# $\begin{array}{c} \mathbf{Modern~Quantum~Chemistry,~Szabo~\&~Ostlund} \\ \mathbf{HW} \end{array}$

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## 5 Pair and Coupled-pair Theories

#### 5.1 The Independent Electron Pair Approximation

Ex 5.1

a.

$${}^{1}E_{\text{corr}}(\text{FO}) = \frac{|\langle 1\bar{1} \parallel 2\bar{2} \rangle|^{2}}{\varepsilon_{1} + \varepsilon_{1} - \varepsilon_{2} - \varepsilon_{2}}$$

$$= \frac{|\langle 1\bar{1} \mid 2\bar{2} \rangle - \langle 1\bar{1} \mid \bar{2}2 \rangle|^{2}}{2\varepsilon_{1} - 2\varepsilon_{2}}$$

$$= \frac{|[12|\bar{1}\bar{2}] - [1\bar{2}|\bar{1}2]|^{2}}{2\varepsilon_{1} - 2\varepsilon_{2}}$$

$$= \frac{K_{12}^{2}}{2(\varepsilon_{1} - \varepsilon_{2})}$$
(5.1.1)

b.

$${}^{1}E_{\text{corr}} = \Delta - \Delta \sqrt{1 + \frac{K_{12}^{2}}{\Delta^{2}}}$$

$$= \Delta - \Delta \left(1 + \frac{K_{12}^{2}}{2\Delta^{2}}\right)$$

$$= -\frac{K_{12}^{2}}{2\Delta}$$

$$\approx \frac{K_{12}^{2}}{2(\varepsilon_{1} - \varepsilon_{2})}$$
(5.1.2)

Ex 5.2 From Eq. 5.9a and 5.9b in the textbook, we get

$$\sum_{t < u} c_{1_i \bar{1}_i}^{tu} \left\langle \Psi_0 \middle| \mathcal{H} \middle| \Psi_{1_i \bar{1}_i}^{tu} \right\rangle = e_{1_i \bar{1}_i}$$
 (5.1.3)

$$\left\langle \Psi^{rs}_{1_{i}\bar{1}_{i}} \middle| \mathcal{H} \middle| \Psi_{0} \right\rangle + \sum_{t < u} \left\langle \Psi^{rs}_{1_{i}\bar{1}_{i}} \middle| \mathcal{H} - E_{0} \middle| \Psi^{tu}_{1_{i}\bar{1}_{i}} \right\rangle c^{tu}_{1_{i}\bar{1}_{i}} = e_{1_{i}\bar{1}_{i}} c^{rs}_{1_{i}\bar{1}_{i}}$$

$$(5.1.4)$$

*:* .

$$c_{1_{i}\bar{1}_{i}}^{2_{i}\bar{2}_{i}}\left\langle \Psi_{0}\left|\,\mathcal{H}\,\right|\Psi_{1_{i}\bar{1}_{i}}^{2_{i}\bar{2}_{i}}\right\rangle =e_{1_{i}\bar{1}_{i}}\tag{5.1.5}$$

$$\left\langle \Psi_{1_{i}\bar{1}_{i}}^{2_{i}\bar{2}_{i}} \middle| \mathcal{H} \middle| \Psi_{0} \right\rangle + \sum_{t \leq u} \left\langle \Psi_{1_{i}\bar{1}_{i}}^{2_{i}\bar{2}_{i}} \middle| \mathcal{H} - E_{0} \middle| \Psi_{1_{i}\bar{1}_{i}}^{tu} \right\rangle c_{1_{i}\bar{1}_{i}}^{tu} = e_{1_{i}\bar{1}_{i}} c_{1_{i}\bar{1}_{i}}^{2_{i}\bar{2}_{i}}$$

$$(5.1.6)$$

(5.1.5) gives

$$K_{12}c_{1_{i}\bar{1}_{i}}^{2_{i}\bar{2}_{i}} = e_{1_{i}\bar{1}_{i}} \tag{5.1.7}$$

(5.1.6) gives

$$K_{12} + \sum_{ik} \left\langle \Psi_{1_i \bar{1}_i}^{2_i \bar{2}_i} \middle| \mathcal{H} - E_0 \middle| \Psi_{1_i \bar{1}_i}^{2_j \bar{2}_k} \right\rangle c_{1_i \bar{1}_i}^{2_j \bar{2}_k} = e_{1_i \bar{1}_i} c_{1_i \bar{1}_i}^{2_i \bar{2}_i}$$

$$(5.1.8)$$

Since

$$\left\langle \Psi_{1_{i}\bar{1}_{i}}^{2_{i}\bar{2}_{i}} \middle| \mathcal{H} - E_{0} \middle| \Psi_{1_{i}\bar{1}_{i}}^{2_{j}\bar{2}_{k}} \right\rangle c_{1_{i}\bar{1}_{i}}^{2_{j}\bar{2}_{k}} = \begin{cases} 2\Delta & j = k = i \\ 0 & j = k \neq i \\ 0 & i = j = \neq k \end{cases}$$
 (5.1.9)

we have

$$K_{12} + 2\Delta c_{1_i\bar{1}_i}^{2_i\bar{2}_i} = e_{1_i\bar{1}_i} c_{1_i\bar{1}_i}^{2_i\bar{2}_i}$$

$$(5.1.10)$$

Ex 5.3

$${}^{2}E_{\text{corr}}(\text{FO}) = \sum_{i} \frac{\left| \langle 1_{i} \bar{1}_{i} \parallel 2_{i} \bar{2}_{i} \rangle \right|^{2}}{\varepsilon_{1} + \varepsilon_{1} - \varepsilon_{2} - \varepsilon_{2}}$$

$$= 2 \times \frac{K_{12}^{2}}{2(\varepsilon_{1} - \varepsilon_{2})}$$

$$= \frac{K_{12}^{2}}{(\varepsilon_{1} - \varepsilon_{2})}$$
(5.1.11)

### 5.1.1 Invariance under Unitary Transformations: An Example

#### Ex 5.4

$$|a\bar{a}b\bar{b}\rangle = 2^{-1/2} \left( |1_1\bar{a}b\bar{b}\rangle + |1_2\bar{a}b\bar{b}\rangle \right)$$

$$= 2^{-1} \left( |1_1\bar{1}_1b\bar{b}\rangle + |1_1\bar{1}_2b\bar{b}\rangle + |1_2\bar{1}_1b\bar{b}\rangle + |1_2\bar{1}_2b\bar{b}\rangle \right)$$

$$= 2^{-2} \left( |1_1\bar{1}_11_1\bar{1}_1\rangle - |1_1\bar{1}_11_1\bar{1}_2\rangle - |1_1\bar{1}_11_2\bar{1}_1\rangle + |1_1\bar{1}_11_2\bar{1}_2\rangle \right)$$

$$+ |1_1\bar{1}_21_1\bar{1}_1\rangle - |1_1\bar{1}_21_1\bar{1}_2\rangle - |1_1\bar{1}_21_2\bar{1}_1\rangle + |1_1\bar{1}_21_2\bar{1}_2\rangle$$

$$+ |1_2\bar{1}_11_1\bar{1}_1\rangle - |1_2\bar{1}_11_1\bar{1}_2\rangle - |1_2\bar{1}_11_2\bar{1}_1\rangle + |1_2\bar{1}_11_2\bar{1}_2\rangle$$

$$+ |1_2\bar{1}_21_1\bar{1}_1\rangle - |1_2\bar{1}_21_1\bar{1}_2\rangle - |1_2\bar{1}_21_2\bar{1}_1\rangle + |1_2\bar{1}_21_2\bar{1}_2\rangle$$

$$+ |1_2\bar{1}_21_1\bar{1}_1\rangle - |1_2\bar{1}_21_1\bar{1}_2\rangle - |1_2\bar{1}_21_2\bar{1}_1\rangle + |1_2\bar{1}_21_2\bar{1}_2\rangle$$

$$= 2^{-2} \left( 2 |1_1\bar{1}_11_1\bar{1}_1\rangle + 2 |1_1\bar{1}_11_2\bar{1}_2\rangle - 2 |1_1\bar{1}_21_1\bar{1}_2\rangle - 2 |1_1\bar{1}_21_2\bar{1}_1\rangle \right)$$

$$= 2^{-2} \left( 2 |1_1\bar{1}_11_2\bar{1}_2\rangle - 2 |1_1\bar{1}_11_2\bar{1}_2\rangle \right)$$

$$= |1_1\bar{1}_12\bar{1}_2\rangle$$

$$(5.1.12)$$

#### Ex 5.5

$$\langle \Psi_0 \mid \mathcal{H} \mid \Psi_{a\bar{a}}^{**} \rangle = 2^{-1/2} \left( \left\langle \Psi_0 \mid \mathcal{H} \mid \Psi_{a\bar{a}}^{r\bar{r}} \right\rangle + \left\langle \Psi_0 \mid \mathcal{H} \mid \Psi_{a\bar{a}}^{s\bar{s}} \right\rangle \right)$$

$$= 2^{-1/2} \left( 2 \times \frac{1}{2} K_{12} \right)$$

$$= 2^{-1/2} K_{12}$$

$$(5.1.13)$$

$$\langle \Psi_{a\bar{a}}^{**} | \mathcal{H} - E_{0} | \Psi_{a\bar{a}}^{**} \rangle = 2^{-1} (\langle \Psi_{a\bar{a}}^{r\bar{r}} | \mathcal{H} - E_{0} | \Psi_{a\bar{a}}^{r\bar{r}} \rangle + \langle \Psi_{a\bar{a}}^{r\bar{r}} | \mathcal{H} - E_{0} | \Psi_{a\bar{a}}^{s\bar{s}} \rangle + \langle \Psi_{a\bar{a}}^{s\bar{s}} | \mathcal{H} - E_{0} | \Psi_{a\bar{a}}^{r\bar{r}} \rangle + \langle \Psi_{a\bar{a}}^{s\bar{s}} | \mathcal{H} - E_{0} | \Psi_{a\bar{a}}^{s\bar{s}} \rangle )$$

$$= 2^{-1} \left[ \left( 2h_{11} + 2h_{22} + \frac{1}{2}J_{11} + \frac{1}{2}J_{22} + 2J_{12} - K_{12} \right) - (4h_{11} + 2J_{11}) \right]$$

$$+ \frac{1}{2}J_{22} + \frac{1}{2}J_{22} + \left( 2h_{11} + 2h_{22} + \frac{1}{2}J_{11} + \frac{1}{2}J_{22} + 2J_{12} - K_{12} \right) - (4h_{11} + 2J_{11}) \right]$$

$$= 2^{-1} \left( -2h_{11} + 2h_{22} - \frac{3}{2}J_{11} + J_{22} + 2J_{12} - K_{12} \right) \times 2$$

$$= -2h_{11} + 2h_{22} - \frac{3}{2}J_{11} + J_{22} + 2J_{12} - K_{12}$$

$$(5.1.14)$$

Since

$$\varepsilon_2 - \varepsilon_1 = h_{22} - h_{11} + 2J_{12} - K_{12} - J_{11} \tag{5.1.15}$$

we have

$$\langle \Psi_{a\bar{a}}^{**} | \mathcal{H} - E_0 | \Psi_{a\bar{a}}^{**} \rangle = 2(\varepsilon_2 - \varepsilon_1) - 2J_{12} + K_{12} + \frac{1}{2}J_{11} + J_{22}$$
 (5.1.16)

**Ex 5.6** Since

$$|\Psi_{a\bar{b}}^{**}\rangle = 2^{-1/2}(|\Psi_{a\bar{b}}^{r\bar{s}}\rangle + |\Psi_{a\bar{b}}^{s\bar{r}}\rangle) \tag{5.1.17}$$

$$\langle \Psi_{0} \mid \mathcal{H} \mid \Psi_{a\bar{b}}^{**} \rangle = 2^{-1/2} \left( \langle \Psi_{0} \mid \mathcal{H} \mid \Psi_{a\bar{b}}^{r\bar{s}} \rangle + \langle \Psi_{0} \mid \mathcal{H} \mid \Psi_{a\bar{b}}^{s\bar{r}} \rangle \right)$$

$$= 2^{-1/2} \left( \langle a\bar{b} \parallel r\bar{s} \rangle + \langle a\bar{b} \parallel s\bar{r} \rangle \right)$$

$$= 2^{-1/2} ((ar|bs) + (as|br))$$

$$= 2^{-1/2} K_{12}$$
(5.1.18)

$$\langle \Psi_{a\bar{b}}^{**} | \mathcal{H} - E_{0} | \Psi_{a\bar{b}}^{**} \rangle = 2^{-1} (\langle \Psi_{a\bar{b}}^{r\bar{s}} | \mathcal{H} - E_{0} | \Psi_{a\bar{b}}^{r\bar{s}} \rangle + \langle \Psi_{a\bar{b}}^{r\bar{s}} | \mathcal{H} - E_{0} | \Psi_{a\bar{b}}^{s\bar{r}} \rangle 
+ \langle \Psi_{a\bar{b}}^{s\bar{r}} | \mathcal{H} - E_{0} | \Psi_{a\bar{b}}^{r\bar{s}} \rangle + \langle \Psi_{a\bar{b}}^{s\bar{r}} | \mathcal{H} - E_{0} | \Psi_{a\bar{b}}^{s\bar{r}} \rangle 
= 2^{-1} \left[ \left( 2h_{11} + 2h_{22} + \frac{1}{2}J_{11} + \frac{1}{2}J_{22} + 2J_{12} - K_{12} \right) - (4h_{11} + 2J_{11}) \right] 
+ \frac{1}{2}J_{22} + \frac{1}{2}J_{22} 
+ \left( 2h_{11} + 2h_{22} + \frac{1}{2}J_{11} + \frac{1}{2}J_{22} + 2J_{12} - K_{12} \right) - (4h_{11} + 2J_{11}) \right] 
= \dots 
= 2(\varepsilon_{2} - \varepsilon_{1}) - 2J_{12} + K_{12} + \frac{1}{2}J_{11} + J_{22} \equiv 2\Delta'$$
(5.1.19)

Thus the equations determining  $e_{a\bar{b}}$  are identical to that of  $e_{a\bar{a}}$ . Similarly,  $e_{\bar{a}b}$  shares the same equations with them.

 $\therefore e_{a\bar{b}} = e_{\bar{a}b} = e_{a\bar{a}}.$ 

#### Ex 5.7

**a.** As shown in Ex 5.5, 5.6

$$\langle \Psi_0 \mid \mathcal{H} \mid \Psi_{a\bar{a}}^{**} \rangle = \langle \Psi_0 \mid \mathcal{H} \mid \Psi_{a\bar{b}}^{**} \rangle = \langle \Psi_0 \mid \mathcal{H} \mid \Psi_{\bar{a}b}^{**} \rangle = 2^{-1/2} K_{12}$$
 (5.1.20)

$$\left\langle \Psi_{a\bar{a}}^{**} \mid \mathcal{H} - E_0 \mid \Psi_{a\bar{a}}^{**} \right\rangle = \left\langle \Psi_{a\bar{b}}^{**} \mid \mathcal{H} - E_0 \mid \Psi_{a\bar{b}}^{**} \right\rangle = \left\langle \Psi_{\bar{a}b}^{**} \mid \mathcal{H} - E_0 \mid \Psi_{\bar{a}b}^{**} \right\rangle = 2\Delta' \tag{5.1.21}$$

Similarly, we get

$$\left\langle \Psi_0 \left| \mathcal{H} \right| \Psi_{b\bar{b}}^{**} \right\rangle = 2^{-1/2} K_{12} \tag{5.1.22}$$

$$\left\langle \Psi_{b\bar{b}}^{**} \middle| \mathcal{H} - E_0 \middle| \Psi_{b\bar{b}}^{**} \right\rangle = 2\Delta' \tag{5.1.23}$$

For the rest,

$$\langle \Psi_{a\bar{a}}^{**} | \mathcal{H} - E_{0} | \Psi_{b\bar{b}}^{**} \rangle = 2^{-1} (\langle \Psi_{a\bar{a}}^{r\bar{r}} | \mathcal{H} - E_{0} | \Psi_{b\bar{b}}^{r\bar{r}} \rangle + \langle \Psi_{a\bar{a}}^{r\bar{r}} | \mathcal{H} - E_{0} | \Psi_{b\bar{b}}^{s\bar{s}} \rangle$$

$$+ \langle \Psi_{a\bar{a}}^{s\bar{s}} | \mathcal{H} - E_{0} | \Psi_{b\bar{b}}^{r\bar{r}} \rangle + \langle \Psi_{a\bar{a}}^{s\bar{s}} | \mathcal{H} - E_{0} | \Psi_{b\bar{b}}^{s\bar{s}} \rangle)$$

$$= 2^{-1} [\langle b\bar{b} | | a\bar{a} \rangle + 0 + 0 + \langle b\bar{b} | | a\bar{a} \rangle]$$

$$= (ab|ab)$$

$$= \frac{1}{2} J_{11}$$

$$(5.1.24)$$

$$\langle \Psi_{a\bar{a}}^{**} | \mathcal{H} - E_{0} | \Psi_{a\bar{b}}^{**} \rangle = 2^{-1} (\langle \Psi_{a\bar{a}}^{r\bar{r}} | \mathcal{H} - E_{0} | \Psi_{a\bar{b}}^{r\bar{s}} \rangle + \langle \Psi_{a\bar{a}}^{r\bar{r}} | \mathcal{H} - E_{0} | \Psi_{a\bar{b}}^{s\bar{r}} \rangle 
+ \langle \Psi_{a\bar{a}}^{s\bar{s}} | \mathcal{H} - E_{0} | \Psi_{a\bar{b}}^{r\bar{s}} \rangle + \langle \Psi_{a\bar{a}}^{r\bar{s}} | \mathcal{H} - E_{0} | \Psi_{a\bar{b}}^{s\bar{r}} \rangle ) 
= 2^{-1} [\langle r\bar{b} | | \bar{a}\bar{s} \rangle - \langle r\bar{b} | | s\bar{a} \rangle + \langle s\bar{b} | | r\bar{a} \rangle - \langle \bar{s}\bar{b} | | \bar{a}\bar{r} \rangle ] 
= 2^{-1} [(ra|bs) - (rs|ba) - (rs|ba) - (sr|ba) + (sa|br) - (sr|ba)] 
= 2^{-1} [(ra|bs) + (sa|br) - 4(ab|sr)] 
= 2^{-1} [2 \times \frac{1}{2} K_{12} - 4 \times \frac{1}{2} J_{12}] 
= \frac{1}{2} K_{12} - J_{12}$$
(5.1.25)

Similarly, we get

$$\left\langle \Psi_{a\bar{b}}^{**} \middle| \mathcal{H} - E_0 \middle| \Psi_{\bar{a}b}^{**} \right\rangle = \frac{1}{2} J_{11}$$
 (5.1.26)

$$\langle \Psi_{a\bar{a}}^{**} | \mathcal{H} - E_0 | \Psi_{\bar{a}b}^{**} \rangle = \langle \Psi_{b\bar{b}}^{**} | \mathcal{H} - E_0 | \Psi_{a\bar{b}}^{**} \rangle = \langle \Psi_{b\bar{b}}^{**} | \mathcal{H} - E_0 | \Psi_{\bar{a}b}^{**} \rangle = \frac{1}{2} K_{12} - J_{12}$$
 (5.1.27)

thus the DCI equation is

$$\begin{pmatrix} 0 & 2^{-1/2}K_{12} & 2^{-1/2}K_{12} & 2^{-1/2}K_{12} & 2^{-1/2}K_{12} & 2^{-1/2}K_{12} \\ 2^{-1/2}K_{12} & 2\Delta' & \frac{1}{2}J_{11} & \frac{1}{2}K_{12} - J_{12} & \frac{1}{2}K_{12} - J_{12} \\ 2^{-1/2}K_{12} & \frac{1}{2}J_{11} & 2\Delta' & \frac{1}{2}K_{12} - J_{12} & \frac{1}{2}K_{12} - J_{12} \\ 2^{-1/2}K_{12} & \frac{1}{2}K_{12} - J_{12} & \frac{1}{2}K_{12} - J_{12} & 2\Delta' & \frac{1}{2}J_{11} \\ 2^{-1/2}K_{12} & \frac{1}{2}K_{12} - J_{12} & \frac{1}{2}K_{12} - J_{12} & \frac{1}{2}J_{11} & 2\Delta' \end{pmatrix} \begin{pmatrix} 1 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = {}^2E_{\text{corr}}(\text{DCI}) \begin{pmatrix} 1 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}$$

$$(5.1.28)$$

b. By solving the DCI equation above (see 5-7.nb), we get

$${}^{2}E_{\text{corr}}(\text{DCI}) = \frac{2\Delta' + \frac{1}{2}J_{11} + 2(\frac{1}{2}K_{12} - J_{12}) - \sqrt{16(2^{-1/2}K_{12})^{2} + [2\Delta' + \frac{1}{2}J_{11} + 2(\frac{1}{2}K_{12} - J_{12})]^{2}}}{2}$$

$$(5.1.29)$$

and

$$c_{1} = c_{2} = c_{3} = c_{4} = \frac{2\Delta' + \frac{1}{2}J_{11} + 2(\frac{1}{2}K_{12} - J_{12}) + \sqrt{16(2^{-1/2}K_{12})^{2} + [2\Delta' + \frac{1}{2}J_{11} + 2(\frac{1}{2}K_{12} - J_{12})]^{2}}}{8 \times 2^{-1/2}K_{12}}$$

$$(5.1.30)$$

Since

$$2\Delta' = 2(\varepsilon_2 - \varepsilon_1) - 2J_{12} + K_{12} + \frac{1}{2}J_{11} + J_{22}$$
(5.1.31)

$$2\Delta = 2(\varepsilon_2 - \varepsilon_1) + J_{11} + J_{22} - 4J_{12} + 2K_{12}$$
(5.1.32)

we have

$$2\Delta = 2\Delta' + \frac{1}{2}J_{11} - 2J_{12} + K_{12}$$
(5.1.33)

٠.

$${}^{2}E_{\text{corr}}(\text{DCI}) = \frac{2\Delta - \sqrt{8K_{12}^{2} + (2\Delta)^{2}}}{2}$$

$$= \Delta - \sqrt{2K_{12}^{2} + \Delta^{2}}$$
(5.1.34)

$$c_1 = c_2 = c_3 = c_4 = \frac{2\Delta + \sqrt{8K_{12}^2 + (2\Delta)^2}}{4\sqrt{2}K_{12}}$$
$$= \frac{\Delta + \sqrt{2K_{12}^2 + \Delta^2}}{2\sqrt{2}K_{12}}$$
(5.1.35)

Ex 5.8

$$\begin{split} E_{\text{corr}}(\text{FO}) &= \sum_{A < B} \sum_{R < S} \frac{|\langle AB \, \| \, RS \rangle|^2}{\varepsilon_A + \varepsilon_B - \varepsilon_R - \varepsilon_S} \\ &= \frac{|\langle a\bar{a} \, \| \, r\bar{r} \rangle|^2 + |\langle a\bar{a} \, \| \, r\bar{s} \rangle|^2 + |\langle a\bar{a} \, \| \, s\bar{r} \rangle|^2 + |\langle a\bar{a} \, \| \, s\bar{s} \rangle|^2}{\varepsilon_1 + \varepsilon_1 - \varepsilon_2 - \varepsilon_2} + \frac{|\langle a\bar{b} \, \| \, r\bar{r} \rangle|^2 + |\langle a\bar{b} \, \| \, r\bar{s} \rangle|^2 + |\langle a\bar{b} \, \| \, s\bar{s} \rangle|^2}{\varepsilon_1 + \varepsilon_1 - \varepsilon_2 - \varepsilon_2} \\ &+ \frac{|\langle b\bar{a} \, \| \, r\bar{r} \rangle|^2 + |\langle b\bar{a} \, \| \, r\bar{s} \rangle|^2 + |\langle b\bar{a} \, \| \, s\bar{r} \rangle|^2 + |\langle b\bar{a} \, \| \, s\bar{s} \rangle|^2}{\varepsilon_1 + \varepsilon_1 - \varepsilon_2 - \varepsilon_2} + \frac{|\langle b\bar{b} \, \| \, r\bar{r} \rangle|^2 + |\langle b\bar{b} \, \| \, s\bar{s} \rangle|^2}{\varepsilon_1 + \varepsilon_1 - \varepsilon_2 - \varepsilon_2} \\ &= \frac{|(ar|ar)|^2 + |(ar|as)|^2 + |(as|ar)|^2 + |(as|as)|^2}{2(\varepsilon_1 - \varepsilon_2)} + \frac{|(ar|br)|^2 + |(ar|bs)|^2 + |(as|br)|^2 + |(as|bs)|^2}{2(\varepsilon_1 - \varepsilon_2)} \\ &+ \frac{|(br|ar)|^2 + |(br|as)|^2 + |(bs|ar)|^2 + |(bs|as)|^2}{2(\varepsilon_1 - \varepsilon_2)} + \frac{|(br|br)|^2 + |(br|bs)|^2 + |(bs|br)|^2 + |(bs|bs)|^2}{2(\varepsilon_1 - \varepsilon_2)} \\ &= \frac{\left|\frac{1}{2}K_{12}\right|^2 + 0 + 0 + \left|\frac{1}{2}K_{12}\right|^2}{2(\varepsilon_1 - \varepsilon_2)} + \frac{0 + \left|\frac{1}{2}K_{12}\right|^2 + \left|\frac{1}{2}K_{12}\right|^2 + 0}{2(\varepsilon_1 - \varepsilon_2)} \\ &+ \frac{0 + 0 + \left|\frac{1}{2}K_{12}\right|^2 + \left|\frac{1}{2}K_{12}\right|^2}{2(\varepsilon_1 - \varepsilon_2)} + \frac{\left|\frac{1}{2}K_{12}\right|^2 + 0 + 0 + \left|\frac{1}{2}K_{12}\right|^2}{2(\varepsilon_1 - \varepsilon_2)} \\ &= \frac{2K_{12}^2}{2(\varepsilon_1 - \varepsilon_2)} \end{aligned}$$

Ex 5.9

a.