi \rangle &= \frac{1}{N} \int d\mathbf{x}_1 \gamma(\mathbf{x}_1, \mathbf{x}_1) \\ &= \int d\mathbf{x}_1 \sum_{ij} \chi_i(\mathbf{x}_1) \gamma_{ij} \chi_j^*(\mathbf{x}_1) \\ &= \frac{1}{N} \sum_{ij} \left[\int d\mathbf{x}_1 \chi_j^*(\mathbf{x}_1) \chi_i(\mathbf{x}_1) \right] \gamma_{ij} \\ &= \frac{1}{N} \sum_{ij} \delta_{ji} \gamma_{ij} \\ &= \frac{1}{N} \text{tr } \gamma \end{aligned} \quad (4.4.3)$$

thus

$$\text{tr } \gamma = N \quad (4.4.4)$$

Ex 4.6

a.

$$\begin{aligned}
\langle \Phi | \mathcal{O}_1 | \Phi \rangle &= \sum_i \langle \Phi | h(\mathbf{x}_1) | \Phi \rangle \\
&= N \int d\mathbf{x}_1 \int d\mathbf{x}_2 \cdots d\mathbf{x}_N \Phi^*(\mathbf{x}_1, \cdots, \mathbf{x}_N) h(\mathbf{x}_1) \Phi(\mathbf{x}_1, \cdots, \mathbf{x}_N) \\
&= N \frac{1}{N} \int d\mathbf{x}_1 [h(\mathbf{x}_1) \gamma(\mathbf{x}_1, \mathbf{x}'_1)]_{\mathbf{x}'_1 = \mathbf{x}_1} \\
&= \int d\mathbf{x}_1 [h(\mathbf{x}_1) \gamma(\mathbf{x}_1, \mathbf{x}'_1)]_{\mathbf{x}'_1 = \mathbf{x}_1}
\end{aligned} \tag{4.4.5}$$

b.

$$\begin{aligned}
\langle \Phi | \mathcal{O}_1 | \Phi \rangle &= \int d\mathbf{x}_1 [h(\mathbf{x}_1) \gamma(\mathbf{x}_1, \mathbf{x}'_1)]_{\mathbf{x}'_1 = \mathbf{x}_1} \\
&= \int d\mathbf{x}_1 [h(\mathbf{x}_1) \sum_{ij} \chi_i(\mathbf{x}_1) \gamma_{ij} \chi_j^*(\mathbf{x}'_1)]_{\mathbf{x}'_1 = \mathbf{x}_1} \\
&= \sum_{ij} \left[\int d\mathbf{x}_1 \chi_j^*(\mathbf{x}_1) h(\mathbf{x}_1) \chi_i(\mathbf{x}_1) \right] \gamma_{ij} \\
&= \sum_{ij} h_{ji} \gamma_{ij} \\
&= \sum_j (\mathbf{h}\boldsymbol{\gamma})_{jj} \\
&= \text{tr}(\mathbf{h}\boldsymbol{\gamma})
\end{aligned} \tag{4.4.6}$$